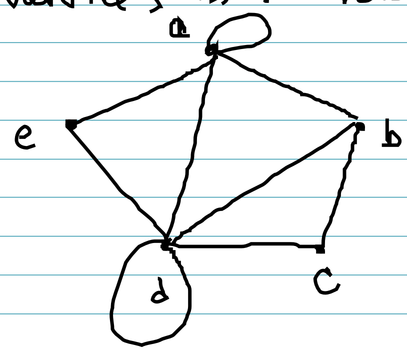
$$\begin{pmatrix} 1 & 2 \\ -5 & 0 \\ 6 & 3 \end{pmatrix}_{3\times 2} \begin{pmatrix} 1 & 5 & -2 \\ 0 & 4 & 6 \end{pmatrix}_{2\times 3} = \begin{pmatrix} 1 & 13 & 10 \\ -5 & -25 & 10 \\ 6 & 42 & 6 \end{pmatrix}_{3\times 3}$$

§8.5 Representations of Graphs

Adjacency matrix.

Given a graph with n vertices, create a square matrix of size n with the name of the vertices as rows and columns (in any order). The entry for the ijth position is ~~1 if there is an edge between~~ The number of edges incident on i and j vertices in the row i and column j and 0 otherwise.



$$A = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 2 & 1 & 0 & 1 & 1 \\ b & 1 & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 1 & 1 & 1 & 2 & 1 \\ e & 1 & 0 & 0 & 1 & 0 \end{array}$$

Note: The sum of rows for a vertex v (or column v) gives the degree of vertex v.

Note: If the vertices of a graph G are labeled 1, 2, 3, ..., the ijth entry in the matrix A^n is equal to the number of paths from i to j of length n.

$$A^2 = A * A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}\begin{array}{c} a \quad b \quad c \quad d \quad e \\ \begin{pmatrix} 7 & 3 & 2 & 6 & 3 \\ 3 & 3 & 1 & 4 & 2 \\ 2 & 1 & 2 & 3 & 1 \\ 6 & 4 & 3 & 8 & 3 \\ 3 & 2 & 1 & 3 & 2 \end{pmatrix} \end{array}$$

b → c {b, d, c}

a → c {a, b, c; a, d, c}

b → d {b, a, d; b, c, d; b, a, d; b, d, d}

c → e {c, d, e}

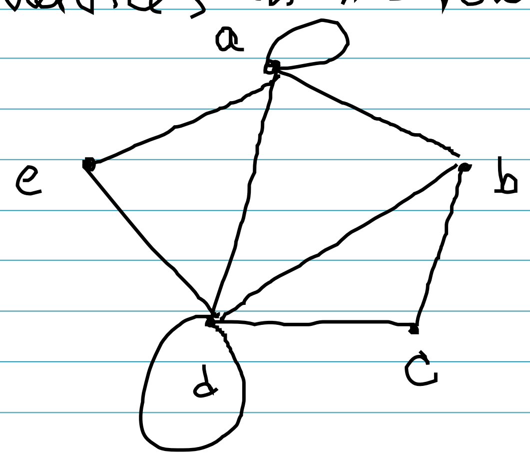
a → d {a, e, d; a, a, d (2); a, d, d (2); a, b, d}

$$\begin{pmatrix} 1 & 2 \\ -5 & 0 \\ 6 & 3 \end{pmatrix}_{3\times 2} \begin{pmatrix} 1 & 5 & -2 \\ 0 & 4 & 6 \end{pmatrix}_{2\times 3} = \begin{pmatrix} 1 & 13 & 10 \\ -5 & -25 & 10 \\ 6 & 42 & 6 \end{pmatrix}_{3\times 3}$$

§8.5 Representations of Graphs

Adjacency matrix.

Given a graph with n vertices, create a square matrix of size n with the name of the vertices as rows and columns (in any order). The entry for the ij th position is ~~1 if there is an edge between~~ the number of edges incident on i and j vertices in the row i and column j and 0 otherwise.



$$A = \begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 2 & 1 & 0 & 1 & 1 \\ b & 1 & 0 & 1 & 1 & 0 \\ c & 0 & 1 & 0 & 1 & 0 \\ d & 1 & 1 & 1 & 2 & 1 \\ e & 1 & 0 & 0 & 1 & 0 \end{array}$$

Note: The sum of rows for a vertex v (or column v) gives the degree of vertex v.

