Control of a string of vehicles

A.Y. 2019/2020
Networked Control
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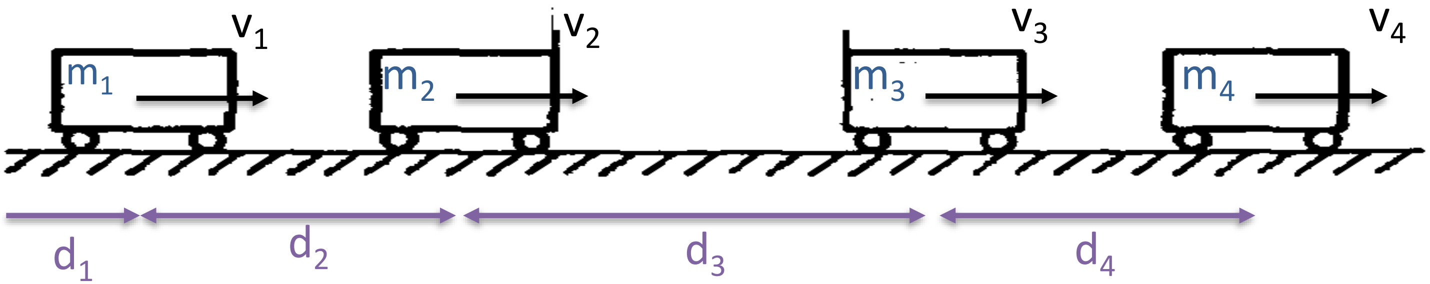
System Description

String of Vehicles



Each vehicle represents a «subsystem» of the overall plant and is characterized by two states:

- d_i: distance between vehicle i and its predecessor i-1 (if i=1, d_i is the vehicle's position).
- v_i: velocity of the vehicle i.

Input u_i: horizontal force to the vehicle i (subsystem i)

Friction force acting on each cart (h friction coefficient = 1) $-h_i v_i$

The dynamic of each cart is described by balance of longitudinal forces : $m_i \ddot{d}_i = m_i \dot{v}_i = -h_i v_i + u_i$

For the vehicle i =1 the dynamics is:

$$\begin{cases} \dot{d}_1 = v_1 \\ \dot{v}_1 = -\dfrac{1}{m_1} v_1 + \dfrac{1}{m_1} u_1 \end{cases}$$

while, for a generic $i > 1$,

$$\begin{cases} \dot{d}_i = v_i - v_{i-1} \\ \dot{v}_i = -\dfrac{1}{m_i} v_i + \dfrac{1}{m_i} u_i \end{cases}$$

State-space representation

Considering N=4 vehicles the system dynamics can be represented as:

$\dot{x}$ =$Ax+B_1u_1+B_2u_2+B_3u_3+B_4u_4$
$y_i=C_ix$ $\quad$ i=1,2,3,4

System Matrices:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B1=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad B2=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad B3=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad B4=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad D=0$$

Discretized state-space representation

Sampling time h=0.1 s

$$\begin{cases} x_{k+1} = F x_k + G u_k \\ y_k = H x_k + L u_k \end{cases} \qquad H = C$$

$$F = e^{Ah} \qquad L = D \qquad G = \int_0^h e^{A\nu} d\nu B$$

$$F=\begin{bmatrix} 1 & 0.0952 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9048 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0952 & 1 & 0.0952 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9048 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.0952 & 1 & 0.0952 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.9048 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.0952 & 1 & 0.0952 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9048 \end{bmatrix} \quad H=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

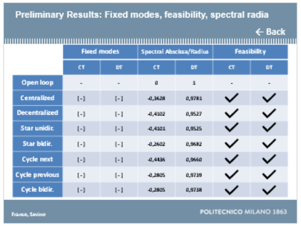
$$G1=\begin{bmatrix} 0.0048 \\ 0.0952 \\ -0.0048 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad G2=\begin{bmatrix} 0 \\ 0 \\ 0.0048 \\ 0.0952 \\ -0.0048 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad G3=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.0048 \\ 0.0952 \\ -0.0048 \\ 0 \end{bmatrix} \quad G4=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.0048 \\ 0.0952 \end{bmatrix} \quad L=0$$

Eigenvalues of the Open-Loop CT and DT systems

Eigenvalues:

$$\text{eigenCT}=\begin{bmatrix}0\\0\\-1\\0\\-1\\0\\-1\\-1\end{bmatrix}\quad \text{eigenDT}=\begin{bmatrix}1\\1\\0.9048\\1\\0.9048\\1\\0.9048\\0.9048\end{bmatrix}$$

Comparisons with closed loops:



Preliminary Results: Fixed modes, feasibility, spectral radia

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	Fixed modes		Spectral Abscissa/Radius		Feasibility	
	CT	DT	CT	DT	CT	DT
Open loop	-	-	0	1	-	-
Centralized	[-]	[-]	-0,3628	0,9781	✓	✓
Decentralized	[-]	[-]	-0,4102	0,9527	✓	✓
Star unidir.	[-]	[-]	-0,4101	0,9525	✓	✓
Star bidir.	[-]	[-]	-0,2602	0,9682	✓	✓
Cycle next	[-]	[-]	-0,4436	0,9660	✓	✓
Cycle previous	[-]	[-]	-0,2805	0,9739	✓	✓
Cycle bidir.	[-]	[-]	-0,2805	0,9738	✓	✓

Franco, Savino

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- Continuous LTI system is asympt. Stable $\Longleftrightarrow$ all eigs of A have strictly negative real part (A Hurwitz stable)
- Continuous LTI system is stable $\Longleftrightarrow$ all eigs of A have modulus smaller than 0 and those equal to 0 have algebraic = geometric multiplicity
- Discrete LTI system is asympt. stable $\Longleftrightarrow$ all eigs of F have modulus strictly smaller than 1 (F Shur stable)
- Discrete LTI system is stable $\Longleftrightarrow$ all eigs of F have modulus smaller than 1 and those equal to 1 have algebraic = geometric multiplicity

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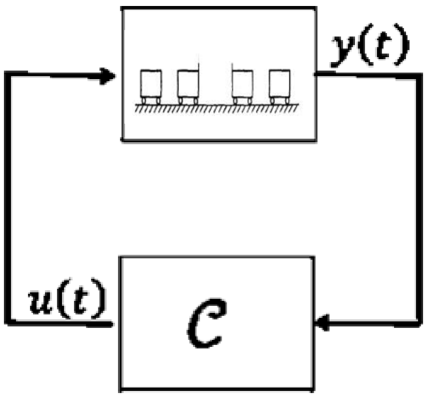
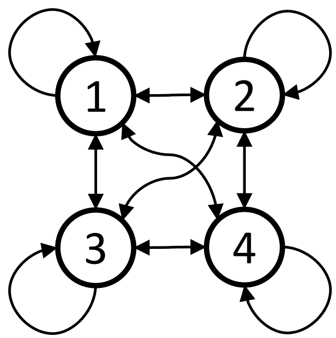
Control Structures

Description of the 7 different control structures developed

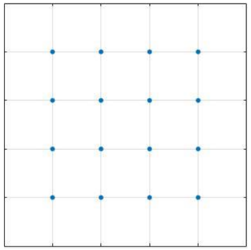
Control Structures: Basic Structures

Centralized Control Structure

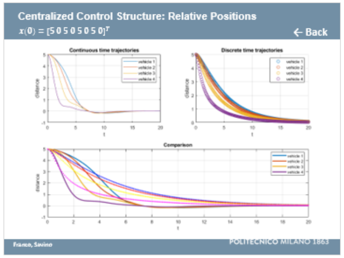




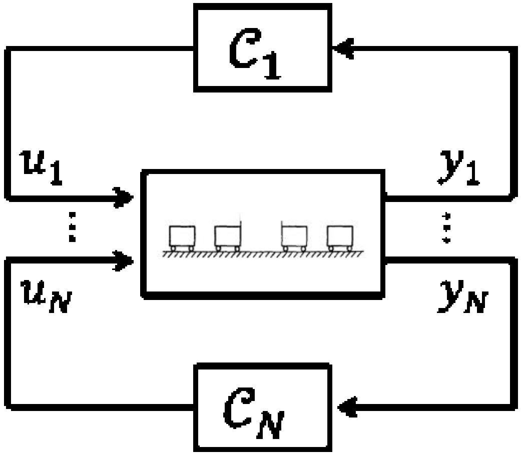
(With the functions available is simplified into a **decentralized control structure** where every node communicate bidirectionally with every other node)

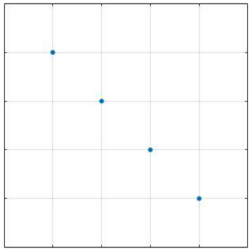


Plots:

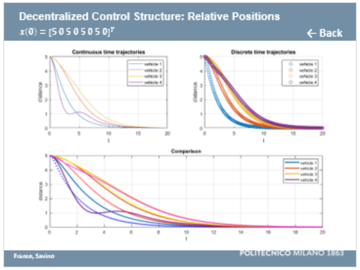


Decentralized Control Structure



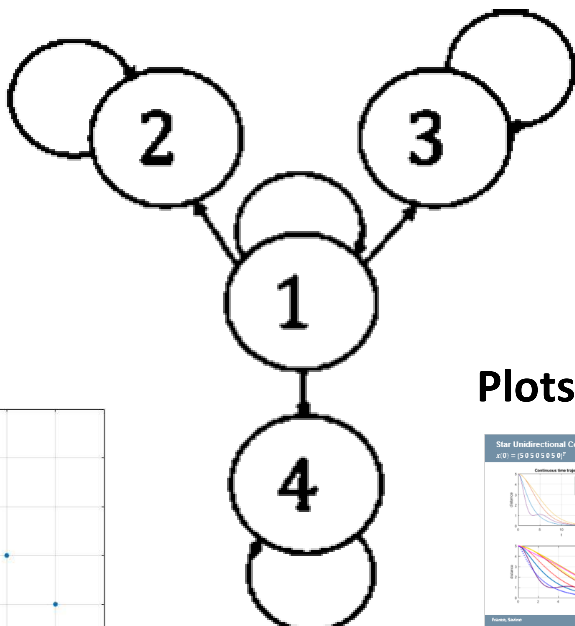


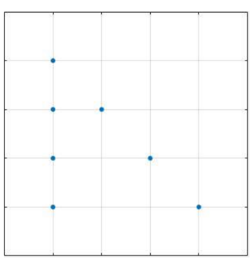
Plots:



