Bouncer Report

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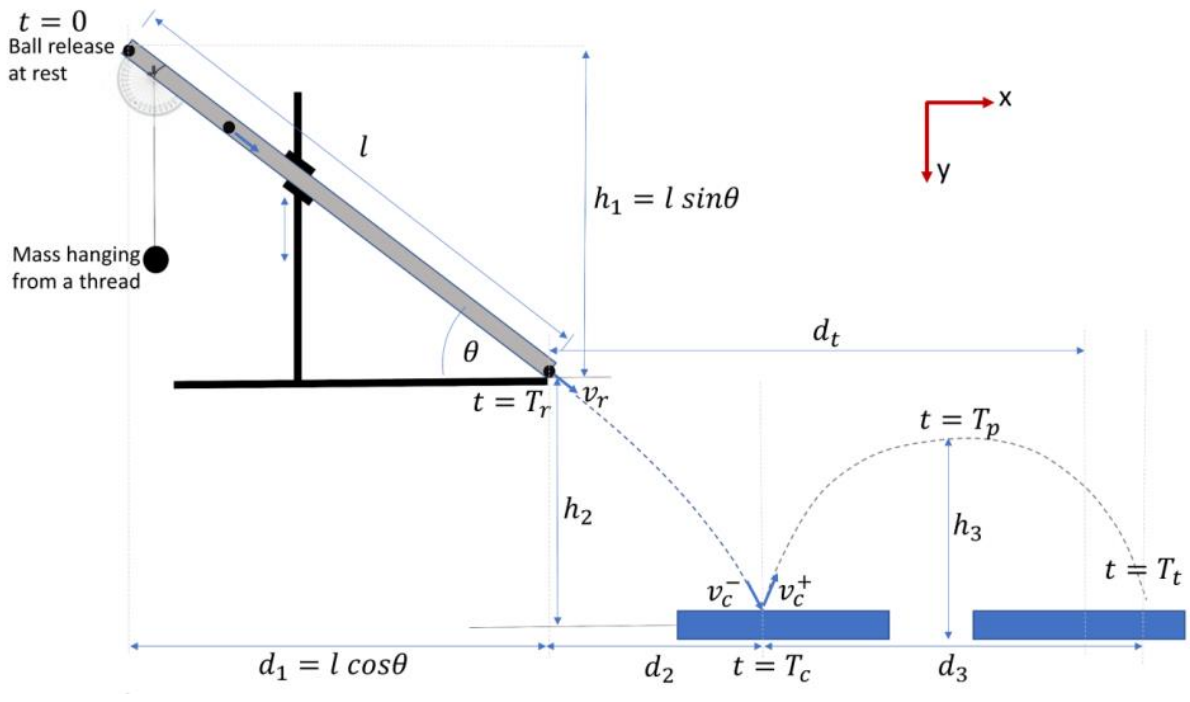
Introduction:

The objective of this report is to calculate the horizontal distance that a ball travels between exiting an inclined tube and its second bounce on the ground below (d_t). As well as analysing the results to determine where any experimental errors may have originated from. Initially, each group was given an angle θ, measured from the horizontal, that the tube would be inclined at. Secondly, they were given a height (h_2), measuring from the bottom of the tube to the horizontal ground below. For our group, these values were 25° and 0.7m.

We began by making initial assumptions about the motion of the ball to simplify the equations and then make an estimate for the distance (d_t) using the given measurements.

- Initial velocity of the ball is 0 m/s
- No friction between the ball and tube
- The ball rolls perfectly inside tube (no slippage)
- The collision with the ground is inelastic
- The ball follows a straight trajectory after exiting the tube

Diagram 1 (1):



Using diagram 1 and the quoted values for the ball's measurements:

θ = 25°

h_2 = 0.7 m

l = 1.5 m

e = 0.4979

radius of the ball = 0.00625 m

mass of the ball = 0.008 kg

gravitational acceleration in London (2) = 9.816 m/s^2

h_1 = $l sin(\theta) = 1.5 \sin(25)$

Method:

The overall summary of the calculations includes:

1. Use energy equations to calculate the velocity (v_r) of the ball at the bottom of the tube
2. Form an equation for the air resistance on the ball and evaluate this using v_r to determine if it should be included in the following calculations.
3. Use equations of motion in both the x and y directions to calculate:
    a. The time taken (t_1) between exiting the tube and the first bounce
    b. The horizontal distance (d_2) travelled by the ball to its first bounce
    c. The vertically velocity (v_{cy}) instantaneously after the first bounce using e.
    d. The time taken (t_2) between the first and second bounce
    e. The horizontal distance (d_3) travelled by the ball between the first and second bounce
    f. The total horizontal distance (d_t) between the bottom of the tube and the second bounce

1: Calculating v_r:

From the top to the bottom of tube:

- Taking the gravitational strength at the bottom of the tube to be 0 kgm/s^2

Initial Kinetic Energy + Initial Rotational Energy + Initial Gravitational Potential Energy

=

Final Kinetic Energy + Final Rotational Energy + Final Gravitational Potential Energy

Rotational Energy = ½ $I\,\omega^2$

I = moment of inertia of the ball = $\frac{2}{5} m\, r^2$

$\omega = v / r$

Therefore, Rotational Energy of the ball = $\frac{1}{2} * \frac{2}{5}\ m\ r^2 * \frac{v^2}{r^2} = \frac{1}{5} m\, v^2$

Substituting this into the energy equation:

(1) $$\frac{1}{2}\ m\,u^2 + \frac{1}{5} m\,u^2 +\ m\,g\,h1\ = \frac{1}{2}\ m\,v^2 + \frac{1}{5} m\,v^2 +\ m\,g *\ 0$$

Simplified and Evaluated: $g\,h_1\ = \frac{7}{10} v^2$

(1) $$v_r\ =\ sqrt\left(10\ *\ 9.816\ *\ 1.5\ * \frac{sin(25)}{7}\right) =\ 2.982\ m/s$$

2. Calculating air resistance:

(2) $$F\ =\ \frac{1}{2}\ C\ \rho\ A\ v^2$$

F = force due to air resistance (3)

ρ = fluid density

A = cross sectional area = $\pi\, r^2$ for a sphere (r = 0.00625 m)

v = velocity

C = Coefficient of drag: This can be found by calculating the Reynolds number (Re) for the ball and then using a graph of Reynolds number vs coefficient of drag

(3) $$Re\ = \frac{\rho v d}{\mu}$$

ρ = fluid density (approximately 1.225 kg/m^3 at sea level and at 15°c (3)

v = velocity (approximately 3 m/s as calculated from the first equation)

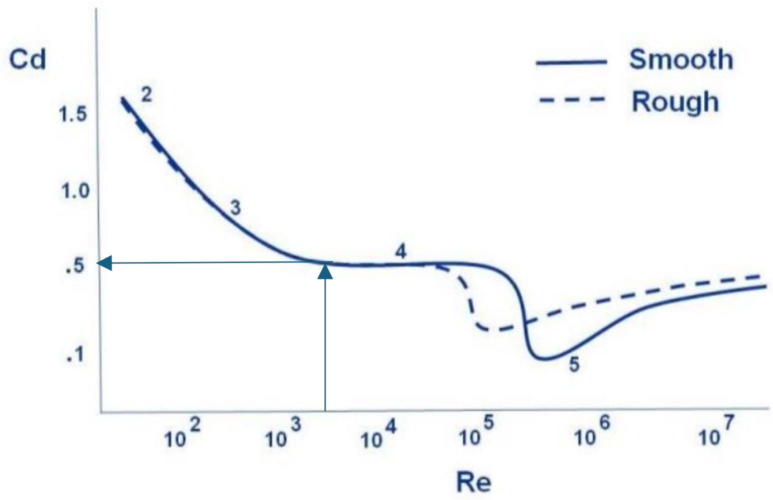
d = diameter of the ball = 0.00125 m

μ = dynamic viscosity of air (approximately $1.81\ *\ 10^{-5}$ Pa at 15°c) (4)

(3) $$Re\ =\ 1.225\ *\ 3\ * \frac{0.00125}{(1.81\ *\ 10^{-5})} =\ 2538$$

Using the graph of Cd vs Re plotted for a sphere in air:

Diagram 2: Graph showing Cd vs Re for a smooth and rough sphere (5)



Using the calculated Reynold's number of 2538, the coefficient of drag was approximately 0.5.

