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§5.1 cont.

Theorem 3: Fundamental Theorem of Arithmetic (5.1.1)
Every positive integer $n > 1$ can be written uniquely as $p_1^{k_1} p_2^{k_2} p_3^{k_3} \cdots p_s^{k_s}$ where $p_1 < p_2 < \cdots < p_s$ are distinct primes that divide $n$ and the $k_i$'s are nonnegative integers giving the number of times that each prime ~~is~~ occurs as a factor of $n$.

Ex: $16 = 2^4$. $18 = 2^1 \cdot 3^2$, $24 = 2^3 \cdot 3^1$, ...

Greatest Common Divisor
The common divisors of 12 and 30 are 2, 3, & 6
Then $GCD(12, 30) = 6$.

Def. If $a$, $b$, and $k$ are positive integers and $k | a$ & $k | b$, we say that $k$ is a common divisor of $a$ and $b$. If $d$ is the largest such $k$, then $d$ is called the greatest common divisor, or GCD of $a$ & $b$, denoted by $GCD(a,b) = d$.

Theorem 4: If $d = GCD(a,b)$, then

i) $d = sa + tb$ for some integers $s$ and $t$
(That mean $d$ can be written as a linear combination of $a$ and $b$).

ii) If $c$ is any other common divisor of $a$ and $b$, then $c | d$.



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§5.1 Cont

Theorem 3: Fundamental Theorem of Arithmetic (5.1.11)
Every positive integer $n > 1$ can be written uniquely as $p_1^{k_1} p_2^{k_2} p_3^{k_3} \cdots p_s^{k_s}$ where $p_1 < p_2 < \cdots < p_s$ are distinct primes that divide $n$ and the $k$'s are nonnegative integers giving the number of times that each prime ~~is~~ occurs as a factor of $n$.

Ex: $16 = 2^4$, $18 = 2^1 \cdot 3^2$, $24 = 2^3 \cdot 3^1$, ...

Greatest Common Divisor.
The common divisors of 12 and 30 are 2, 3, & 6.

Thus $GCD(12, 30) = 6$.

Def: If $a$, $b$, and $k$ are positive integers and $k \mid a$ & $k \mid b$, we say that $k$ is a common divisor of $a$ and $b$. If $d$ is the largest such $k$, then $d$ is called the greatest common divisor, or GCD of $a$ & $b$, denoted by $GCD(a, b) = d$.

Theorem 4: If $d = GCD(a, b)$, then

i) $d = sa + tb$ for some integers $s$ and $t$.
(That mean $d$ can be written as a linear combination of $a$ and $b$).

ii) If $c$ is any other common divisor of $a$ and $b$, then $c \mid d$.

(5.1.17) positive

Theorem 5: Let $m$ and $n$ ~~be~~ integers with prime factorization $m = p_1^{k_1} p_2^{k_2} \cdots p_s^{k_s}$ and $n = p_1^{t_1} p_2^{t_2} \cdots p_s^{t_s}$. Then

$$GCD(m,n) = p_1^{\min(k_1,t_1)} p_2^{\min(k_2,t_2)} \cdots p_s^{\min(k_s,t_s)}.$$

Ex: Find GCD(540, 504) using prime factorization.

$$540 = 2^2 \cdot 3^3 \cdot 5^1 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 = \boxed{2^2 \cdot 3^3 \cdot 5^1 \cdot 7^0}$$

$$504 = 2^3 \cdot 3^2 \cdot 7^1 = \boxed{2^3 \cdot 3^2 \cdot 5^0 \cdot 7^1}$$

$$GCD(540,504) = 2^{\min(2,3)} \cdot 3^{\min(3,2)} \cdot 5^{\min(1,0)} \cdot 7^{\min(0,1)}$$

$$= 2^2 \cdot 3^2 \cdot 5^0 \cdot 7^0$$

$$= 4 \cdot 9 = 36$$

$$LCM(540,504) = 2^{\max(2,3)} \cdot 3^{\max(3,2)} \cdot 5^{\max(1,0)} \cdot 7^{\max(0,1)}$$

$$= 2^3 \cdot 3^3 \cdot 5^1 \cdot 7^1$$

$$= (8)(27)(5)(7) = 7560$$

$$(540)(504) = 272160$$

(5.1.25)

Theorem 5: If $a$ and $b$ are positive integers

$$a \cdot b = GCD(a,b) \cdot LCM(a,b)$$

Def LCM: Let $m$ and $n$ be positive integers. A common multiple of $m$ and $n$ is an integer that is divisible by both $m$ and $n$

The least common multiple LCM$(m,n)$ is the smallest of the common multiples.

~~Theorem~~ §5.1 HWK: 6, 15, 16, 17, 20, 27, 28

## 5.2 Integer Representations

$$5632_{10} = 5\cdot 10^3 + 6\cdot 10^2 + 3\ 10^1 + 2\cdot 10^0$$

$$1101101_2 = 1\cdot 2^6 + 1\cdot 2^5 + 0\cdot 2^4 + 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0$$

$$= 64+32+8+4+1 = 109_{10}$$

Express in decimal form

$$11011_2 = 1\cdot 2^4 + 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$$

$$= 16 + 8 + 2 + 1 = 27_{10}$$