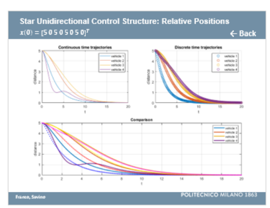
Control Structures: Star Node

Star node with unidirectional communication

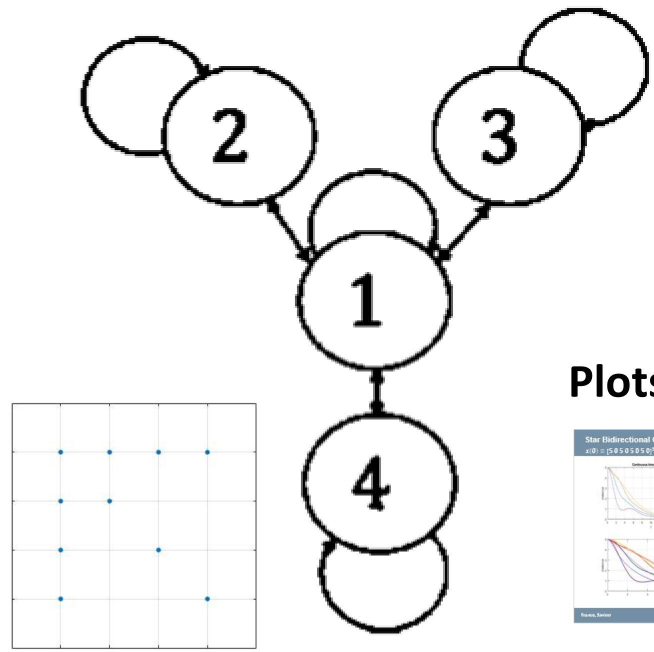




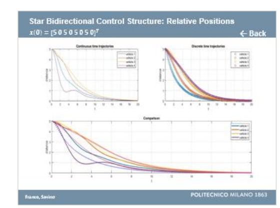
Plots:



Star node with bidirectional communication



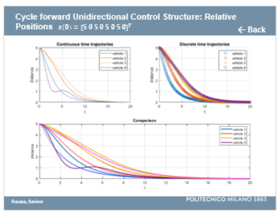
Plots:

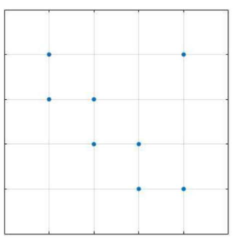


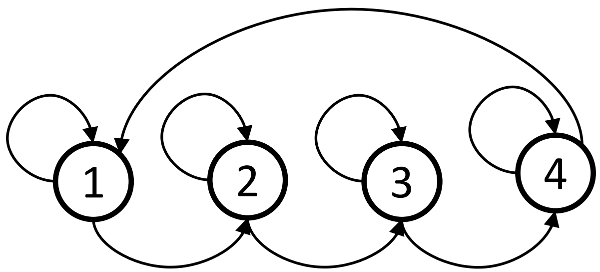
Control Structures: Cycles

Cycle structure unidirectional towards next vehicle

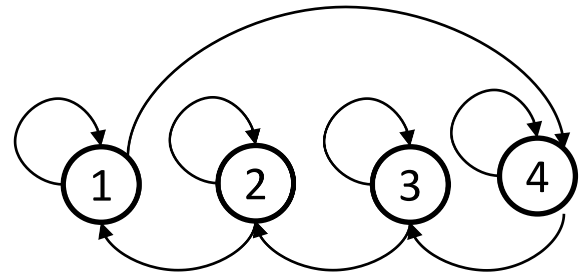
Plots:



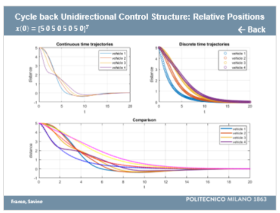


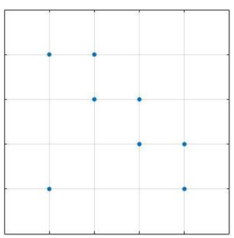


Cycle structure unidirectional towards previous vehicle

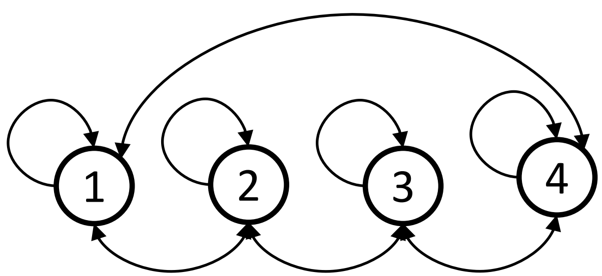


Plots:

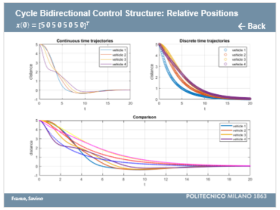


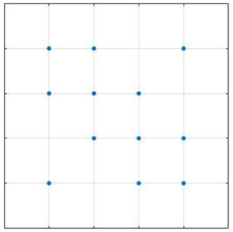


Cycle structure bidirectional



Plots:





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Preliminary Results

Fixed modes, feasibility, spectral abscissa/radia

Preliminary Results: Fixed modes, feasibility, spectral radia

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Cycle bidir.	[-]	[-]	-0,2805	0,9738	✓	✓

Considerations on Preliminary Results

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- The open loop eigenvalues do not guarantee asymptotical stability, but applying a state feedback controller makes the system asympotitically stable, as seen from the spectral abscissa
- System becomes asymptotically stable both in CT and DT;
- None of the obtained closed loop systems presents Fixed Modes;
- Performances in terms of settling time are similar for every control scheme ($t_{set} \approx 10/15s$);
- The couples Decentralized/Star Unidirectional and Cycle Previous/Cycle Bidirectional present very similar behaviours due to similar closed loop matrices that only differ in some small elements outside the diagonal.

Plots

$$\boldsymbol{x}(\mathbf{0}) = [\mathbf{5\ 0\ 5\ 0\ 5\ 0\ 5\ 0}]^{T}$$

$$\boldsymbol{x}_{\boldsymbol{ref}} = [\mathbf{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0}]^{T}$$

Continuous and Discrete Time comparison of relative positions

Continuous and Discrete Time comparison of absolute positions

Examples of Speed Profiles with Simulink plots

Centralized Control Structure: Relative Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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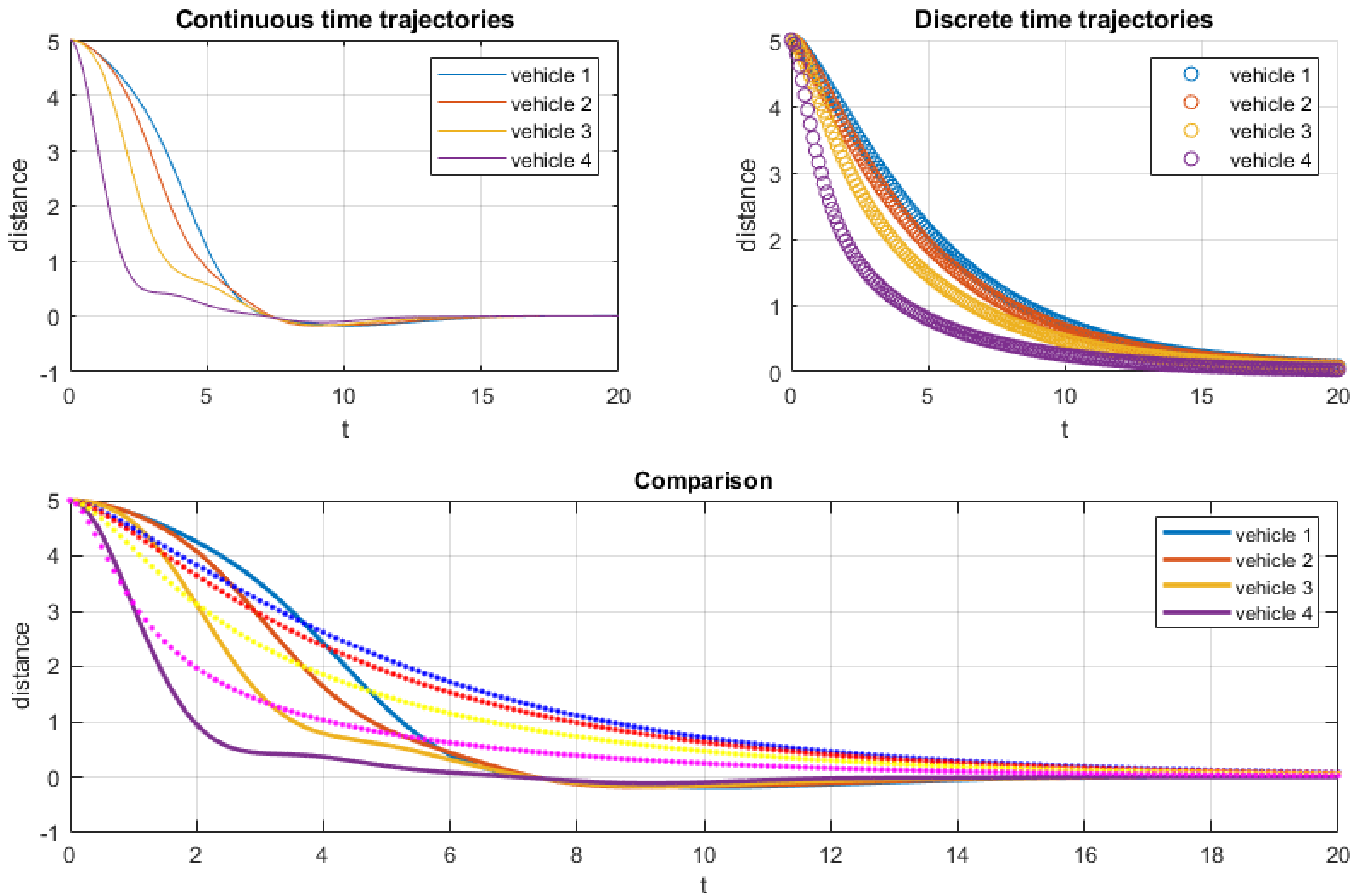

