

The problems for the 3rd set of HWK are § 7.2: # 9, 11-14. I have also asked them to include an induction proof to show their two equations produce the same sequence of values.

please grade # 13 for 6 pts; 3 pts for the derivation and 3 pts for the proof. the rest may have 1 pt each without grading. Total = 10. Thank you.

$$\begin{aligned}
 a_n &= a_{n-1} + n & a_0 &= 0 \\
 &= (a_{n-2} + (n-1)) + n \\
 &= a_{n-2} + (n-1) + n \\
 &= (a_{n-3} + (n-2)) + (n-1) + n \\
 &= a_{n-3} + (n-2) + (n-1) + n \\
 &\vdots \\
 &= \underbrace{a_{n-n}}_{=0} + 1 + 2 + 3 + \dots + (n-2) + (n-1) + n \\
 &= 1 + 2 + \dots + n = \frac{n(n+1)}{2} = a_n
 \end{aligned}$$

To show that  $a_n = a_{n-1} + n$  and  $a_n = \frac{n(n+1)}{2}$  produce the same values note that

PF (induction)

Base Case:  $a_1 = 0 + 1 = \frac{1(1+1)}{2} = 1$  ✓ or  $a_0 = 0 = \frac{0(1)}{2} = 0$  ✓

Induction Case: Assume  $a_k = \frac{k(k+1)}{2}$  for some  $k \geq 1$ .  
We need to show that  $a_{k+1} = \frac{(k+1)(k+2)}{2}$ .

Principle  
of Mathematical  
Induction

note that  $a_{k+1} = a_k + (k+1)$  by the recurrence relation

$$\begin{aligned}
 &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} && \text{by the induction hypothesis} \\
 &= \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}
 \end{aligned}$$

Therefore, by the PMI, the two equations produce the same sequence.

The first thing I noticed when I stepped out of the car was the cold. It was a sharp contrast to the warm blanket I had been sitting under. I looked up at the sky, which was a pale, overcast grey. The air felt heavy, like a thick blanket. I took a deep breath, trying to clear my head. The sound of the city was in the background, a distant hum of activity. I felt a little disoriented, like I had been dropped into a new world. I looked around, trying to make sense of my surroundings. The buildings were tall and modern, their windows reflecting the light. The streets were wide and clean, with a few cars driving by. I felt a sense of wonder, like I had discovered a new place. I took a few more steps, feeling the ground beneath my feet. The air was crisp and clean, a welcome change from the stuffy car. I felt a sense of freedom, like I had been released from a cage. I looked up at the sky again, feeling a sense of peace. The clouds were soft and white, like a blanket. I felt a sense of hope, like I had found a new beginning. I took a deep breath, feeling the air fill my lungs. I felt a sense of joy, like I had found a new home. I looked around one last time, feeling a sense of contentment. The city was beautiful, and I was glad to be here. I took a few more steps, feeling the ground beneath my feet. The air was crisp and clean, a welcome change from the stuffy car. I felt a sense of freedom, like I had been released from a cage. I looked up at the sky again, feeling a sense of peace. The clouds were soft and white, like a blanket. I felt a sense of hope, like I had found a new beginning. I took a deep breath, feeling the air fill my lungs. I felt a sense of joy, like I had found a new home. I looked around one last time, feeling a sense of contentment. The city was beautiful, and I was glad to be here.