§8.3: #1–11, 16, 17

§8.4: 2, 3, 5

For §8.3, Please grade #4, 7, 16, and 17 for 1 point each the rest (1, 2, 3, 5, 6, 8, 9, 10, 11) may have $\frac{1}{3}$ point without grading.

For §8.4, please grade #2 for 1 point. As long as they have some work and the path and length is correct, you could give them the point.

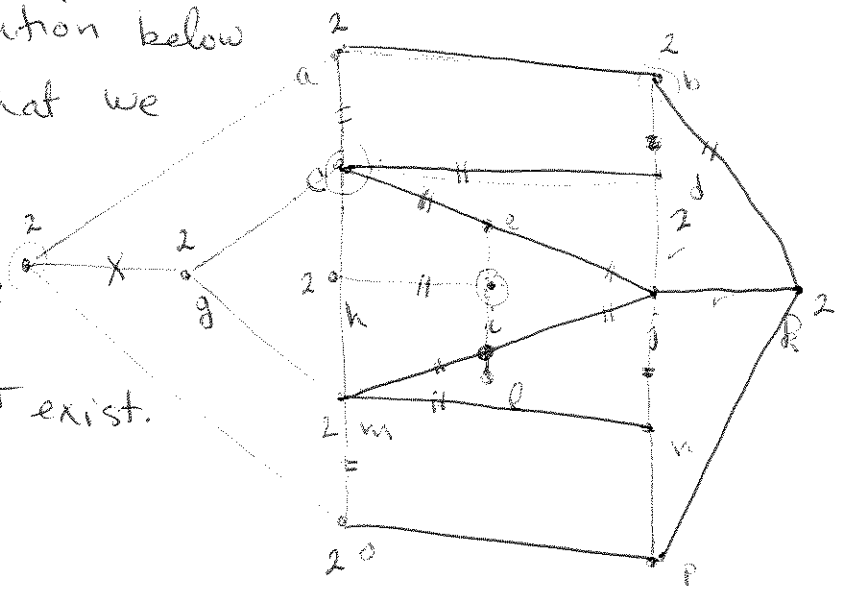
The other 2 problems can have $\frac{1}{2}$ point without grading

$$\text{total} = \left(4 + \frac{9}{3}\right) + (1+1) = 9$$

thank you.

§8.3

#4) Assume the given graph has a Hamiltonian cycle. We eliminate edges from vertices that have degree bigger than 2. See below for 1 example. Since there are 16 vertices, we need to have 16 edges in a Hamiltonian cycle. The elimination below shows that we have 15 edges only. Therefore a HC does not exist.



c, j, m

f 1

b 1

i 1

c 111

m 111

j 111

12

$27 - 12 = 15$ edges

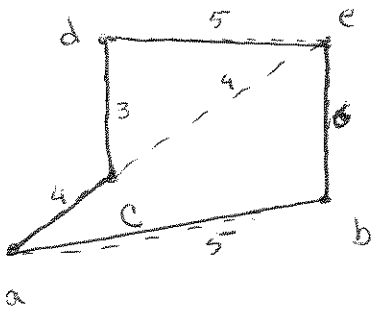
16 vertices

#7) one possible Hamiltonian cycle is

(a, b, c, g, l, m, r, q, p, k, j, f, e, i, n, o, t, s, h, d, a)

#16) The edges with smallest weights 3, 4, 4, 5, 6, which gives $3+4+4+5+6 = 21$ (minimum). However, edges with weights 3, 4, 4 are all incident on c. Hence not all can be in a MST. We want to avoid edges w/ weights 7 & 8 so we delete edges (a, e), (b, c), and (b, d).

Replace edge (e, c) with the next available edge that contributes to a cycle with edge w/ weight 5. That would be (e, b) w/ weight 6.



total weight = 23.

Recall that we need 5 edges only.

#17) Best case: 2, 3, 4, 4, 5 $\Rightarrow$ $2+3+4+4+5 = 18$.