Centralized Control Structure: Absolute Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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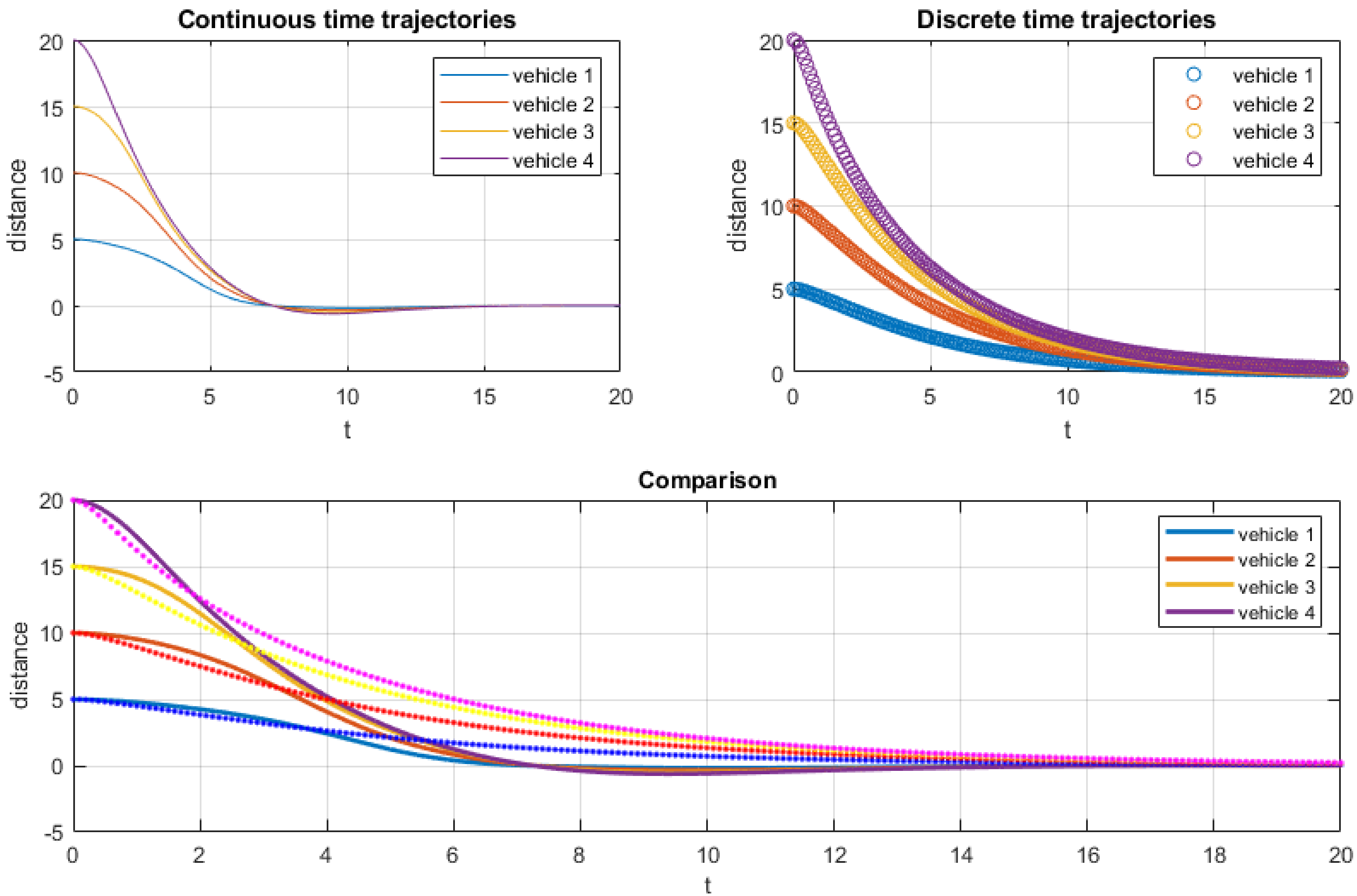

Centralized Control Structure: CT Speed Profiles

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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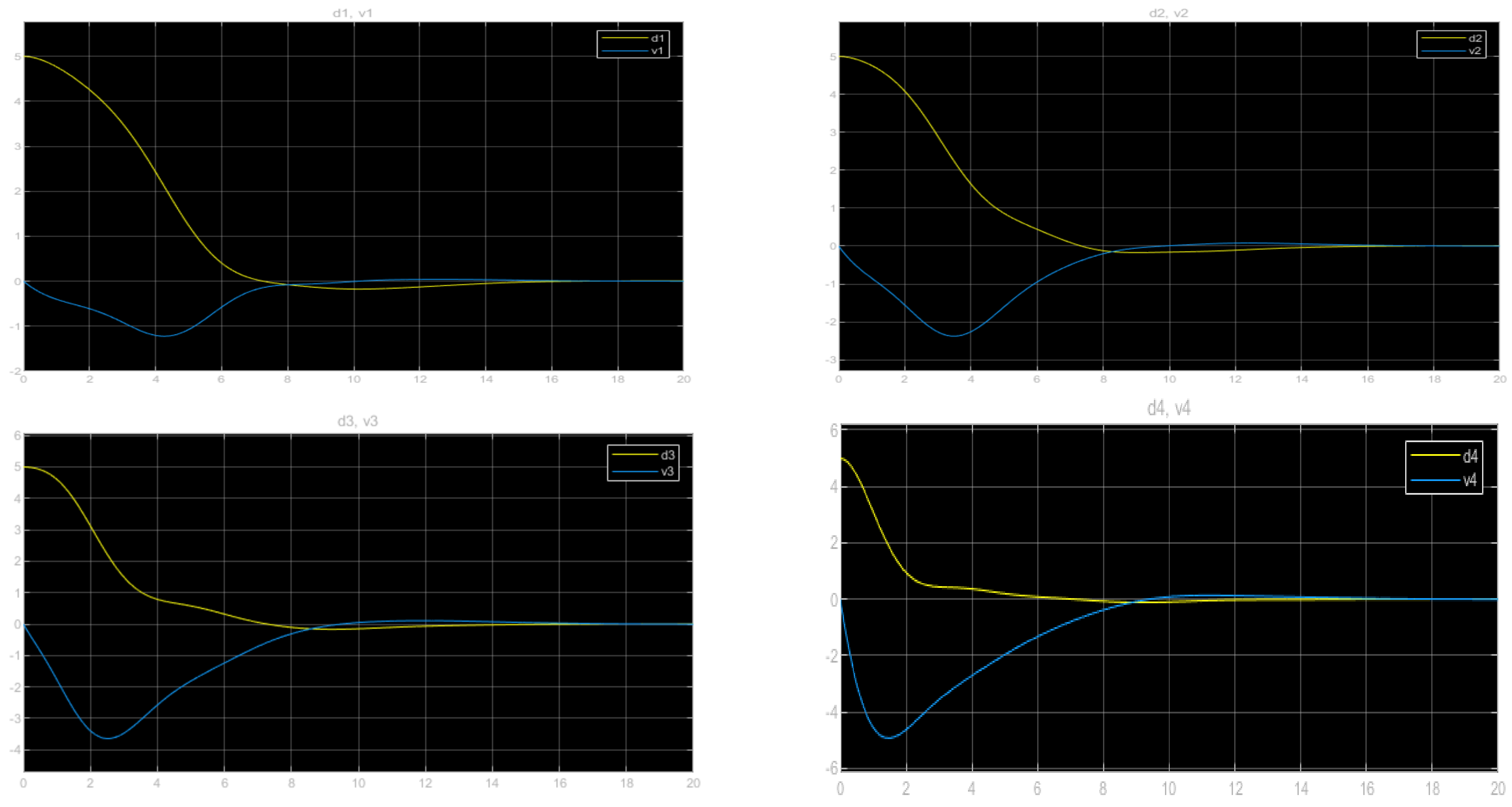

Decentralized Control Structure: Relative Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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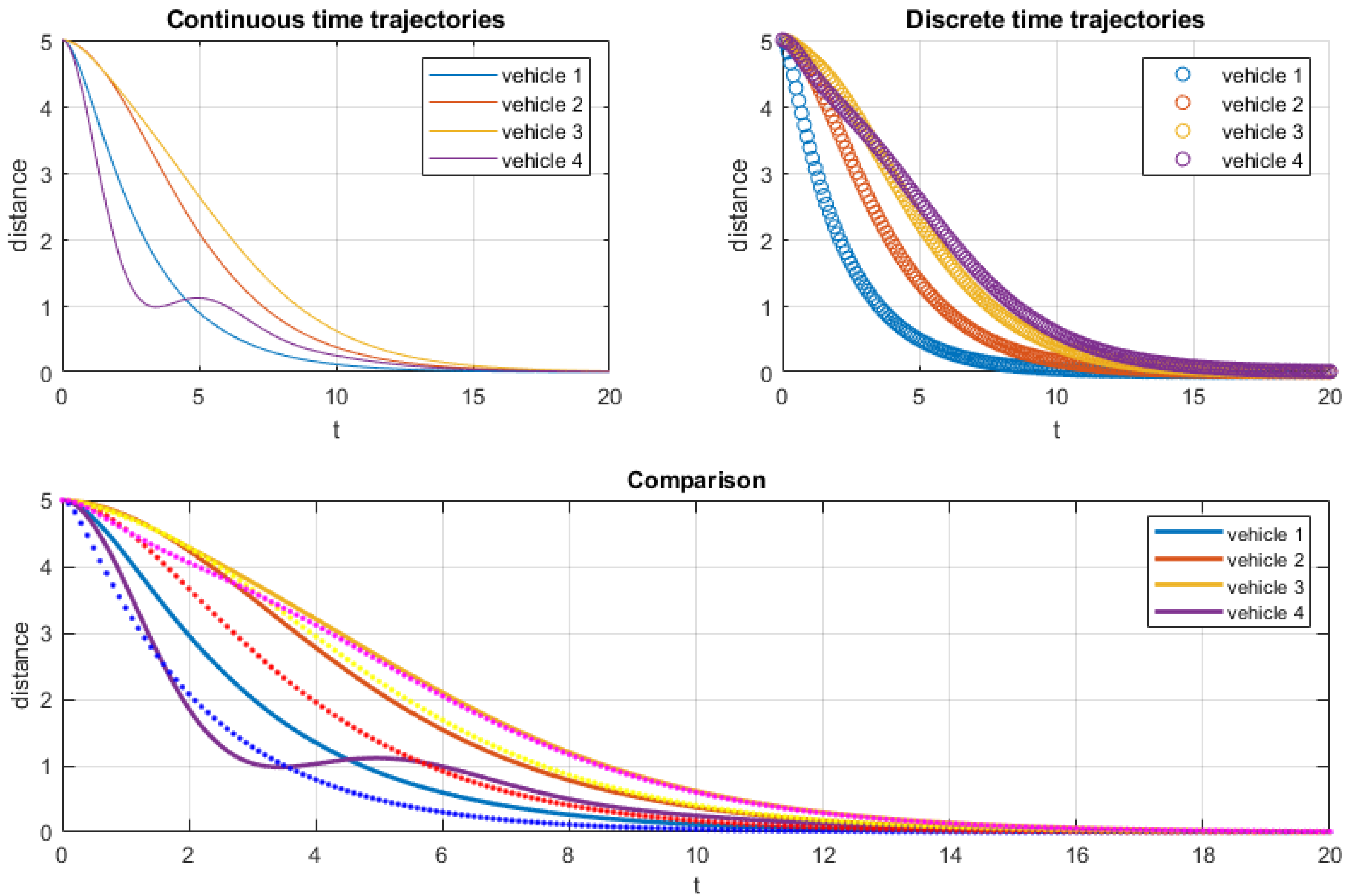

