

# Estudo de um modelo simples para o diodo térmico



# 1. Introdução

## 1.1. Motivação

## 1.2. Fundamentos

# 2. Modelo matemático

# 3. Metodologia

## 3.1. Análise

## 3.2. Métodos Numéricos

# 4. Resultados

# 5. Conclusão

# 1. Introdução

## 1. Introdução

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# Solid-State Thermal Rectifier

C. W. Chang,<sup>1,4</sup> D. Okawa,<sup>1</sup> A. Majumdar,<sup>2,3,4</sup> A. Zettl<sup>1,3,4\*</sup>

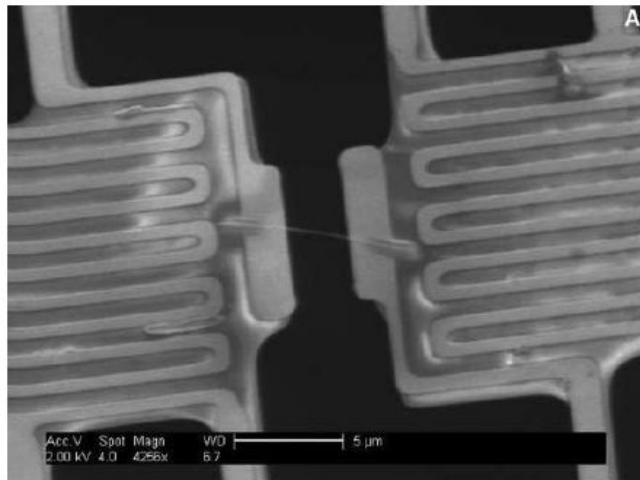
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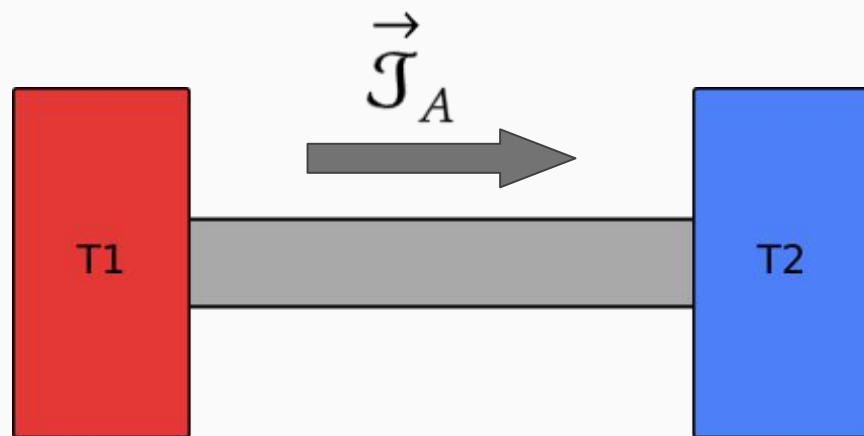
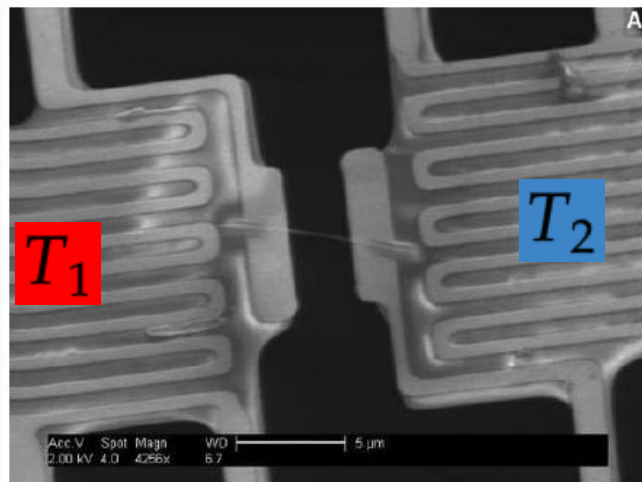


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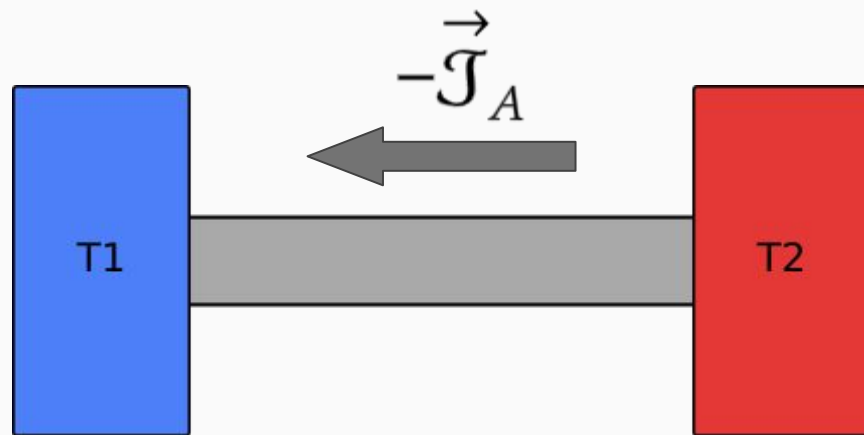
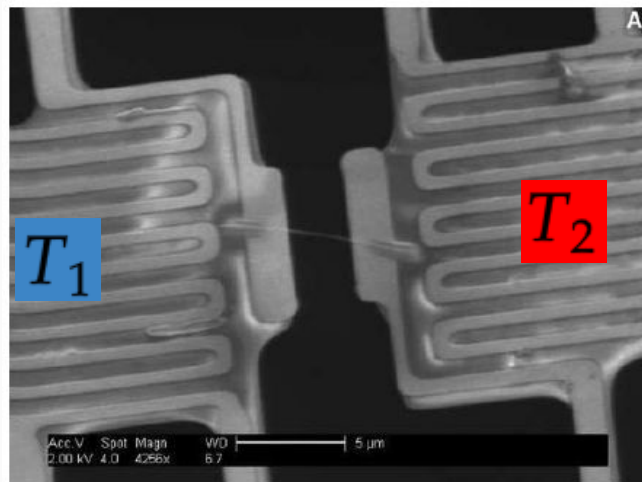


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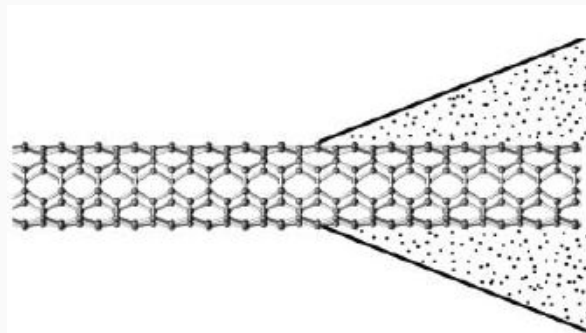
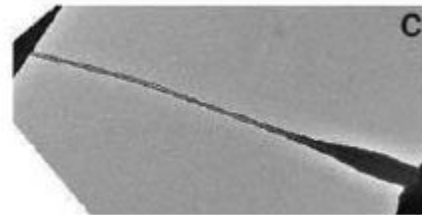
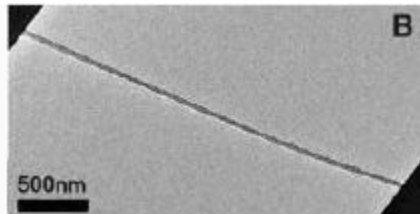


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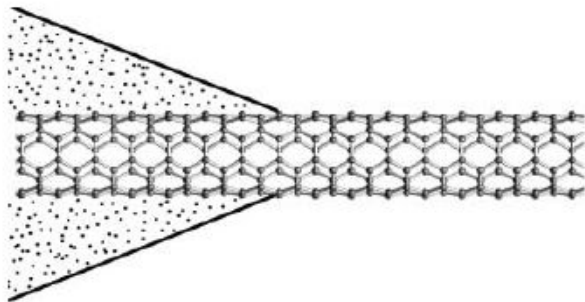


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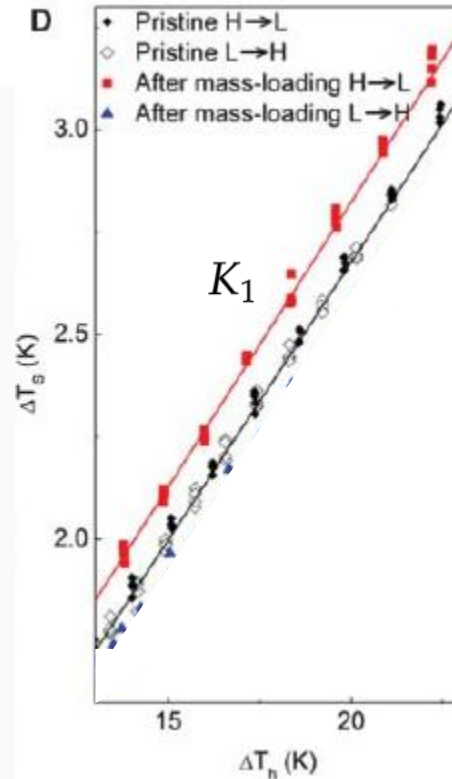
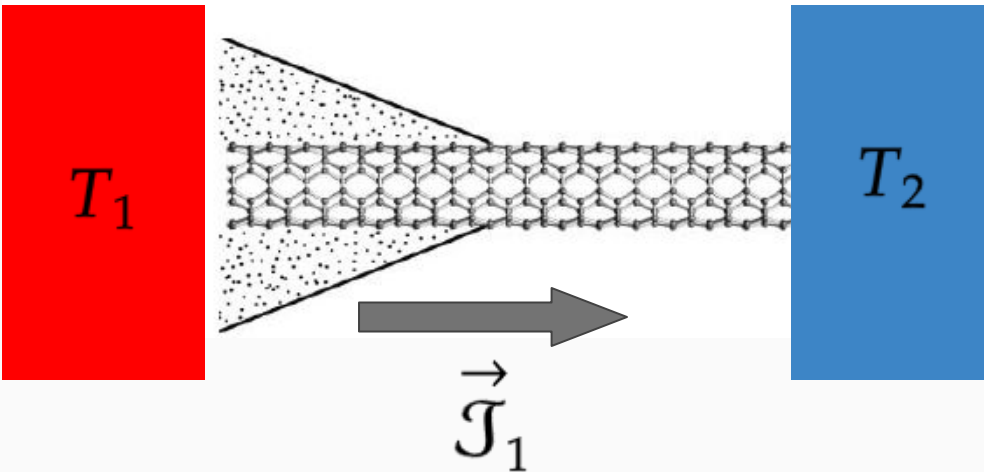


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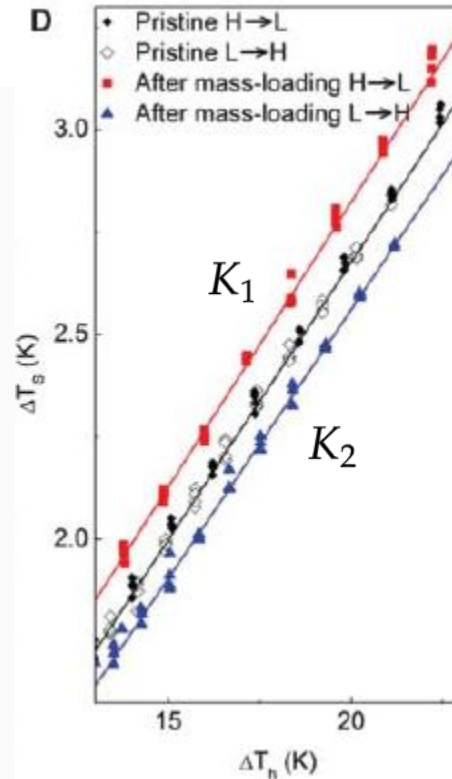
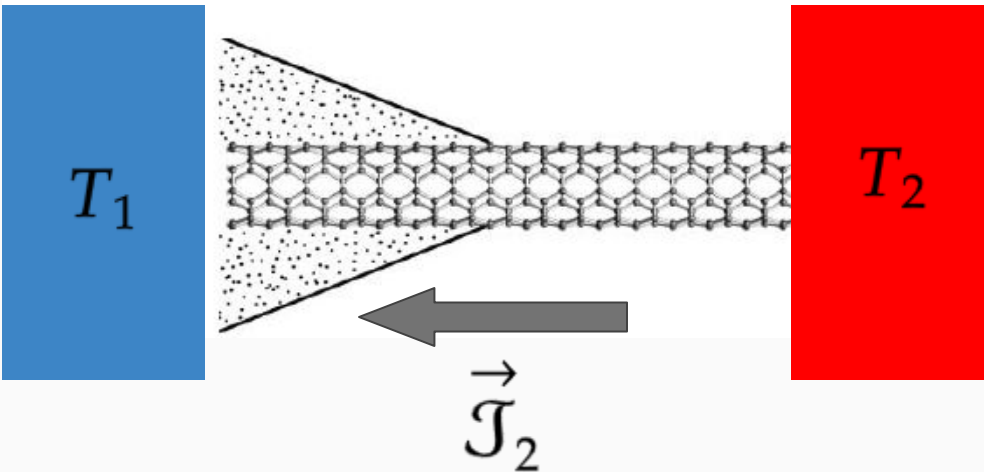