Note: If the vertices of a graph G are labeled 1, 2, 3, --, the ij th entry in the matrix A^n is equal to the number of paths from i to j of length n.

$$A^2 = A * A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 7 & 3 & 2 & 6 & 3 \\ 3 & 3 & 1 & 4 & 2 \\ 2 & 1 & 2 & 3 & 1 \\ 6 & 4 & 3 & 8 & 3 \\ 3 & 2 & 1 & 3 & 2 \end{pmatrix} \end{array}$$

$b \to c$ {b, d, c}

$a \to c$ {a, b, c; a, d, c}

$b \to d$ {b, a, d; b, c, d; b, a, d; b, d, d}

$c \to e$ {c, d, e}

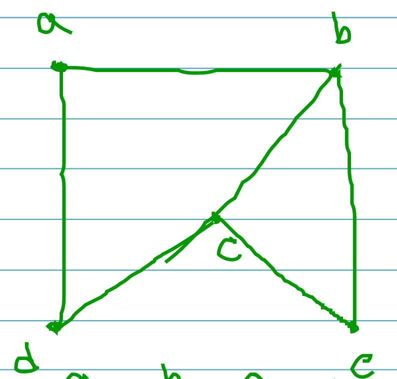
$a \to d$ {a, e, d; a, a, d (2) (2); a, ; a, b, d}

Note: The entries on the main diagonal of A^2 give the degrees of the vertices (when the graph is simple)

Theorem 8.5.3: If A is the adjacency matrix of a simple graph, the ijth entry of A^n is equal to the number of paths of length n from vertex i to vertex j

$n = 1, 2, \ldots$

Ex:



$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 2 & 0 & 2 & 0 & 1 \\ 0 & 3 & 1 & 2 & 1 \\ 2 & 1 & 3 & 0 & 1 \\ 0 & 2 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix} \end{array}$$

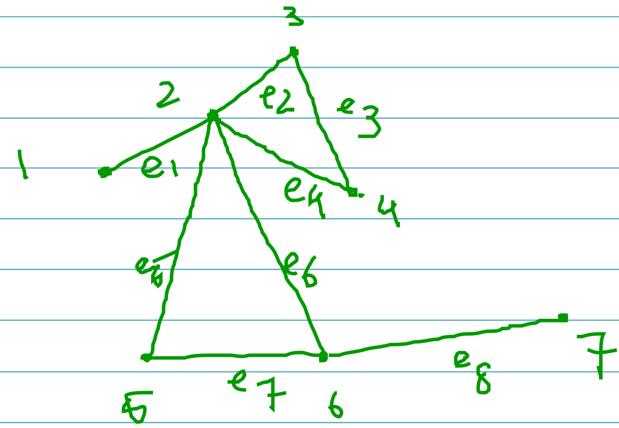
$d \to b \begin{Bmatrix} d, a, b \\ d, c, b \end{Bmatrix}$

path of length 4 (see A^4 in book)

$d \xrightarrow{6} e$ $\{$ d, a, b, c, e ✓ | d, a, d, c, e ✓
d, c, e, b, e ✓ | d, c, e, c, e ✓
d, c, b, c, e ✓ | d, c, d, c, e ✓

Incidence Matrix: To create an incidence matrix label the rows with the vertices and the columns with the edges (in any order). The entry for row and column is 1 if e is incident on v and 0 otherwise.

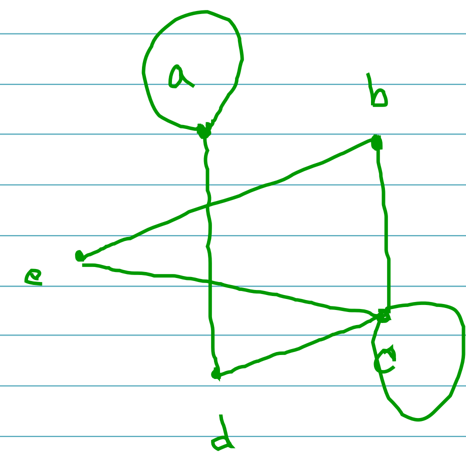
#10 HWK Find the incidence matrix for graph 8.2.1



	e_1	e_2	e_3	e_4	e_5	e_6		e_8
1	1	0	0	0	0	0	0	0
2	1	1	0	1	1	1	0	0
3	0	1	1	0	0	0	0	0
4	0	0	1	1	0	0	0	0
5	0	0	0	0	1	0	1	0
6	0	0	0	0	0	1	1	1
7	0	0	0	0	0	0	0	1

#13)

	a	b	c	d	e	
a	2	0	0	1	0	
b	0	2	1	0	1	0
c	0	1	2	1	1	
d	1	0	1	0	0	
e	0	1	1	0	0	



#24)

$$
\begin{array}{c}
a \\ b \\ c \\ d \\ e \\
\end{array}
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 \\
 & & 1 & & 1 &
\end{pmatrix}
$$