Decentralized Control Structure: Absolute Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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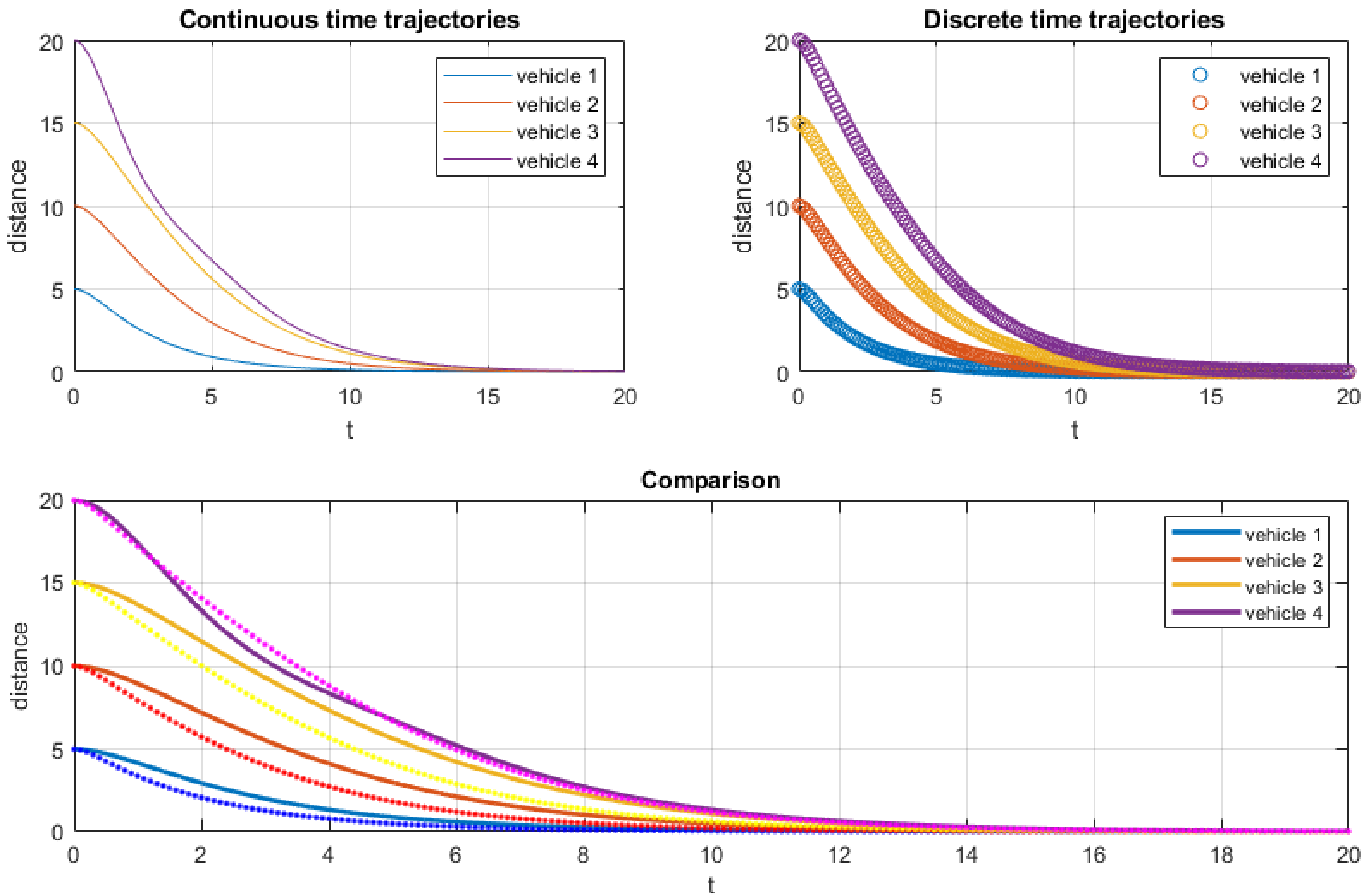

Decentralized Control Structure: DT Speed Profiles

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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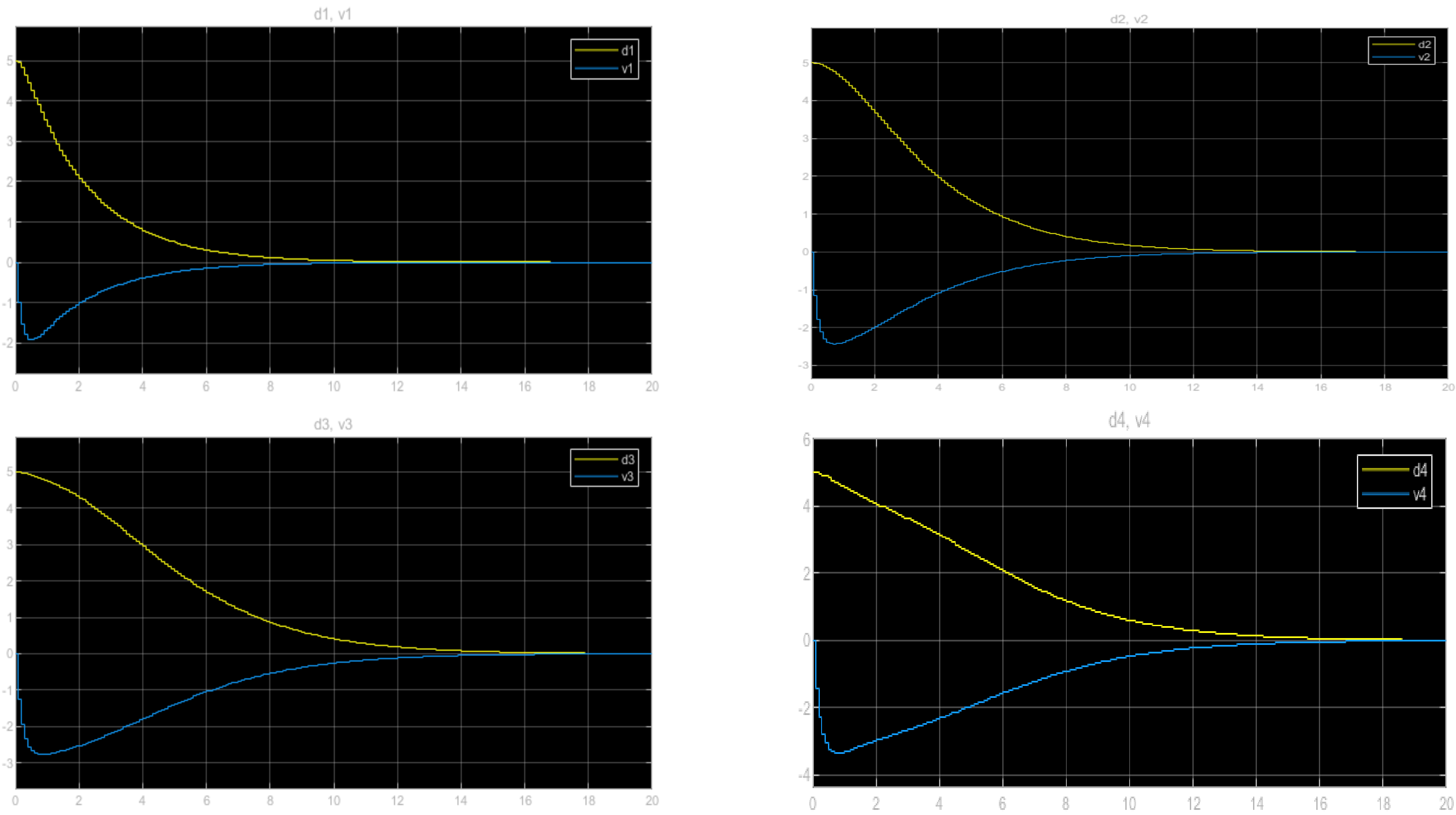