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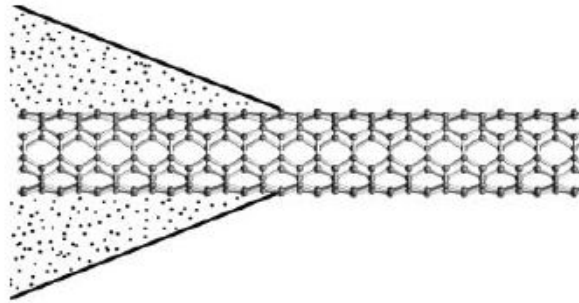


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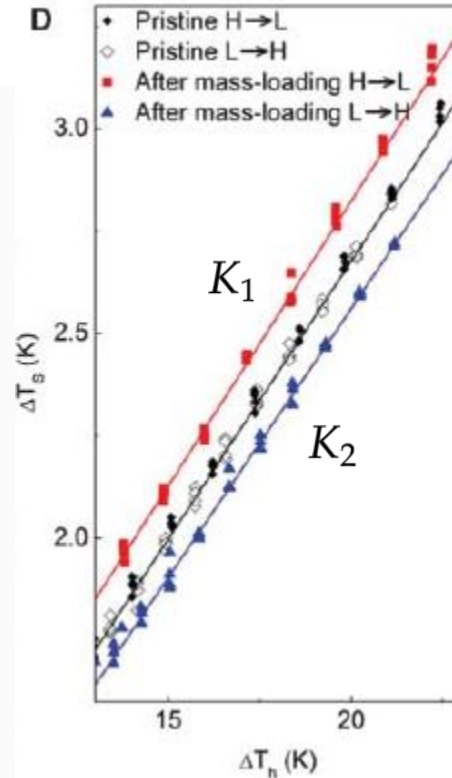
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$$\frac{K_1 - K_2}{K_2} \approx 7\%$$



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APPLIED PHYSICS LETTERS **95**, 171905 (2009)

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## Rectification of electronic heat current by a hybrid thermal diode

Maria José Martínez-Pérez, Antonio Fornieri and Francesco Giazotto\*

nature  
nanotechnology

LETTERS

PUBLISHED ONLINE: 23 FEBRUARY 2015 | DOI: 10.1038/NNANO.2015.11

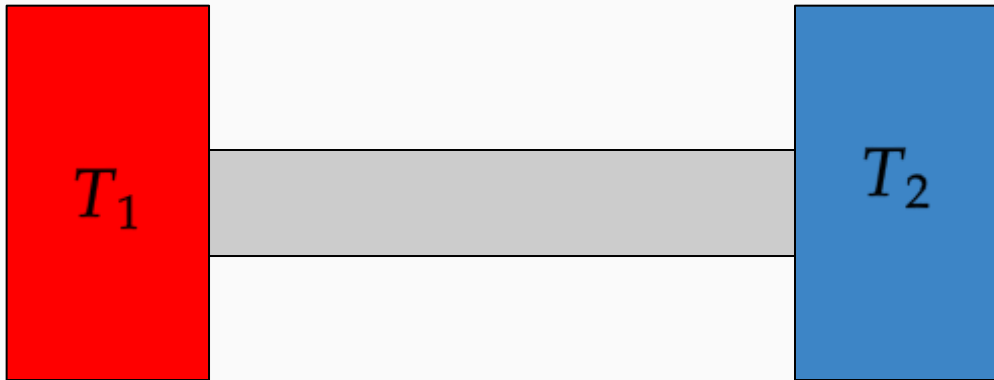
## 1. Introdução

# Equação do Calor



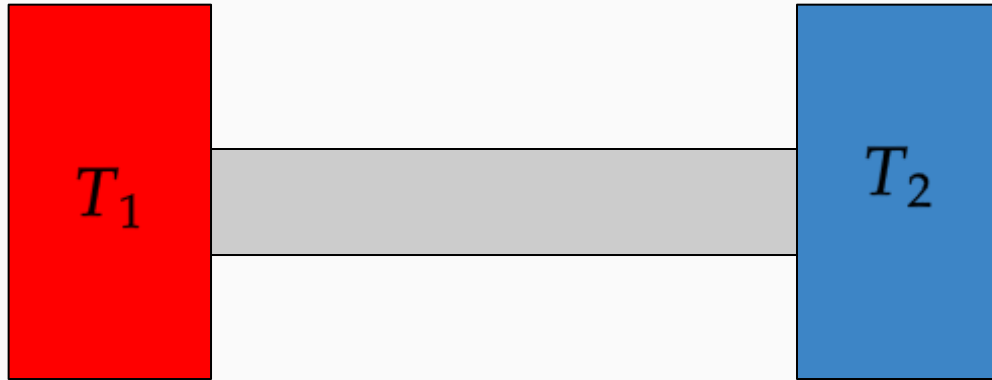
## 1. Introdução

### Equação do Calor



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## Equação do Calor



1ª lei da  
termodinâmica

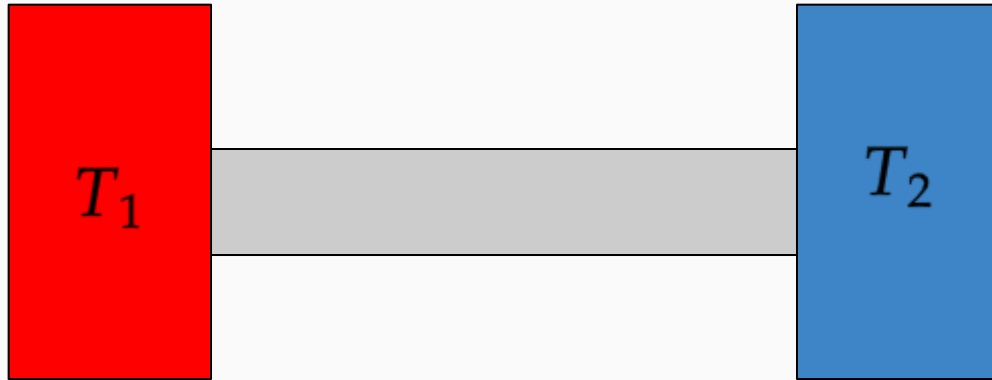


Equação da  
Continuidade

$$-\frac{\partial \dot{Q}}{\partial x} = c_p \frac{\partial T}{\partial t}$$

# 1. Introdução

## Equação do Calor



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Equação da  
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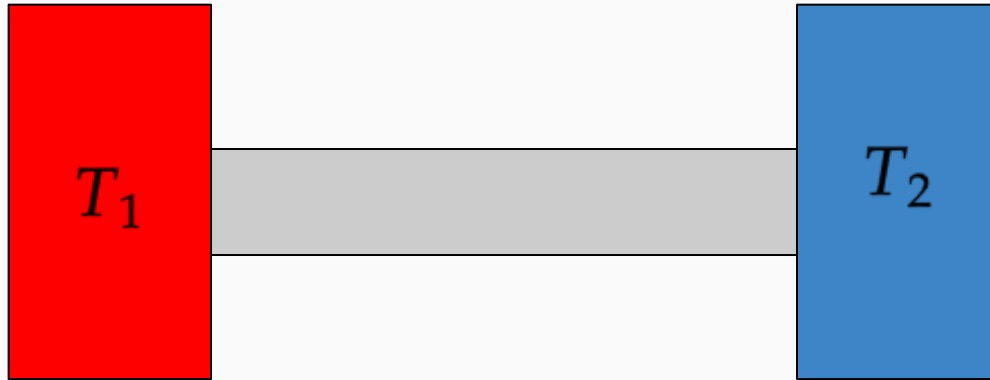
$$-\frac{\partial \dot{Q}}{\partial x} = C\rho \frac{\partial T}{\partial t}$$

Lei de Fourier

$$\dot{Q} = -\kappa \frac{dT}{dx}$$

# 1. Introdução

## Equação do Calor



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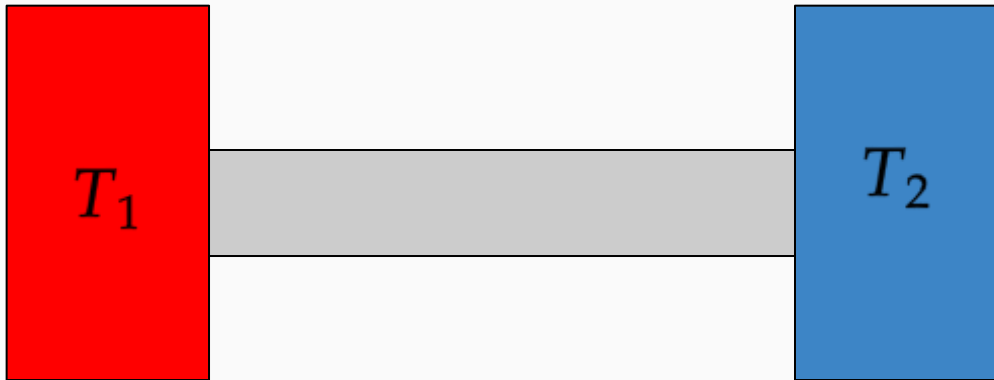
$$\dot{Q} = -\kappa \frac{dT}{dx}$$

Equação do Calor

$$\frac{\partial T}{\partial t} = \alpha^2 \frac{\partial^2 T}{\partial x^2}; \quad \alpha^2 = \frac{\kappa}{C\rho}$$

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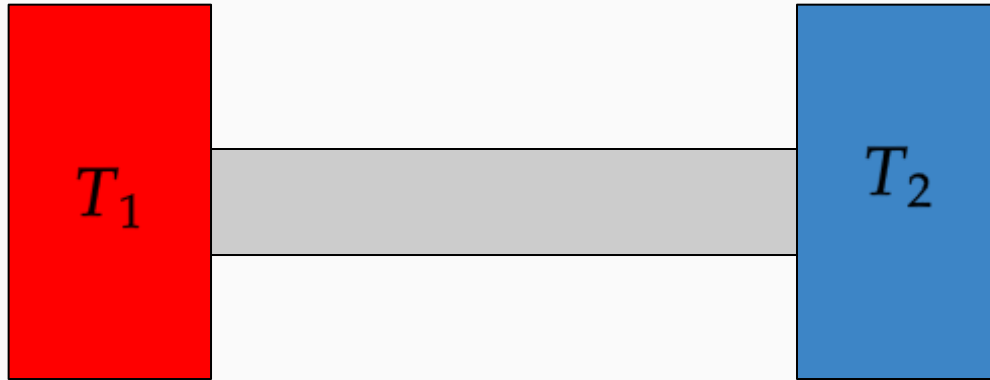


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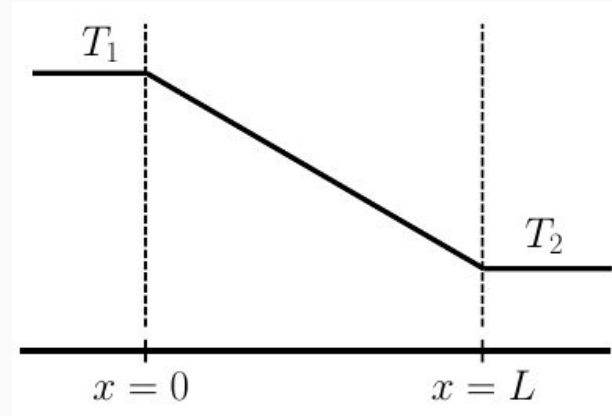


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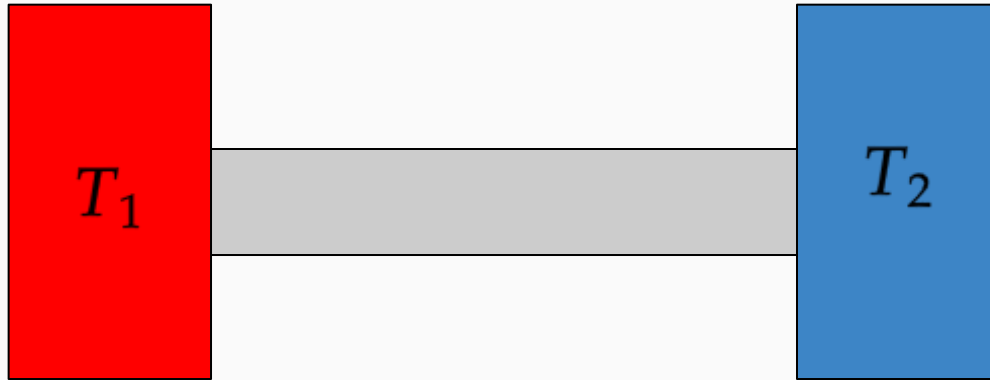
Estado Estacionário

$$T = -\frac{(T_1 - T_2)}{L} \cdot x + T_1$$



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## Equação do Calor

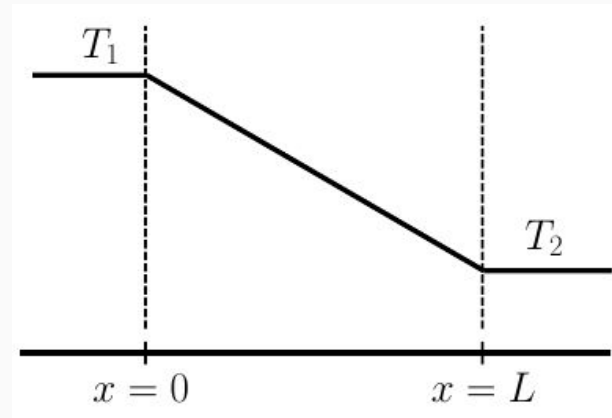


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$$T = -\frac{(T_1 - T_2)}{L} \cdot x + T_1 \quad \Bigg| \quad \frac{\dot{Q}}{\Delta T} = -\frac{\kappa}{L}$$