(e, d, a, b, c, e) $\rightarrow$ $2+4+3+4+7 = 20$

(e, d, a, c, b, e) $\rightarrow$ $2+4+6+4+5 = 21$

(e, d, c, a, b, e) $\rightarrow$ $2+7+6+3+5 = 23$

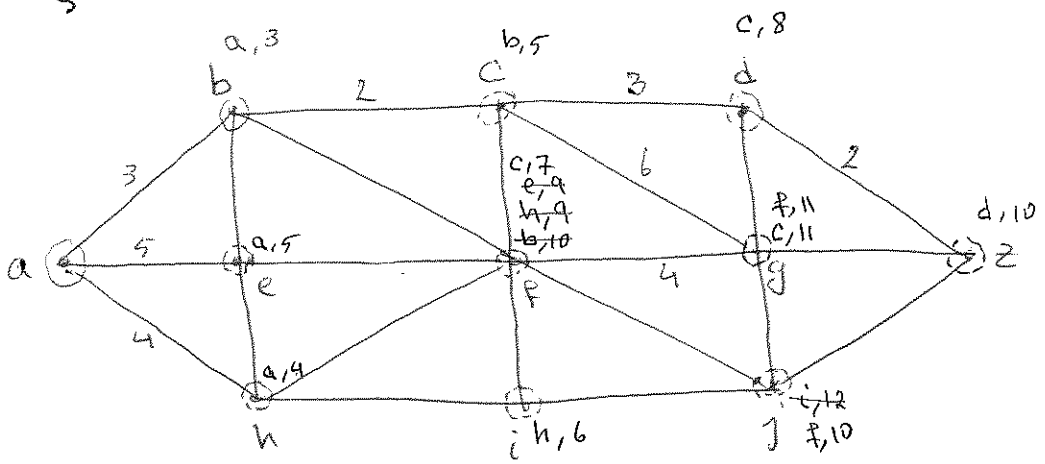
(a, d, e, b, c, a)

$= 4+2+5+4+6$

$= 21$

§ 8.4

#2 a, g



Initial step

$L(a)=0,\ L(b)=\cdots=L(z)=\infty$

1st iteration
circle vertex a

$L(b)=3,\ L(e)=5,\ L(h)=4$

2nd iteration
circle vertex b

$L(c)=5,\ L(e)=5,\ L(f)=10$

3rd iteration
circle vertex h

$L(e)=5,\ L(i)=6,\ L(f)=9$

4th iteration
circle vertex e

$L(f)=9$

5th iteration
circle vertex c

$L(f)=7,\ L(d)=8,\ L(g)=11$

6th iteration
circle vertex i

$L(f)=7,\ L(j)=12$

7th iteration
circle vertex f

$L(g)=11,\ L(j)=10$

8th iteration
circle vertex d

$L(g)=11,\ L(z)=10$

9th iteration
circle vertex j

$L(g)=11,\ L(z)=10$

10th iteration
circle vertex z

$L(g)=11$

11th iteration
circle vertex g

path: $\begin{cases}(a,b,c,g) \leftarrow \text{shortest path.}\\ (a,b,c,f,g)\end{cases}$

length: 11

h, d

#5)

initial step

$L(h)=0,\ L(a)=\dots=L(z)=\infty$

1st iteration

circle vertex h

$L(a)=4,\ L(e)=7,\ L(f)=5,\ L(i)=2$

2nd iteration

circle vertex i

$L(f)=5,\ L(j)=8$

3rd iteration

circle vertex a

$L(b)=7,\ L(e)=7$

4th iteration

circle vertex f

$L(e)=7,\ L(b)=7,\ L(c)=7,\ L(g)=9$

$L(j)=8$

5th iteration

circle vertex c

$L(b)=7,\ L(g)=9,\ L(d)=10$

6th iteration

circle vertex b

$L(e)=7,$

7th iteration

circle vertex e

8th iteration

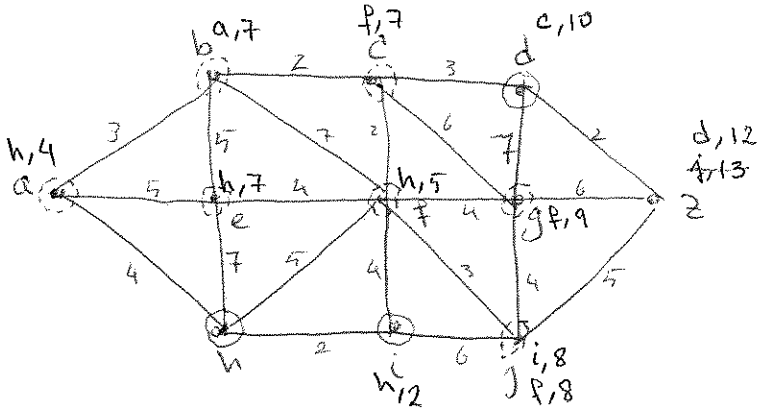
circle vertex j

$L(g)=9,\ L(z)=13$

9th iteration

circle vertex g

$L(d)=10,\ L(z)=13$



10th iteration

circle vertex d

$L(z)=12$

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path: (h, f, c, d)

length: 10