Example 7.1.6

Let S_n denote the number of n-bit strings that do not contain the pattern 111. Develop a RR for $S_1, S_2, \ldots$ and initial conditions that define the sequence S

Sol: To find a RR for $S_1, S_2, \ldots$ that do not contain the pattern 111, we consider

a) strings that begin with 0

b) " " " " 10

c) " " " " 11

These cases are disjoint and hence the addition principle can be applied to give

S_n = # of strings of type a + # of strings of type b + # of strings of type c

For type (a), suppose that an n-bit string starts with 0 and does not contain the pattern 111. Then the (n-1) bit string following 0 does not contain the pattern 111 and hence by definition there are S_{n-1} strings of type a, since any (n-1) bit string not containing 111 can follow the initial 0. Similarly for type b, we have S_{n-2} strings. For type c, we have S_{n-3} since the (n-2) bit string can not ~~follow~~ start with 1.

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§ 5.1

#18) Let S_n denote the number of n-bit strings that do not contain the pattern 000.

a) 1 $\longrightarrow S_{n-1}$

b) 01 $\longrightarrow S_{n-2}$

c) 00 $\longrightarrow S_{n-3}$

$$S_n = S_{n-1} + S_{n-2} + S_{n-3} \quad RR$$

$$S_1 = 2, \quad S_2 = 4, \quad S_3 = 7$$