# 1. Introdução

## Sólidos

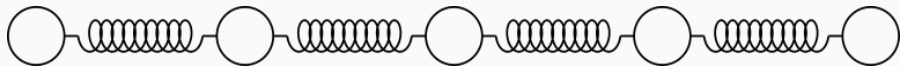


$$H = \sum_n \frac{m}{2} v^2 + \frac{K}{2} (u_n - u_{n+1})^2$$



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## Sólidos



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## Condutores | Isolantes

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## Condutores | Isolantes

Fônons

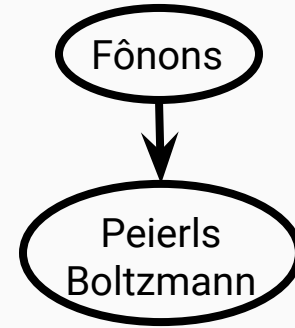
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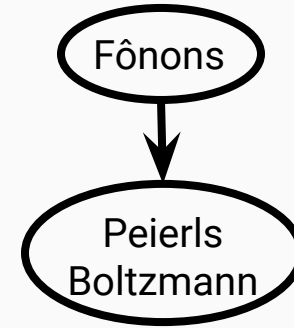
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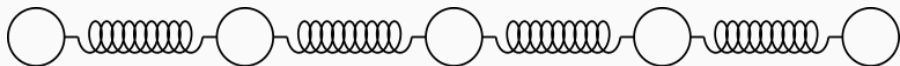
- Resistividade elétrica ☒
- Calor específico ☒
- Condutividade térmica ☒

## Condutores | Isolantes



# 1. Introdução

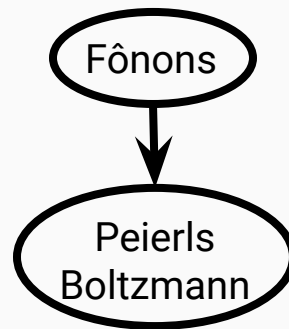
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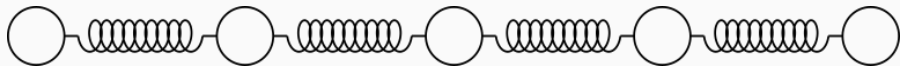
## Condutores | Isolantes



$$U^{anh} = \sum_{i,j,k} \frac{1}{3!} \left. \frac{\partial^3 U}{\partial u_i \partial u_j \partial u_k} \right|_{\mathbf{u}=0} \cdot u_i u_j u_k + \sum_{i,j,k,l} \frac{1}{4!} \left. \frac{\partial^4 U}{\partial u_i \partial u_j \partial u_k \partial u_l} \right|_{\mathbf{u}=0} \cdot u_i u_j u_k u_l + \dots$$

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## Sólidos

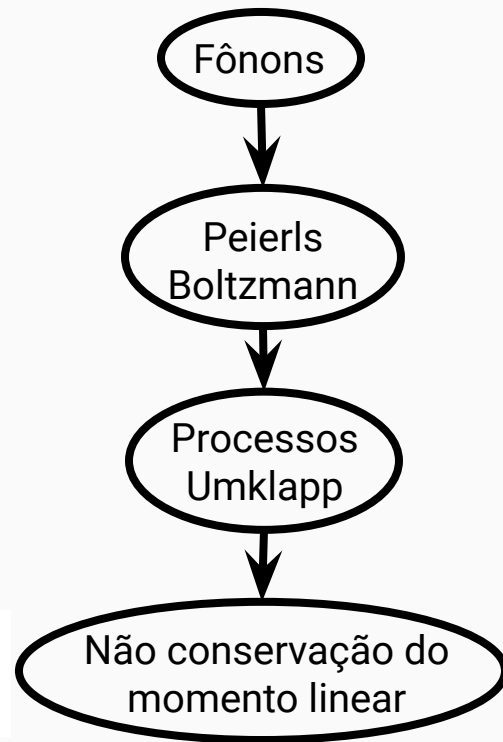


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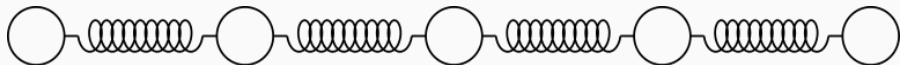
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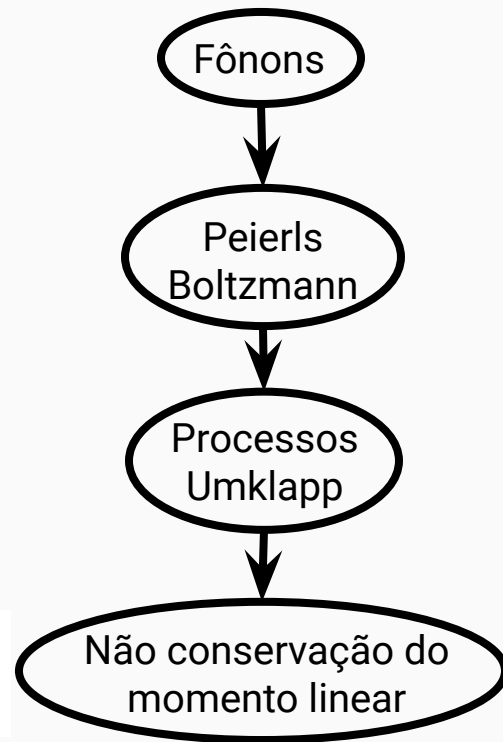


$$H = \sum_n \frac{m}{2} \dot{u}_n^2 + \frac{K}{2} (u_n - u_{n+1})^2$$

- Resistividade elétrica ☒
- Calor específico ☒
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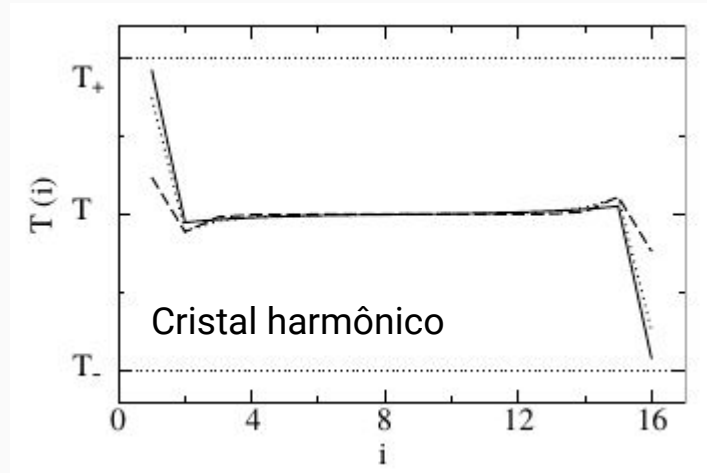
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cálculo da **condutividade térmica**  
X  
**relaxação** para o estado estacionário



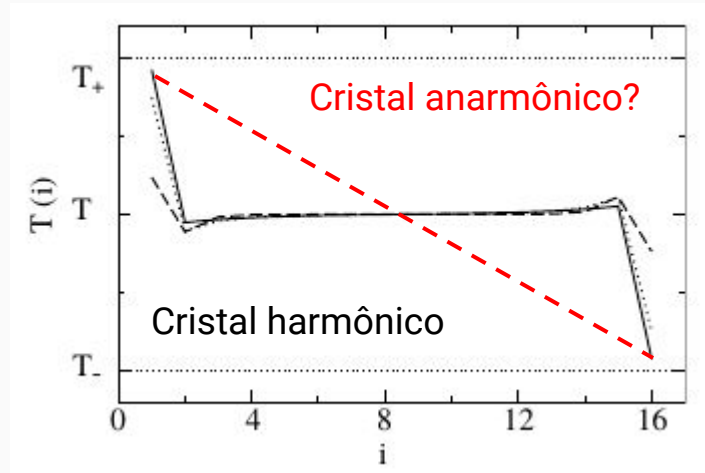
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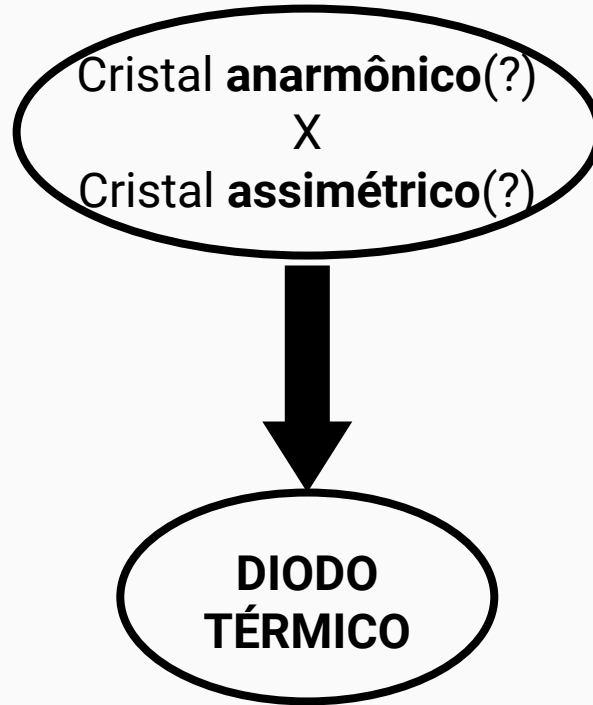


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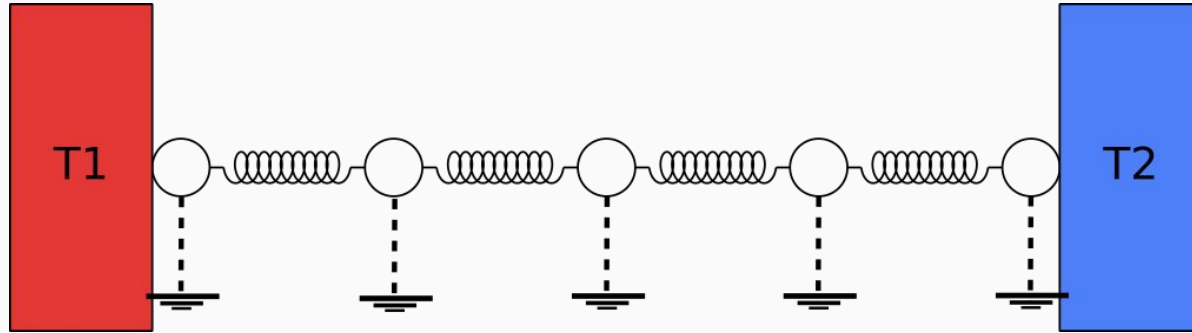


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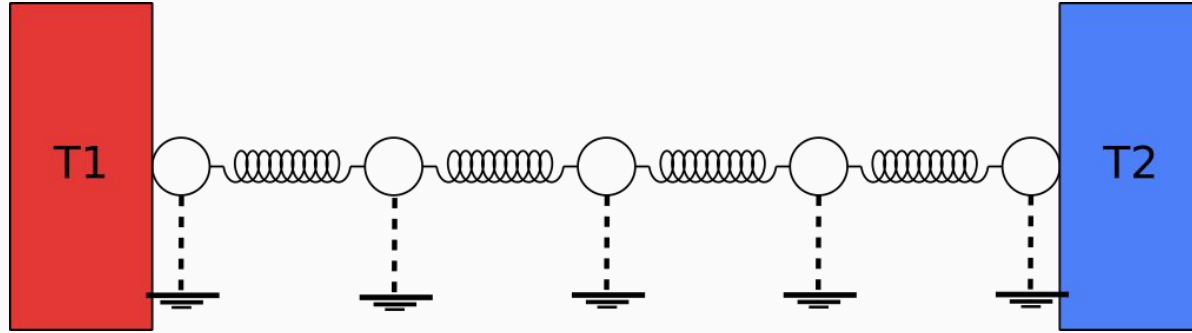


