Using induction, show that $b_n = 4b_{n-1} + 5$ and $b_n = \left(\frac{11}{3}\right)4^{n-1} - \frac{5}{3}$ produce the same sequence.

Pf (induction)

BC: $b_1 = \frac{11}{3} 4^{1-1} - \frac{5}{3} = \frac{11}{3} \underbrace{4^0}_{=1} - \frac{5}{3} = \frac{11}{3} - \frac{5}{3} = \frac{6}{3} = 2 = b_1$ ✓

IC: Assume $b_k = \left(\frac{11}{3}\right)4^{k-1} - \frac{5}{3}$ for some $k \geq 1$.

We need to show $\boxed{b_{k+1} = \left(\frac{11}{3}\right)4^{k} - \frac{5}{3}.}$

We note that $b_{k+1} = 4b_k + 5$ by RR

$= 4\left(\frac{11}{3} 4^{k-1} - \frac{5}{3}\right) + 5$ by IH

$= \left(\frac{11}{3}\right)4^k - 4\left(\frac{5}{3}\right) + 5$

$= \left(\frac{11}{3}\right)4^k - \frac{20}{3} + \frac{15}{3}$

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