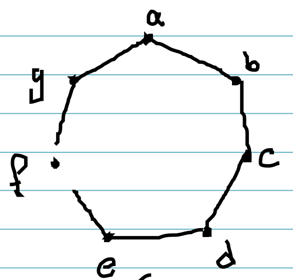
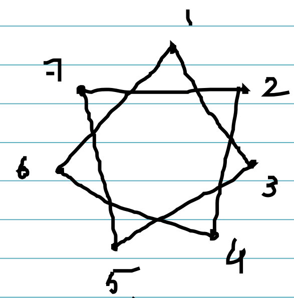


G_1



$a \mapsto 1$
$b \mapsto 3$
$c \mapsto 5$
$d \mapsto 7$
$e \mapsto 2$
$f \mapsto 4$
$g \mapsto 6$

	a	b	c	d	e	f	g
a	0	1	0	0	0	0	1
b	1	0	1	0	0	0	0
c	0	1	0	1	0	0	0
d	0	0	1	0	1	0	0
e	0	0	0	1	0	1	0
f	0	0	0	0	1	0	1
g	1	0	0	0	0	1	0

	1	3	5	7	2	4	6
1	0	1	0	0	0	0	1
3	1	0	1	0	0	0	0
5	0	1	0	1	0	0	0
7	0	0	1	0	1	0	0
2	0	0	0	1	0	1	0
4	0	0	0	0	1	0	1
6	1	0	0	0	0	1	0

Corollary 8.6.4. Let G_1 & G_2 be simple graphs. The following are equivalent.

a) G_1 & G_2 are isomorphic.

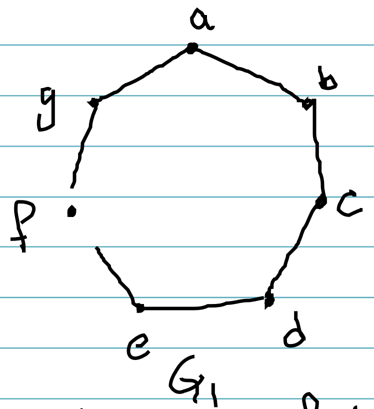
vertix set of G_1 — vertix set of G_2

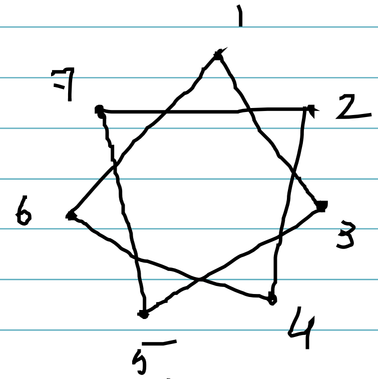
b) There exists $\exists$ a bijection $f: V_{G_1} \to V_{G_2}$ satisfying the following: Vertices v & w are adjacent in $G_1 \Leftrightarrow$ the vertices $f(v)$ and $f(w)$ are adjacent in G_2.

Note: To show that 2 simple graphs G_1 & G_2 are not isomorphic, find a property of G_1 that G_2 does not have, but that G_2 would have if G_1 and G_2 were isomorphic.

Such a property is called <u>invariant</u>.

<u>Def.</u> A property P is an invariant if whenever G_1 & G_2 are isomorphic graphs:
If G_1 has property P, G_2 has property P





$a \mapsto 1$
$b \mapsto 3$
$c \mapsto 5$
$d \mapsto 7$
$e \mapsto 2$
$f \mapsto 4$
$g \mapsto 6$

	a	b	c	d	e	f	g
a	0	1	0	0	0	0	1
b	1	0	1	0	0	0	0
c	0	1	0	1	0	0	0
d	0	0	1	0	1	0	0
e	0	0	0	1	0	1	0
f	0	0	0	0	1	0	1
g	1	0	0	0	0	1	0

	1	3	5	7	2	4	6
1	0	1	0	0	0	0	1
3	1	0	1	0	0	0	0
5	0	1	0	1	0	0	0
7	0	0	1	0	1	0	0
2	0	0	0	1	0	1	0
4	0	0	0	0	0	0	1
6	1	0	0	0	0	1	0

Corollary 8.6.5. Let G_1 & G_2 be simple graphs. The following are equivalent.

vertex set of G_1

vertex set of G_2

a) G_1 & G_2 are isomorphic.

b) $\exists$ a bijection $f : V_{G_1} \longrightarrow V_{G_2}$ satisfying the following:

There exists

Vertices v & w are adjacent in $G_1 \Leftrightarrow$ the vertices $f(v)$ and $f(w)$ are adjacent in G_2.