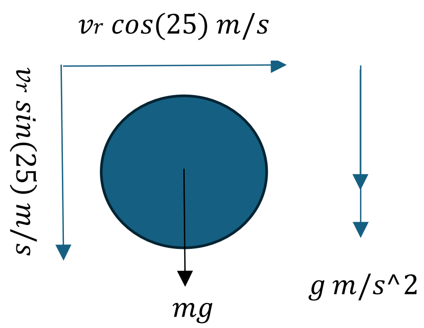
Therefore, by returning to the equation for the force due to air resistance:

(2) $$F = ½ * ½ * 1.225 * \pi * 0.00625^2 * 3^2 = 3.382 * 10^{-4}\, N$$

Deceleration due to air resistance = $a = \frac{F}{m} = 3.382 * \frac{10^{-4}}{0.008} = 0.0423\, m/s^2$ (4)

This value is negligible compared to the gravitational acceleration and therefore air resistance will not be included in the equations of motion for the ball.

3a. Calculating t_1



From bottom of the tube to the first bounce:

Updated assumptions:

- No air resistance
- No spin effects

Using the y components to calculate:

1. The vertically velocity v_{cy-} as the ball hits the ground on the first bounce
2. The time t_1

Initial Velocity = $u = -v_r \sin(25) = -2.982 \sin(25)\, m/s$

Acceleration = $a = -9.816\, m/s^2$

Displacement = $-h_2 = -0.7\, m$

(5) $$v^2 = u^2 + 2as$$

$$v_{cy-} = -sqrt((-2.982 \sin(25))\text{^}2 + 2 * -9.816 * -0.7)) = -3.915\, m/s$$

(6)

$$s = ut + \frac{1}{2} a t^2$$

$$\frac{1}{2} * -9.816\, t^2 - 2.982 \sin(25)\, t + 0.7 = 0$$

$$t_1 = -0.5272\, s \text{ or } 0.2705\, s$$

Therefore t_1 = 0.2705 s

3b. Calculating d_2

From the bottom of the tube to the first bounce using the x components:

Acceleration = a = $0\, m/s^2$

Initial Velocity = $u = v_r \cos(25) = 2.982 \cos(25)\, m/s$

Time = $t_1 = 0.2705\, s$

Displacement = $s = d_2$

(7)

$$s = ut + ½\, a t^2$$

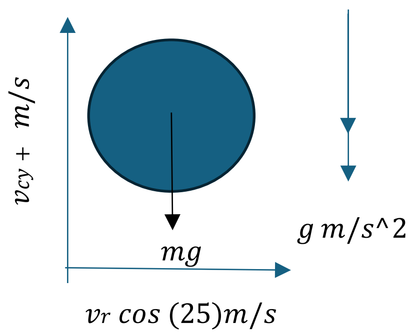
$$d_2 = 2.982 \cos(25) * 0.2705 = 0.7310\, m$$

3c. Calculating v_{cy+} Using e

Velocity after bounce = $v_{cy+} = -e v_{cy-} = -0.4979 * -3.915 = 1.949\, m/s$ (8)

3d. Calculating t_2

From first bounce to second bounce using y components:



Displacement = $s = 0\, m$

Acceleration = $a = -9.816\, m/s^2$

Initial Velocity = $u = v_{cy+} = 1.949\, m/s$

(9)

$$s = ut + \frac{1}{2} a t^2$$

$$0 = 1.949t + \frac{1}{2} * -9.816 * t^2$$

$$t_2 = 0\, s \text{ or } 0.3972\, s$$

Therefore t_2 = 0.3972 s

3e. Calculating d_3

From first bounce to second bounce using x components:

Displacement = $s = d_3$

Initial Velocity = $u = v_r \, cos(25) = 2.982 \, cos(25) \; m/s$

Acceleration = $a = 0 \; m/s^2$

Time = $t_2 = 0.3972 \; s$

(10)
$$s = ut + \frac{1}{2} a t^2$$

$$d_3 = 2.982 \cos(25) * 0.3972 = 1.073 \; m$$

3f. Calculating d_t using d_2 and d_3

(11)
$$d_t = d_2 + d_3 = 0.7310 + 1.073 = 1.804 \; m$$

Results:

The calculated horizontal distance from the lower end of the tube to the second bounce was 180 cm. This result was based on the assumptions of:

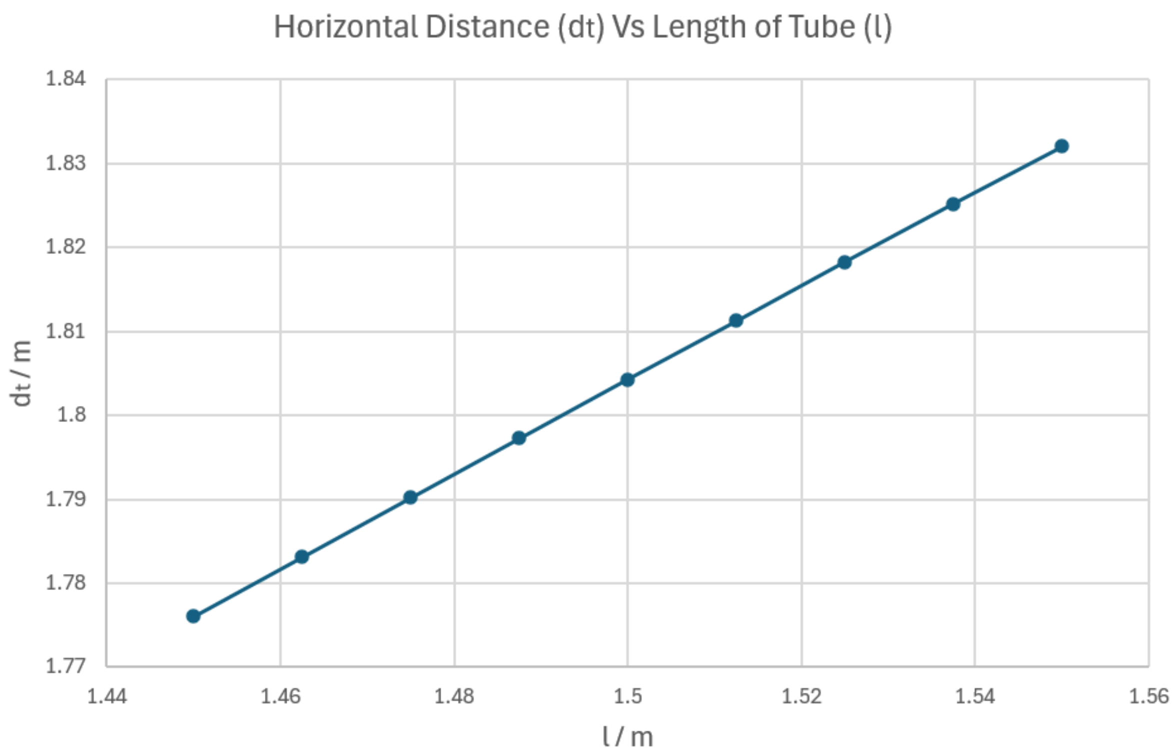
- No friction between the ball and the tube
- No air resistance on the ball
- No friction at the collision between the ball and the ground

