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§5.2 (cont)

The number of bits necessary to represent a positive integer n.

Given $n = 1 \cdot 2^k + b_{k-1} 2^{k-1} + b_{k-2} 2^{k-2} + \cdots + b_0 2^0$

Since not all of the b_i's are zero, we have

$n \geq 2^k$ (1). Also

$n = 1 \cdot 2^k + b_{k-1} 2^{k-1} + b_{k-2} 2^{k-2} + \cdots + b_0 2^0$

$\leq 1 \cdot 2^k + 1 \cdot 2^{k-1} + 1 \cdot 2^{k-2} + \cdots + 1 \cdot 2^0$

$= 2^0 + 2^1 + 2^2 + \cdots + 2^{k-2} + 2^{k-1} + 2^k = \dfrac{2^{k+1} - 1}{2-1} = 2^{k+1} - 1.$

Finite geometric series: $n < 2^{k+1}$ (2)

$r^0 + r^1 + \cdots + r^n = \dfrac{r^{n+1} - 1}{r-1}$

Taking log base 2 of both sides of (1) and (2), we get

$\lg n \geq \lg 2^k = \log_2 2^k = k$ so $\lg n \geq k$ (1')

$\lg n < \lg 2^{k+1} = \log_2 2^{k+1} = k+1$ so $\lg n < k+1$ (2')

$k+1 \overset{(1')}{\leq} \underline{1 + \lg n} \overset{(2')}{<} k+2$

Therefore, $k+1 = \lfloor 1 + \lg n \rfloor$

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§5.1 (cont)

The number of bits necessary to represent a positive integer n.

Given $n = 1\cdot 2^{k} + b_{k-1} 2^{k-1} + b_{k-2} 2^{k-2} + \cdots + b_0 2^0$

Since not all of the b_i's are zero, we have

$n \geq 2^{k}$ ⋅ (1). Also

$$n = 1\cdot 2^{k} + b_{k-1} 2^{k-1} + b_{k-2} 2^{k-2} + \cdots + b_0 2^0$$
$$\leq 1\cdot 2^{k} + 1\cdot 2^{k-1} + 1\cdot 2^{k-2} + \cdots + 1 2^0$$
$$= 2^0 + 2^1 + 2^2 + \cdots + 2^{k-2} + 2^{k-1} + 2^{k} = \frac{2^{k+1}-1}{2-1} = 2^{k+1} - 1.$$

$n < 2^{k+1}$ (2)

Finite geometric series:

$$r^0 + r^1 + \cdots + r^n = \frac{r^{n+1}-1}{r-1}$$

Taking log base 2 of both sides of (1) and (2), we get

$$\lg n \geq \lg 2^{k} = \log_2 2^{k} = k \quad \text{so} \quad \lg n \geq k \quad (1')$$
$$\lg n < \lg 2^{k+1} = \log_2 2^{k+1} = k+1 \quad \text{so} \quad \lg n < k+1 \quad (2')$$

$$k+1 \overset{(1')}{\leq} \underline{1 + \lg n} \overset{(2')}{<} k+2$$

Therefore, $k+1 = \lfloor 1 + \lg n \rfloor$

Find the number of bits for

#3 $n = 209$

$$k+1 = \lfloor 1 + \lg 209 \rfloor = \lfloor 1 + \frac{\log 209}{\log 2} \rfloor = 8$$

log base conversion = $\log_a b = \frac{\ln b}{\ln a} = \frac{\log b}{\log a}$

$$\lg 5 = \frac{\log 5}{\log 2} = \frac{\ln 5}{\ln 2}$$

#5) $n = 1007$

$$k+1 = \lfloor 1 + \lg 1007 \rfloor = 10$$

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binary $\longrightarrow$ base 10

Hexadecimal symbols: 0, 1, ... 9, A, B, C, D, E, F
(A = 10, B = 11, C = 12, D = 13, E = 14, F = 15)

Hex $\longrightarrow$ base 10

$$C5D_{16} = 12 \cdot 16^2 + 5 \cdot 16^1 + 13 \cdot 16^0$$
$$= 3165_{10}$$

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base 10 $\longrightarrow$ binary

Ex: $n = 223$

$223 = 2(111) + 1$	1's bit 2^0
$111 = 2(55) + 1$	2's bit 2^1
$55 = 2(27) + 1$	4's bit 2^2
$27 = 2(13) + 1$	8's bit 2^3
$13 = 2(6) + 1$	16's bit 2^4
$6 = 2(3) + 0$	32's bit 2^5
$3 = 2(1) + 1$	64's bit 2^6
$1 = 2(0) + 1$	128's bit 2^7

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