## 2. Modelo matemático

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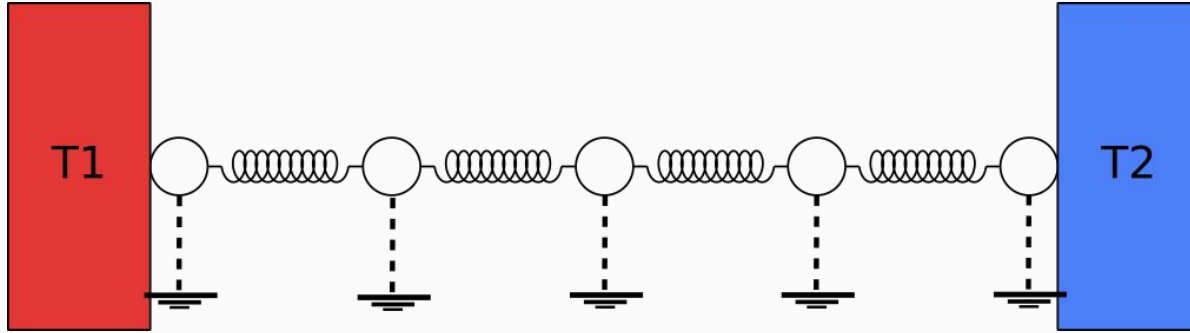


## 2. Modelo matemático



$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + U_{int}(x_{i+1}, x_i) + U_{ext,i}(x_i)$$

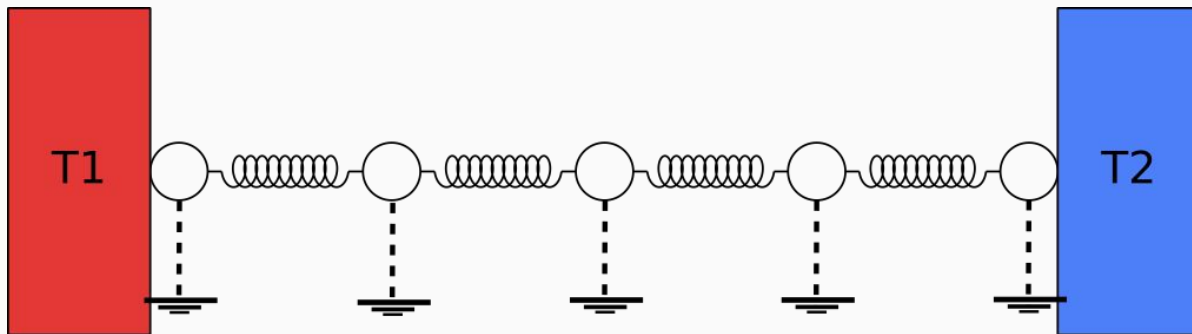
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**Banho Térmico?**

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### Banho Térmico?

Alterações nas equações de movimento para as partículas 1 e  $N$



## 2. Modelo matemático

Como interpretar o modelo em termos “termodinâmicos”?

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Como interpretar o modelo em termos “termodinâmicos”?

**Temperatura “Cinética”**

$$T_j = \left\langle \frac{p_j^2}{m} \right\rangle$$

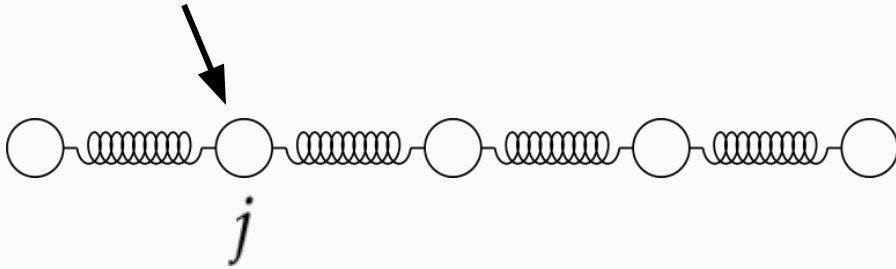


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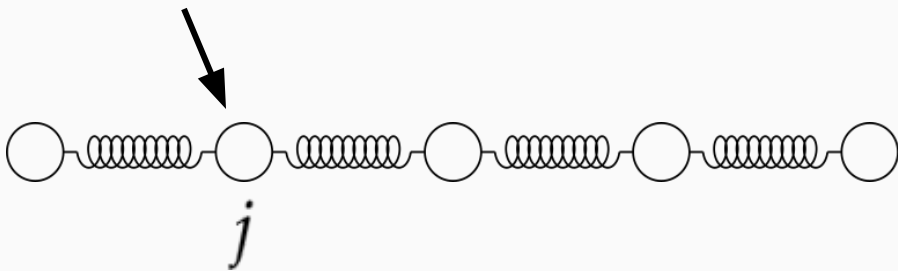


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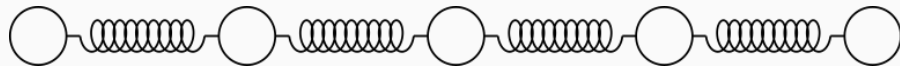
**Temperatura “Cinética”**

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**Fluxo de Calor**

$$\mathcal{J}_n = \frac{1}{2}(\dot{x}_{n+1} + \dot{x}_n) \cdot F(x_{n+1} - x_n)$$

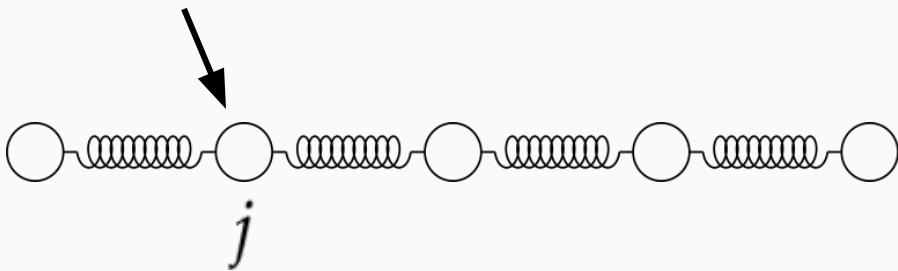


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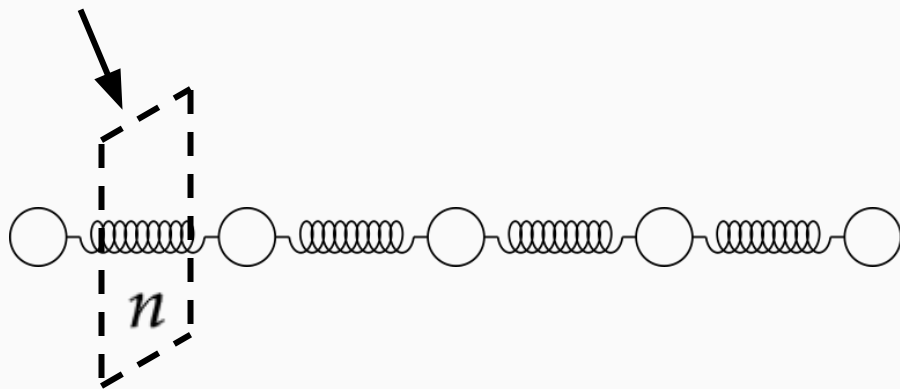
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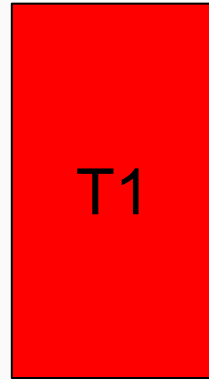
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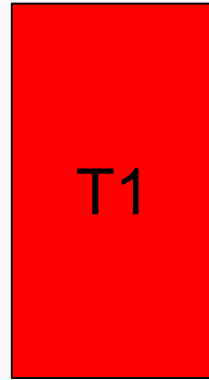
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### Banho Térmico



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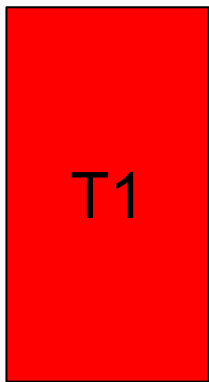
### Banho Térmico



Como descrever um banho térmico?

## 2. Modelo matemático

### Banho Térmico



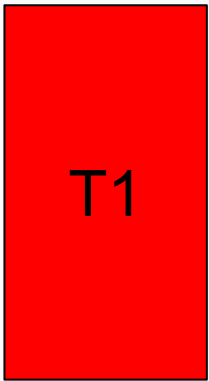
Temperatura constante  $\Rightarrow$  Ensemble canônico  $\Rightarrow$  Distribuição de Maxwell-Boltzmann

Como descrever um  
banho térmico?



## 2. Modelo matemático

### Banho Térmico

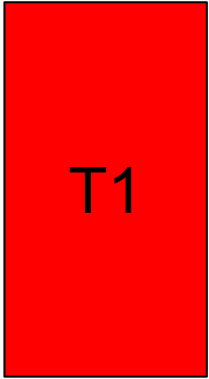


Como descrever um  
banho térmico?

### Distribuição de Maxwell-Boltzmann

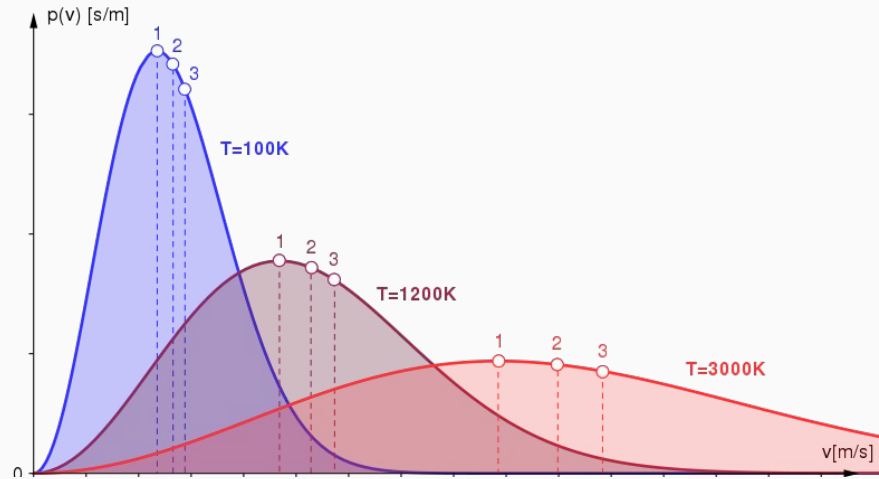
## 2. Modelo matemático

### Banho Térmico



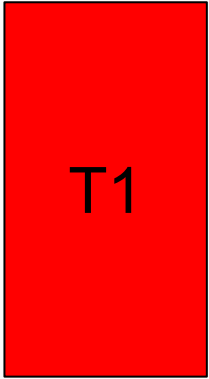
Como descrever um  
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### Distribuição de Maxwell-Boltzmann



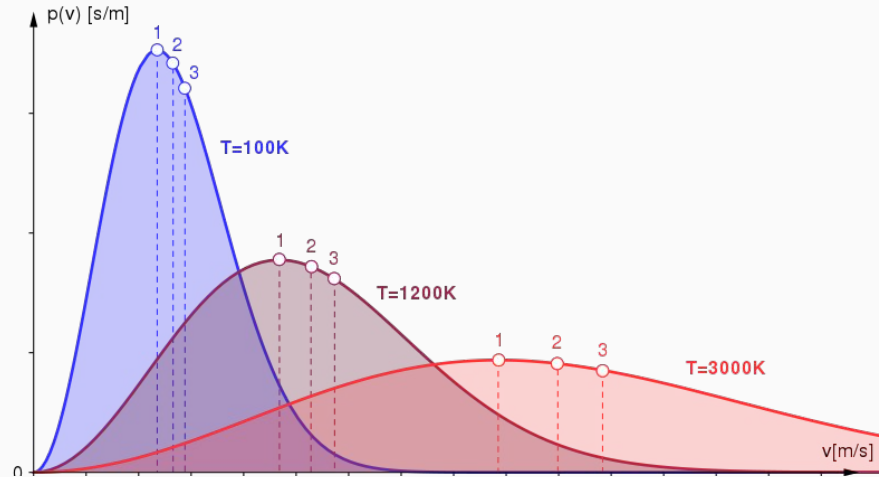
## 2. Modelo matemático

### Banho Térmico



Como descrever um  
banho térmico?