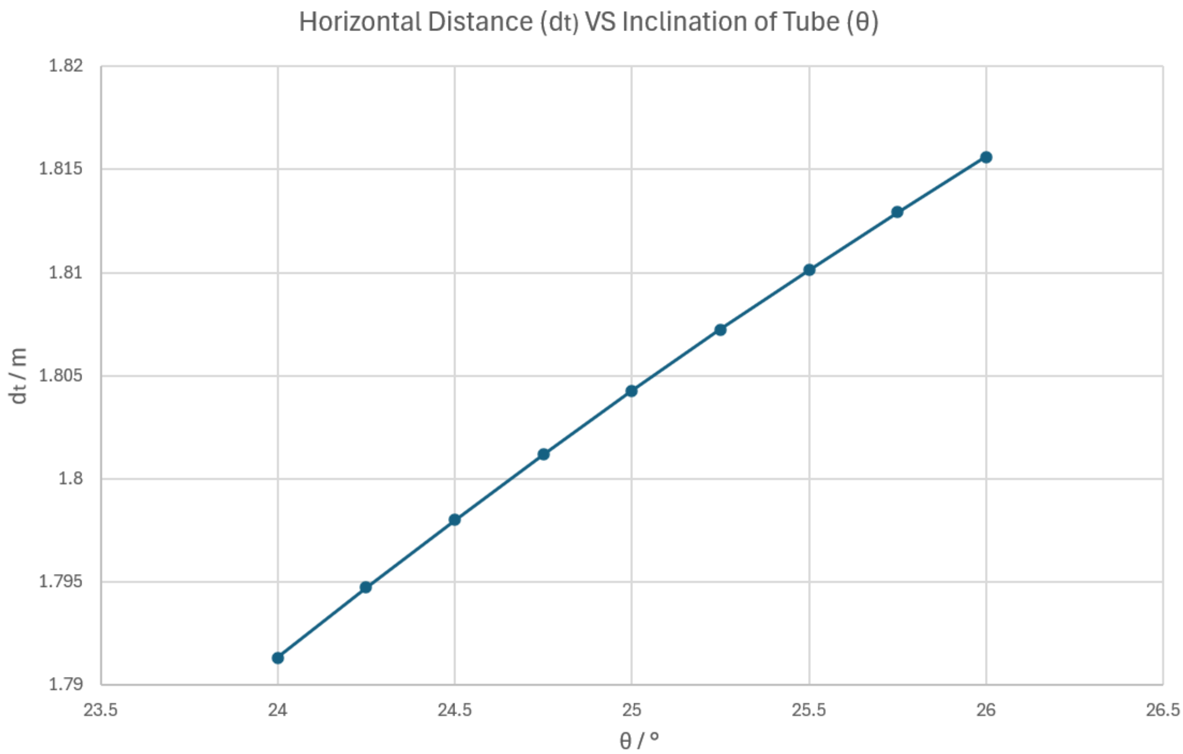
Sensitivity Analysis:

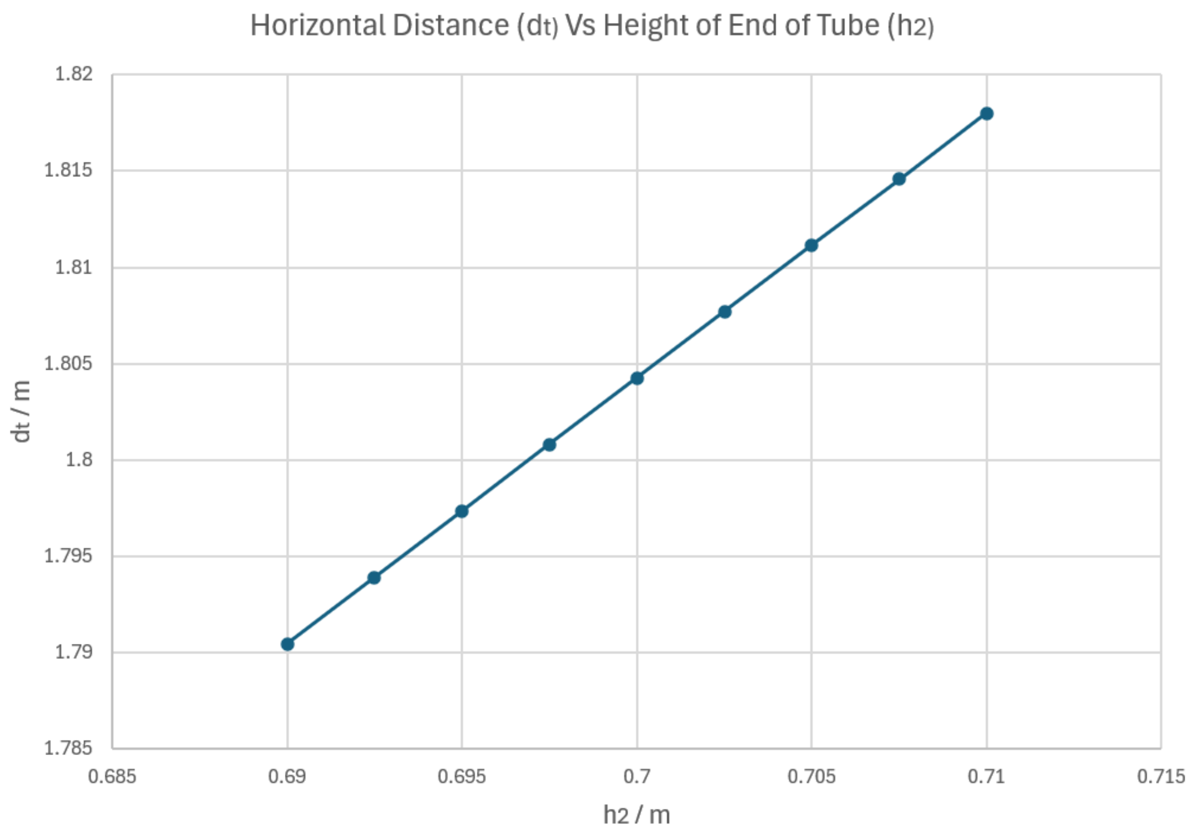
By producing an expression for d_t in terms of only the independent variables g, l, θ, e and h_2, each of the sensitivity analysis graphs were produced.

$$dt = \frac{\cos(\theta)\sqrt{\frac{10}{7} gl \sin(\theta)}\left(\sin(\theta)\sqrt{\frac{10}{7} gl \sin(\theta)} - \sqrt{\frac{10}{7} gl (\sin(\theta))^3 + 2gh_2}\right)}{-g} + \frac{2e\cos(\theta)\sqrt{\frac{10}{7} gl \sin(\theta)}\sqrt{\frac{10}{7} gl (\sin(\theta))^3 + 2gh_2}}{g}$$

The uncertainty in the value of g was $1 * 10^{\wedge} - 4 \; m/s^2$ which can be considered negligible and therefore the sensitivity analysis was not plotted for this variable. The uncertainty used in the other variables were l = 1.5 ± 0.05 m, θ = 25 ± 1°, h_2 = 0.7 ± 0.01 m, e = 0.4979 ± 0.0142.


Horizontal Distance (dt) Vs Length of Tube (l)
1.84
1.83
1.82
1.81
1.8
1.79
1.78
1.77
dt / m
1.44
1.46
1.48
1.5
1.52
1.54
1.56
l / m


Horizontal Distance (dt) VS Inclination of Tube (θ)
1.82
1.815
1.81
1.805
1.8
1.795
1.79
dt / m
23.5
24
24.5
25
25.5
26
26.5
θ / °


Horizontal Distance (dt) Vs Height of End of Tube (h2)
1.82
1.815
1.81
1.805
1.8
1.795
1.79
1.785
dt / m
0.685
0.69
0.695
0.7
0.705
0.71
0.715
h2 / m


Horizontal Distance (dt) Vs Coefficient of Restitution (e)
1.84
1.83
1.82
1.81
1.8
1.79
1.78
1.77
dt / m
0.48
0.485
0.49
0.495
0.5
0.505
0.51
0.515
Coefficient of Restitution (e)

Using the graphs, the sensitivity coefficient for each variable was produced by plotting a linear line of best fit at the base value (those used for the final calculation of d_t) and reading the gradient of such.

