MTH 354 10-11-17

$c_n = c_{n-1} + n \quad c_1 = 4$

$= (c_{n-2} + (n-1)) + n$

$= c_{n-2} + (n-1) + n$

$= c_{n-3} + (n-2) + (n-1) + n$

$\vdots$

$= c_{\underbrace{n-(n-1)}_{=1}} + 2 + 3 + \dots + (n-2) + (n-1) + n$

$= 4 + (2 + 3 + \dots + n) = 4 + \frac{n(n+1)}{2} - 1$

$= 3 + \frac{n(n+1)}{2}$

$c_1 = 4$
$c_2 = c_1 + 2 = 6$
$c_3 = c_2 + 3 = 9$
$c_4 = c_3 + 4 = 13$
$\vdots$
$c_{n-1} = c_{n-2} + (n-1)$
$c_{n-2} = c_{n-3} + (n-2$

$c_1 = 3 + \frac{1(2)}{2} = 4$

$c_2 = 3 + \frac{2(2+1)}{2} = 6$

$c_3 = 3 + \frac{3(4)}{2} = 9$

$c_4 = 3 + \frac{4(5)}{2} = 13$

Show $c_n = c_{n-1} + n$ and $c_n = 3 + \frac{n(n+1)}{2}$ produce the same sequence of values.

Pf (induction):

BC. ✓

IC: Assume $c_k = 3 + \frac{k(k+1)}{2}$ for some $k \geq 1$.

We need to show $c_{k+1} = 3 + \frac{(k+1)(k+2)}{2}$.

note $c_{k+1} = c_k + (k+1)$ by RR

$= 3 + \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$

$= 3 + \frac{k(k+1) + 2(k+1)}{2}$

$= 3 + \frac{(k+1)(k+2)}{2}$

by The PMI the sequence ... seq.

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$$c_n = c_{n-1} + n \quad c_1 = 4$$
$$= \left(c_{n-2} + (n-1)\right) + n$$
$$= c_{n-2} + (n-1) + n$$
$$= c_{n-3} + (n-2) + (n-1) + n$$
$$\vdots$$
$$= c_{\underbrace{n-(n-1)}_{=1}} + 2 + 3 + \cdots + (n-2) + (n-1) + n$$
$$= 4 + (2 + 3 + \cdots + n) = 4 + \frac{n(n+1)}{2} - 1$$
$$= 3 + \frac{n(n+1)}{2}$$

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Show $c_n = c_{n-1} + n$ and $c_n = 3 + \frac{n(n+1)}{2}$ produce the same sequence of values.

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We need to show $c_{k+1} = 3 + \frac{(k+1)(k+2)}{2}$.

Note $c_{k+1} = c_k + (k+1)$ by RR

$$= 3 + \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$
$$= 3 + \frac{k(k+1) + 2(k+1)}{2}$$
$$= 3 + \frac{(k+1)(k+2)}{2}$$

∴ by the PMI. The sequence ... seq.

#14, $a_n = 2^n a_{n-1}$, $a_0 = 1$

$$= 2^n(2^{n-1} a_{n-2})$$

$$= 2^n 2^{n-1}(2^{n-2} a_{n-3})$$

$$= 2^n 2^{n-1} 2^{n-2} a_{n-3}$$

$\vdots$

$$= 2^n 2^{n-1} 2^{n-2} \cdots 2^1 a_{\underbrace{n-n}_{0}}$$

$$= 2^{n+(n-1)+(n-2)+\cdots+2+1}$$

$$a_n = 2^{\frac{n(n+1)}{2}}$$

$a_0 = 1$

$a_1 = 2^{(1)} a_0 = 2$

$a_2 = 2^2 a_1 = 8$

$a_3 = 2^3 a_2 = 64$

$a_{n-1} = 2^{n-1} a_{n-2}$

$a_{n-2} = 2^{n-2} a_{n-3}$

Show $a_n = 2^n a_{n-1}$ and

$a_n = 2^{\frac{n(n+1)}{2}}$ produce the same sequence of values

$a_0 = 2^{\frac{0(1)}{2}} = 2^0 = 1$

$a_1 = 2^{1(2)/2} = 2^1 = 2$

$a_2 = 2^{2(3)/2} = 8$

$a_3 = 2^{3(4/2)} = 64$

Pf (induction)

B.C ✓

I.C: Assume $a_k = 2^{\frac{k(k+1)}{2}}$ for some $k \geq 0$

We need to show $a_{k+1} = 2^{(k+1)(k+2)/2}$.

note that $a_{k+1} = 2^{k+1} a_k$ by RR

$$= 2^{k+1}\left(2^{\frac{k(k+1)}{2}}\right)$$ by I.H

$$= 2^{\frac{2(k+1)}{2} + \frac{k(k+1)}{2}}$$

$$= 2^{\frac{2(k+1)+k(k+1)}{2}}$$

$$= 2^{\frac{(k+1)(k+2)}{2}}$$ ✓ ∎

Thm: Consider a linear recurrence relation of order 1, which is nonhomogeneous, i.e., $a_n = r a_{n-1} + b_n$ where $b_n \neq 0$ is a function of n (b_n could be a constant.)

Then $a_n = r^{n-1} a_1 + \sum_{i=2}^{n} r^{n-i} b_i$

Pf: use backtracking

$$a_n = r a_{n-1} + b_n \qquad a_{n-1} = r a_{n-2} + b_{n-1}$$

$$= r(r a_{n-2} + b_{n-1}) + b_n$$

$$= r^2 a_{n-2} + r b_{n-1} + b_n \qquad a_{n-2} = r a_{n-3} + b_{n-2}$$

$$= r^2 (r a_{n-3} + b_{n-2}) + r b_{n-1} + b_n$$

$$= r^3 a_{n-3} + r^2 b_{n-2} + r b_{n-1} + b_n$$

$\vdots$

$$= r^{n-1} a_{\underbrace{n-(n-1)}_{1}} + r^{n-2} b_2 + r^{n-3} b_3 + \dots + r^2 b_{n-2} + r b_{n-1} + b_n$$

$$a_n = r^{n-1} a_1 + \sum_{i=2}^{n} r^{n-i} b_i \qquad \blacksquare$$

Ex: $S_n = 2 S_{n-1} + 3 \qquad S_1 = 4$

$r = 2,\ b_n = 3$

$$S_n = 2^{n-1}(4) + \sum_{i=2}^{n} 2^{n-i}(3)$$

$$= 2^{n+1} + 3 \sum_{i=2}^{n} 2^{n-i}$$

Finite geom series $r = 2$

$$= 2^{n+1} + 3(2^{n-2} + 2^{n-3} + 2^{n-4} + \dots + 2^0)$$

$$2^{n-2}+2^{n-3}+\cdots+2^1+1$$

$$S_n = 2^{n+1} + 3\left(\frac{2^{n-1}-1}{2-1}\right)$$

$$S_n = 2^{n+1} + 3(2^{n-1}-1)$$

$S_1 = 4$

$S_2 = 2(4)+3 = 11$ ✓

$S_3 = 2(11)+3 = 25$ ✓

$S_1 = 2^2 + 3\cancel{(2^0-1)} \to 0 = 4$

$S_2 = 2^3 + 3(2^1-1) = 11$

$S_3 = 2^4 + 3(2^2-1) =$ 25

$(16)+(9) = 25$

To use this Theorem, you need to know FGS, and Gauss's formula.

$$1+r+r^2+\cdots+r^n = \frac{r^{n+1}-1}{r-1}$$

$$1+2+\cdots+n = \frac{n(n+1)}{2}$$