Solving Recurrence Relations Part1 **Worksheet 5**

Name: **Date:**

Consider the following recurrence relation: $A_n = 3A_{n-1} + 5$, $A_1 = 2$

1. Find A_1, A_2, A_3, A_4, and A_5.

$A_1 = 2$

$A_2 = 11$

$A_3 = 38$

$A_4 = 119$

$A_5 = 362$

2. Using backtracking (iteration) solve the given recurrence relation.

$A_n = 3A_{n-1} + 5$ $\qquad A_{n-1} = 3A_{n-2} + 5$

$= 3(3A_{n-2} + 5) + 5$

$= 3^2 A_{n-2} + 3(5) + 5 \qquad A_{n-2} = 3A_{n-3} + 5$

$= 3^2(3A_{n-3} + 5) + 3(5) + 5$

$= 3^{3} A_{n-3} + 3^2(5) + 3(5) + 1(5)$

$\vdots$

$= 3^{n-1} A_{\underbrace{n-(n-1)}_{1}} + 3^{n-2}(5) + 3^{n-3}(5) + \cdots + 3^2(5) + 3(5) + 1(5)$

$= 3^{n-1} A_1 + 3^{n-2}(5) + \cdots + 1(5)$

$= 3^{n-1}(2) + 5(3^{n-2} + 3^{n-3} + \cdots + 3^2 + 3^1 + 1)$

$= 3^{n-1}(2) + 5\left(\frac{3^{n-1} - 1}{\underbrace{3-1}_{=2}}\right)$

$= 3^{n-1}(2) + \frac{5}{2}(3^{n-1} - 1) = 3^{n-1}(2) + 3^{n-1}(5/2) - \frac{5}{2}$

$= 3^{n-1}(2 + 2.5) - 2.5 = \frac{9}{2}\, 3^{n-1} - \frac{5}{2}$

3. Find A_1, A_2, A_3, A_4, and A_5.

$A_1 = 2$

$A_2 = 11$

$A_3 = 38$

$A_4 = 119$

$A_5 = 362$

4. Using induction show that the formulas produce the same sequence.

show $A_n = 3A_{n-1} + 5$ and $A_n = \frac{9}{2} 3^{n-1} - \frac{5}{2}$ produce the same sequence of values.

Pf (induction)

BC: Done by above.

IC: Assume $A_k = \frac{9}{2} 3^{k-1} - \frac{5}{2}$ for some $k \geq 1$.

We need to show $\boxed{A_{k+1} = \frac{9}{2} 3^{k} - \frac{5}{2}}$ is also true.

We note that
$$A_{k+1} \overset{RR}{=} 3A_k + 5$$
$$= 3\left(\frac{9}{2} 3^{k-1} - \frac{5}{2}\right) + 5 \quad \text{by I.H}$$
$$= \frac{9}{2} 3^{k} - \frac{15}{2} + \frac{10}{2}$$
$$\boxed{A_{k+1} = \frac{9}{2} 3^{k} - \frac{5}{2}}$$