MTH 354 9-28-17

Thm 6: (5.3.2) If a is a nonnegative integer, b is a positive integer and r = a mod b, then

$$\gcd(a,b) = \gcd(b,r)$$

pf:

MTH 354

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Pf:

Use Euclidean algo to find GCD(190, 34)

① $190 = 34(5) + 20$
② $34 = 20(1) + 14$ ✓
③ $20 = 14(1) + 6$ ✓
④ $14 = 6(2) + 2$ ✓
⑤ $6 = 2(3) + 0$

GCD(190, 34) = 2

Find GCD(128, 77)

$128 = 77(1) + 51$
$77 = 51(1) + 26$
$51 = 26(1) + 25$
$26 = 25(1) + 1$
$25 = 1(25) + 0$

GCD(128, 77) = 1

Relatively prime

Find GCD(190, 34) = 2 $= 14 - 2(6)$ by ④
$= 14 - 2[20 - 1(14)]$ by ③
$= 14 - 2(20) + 2(14)$
$= 3(14) - 2(20)$
$= 3[34 - 1(20)] - 2(20)$ by ②
$= 3(34) - 3(20) - 2(20)$
$= 3(34) - 5(20)$
$= 3(34) - 5[190 - 5(34)]$ by ①
$= 3(34) - 5(190) + 25(34)$
$= 28(34) - 5(190)$