§7.2

#15) $a_n = 6a_{n-1} - 8a_{n-2}$, $a_0 = 1$, $a_1 = 0$

$t^2 - 6t + 8 = (t-2)(t-4) = 0 \Rightarrow r_1 = 2, r_2 = 4$

$a_n = P2^n + Q4^n$ where $\left.\begin{array}{l} P+Q=1 \\ 2P+4Q=0 \end{array}\right\} \Rightarrow \begin{array}{r} -2P-2Q=-2 \\ 2P+4Q=0 \\ \hline 2Q=-2 \end{array}$

Therefore $Q=-1$ and $P=2$. Hence

$a_n = 2(2)^n - 4^n = 2^{n+1} - 4^n$

Using induction, show that $a_n = 6a_{n-1} - 8a_{n-2}$ and $a_n = 2^{n+1} - 4^n$ produce the same sequence of values.

Pf (induction)

BC: $a_0 = 2^{0+1} - 4^0 = 2 - 1 = 1$, $a_1 = 2^2 - 4 = 4 - 4 = 0$

IC: Assume $a_r = 2^{r+1} - 4^r$, for all r, k ... $r \le k$, for some $k \ge 1$. We need to show $a_{k+1} = 2^{k+2} - 4^{k+1}$.

note that
$$\begin{aligned} a_{k+1} &= 6a_k - 8a_{k-1} \quad \text{by RR} \\ &= 6(2^{k+1} - 4^k) - 8(2^k - 4^{k-1}) \quad \text{by I.H} \\ &= \frac{6}{2}2^{k+2} - \frac{6}{4}4^{k+1} - \frac{8}{4}2^{k+2} + \frac{8}{16}4^{k+1} \\ &= (3-2)2^{k+2} - \left(\frac{3}{2} - \frac{1}{2}\right)4^{k+1} \\ &= 2^{k+2} - 4^{k+1}. \end{aligned}$$

Therefore, by the PMI, the two formulas produce the same sequence of values. ∎

#16) $a_n = 7a_{n-1} - 10a_{n-2}$, $a_0 = 5$, $a_1 = 16$

$t^2 - 7t + 10 = (t-2)(t-5) = 0 \Rightarrow r_1 = 2, r_2 = 5$

$a_n = P2^n + Q5^n$ where $\left.\begin{array}{l} P+Q=5 \\ 2P+5Q=16 \end{array}\right\} \Rightarrow \begin{array}{r} \cancel{-2P} - 2Q = -10 \\ \cancel{2P} + 5Q = 16 \\ \hline 3Q = 6 \end{array}$

Hence $Q = 2$ and $P = 3$. Therefore

$a_n = 3(2^n) + 2(5^n)$.

using induction, show that $a_n = 7a_{n-1} - 10a_{n-2}$ and $a_n = 3(2^n) + 2(5^n)$ produce the same sequence of values.

BC: $a_0 = 3(2^0) + 2(5^0) = 3 + 2 = 5$ ✓

$a_1 = 3(2) + 2(5) = 6 + 10 = 16$ ✓

IC: Assume $a_r = 3(2^r) + 2(5^r)$ for all r, $1 \le r \le k$, for some $k \geq 1$. Need to show $a_{k+1} = 3(2^{k+1}) + 2(5^{k+1})$.

note that

$$\begin{aligned} a_{k+1} &= 7a_k - 10a_{k-1} \quad \text{by RR} \\ &= 7\left(3(2^k) + 2(5^k)\right) - 10\left(3(2^{k-1}) + 2(5)^{k-1}\right) \\ &= 3\left(\tfrac{7}{2}\right)2^{k+1} + 2\left(\tfrac{7}{5}\right)5^{k+1} - 3\left(\tfrac{10}{4}\right)2^{k+1} - 2\left(\tfrac{10}{25}\right)5^{k+1} \\ &= 3(2^{k+1})\underbrace{\left(\tfrac{7}{2} - \tfrac{10}{4}\right)}_{3.5-2.5=1} + 2(5^{k+1})\underbrace{\left(\tfrac{7}{5} - \tfrac{10}{25}\right)}_{\frac{35-10}{25}=1} \\ &= 3(2^{k+1}) + 2(5^{k+1}). \end{aligned}$$

Therefore, by the PMI, the two formulas produce the same sequence of values. $\boxdot$

#17 & 19 are on Pencast.