Variable	l	θ	h_2	e
Sensitivity Coefficient	0.5601	0.0121	1.3774	2.1557

This shows that the variable with the highest sensitivity is the value of the coefficient of restitution. However, this is not a parameter that can be controlled effectively during the experiment. The variable with the second highest sensitivity coefficient is h_2 which can be controlled. Therefore, during the experiment, the height of the end of the tube from the ground should be regarded as the most important variable to be controlled and monitored as it has the highest effect on the accuracy of d_t.

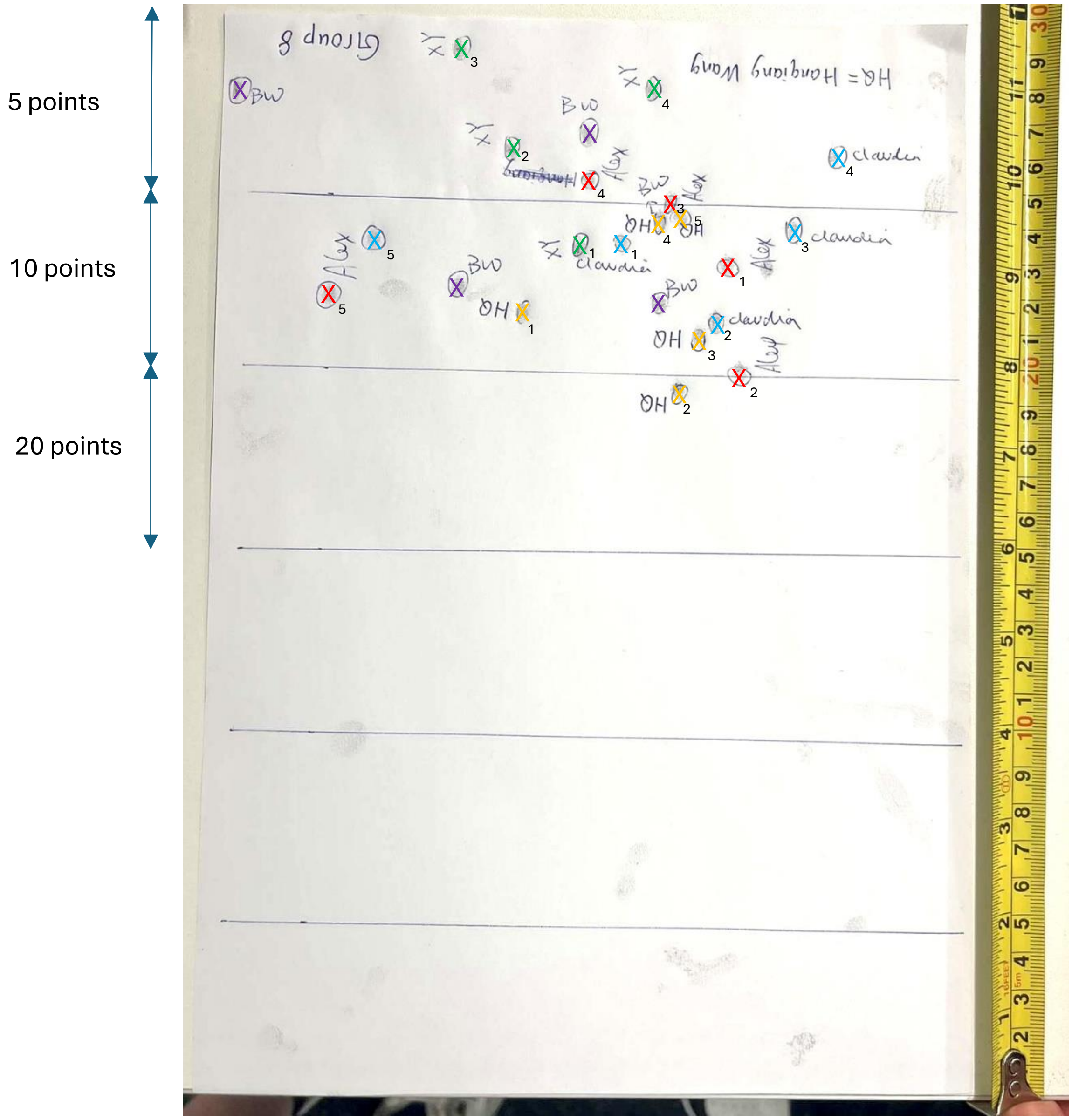


Figure 1: Paper with ball collision markings

Person	Alex	Claudia	Bill	Robert	Xiaoyun
Marker Colour	X	X	X	X	X
Estimate /cm	180	180	179	180	180
Mean /cm	171.8	171.3	170.1	173	163.4
Score	5, 10, 10, 10, 20	5, 10, 10, 10, 10	0, 5, 5, 10, 10	10, 10, 10, 10, 20	0, 5, 5, 5, 10
Mean Score	11	9	6	12	5

Discussion:

The outcome of the experiment revealed a consistent trend across all team members, where the estimated distances were systematically around 10 cm greater than the mean experimental distances observed. This discrepancy could be due to a variety of factors, including the potential measurement inaccuracies of key variables such as l, e, θ or h_2. A sensitivity analysis conducted to assess the impact of these variables suggested that even with a reasonable margin of experimental error (around 5%), the variations in the distance (d_t) were minimal at around 1 cm. This suggests that the primary sources of the observed discrepancy are more likely the assumptions made within the model. After reflecting with the group, it became apparent that none of the group members had accounted for energy losses from friction with the tube, air resistance, the rotational energy of the ball while airborne or energy losses during the collision with the ground. The difference between the scores for each team member is likely due to their individual release techniques for the ball, as well as the calibration of the setup for each student.

As shown in the method part 2, the deceleration due to air resistance was approximated and this value was negligible, suggesting that it could not have accounted for a decrease in the distance (d_t) by upwards of 10 cm. Alternatively, whilst conducting the experiment, we noticed that the ball produced a sound when travelling down the PVC tube, suggesting that it did not perfectly roll, as assumed by the model, but rather slipped down due to the low coefficient of friction between the two surfaces. This energy would have been lost to heat and sound, therefore decreasing the total horizontal distance travelled. It would also have resulted in an inaccurate calculation for the final rotational energy of the ball at the end of the tube. Perhaps in future, a tube with a higher coefficient of friction could be used to ensure that the ball perfectly rolls down it.