Star Unidirectional Control Structure: Relative Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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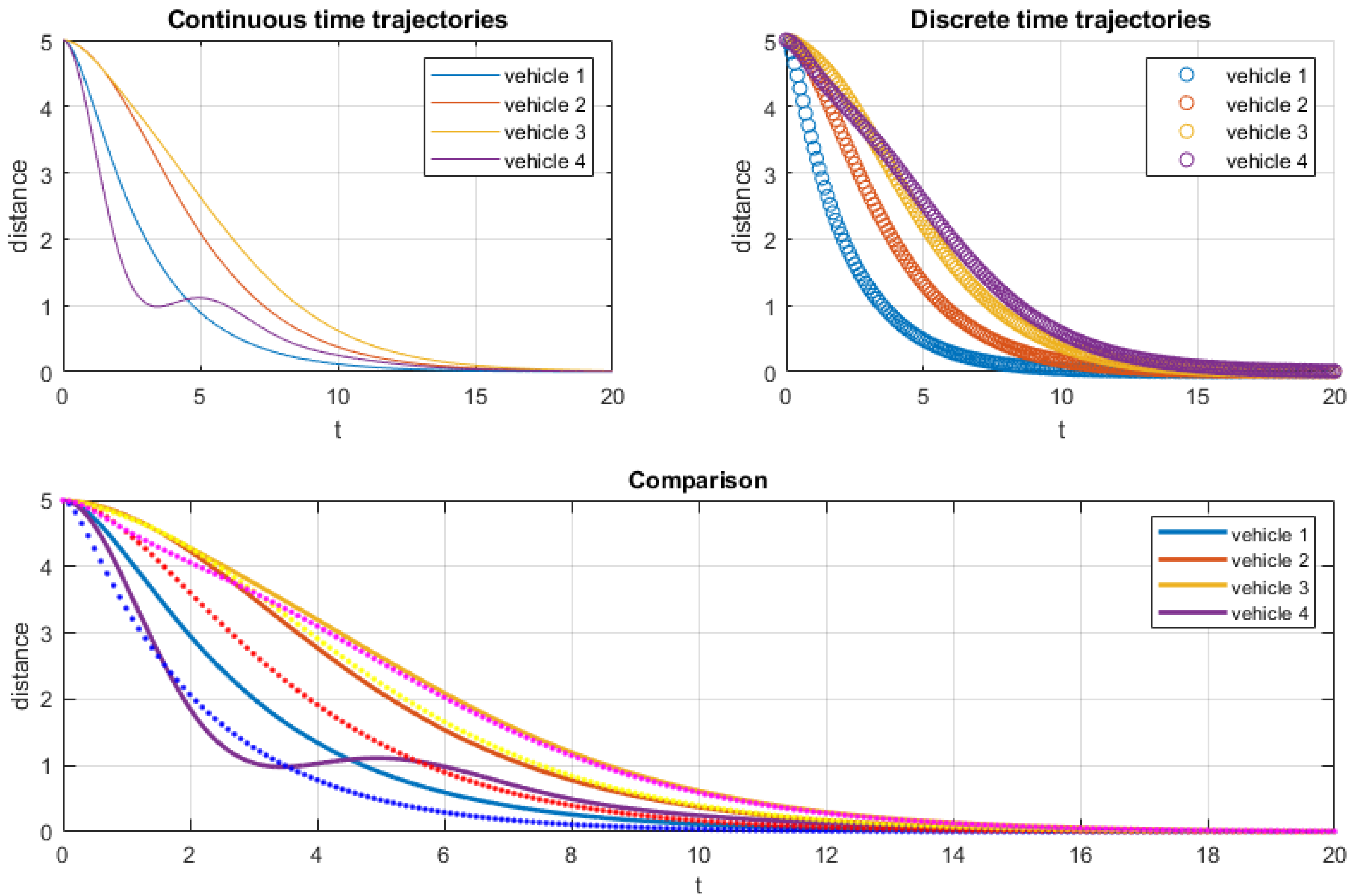
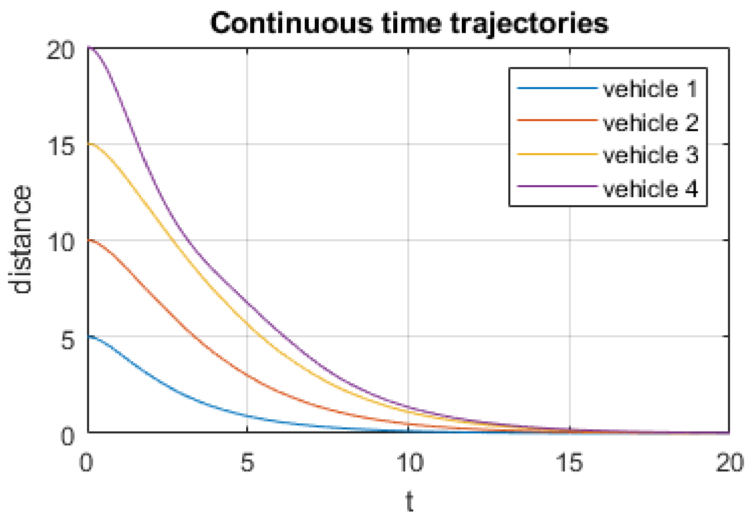
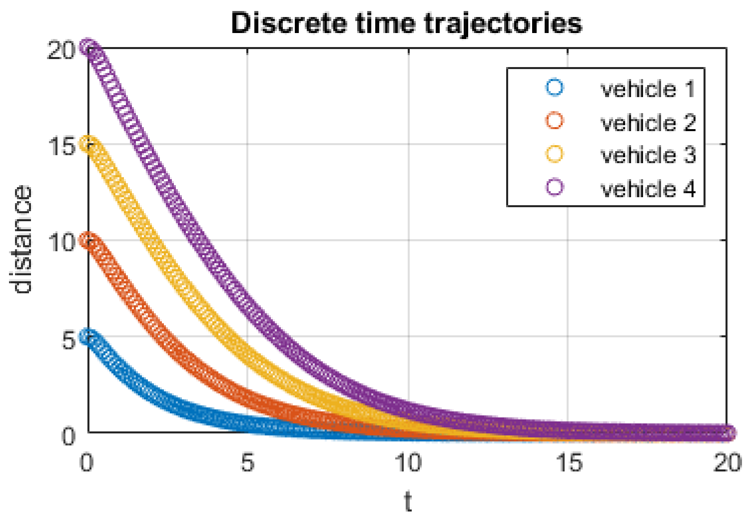


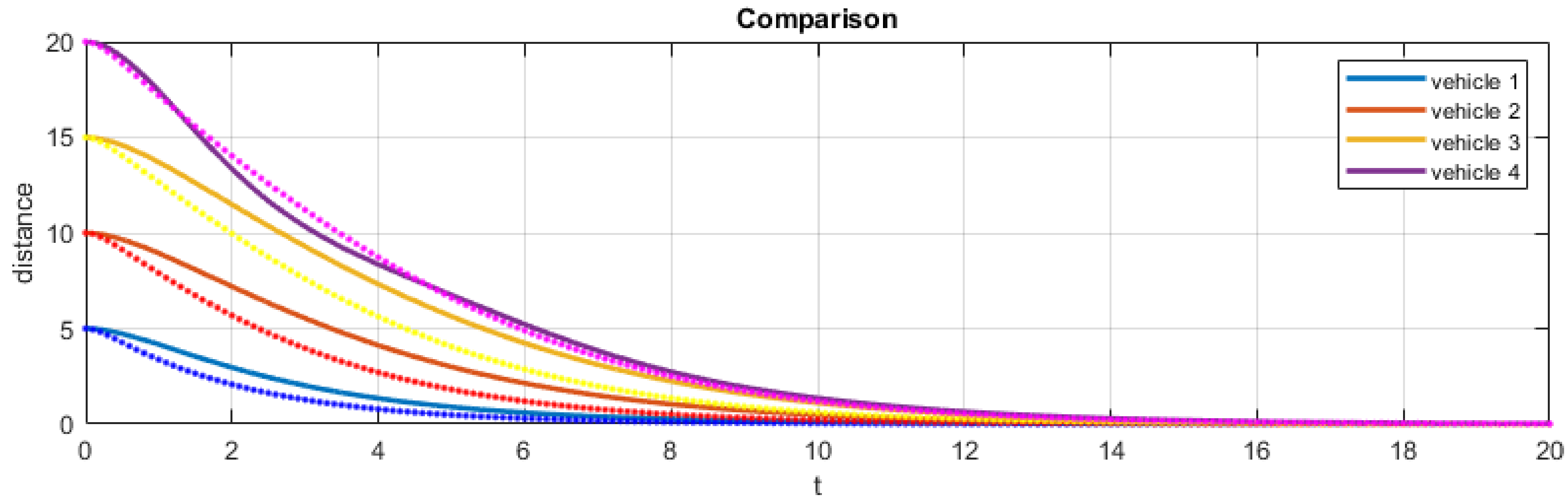
Star Unidirectional Control Structure: Absolute Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

← Back







Star Bidirectional Control Structure: Relative Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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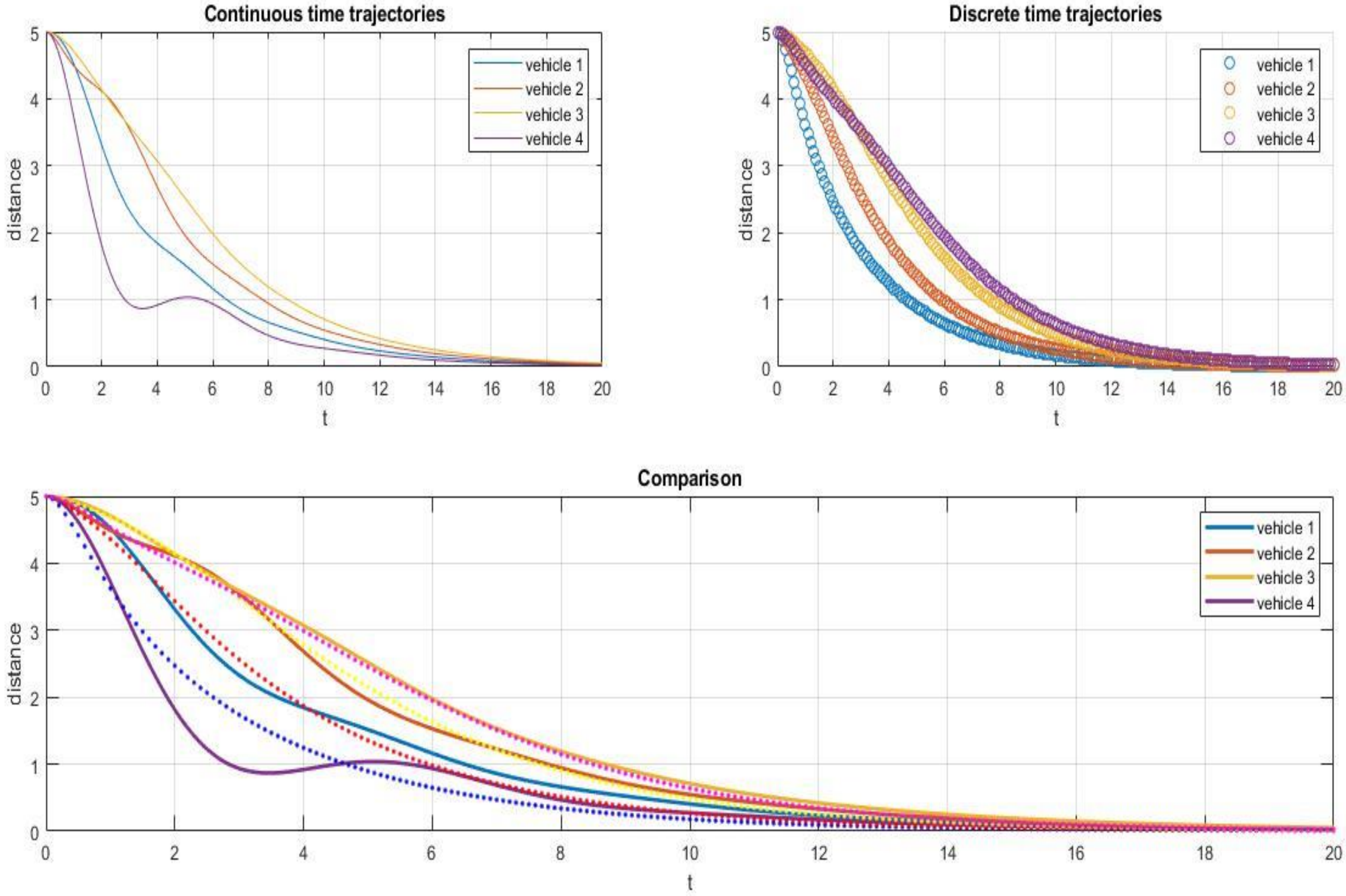

