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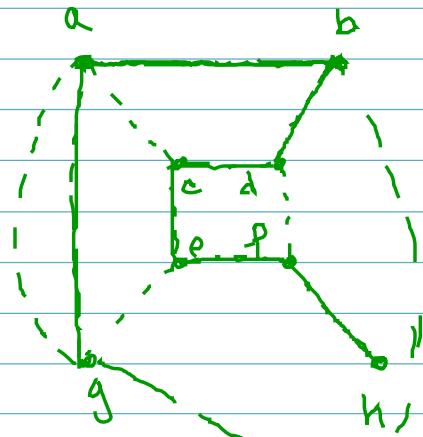
<p>MTH 354</p> <p>§ 9.3 Spanning Tree</p> <p>Def 9.3.1: A tree <math>T</math> is a spanning tree of a graph <math>G</math> if <math>T</math> is a subgraph of <math>G</math> that contains all of the vertices of <math>G</math>.</p>	<p>11/14/17</p> <p>Can we find a spanning tree <math>T</math> of the graph with root <math>a</math>?  <math>abcde\cancel{f}gh</math></p> <p>Maybe <math>\rightarrow</math> Depth-First search</p> <p>Breadth-First search</p> <p>if b</p> <p>Theorem 9.3.4: A graph <math>G</math> has a spanning tree <u>if and only if</u> <math>G</math> is connected.</p> <p>Breadth-First Search algo 9.3.6      Consider the graph above with the vertex ordering of <math>abcde\cancel{f}gh</math>.</p> <table border="0" style="margin-left: 20px;"> <tr> <td style="vertical-align: top;"> <math>T: a</math> (root)          Level 0: add edges(s): <math>(a,b)</math>, <math>(a,c)</math>, <math>(a,g)</math>          Level 1: " " <math>b: (b,d)</math>, <math>\cancel{(b,g)}</math> ← do not include otherwise we create cycle.  <math>c: (c,d)</math>  <math>g: \text{none}</math> </td> <td style="vertical-align: bottom; padding-left: 20px;"> <math>d: (d,f)</math>  <math>e: \text{none}</math> </td> </tr> </table>	$T: a$ (root) Level 0: add edges(s): $(a,b)$ , $(a,c)$ , $(a,g)$ Level 1: " " $b: (b,d)$ , $\cancel{(b,g)}$ ← do not include otherwise we create cycle. $c: (c,d)$ $g: \text{none}$	$d: (d,f)$ $e: \text{none}$
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11.14.2017 2:10p

11/14/17, 5:13 AM, 36m 31s

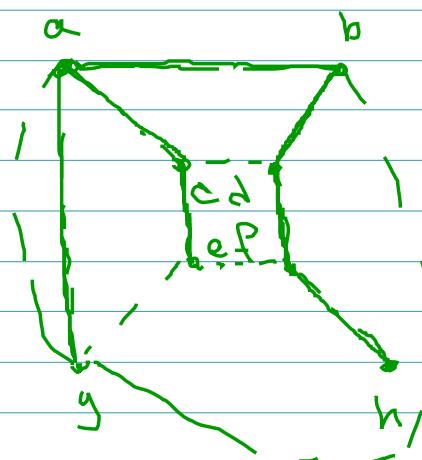
### § 9.3 Spanning Tree

**Def 9.3.1:** A tree  $T$  is a spanning tree of a graph  $G$  if  $T$  is a subgraph of  $G$  that contains all of the vertices of  $G$ .



Can we find a spanning tree  $T$  of the graph with root  $a$ ?  
abcde~~fgh~~

Maybe  $\rightarrow$  Depth-First search



Breadth-First search

if b

**Theorem 9.3.4:** A graph  $G$  has a spanning tree if and only if  $G$  is connected.

Breadth-First search algo 9.3.6

Consider the graph above with the vertex ordering of abcde~~fgh~~.

$T: a$  (root)  
Level 0: add edges(s):  $(a,b)$ ,  $(a,c)$ ,  $(a,g)$

Level 1, " "  $b: (b,d)$ ,  $\cancel{(b,g)}$  ← do not include otherwise we  
 $c: (c,e)$  create a cycle.  
 $g: \text{none}$

Level 2,

$d: (d,f)$

$e: \text{none}$

... =  $f$

#1) hgfedcba

T: h (root)

level 0; h: include (h,f)

level 1; f: " (f,d), (f,e)

level 2; e: " (e,c), (e,g)

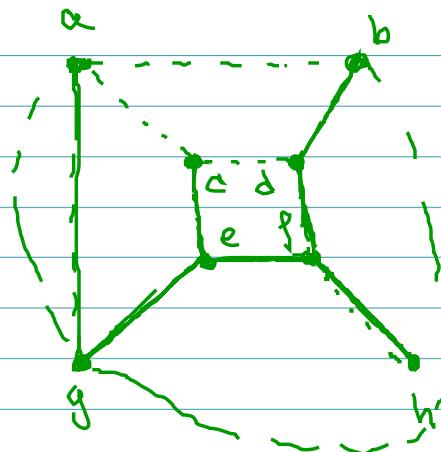
d: "

level 3; g: " (g,a)

c: none

b: "

End.



#2) hfdbgeca

T: h (root)

level 0; h: include (h,f)

level 1; f: " (f,d), (f,e)

level 2; d: " (d,b), (d,c)

e: " (e,g)

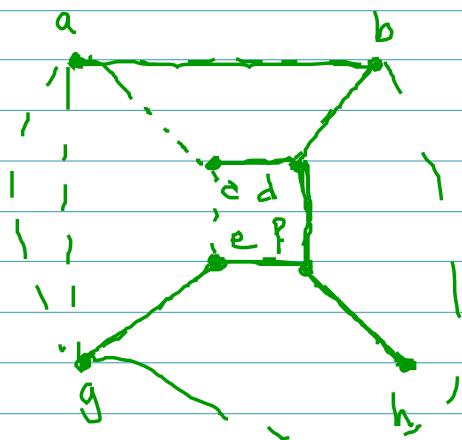
level 3; b: " (b,a)

g: " none

c: " "

level 4; a: ~~none~~ none

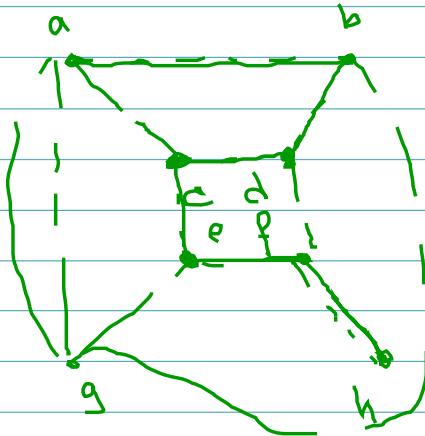
end.



## Depth-First search algo.

This method proceeds to successive levels via a tree at the earliest opportunity.

Ex: Try figure 9.3.1 using ordering abcdefgh



T: a (root)

add edge(s): (a,b)

" " : (b,d), (d,c), (c,e), (e,f),  
(f,h)

back track to f. There are  
no edges to add.

back track to e. add edge  
(e,g).

No more edges to be added.