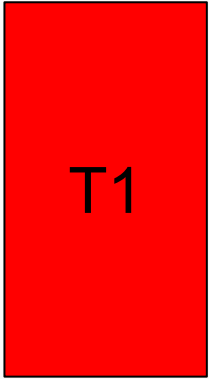
### Distribuição de Maxwell-Boltzmann



$$p(v) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} 4\pi v^2 \exp\left( -\frac{mv^2}{2k_B T} \right)$$

## 2. Modelo matemático

### Banho Térmico



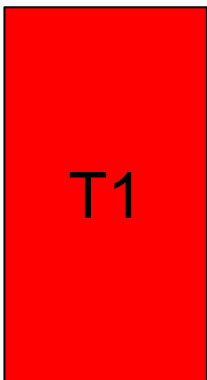
### Distribuição de Maxwell-Boltzmann

$$p(v) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} 4\pi v^2 \exp\left( -\frac{mv^2}{2k_B T} \right)$$



## 2. Modelo matemático

### Banho Térmico



T1

### Distribuição de Maxwell-Boltzmann

$$p(v) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} 4\pi v^2 \exp\left( -\frac{mv^2}{2k_B T} \right)$$

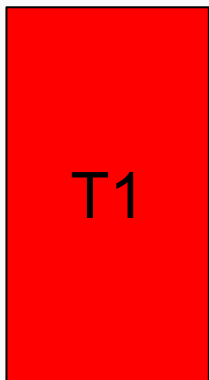
### Modelo determinístico

Termostato de  
Nosè-Hoover



## 2. Modelo matemático

### Banho Térmico



### Distribuição de Maxwell-Boltzmann

$$p(v) = \left( \frac{m}{2\pi k_B T} \right)^{3/2} 4\pi v^2 \exp\left( -\frac{mv^2}{2k_B T} \right)$$

#### Modelo determinístico

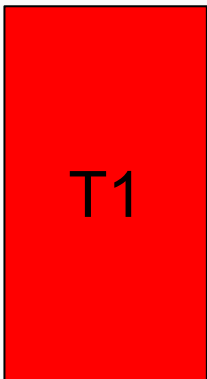
Termostato de  
Nosè-Hoover

#### Modelo estocástico

Banho de Langevin

## 2. Modelo matemático

### Banho Térmico

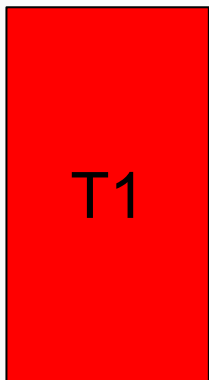


### Termostato Nosè-Hoover

$$H_N = \sum_{i=1}^N \left( \frac{\mathbf{p}_i^2}{2m_i s^2} + \phi(\mathbf{q}) \right) + g k_B T \cdot \ln(s) + \frac{p_s^2}{2Q} ; \quad \begin{aligned} Q &= 3Nk_B T \tau^2 \\ g &= 3N \end{aligned}$$

## 2. Modelo matemático

### Banho Térmico



### Termostato Nosè-Hoover

$$H_N = \sum_{i=1}^N \left( \frac{\mathbf{p}_i^2}{2m_i s^2} + \phi(\mathbf{q}) \right) + g k_B T \cdot \ln(s) + \frac{p_s^2}{2Q} ; \quad \begin{matrix} Q = 3Nk_B T \tau^2 \\ g = 3N \end{matrix} ; \quad \xi = \frac{p_s}{Q}$$

$$\frac{d\mathbf{q}_i}{dt} = \frac{\mathbf{p}_i}{m}$$

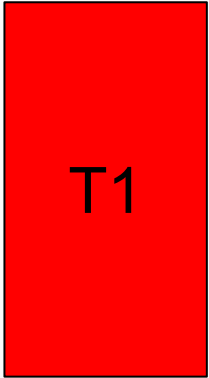
$$\frac{d\mathbf{p}_i}{dt} = \mathbf{F}_i - \xi \mathbf{p}_i$$

$$\frac{d\xi}{dt} = \frac{1}{\tau^2} \left[ \frac{1}{mk_B T N} \sum_{i=1}^N \mathbf{p}_i^2 - 1 \right]$$



## 2. Modelo matemático

### Banho Térmico

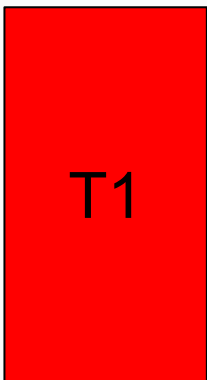


### Banho de Langevin

$$m \frac{d\dot{q}}{dt} = -\gamma \dot{q} + \eta(t) \quad ; \quad \begin{aligned} \langle \eta(t) \rangle &= 0 \\ \langle \eta(t) \eta(t') \rangle &= \Gamma \delta(t - t') \end{aligned}$$

## 2. Modelo matemático

### Banho Térmico



### Banho de Langevin

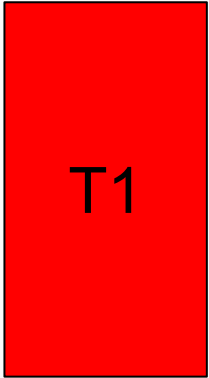
$$m \frac{d\dot{q}}{dt} = \underbrace{-\gamma \dot{q}}_{\text{Graus de liberdade rápidos}} + \underbrace{\eta(t)}_{\text{Graus de liberdade lentos}} ; \quad \langle \eta(t) \rangle = 0$$
$$\langle \eta(t) \eta(t') \rangle = \Gamma \delta(t - t')$$

Graus de liberdade rápidos

Graus de liberdade lentos

## 2. Modelo matemático

### Banho Térmico

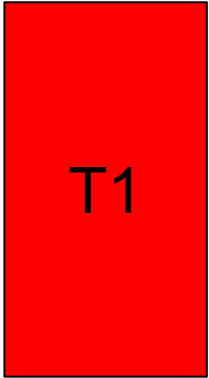


### Banho de Langevin

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## 2. Modelo matemático

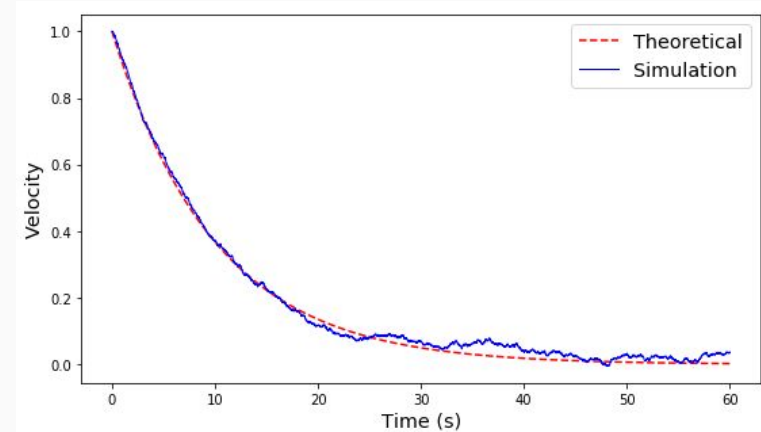
### Banho Térmico



### Banho de Langevin

$$m \frac{d\dot{q}}{dt} = -\gamma \dot{q} + \eta(t) \quad ; \quad \begin{aligned} \langle \eta(t) \rangle &= 0 \\ \langle \eta(t) \eta(t') \rangle &= \Gamma \delta(t - t') \end{aligned}$$

$$\langle \dot{q} \rangle = v_0 e^{-\gamma t}$$



## 2. Modelo matemático

### Banho Térmico

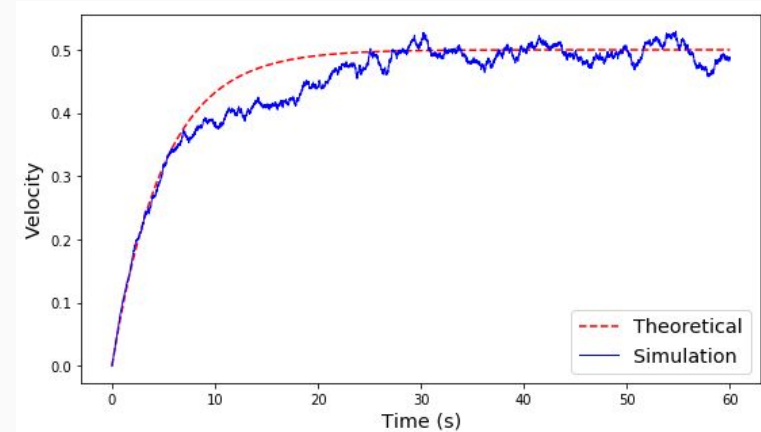
T1

### Banho de Langevin

$$m \frac{d\dot{q}}{dt} = -\gamma \dot{q} + \eta(t) \quad ; \quad \begin{aligned} \langle \eta(t) \rangle &= 0 \\ \langle \eta(t) \eta(t') \rangle &= \Gamma \delta(t - t') \end{aligned}$$

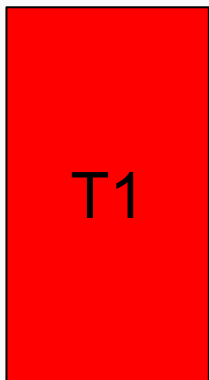
$$\langle \dot{q} \rangle = v_0 e^{-\gamma t}$$

$$\langle \dot{q}^2 \rangle - \langle \dot{q} \rangle^2 = \frac{\Gamma}{2\gamma} (1 - e^{-2\gamma t})$$



## 2. Modelo matemático

### Banho Térmico



### Banho de Langevin

$$m \frac{d\dot{q}}{dt} = -\gamma \dot{q} + \eta(t) \quad ; \quad \begin{aligned} \langle \eta(t) \rangle &= 0 \\ \langle \eta(t) \eta(t') \rangle &= \Gamma \delta(t - t') \end{aligned}$$

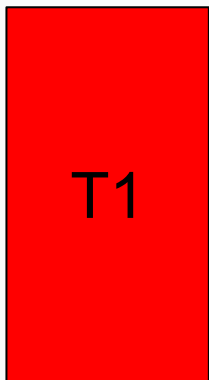
$$\langle \dot{q} \rangle = v_0 e^{-\gamma t}$$

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$$\Gamma = \frac{2\gamma k_B T}{m}$$

## 2. Modelo matemático

### Banho Térmico



### Banho de Langevin

$$m \frac{d\dot{q}}{dt} = -\gamma \dot{q} + \frac{2\gamma k_B T}{m} \epsilon(t) \quad ; \quad \begin{aligned} \langle \epsilon(t) \rangle &= 0 \\ \langle \epsilon(t) \epsilon(t') \rangle &= \delta(t - t') \end{aligned}$$

$$\langle \dot{q} \rangle = v_0 e^{-\gamma t}$$

$$\langle \dot{q}^2 \rangle - \langle \dot{q} \rangle^2 = \frac{\Gamma}{2\gamma} (1 - e^{-2\gamma t})$$

$$\Gamma = \frac{2\gamma k_B T}{m}$$

## 2. Modelo matemático

### **Cadeia de Partículas**





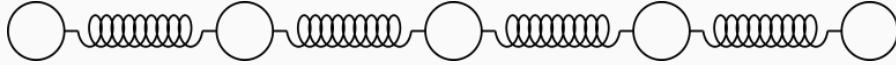
## 2. Modelo matemático

### Cadeia de Partículas



## 2. Modelo matemático

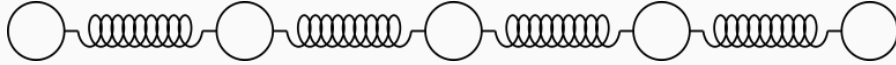
### Cadeia de Partículas



$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2$$

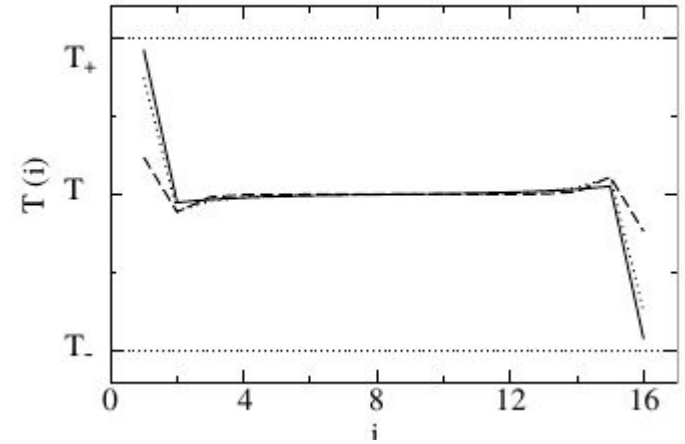
## 2. Modelo matemático

### Cadeia de Partículas



$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2$$

### Modelo harmônico



## 2. Modelo matemático

### Cadeia de Partículas



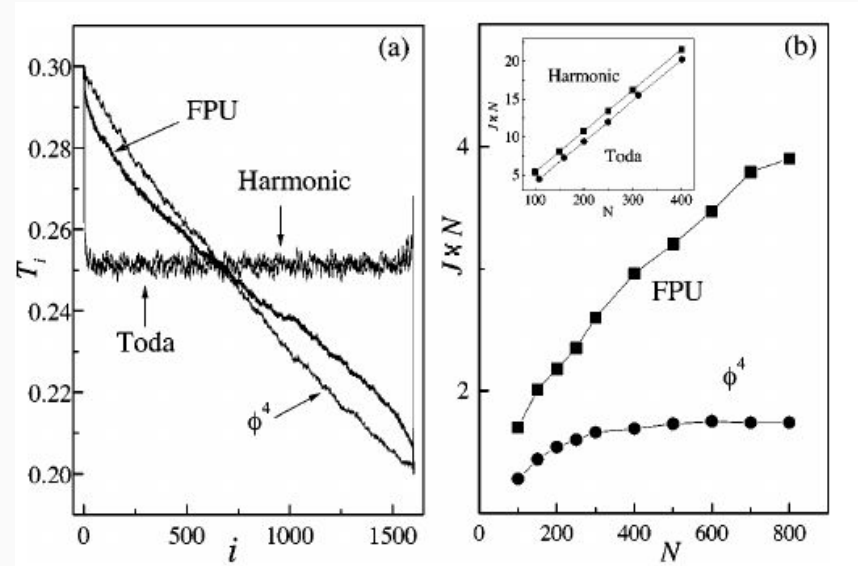
$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2 + U_{anh,i}$$