Star Bidirectional Control Structure: Absolute Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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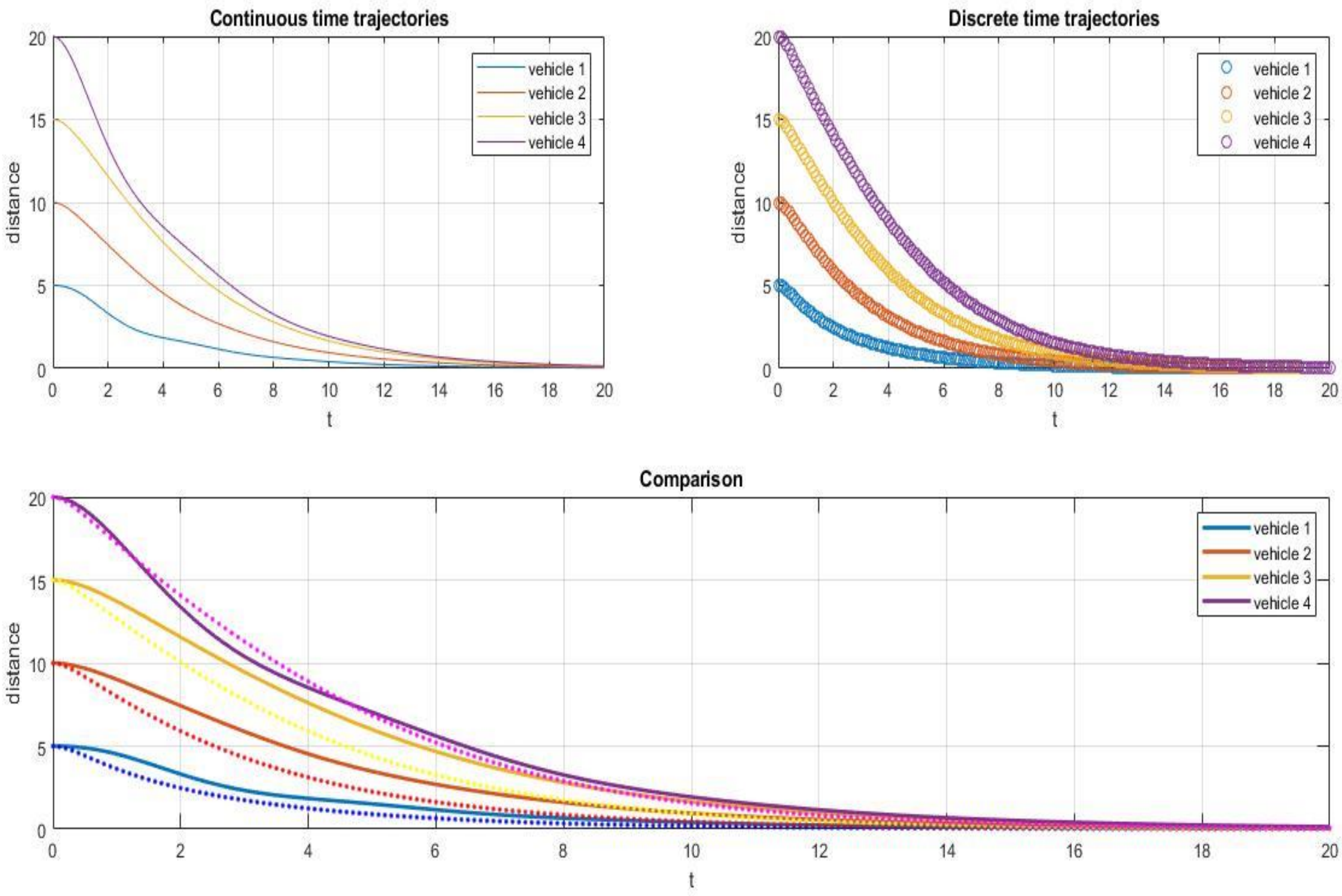

Cycle forward Unidirectional Control Structure: Relative Positions $x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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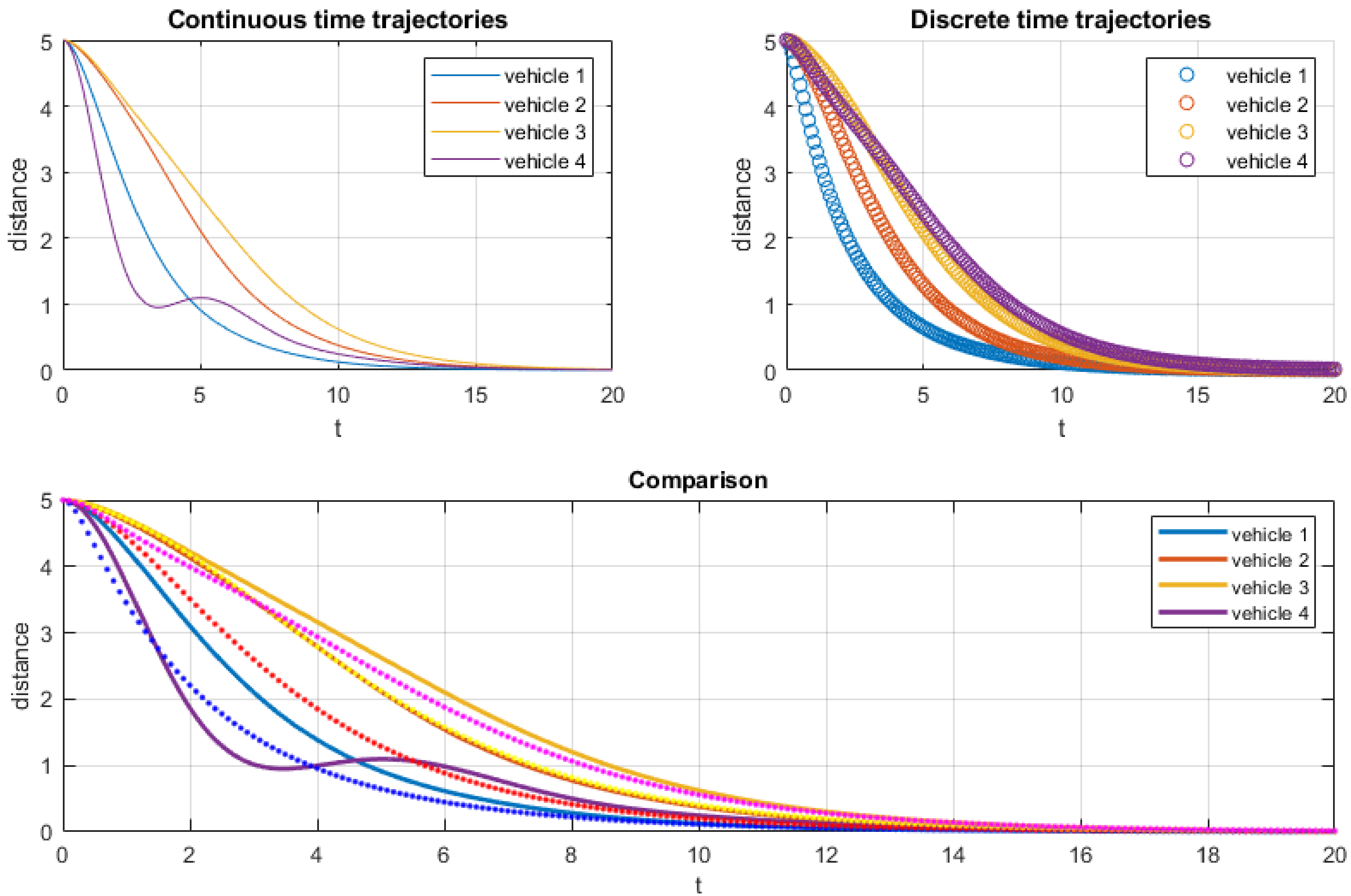
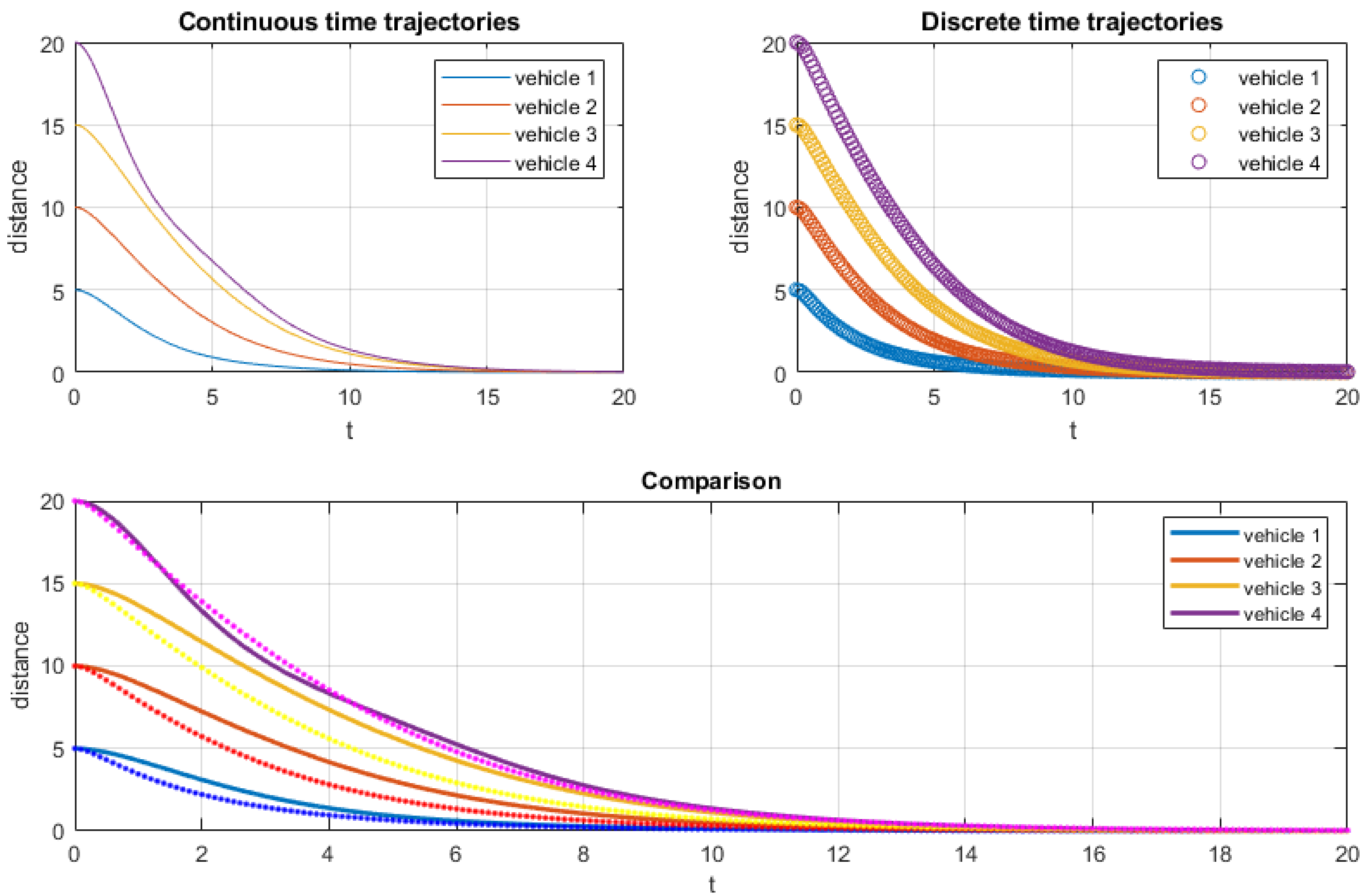