Note: To show that 2 simple graphs G_1 & G_2 are not isomorphic, find a property of G_1 that G_2 does not have, but that G_2 would have if G_1 and G_2 were isomorphic.

Such a property is called <u>invariant</u>.

<u>Def</u>: A property P is an invariant if whenever G_1 & G are isomorphic graphs:

If G_1 has property P, G_2 has property P.

Examples of invariants

1: If e & n are nonnegative integers, then the properties "has e edges" & "has n vertices" are invariant.

2: If $k \in \mathbb{Z}^+$, then "has a vertex of degree k" is an invariant

3: If $k \in \mathbb{Z}^+$, then "has a simple cycle of length k" is an invariant.

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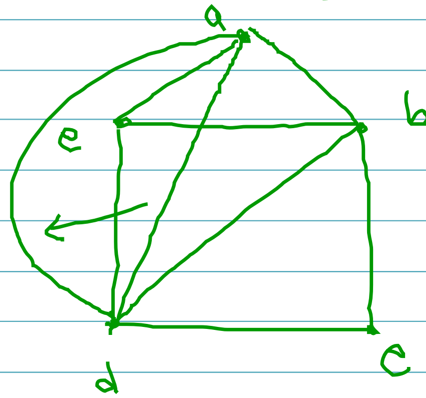
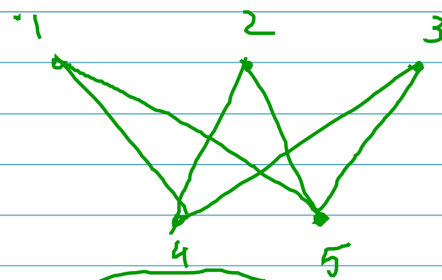
§ 8.7 planar Graphs

Def: A graph is planar if it can be drawn in the plane without its edges crossing.

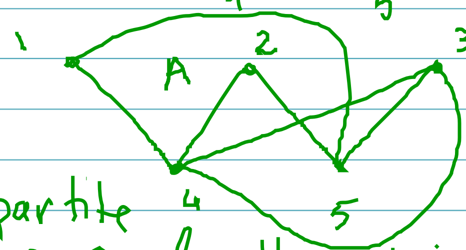


a planar graph.

Consider $K_{2,3}$.



Can we redraw the edges so that the graph is planar?

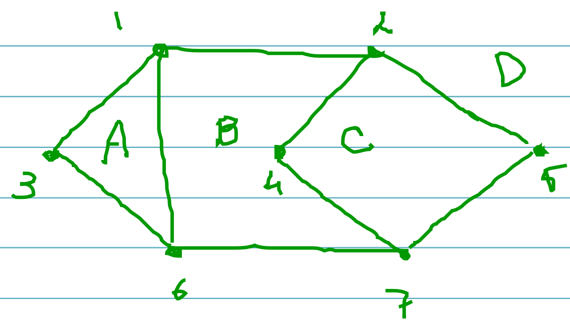


planar

Note:
In any complete bipartite graph that can have a cycle, the minimum length for such cycle is 4.

Def: A face of a planar graph is a maximal section of a plane s.t any two points within the section maybe connected by a curve which does not cross an edge

Ex:



Recall that faces are characterized by the cycle that forms its boundary

$A \longrightarrow (1,3,6,1)$
$B \longrightarrow (1,2,4,7,6,1)$
$C \rightarrow (2,4,7,5,2)$
$D \rightarrow (1,2,5,7,6,3,1)$

Note that in the graph above, we have

$f=4, \quad e=9, \quad V=7. \quad f=e-V+2$ Euler's equation for planar graph.

$e-v+2=9-7+2=2+2=4=f.$

Euler's formula for planar graphs

Thm 8.7.9: If G is a connected planar graph with v vertices, e edges, and f faces, then $f=e-v+2$.

Is $K_{3,3}$ a planar graph?

NO. $K_{3,3}$ is not a planar graph.