MTH 354 11/2/17

§8.5 Representations of Graphs

short review of matrices

$$\begin{aligned} 2x - 5y &= 2 \\ 3x + 4y &= 6 \end{aligned} \rightarrow \underbrace{\begin{pmatrix} 2 & -5 \\ 3 & 4 \end{pmatrix}}_{2\times 2} \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{2\times 1} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

size of this matrix is 2×1 (# of rows × # of columns)

$$B = \begin{pmatrix} 1 & 2 \\ -6 & 0 \\ 7 & 2 \end{pmatrix}_{3\times 2} \qquad A = \begin{pmatrix} 1 & 5 & 6 & 7 \end{pmatrix}_{1\times 4}$$

Matrix operations

+, constant multiplication, matrix multiplication

$$A = \begin{pmatrix} 1 & 5 \\ 6 & -2 \end{pmatrix}_{2\times 2}, \quad B = \begin{pmatrix} 2 & 0 \\ -4 & 5 \end{pmatrix}_{2\times 2}$$

$$A + B = \begin{pmatrix} 3 = 1+2 & 5 = 5+0 \\ 2 = 6+(-4) & 3 = -2+5 \end{pmatrix}$$

$$-1B = \begin{pmatrix} -2 & 0 \\ 4 & -5 \end{pmatrix} \longrightarrow A - B = A + (-B) = \begin{pmatrix} 1 & 5 \\ 6 & -2 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 4 & -5 \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 10 & -7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 5 \\ 6 & -2 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} (1)(2)+(5)(-4) & (1)(0)+(5)(5) \\ (6)(2)+(-2)(-4) & (6)(0)+(-2)(5) \end{pmatrix}$$

$$= \begin{pmatrix} -18 & 25 \\ 20 & -10 \end{pmatrix}$$

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size of the matrix is 2×2

\# of rows, \# of columns

$$B = \begin{pmatrix} 1 & 2 \\ -6 & 0 \\ 7 & 2 \end{pmatrix}_{3\times 2} \qquad A = \begin{pmatrix} 1 & 5 & 6 & 7 \end{pmatrix}_{1\times 4}$$

Matrix operations

+, constant multiplication, matrix multiplication

$$A = \begin{pmatrix} 1 & 5 \\ 6 & -2 \end{pmatrix}_{2\times 2}, \quad B = \begin{pmatrix} 2 & 0 \\ -4 & 5 \end{pmatrix}_{2\times 2}$$

$$A+B = \begin{pmatrix} 3=1+2 & 5=5+0 \\ 2=6+(-4) & 3=-2+5 \end{pmatrix}$$

$$-1B = \begin{pmatrix} -2 & 0 \\ 4 & -5 \end{pmatrix} \rightarrow A-B = A+(-B) =$$

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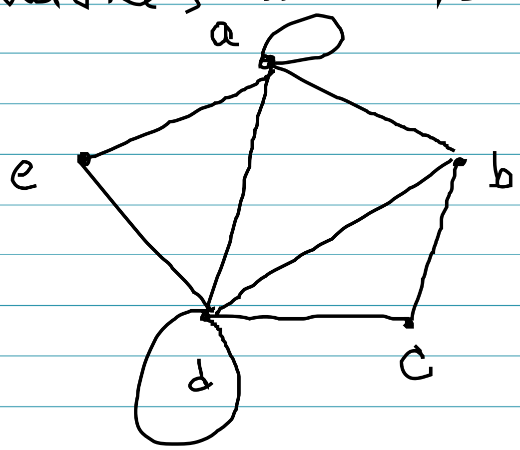
$$= \begin{pmatrix} -18 & 25 \\ 20 & -10 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -5 & 0 \\ 6 & 3 \end{pmatrix}_{3\times 2} \begin{pmatrix} 1 & 5 & -2 \\ 0 & 4 & 6 \end{pmatrix}_{2\times 3} = \begin{pmatrix} 1 & 13 & 10 \\ -5 & -25 & 10 \\ 6 & 42 & 6 \end{pmatrix}_{3\times 3}$$

§8.5 Representations of Graphs

Adjacency matrix.

Given a graph with n vertices, create a square matrix of size n with the name of the vertices as rows and columns (in any order). The entry for the ij th position is ~~1 if there is an edge between~~ The number of edges incident on i and j vertices in the row i and column j and 0 otherwise.



$$A = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 2 & 1 & 0 & 1 & 1 \\ b & 1 & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 1 & 1 & 1 & 2 & 1 \\ e & 1 & 0 & 0 & 1 & 0 \end{array}$$

Note: The sum of rows for a vertex v (or column v) gives the degree of vertex v.

Note: If the vertices of a graph G are labeled 1, 2, 3, ---, the ijth entry in the matrix A^n is equal to the number of paths from i to j of length n.

$$A^2 = A * A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} a \quad b \quad c \quad d \quad e \\ \begin{pmatrix} 7 & 3 & 2 & 6 & 3 \\ 3 & 3 & 1 & 4 & 2 \\ 2 & 1 & 2 & 3 & 1 \\ 6 & 4 & 3 & 8 & 3 \\ 3 & 2 & 1 & 3 & 2 \end{pmatrix} \end{array}$$

$b \to c$ {b, d, c}

$a \to d$ {a, e, d; a, a, d (2); a, d, d (2); a, b, d}

$a \to c$ {a, b, c; a, d, c}

$b \to d$ {b, a, d; b, c, d; b, a, d; b, d, d}

$c \to e$ {c, d, e}