Cycle forward Unidirectional Control Structure: Absolute Positions $x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

← Back



Cycle back Unidirectional Control Structure: Relative Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

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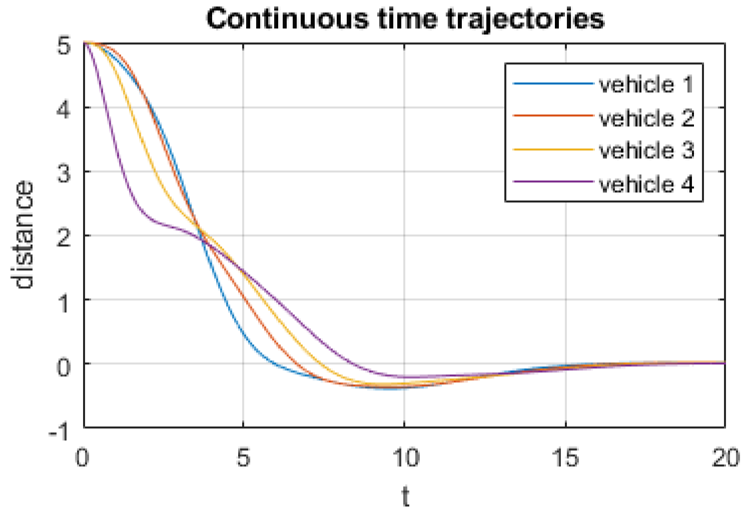
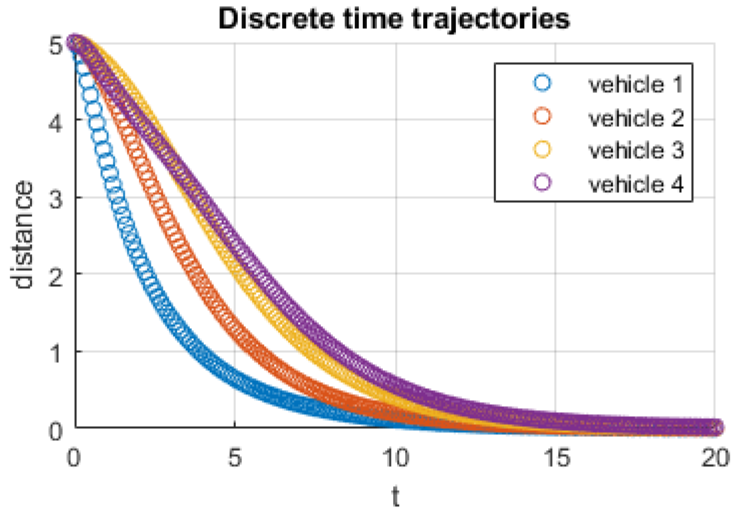
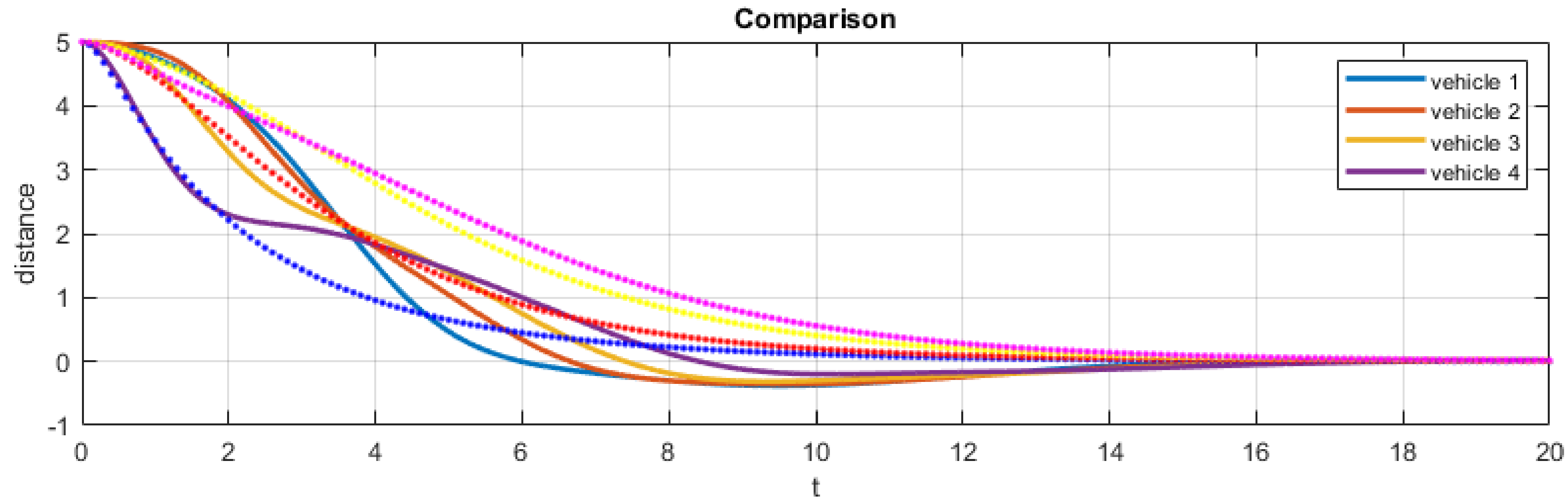
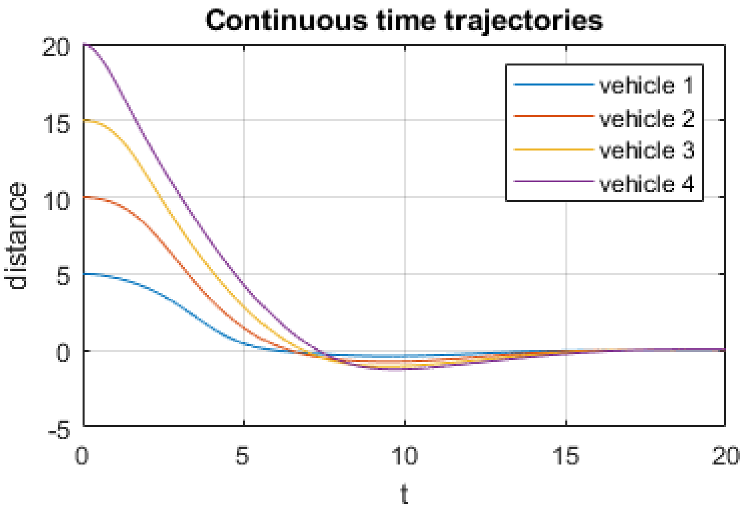