## 2. Modelo matemático

### Cadeia de Partículas

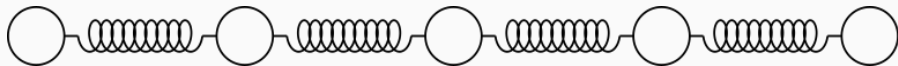


$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2 + U_{anh,i}$$

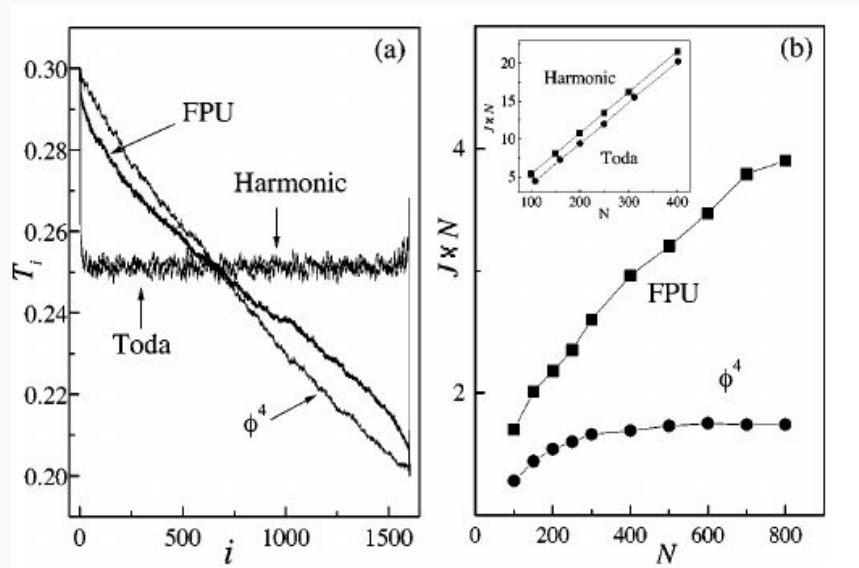


## 2. Modelo matemático

### Cadeia de Partículas

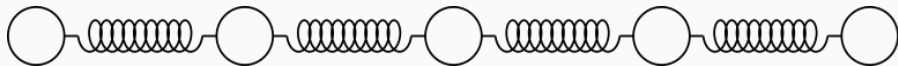


**Toda** 
$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \exp(-(x_{i+1} - x_i)) + (x_{i+1} - x_i) - 1$$



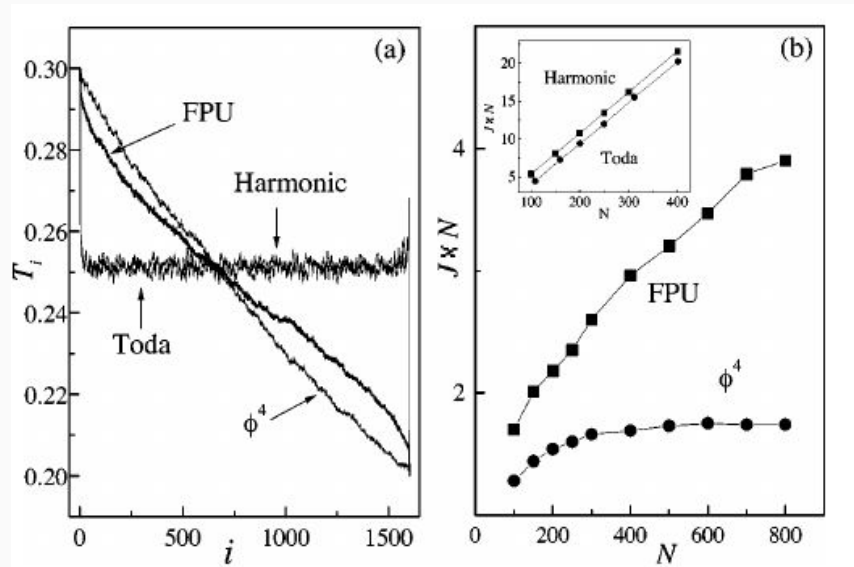
## 2. Modelo matemático

### Cadeia de Partículas



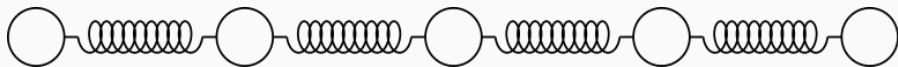
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**FPUT** 
$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2 + \frac{k'}{4}(x_i - x_{i+1})^4$$



## 2. Modelo matemático

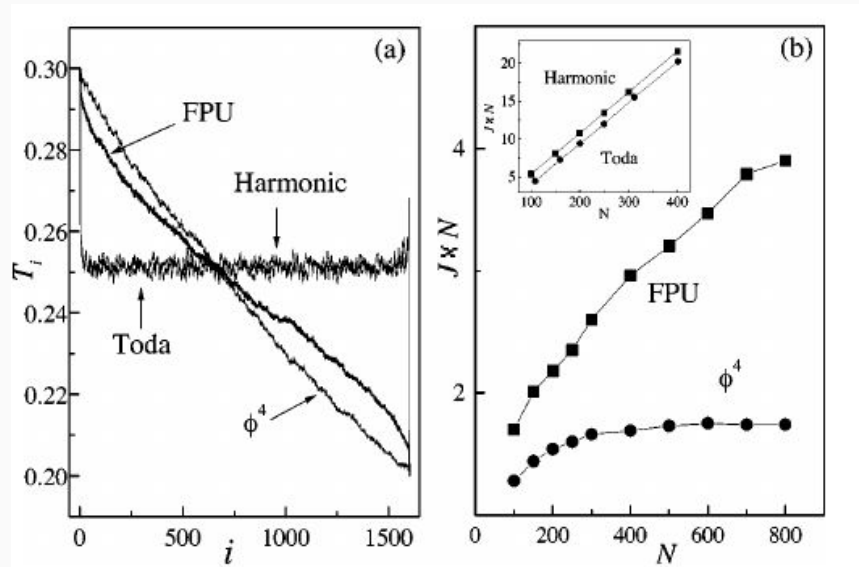
### Cadeia de Partículas



**Toda** 
$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \exp(-(x_{i+1} - x_i)) + (x_{i+1} - x_i) - 1$$

**FPUT** 
$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2 + \frac{k'}{4}(x_i - x_{i+1})^4$$

$\phi^4$  
$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2 + \frac{k''}{4}x_i^4$$





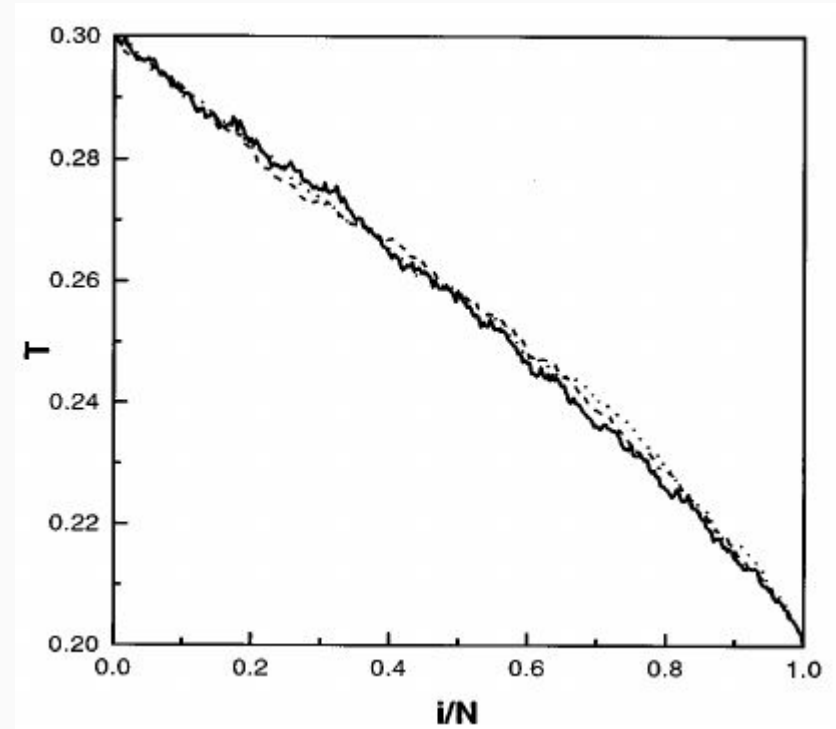
## 2. Modelo matemático

### Cadeia de Partículas



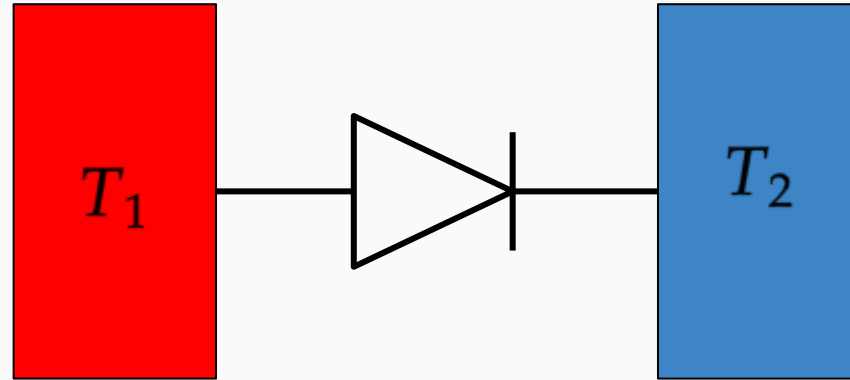
### Frenkel-Kontorova

$$\mathcal{H} = \sum_{i=1}^N \frac{p_i^2}{2m_i} + \frac{k}{2}(x_i - x_{i+1})^2 + \frac{k'}{4}(x_i - x_{i+1})^4$$