The experiment was carried out in a carpeted room. This would have meant that the ball sunk into the material slightly during the collision which was therefore not perfectly elastic in the x direction – as assumed by the model. This means that kinetic energy would have been lost during the collision and not accounted for, decreasing the horizontal distance (d_t) further.

The model assumes that after exiting the tube, the rotational energy of the ball remains constant due to the conservation of angular momentum. However, the angular velocity would have changed multiple times. After leaving the tube, the air resistance may have acted unevenly on it, caused an external torque which would affect the angular velocity. Upon collision with the ground, the change in rotational energy would depend on factors such as the coefficient of restitution, the surface texture and how the ball's point of contact moves relative to its centre of mass. Any change in the rotational energy of the ball would affect the horizontal distance to the second bounce.

Other assumptions made in the model include the ball being perfectly spherical and the tube being perfectly uniform. In reality, this is unlikely to be the case. Both these factors would cause an uneven roll as the ball travels down the tube, losing energy as it potentially travels side to side. An uneven roll would also affect the ball's rotational inertia and could result in it not following a straight trajectory, overall decreasing the horizontal distance.

During the setup of the experiment, the height of the tube (h_2) was measured from the ground using a metre rule. However, it was difficult to achieve the exact height as the rule was unable to contact the bottom centre of the tube due to the incline of the tube causing an overhang above. The angle of inclination was measured using a digital protractor and was simply held in place at the correct angle. To improve the accuracy of these features in future, the tube could be cut at an angle to allow the rule to contact the point at which it measures from, as well as using a bracket that can be adjusted to hold the tube at the required angle.

References:

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