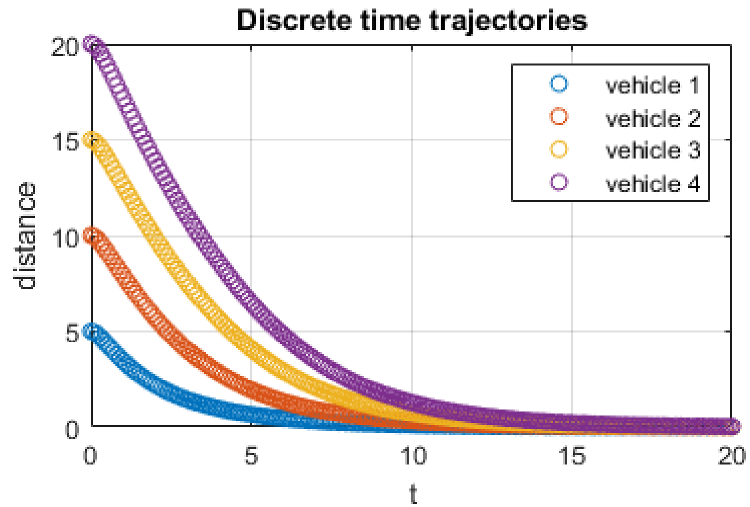


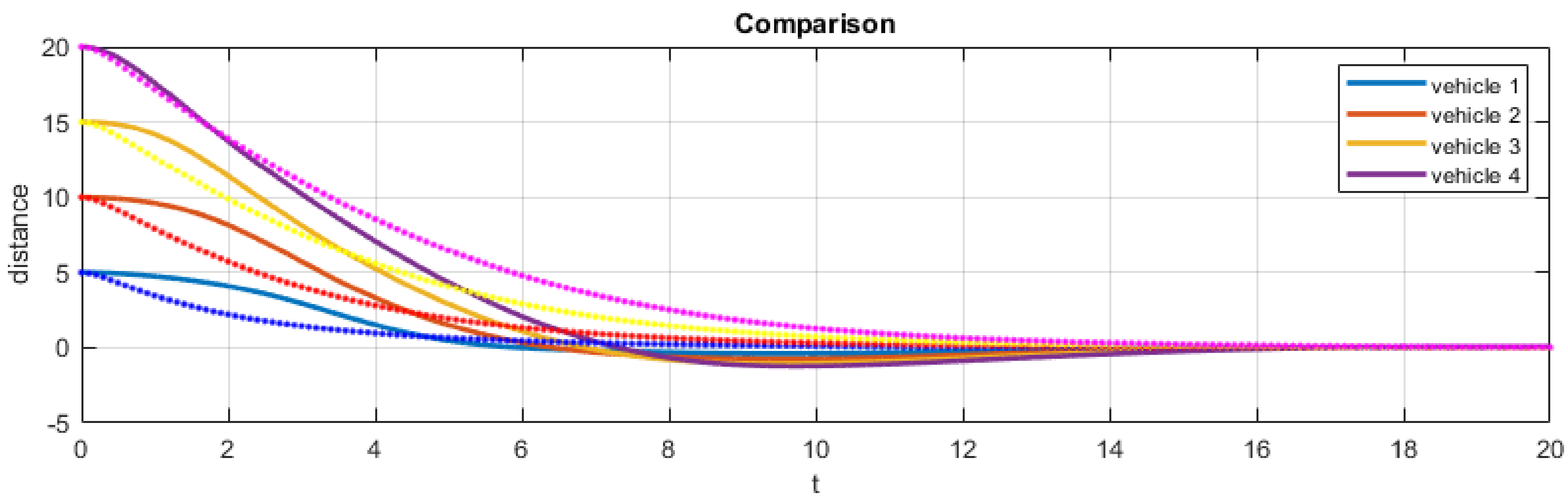


Cycle back Unidirectional Control Structure: Absolute Positions $x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

← Back



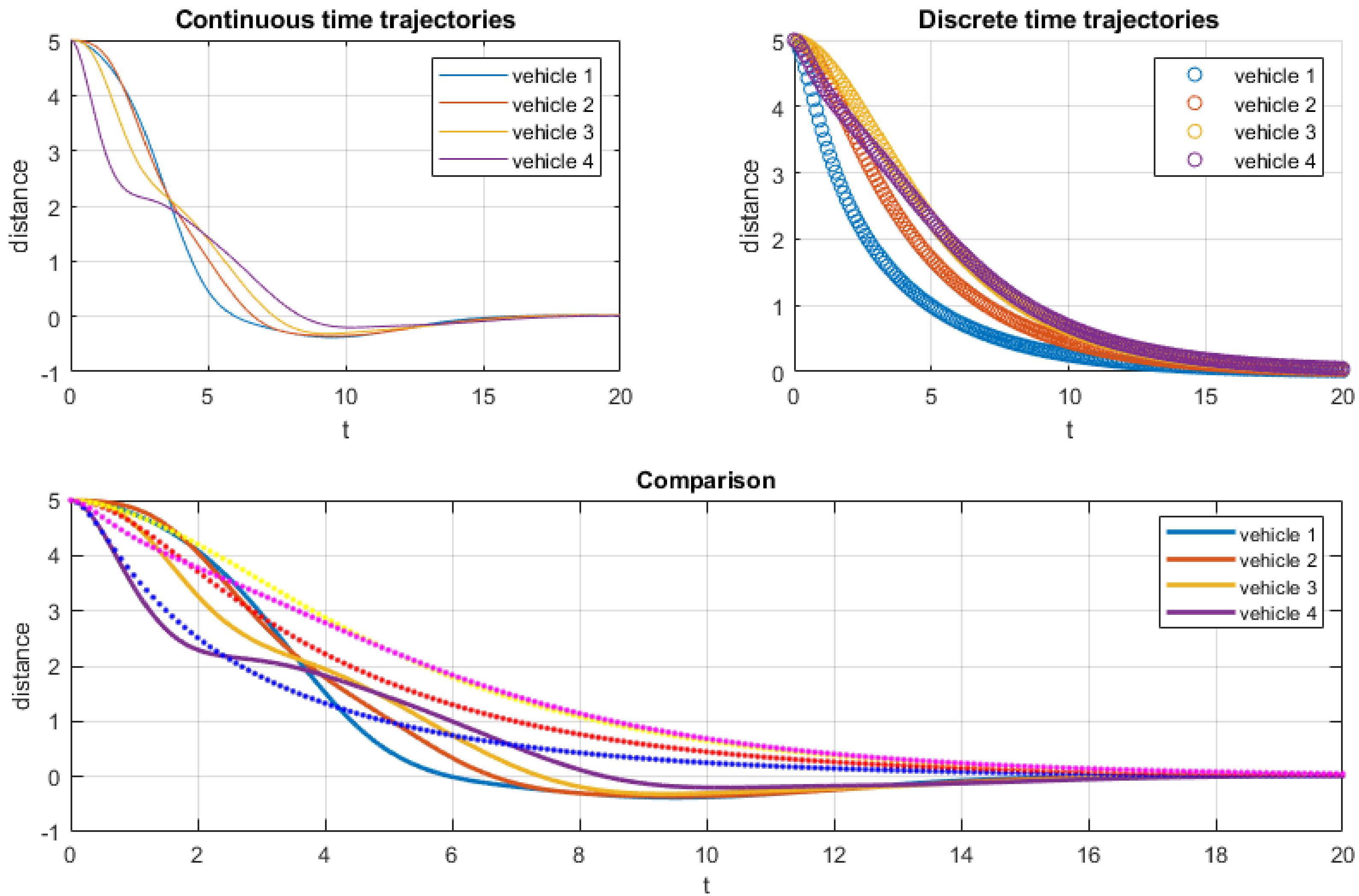




Cycle Bidirectional Control Structure: Relative Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

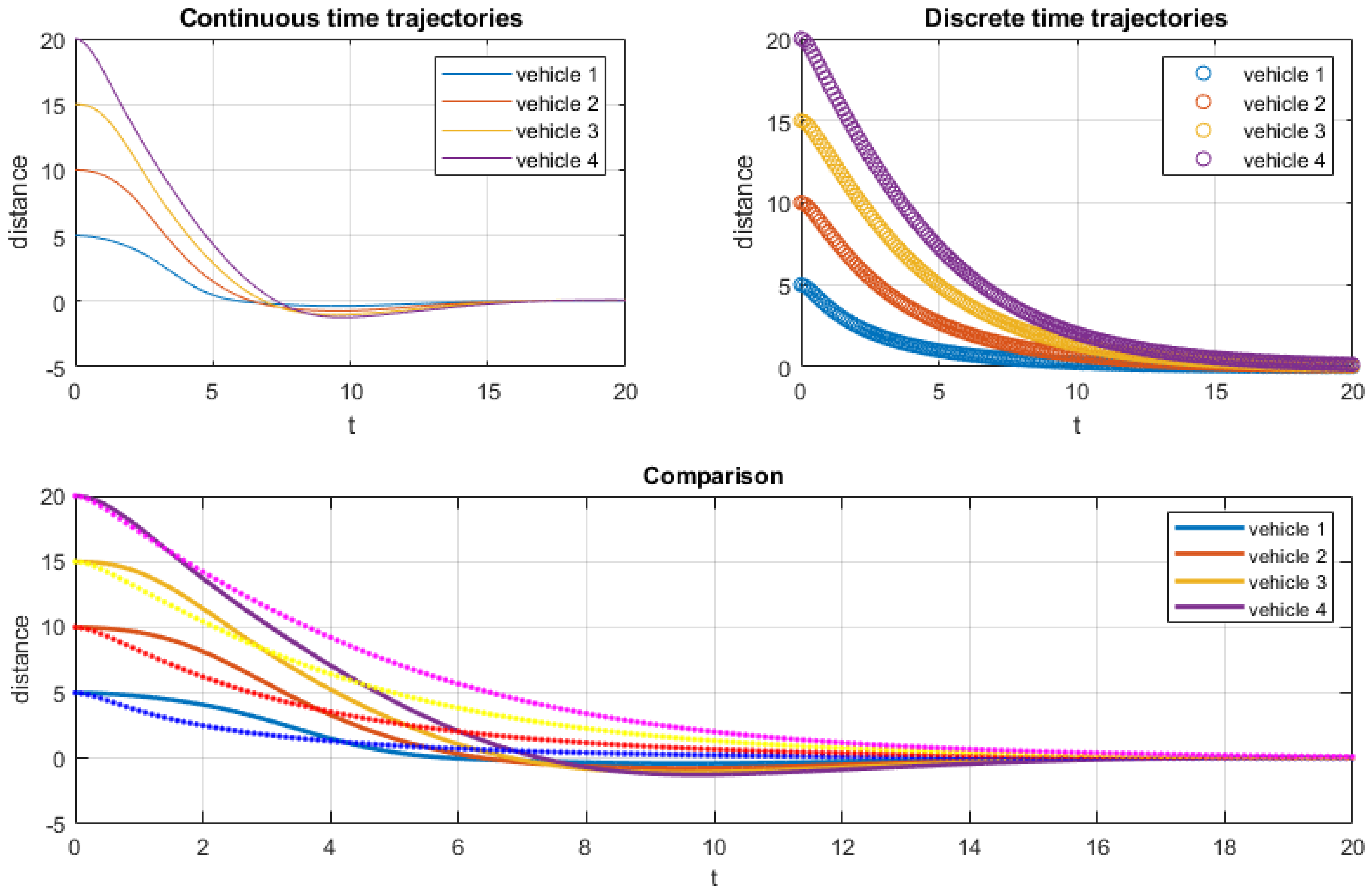
← Back



Cycle Bidirectional Control Structure: Absolute Positions

$x(0) = [5\ 0\ 5\ 0\ 5\ 0\ 5\ 0]^T$

← Back



Conclusions

- The Centralized scheme in CT is the faster in reaching the stability: all vehicles go to position 0 in 7,5 s;
- Apart from the first and the last two cases, Discrete Time system have better performaces with respect to their Continuous Time twin;
- Generally, Discrete Time systems initially react with a larger input signal which gives a largest negative velocity to the vehicles in order to push them towards the position 0. Then, in the cases where CT systems are more performing, the speed profile in DT becomes smoother while it gets more sloping in CT.