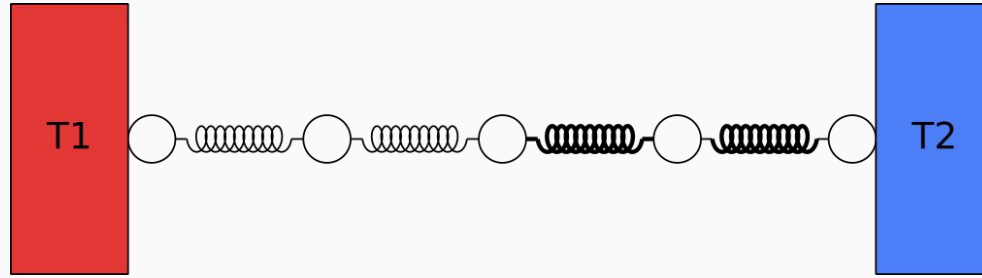
## 2. Modelo matemático

### Diodo Térmico



## 2. Modelo matemático

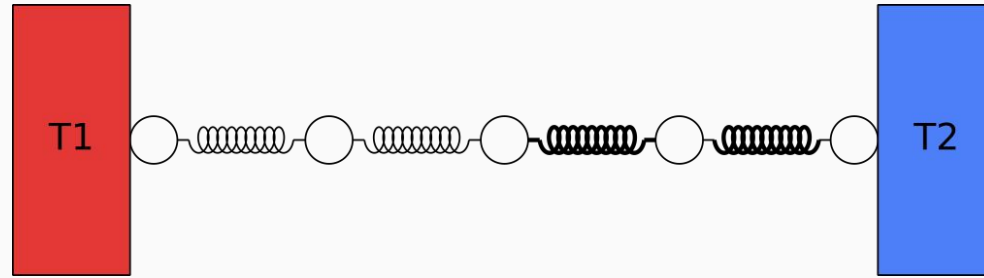
### Diodo Térmico



## 2. Modelo matemático

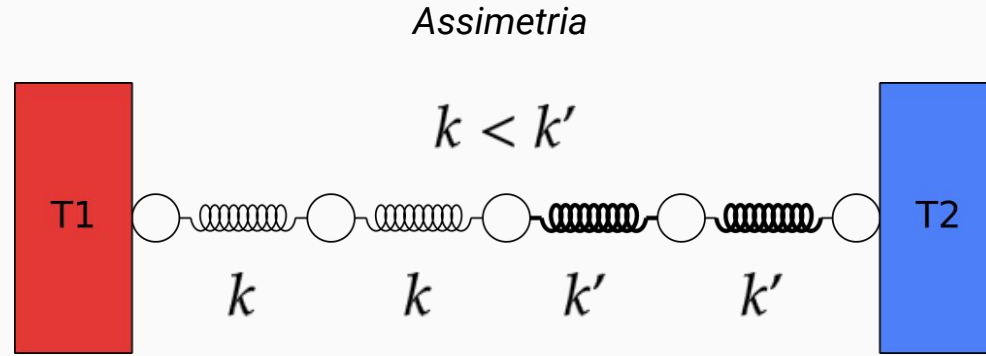
### Diodo Térmico

*Assimetria*



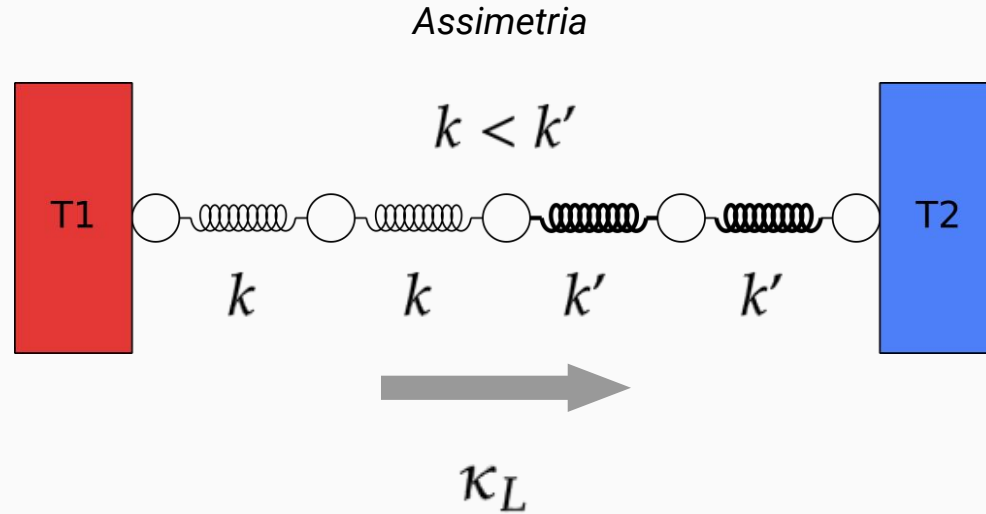
## 2. Modelo matemático

### Diodo Térmico



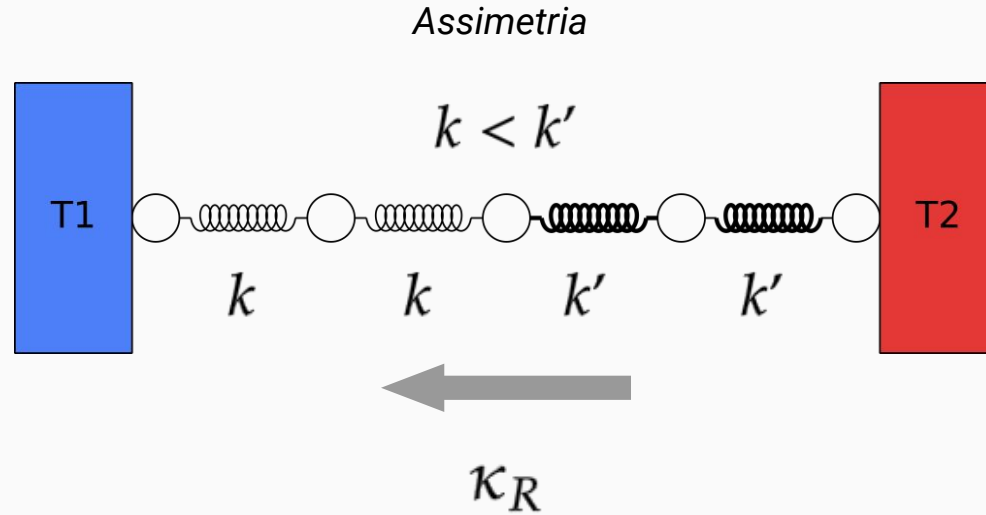
## 2. Modelo matemático

### Diodo Térmico



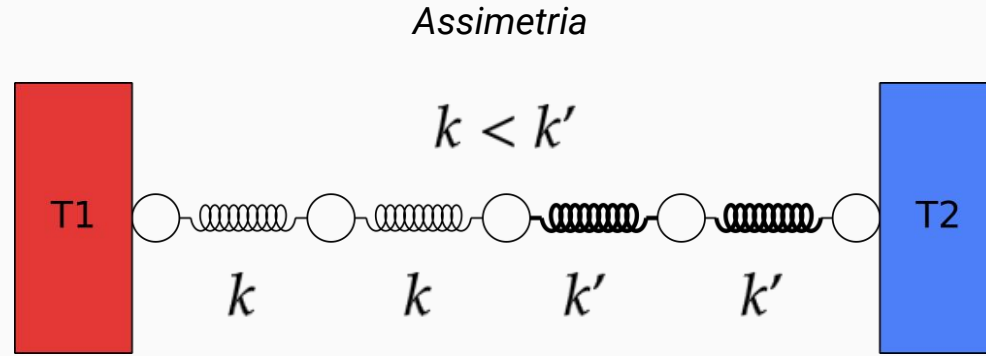
## 2. Modelo matemático

### Diodo Térmico



## 2. Modelo matemático

### Diodo Térmico

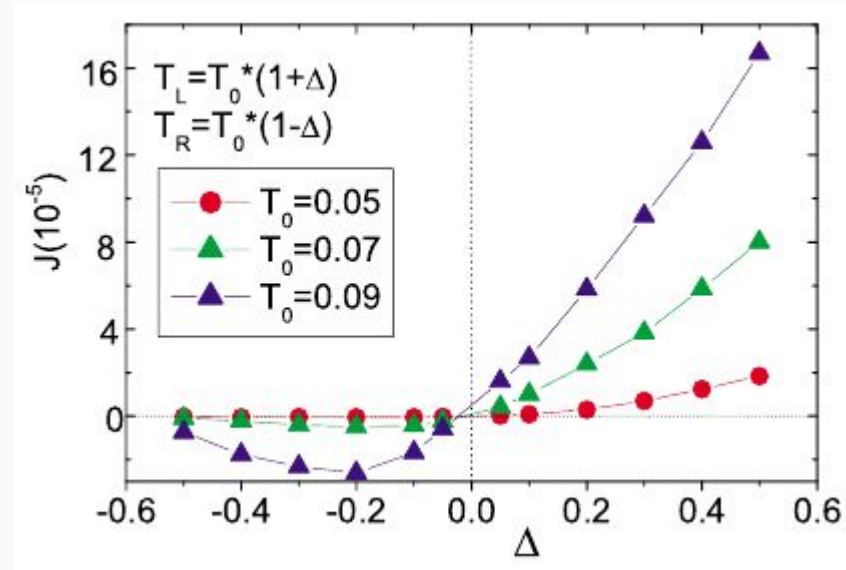


$$R = \frac{\kappa_R - \kappa_L}{\kappa_L}$$



## 2. Modelo matemático

### Diodo Térmico



# 3. Metodologia

### 3. Metodologia

#### Métodos Numéricos Estocásticos

Processo Ornstein-Uhlenbeck => Cálculo Estocástico => Revisar nossos métodos numéricos

Pulo do Gato: Variar só um pouco os determinísticos pra obter os estocásticos. É mais fácil.

#### B.2

##### Integração numérica de equações diferenciais estocásticas

Para uma equação diferencial estocástica da forma

$$\frac{dx}{dt} = a(x, t) + b(x, t)\xi(t), \quad (\text{B-5})$$

não existe uma adaptação simples dos métodos numéricos de equações diferenciais ordinárias [22] e isto é porque a idealização do ruído é não diferenciável.

Portanto, a solução

$$x(t) = x_0 + \int_{t_0}^t a(x(s))ds + \int_{t_0}^t b(x(s))dW(s), \quad (\text{B-6})$$

### 3. Metodologia

o qual permite usar os métodos de integração numérica usuais e acrescentar o termo do ruído a cada passo de tempo.

### 3. Metodologia

#### Euler

The Euler's Method consists of approaching the evolution of the system by a Taylor series truncated at the linear terms at each step

$$\mathbf{u}(t_n + \tau) \approx \mathbf{u}(t_n) + \tau \left( \frac{d\mathbf{u}}{dt} \right)_{t=t_n},$$

$$v_{n+1} = v_n - \tau \gamma v_n + \sqrt{\tau \Gamma} \xi_n$$

$$x_{n+1} = x_n + \tau v_n$$

### 3. Metodologia

#### Verlet

$$e^{i\mathcal{L}\delta t} \approx e^{i\mathcal{L}_2\delta t/2} e^{i\mathcal{L}_1\delta t} e^{i\mathcal{L}_2\delta t/2}.$$

Using  $\mathcal{A}$  as being the vector in the phase space and applying the approximated evolution operator, one gets the system of equations:

$$\mathbf{p}(t + \delta t/2) = \mathbf{p}(t) + \frac{\delta t}{2} \frac{d\mathbf{p}(t)}{dt}$$

$$\mathbf{q}(t) = \mathbf{q}(t) + \delta t \frac{1}{m} \mathbf{p}(t)$$

$$\mathbf{p}(t + \delta t) = \mathbf{p}(t + \delta t/2) + \frac{\delta t}{2} \frac{d\mathbf{p}(t + \delta t)}{dt}$$

### 3. Metodologia

#### Runge Kutta

##### **Runge Kutta**

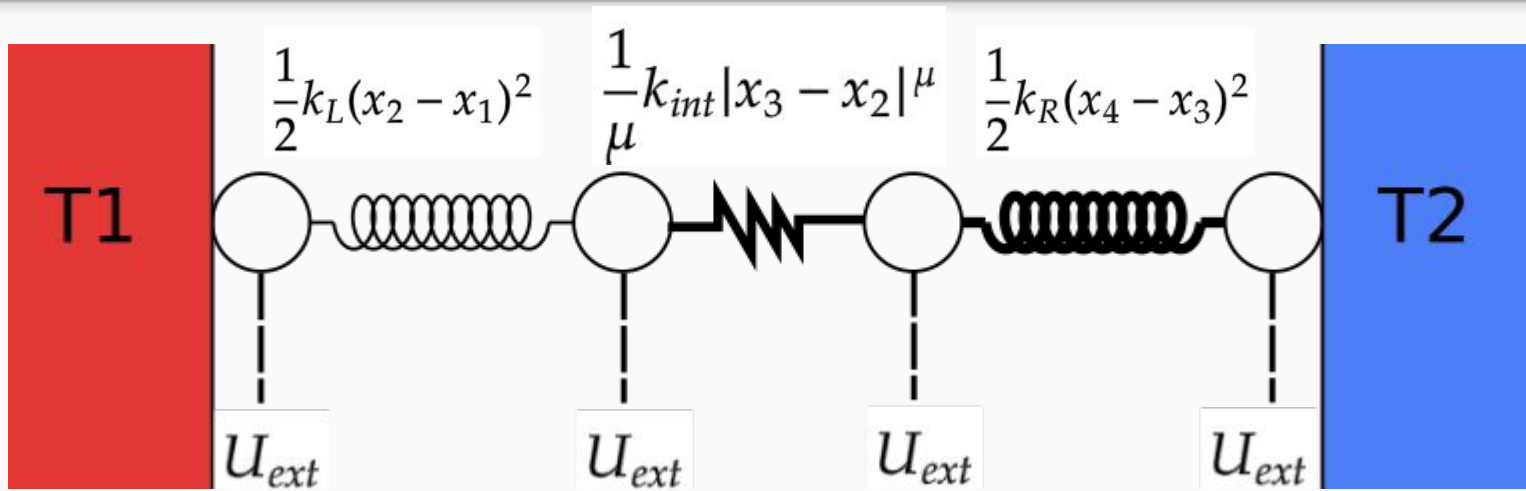
Another commonly used algorithm is the Runge Kutta family of methods, mostly the classical 2nd order and 4th order methods. In this method, the increment is approximated by a series.

$$\phi = \sum_{i=0}^m (a_i k_i)$$

## 4. Resultados



## 4. Resultados



**Modelo de Frenkel-Kontorova**

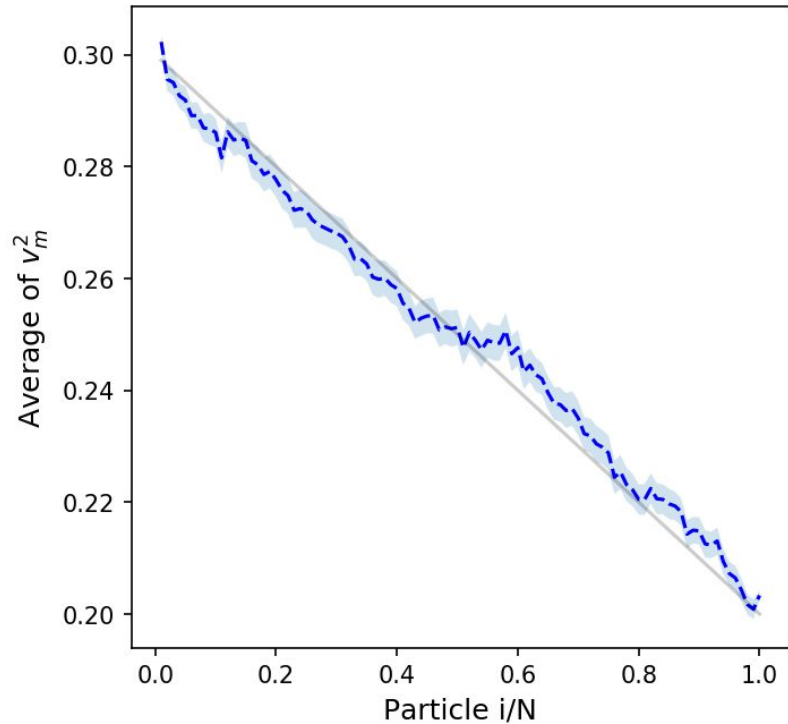
$$U_{ext}(x_i) = A \cdot \left( 1 - \cos\left(\frac{2\pi x_i}{a_S}\right) \right)$$

