MTH 354 10-17-17

§7.2

#19) Find a closed form $a_n = a_{n-1} + 1 + 2^{n-1}$ $a_0 = 0$

<u>sol:</u> $a_n = a_{n-1} + 1 + 2^{n-1}$ $a_{n-1} = a_{n-2} + 1 + 2^{n-2}$

$= (a_{n-2} + 1 + 2^{n-2}) + 1 + 2^{n-1}$

$= a_{n-2} + 2 + 2^{n-2} + 2^{n-1}$ $a_{n-2} = a_{n-3} + 1 + 2^{n-3}$

$= (a_{n-3} + 1 + 2^{n-3}) + 2 + 2^{n-2} + 2^{n-1}$

$= a_{n-3} + 3 + 2^{n-3} + 2^{n-2} + 2^{n-1}$

$\vdots$

$= a_{n-10} + 10 + 2^{n-10} + 2^{n-9} + \dots + 2^{n-1}$

$\vdots$

$= a_{n-n} + n + \underbrace{2^0 + 2^1 + 2^2 + \dots + 2^{n-1}}_{= \frac{2^n - 1}{2-1}}$ $1 + r + \dots + r^n = \frac{r^{n+1} - 1}{r - 1}$

$= \cancel{a_0}^{0} + n + 2^n - 1$

$a_n = n + 2^n - 1$

| <u>RR</u> | <u>closed form</u> |
|---|---|
| $a_0 = 0$ | $a_0 = 0 + 2^0 - 1 = 0$ |
| $a_1 = a_0 + 1 + 2^0 = 2$ | $a_1 = 1 + 2^1 - 1 = 2$ |
| $a_2 = a_1 + 1 + 2^1 = 5$ | $a_2 = 2 + 2^2 - 1 = 5$ |
| $a_3 = a_2 + 1 + 2^2 = 10$ | $a_3 = 3 + 2^3 - 1 = 10$ |
| $a_4 = a_3 + 1 + 2^3 = 19$ | $a_4 = 4 + 2^4 - 1 = 19$ |

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$= a_{n-2} + 2 + 2^{n-2} + 2^{n-1}$ $a_{n-2} = a_{n-3} + 1 + 2^{n-3}$

$= (a_{n-3} + 1 + 2^{n-3}) + 2 + 2^{n-2} + 2^{n-1}$

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$= \cancel{a_0}^{\,0} + n + 2^n - 1$

$a_n = n + 2^n - 1$

<u>RR</u>

$a_0 = 0$

$a_1 = a_0 + 1 + 2^0 = 2$

$a_2 = a_1 + 1 + 2^1 = 5$

$a_3 = a_2 + 1 + 2^2 = 10$

$a_4 = a_3 + 1 + 2^3 = 19$

<u>closed form</u>

$a_0 = 0 + 2^0 - 1 = 0$

$a_1 = 1 + 2^1 - 1 = 2$

$a_2 = 2 + 2^2 - 1 = 5$

$a_3 = 3 + 2^3 - 1 = 10$

$a_4 = 4 + 2^4 - 1 = 19$

Selection Sort

Goal: Find a $\Theta$ notation for the time required for the code to run.

Define $b_n :=$ The number of comparisons at line 6 to sort $n$ items.

Initial condition. $n=1$. $b_1 = 0$.

For an arbitrary sequence of length bigger than 1, count the number of comparisons at each line and add them to find the total number of comparisons $b_n$.

lines 1–5 has 0 comparisons.

line 6 has $n-1$ comparisons (line 5 causes line 6 to execute $n-1$ times.)

lines 7 and 8 have 0 comparisons.

line 9 has $b_{n-1}$ comparisons (by definition) recursive call

~~Hence~~ $b_n = (n-1) + b_{n-1}$, $b_1 = 0$

solve this.

$$b_n = b_{n-1} + n - 1$$

$$= b_{n-2} + (n-2) + (n-1) \qquad b_{n-1} = b_{n-2} + (n-1) - 1$$

$$= b_{n-3} + (n-3) + (n-2) + (n-1) \qquad b_{n-2} = b_{n-3} + (n-2) - 1$$

$$\vdots$$

$$= b_{n-10} + (n-10) + (n-9) + \cdots + (n-1)$$

$$\vdots$$

$$= \boxed{b_{n-(n-1)}} + 1 + 2 + 3 + \cdots + (n-1) \qquad \rightarrow b_1 = 0$$

$$= 1 + 2 + 3 + \cdots + (n-1) = \frac{(n-1)(n)}{2} = \frac{1}{2}(n^2 - n)$$

$$\Theta(n^2)$$

Example of binary search

Input: $s_1 = $ 'C', $s_2 = $ 'G', $s_3 = $ 'J', $s_4 = $ 'M', key = 'J' $i=1, j=4$

At line 2 $i > j$ is false, go to line 4, where $k = 2$

At line 5, since key ('J') $\neq s_2$ ('G'), proceed to line 7

At line 7, key $< s_2$ ('J' < 'G'), is false, so set

$i = 3$ at line 10.

call the algorithm with $i=3, j=4$

At line 2, $i > j$ is false, go to line 4 where $k := 3$

At line 5, since key ('J') $= s_3$ ('J'), return 3.