**Modelo  $\phi^4$**

$$U_{ext}(x_i) = A \cdot x_i^4$$

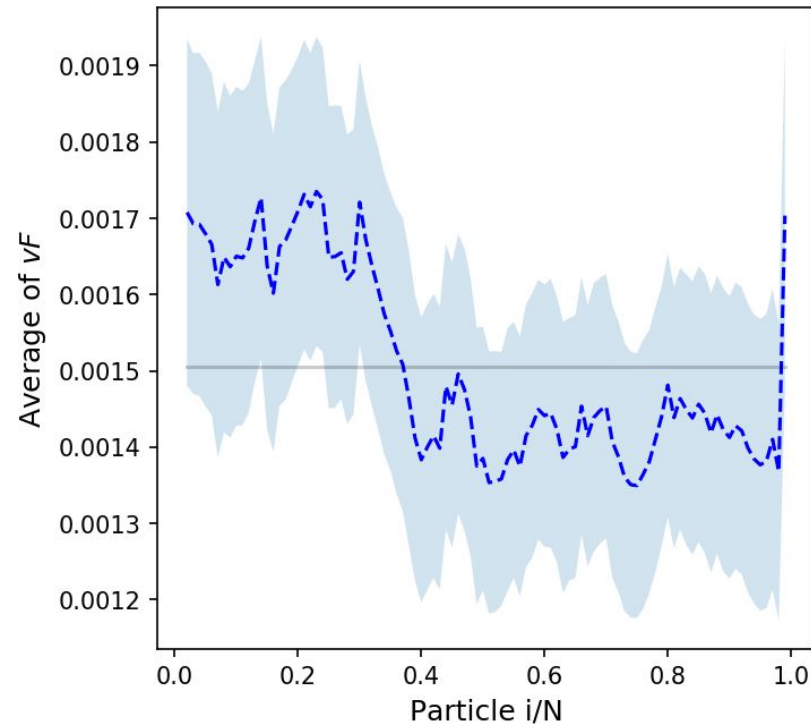
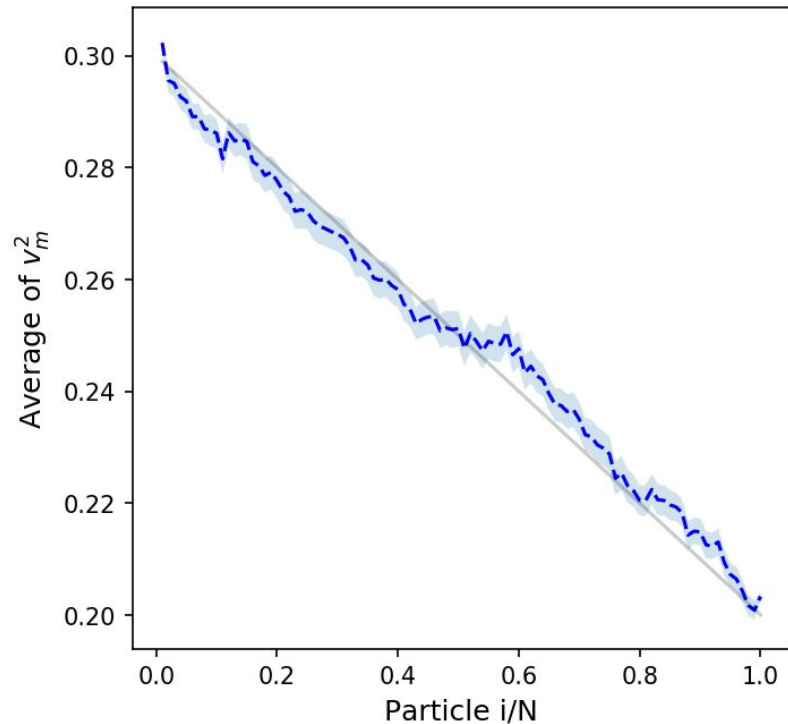
## 4. Resultados

### Modelo Frenkel-Kontorova



## 4. Resultados

### Modelo Frenkel-Kontorova

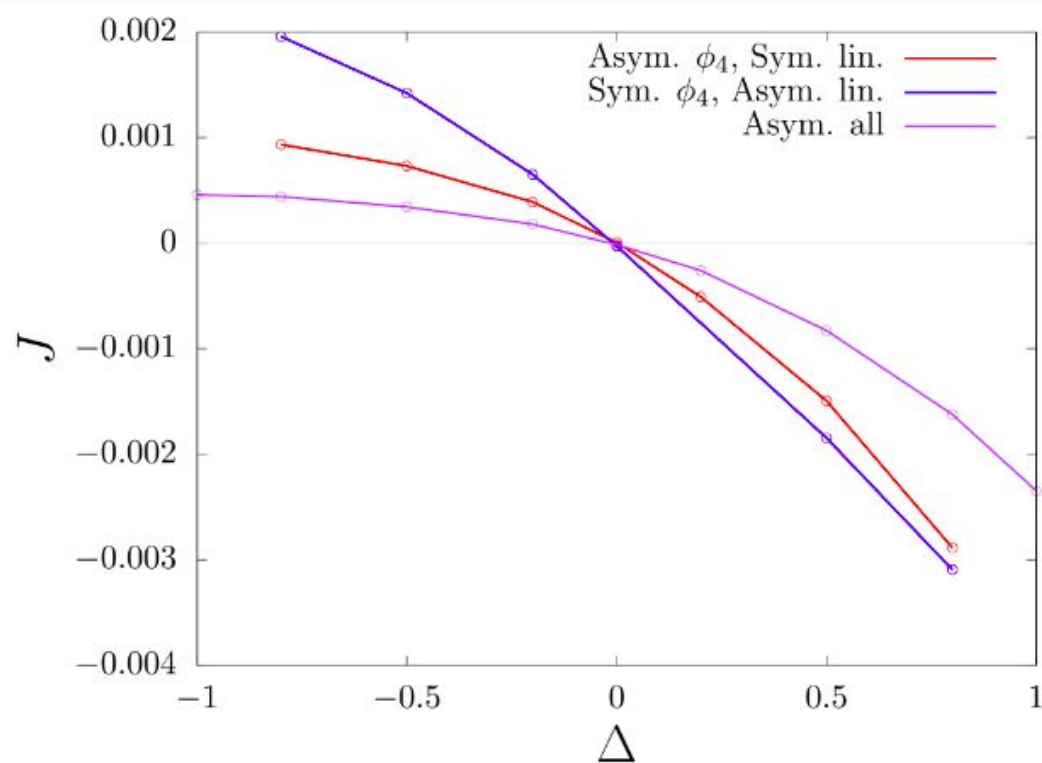


### **Modelo Frenkel-Kontorova x Modelo $\phi^4$**

Dificuldade do modelo Frenkel Kontorova para convergência

## 4. Resultados

### Modelo $\phi^4$



## 4. Resultados

Número de amostras: 200

Número de partículas na cadeia: 4

Potencial:

Passo de tempo: 0.001

Tempo total da simulação: 100,000.0

Tempo transiente: 1,000.0

Temperatura média: 1.0

Constante elástica da esquerda: 1.0

Amplitude do potencial esquerdo: 1.0

Constante elástica da direita: 5.0

Amplitude do potencial direita: 5.0

Coeficiente de arraste: 1.0

Constante elástica da interfase: 0.1

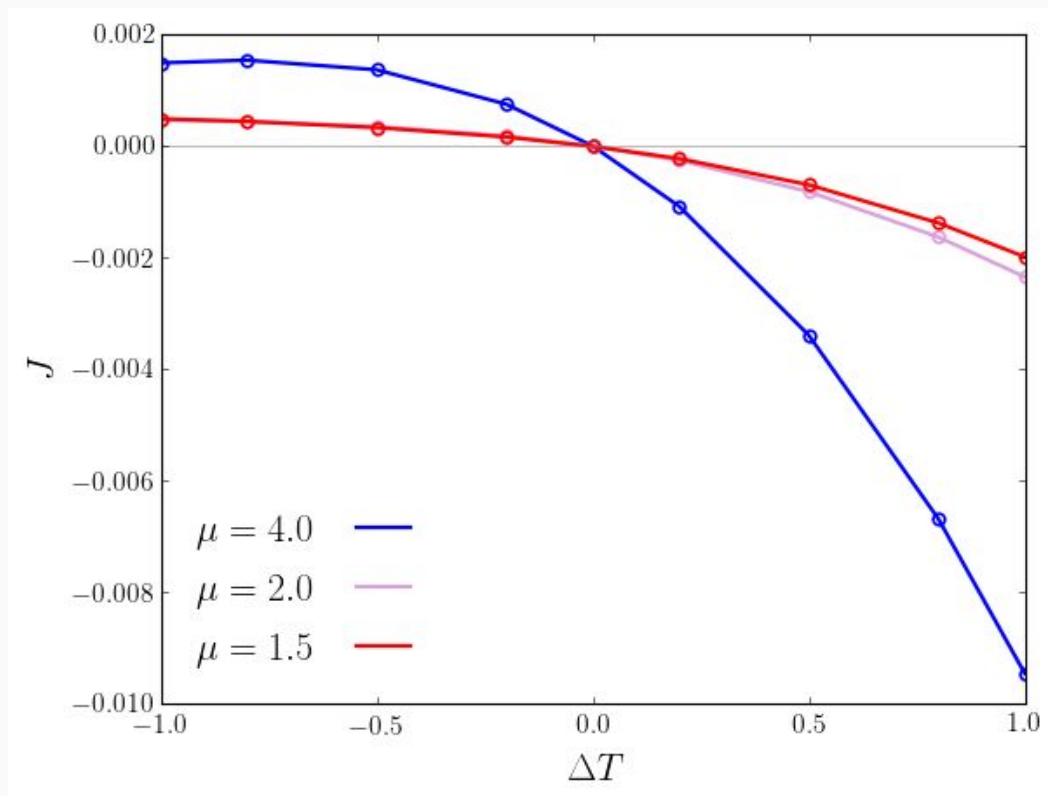
A razão  $k/A$  fica constante na esquerda e na direita.

> Variar  $\mu$

> Variar  $dT$  ou  $k_{int}$

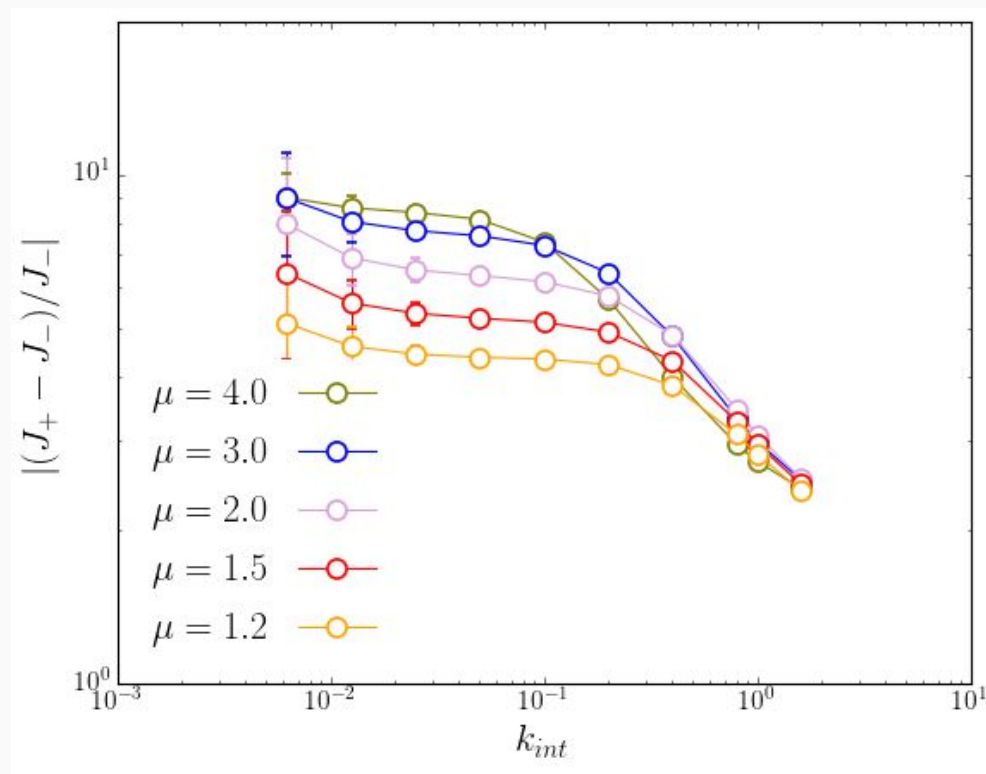
## 4. Resultados

### Diodo Térmico



## 4. Resultados

### Eficiência





## 5. Conclusão

## 5. Conclusões

### Discussão de resultados

- $\mu$ : um tipo de interação de interfase. Quanto maior, melhor.

## 5. Conclusões

Novos trabalhos

- Tempo até estacionaridade

# REFERÊNCIAS

Obrigado!

# 1. Introdução