

MTH 354

10-19-17

§7.2

$$\#15) a_n = 6a_{n-1} - 8a_{n-2}, \quad a_0 = 1, a_1 = 0$$

closed form $a_n = 2(2^n) - 4^n = 2^{n+1} - 4^n$

pf (induction)

BC ✓

IC Assume $a_r = 2^{r+1} - 4^r$ for all r , $1 \leq r \leq k$ for some $k \geq 1$
 Need to show $a_{k+1} = 2^{k+2} - 4^{k+1}$.

$$\begin{aligned} \text{Note that } a_{k+1} &\stackrel{IC}{=} 6a_k - 8a_{k-1} \\ &= 6(2^{k+1} - 4^k) - 8(2^k - 4^{k-1}) \\ &= \frac{6}{2} 2^{k+2} - \frac{6}{4} 4^{k+1} - \frac{8}{2^2} 2^{k+2} + \frac{8}{4^2} 4^{k+1} \\ &= 2^{k+2} (3 - 2) - 4^{k+1} \left(\frac{3}{2} - \frac{1}{2} \right) \\ &= 2^{k+2} \underbrace{(3-2)}_{=1} - 4^{k+1} \underbrace{\left(\frac{3}{2} - \frac{1}{2} \right)}_{=1} \\ a_{k+1} &= 2^{k+2} - 4^{k+1} \end{aligned}$$

Therefore, by the PMI, the two formulas produce the same sequence of values. ■

§ 7.2

#15) $a_n = 6a_{n-1} - 8a_{n-2}$, $a_0 = 1, a_1 = 0$

closed $a_n = 2(2^n) - 4^n = 2^{n+1} - 4^n$

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Pf (induction)

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IC: Assume $a_r = 2^{r+1} - \frac{r}{4}$ for all r , $1 \leq r \leq k$ for some $k \geq 1$

Need to show $a_{k+1} = 2^{k+2} - 4$.

note that $a_{k+1} \stackrel{IR}{=} 6a_k - 8a_{k-1}$

$$= 6 \left(2^k - 4^k \right) - 8 \left(2^{k-1} - 4^{k-1} \right)$$

$$= \frac{6}{2} 2^{k+1} - \frac{6}{4} 4^{k+1} - \frac{8}{2^2} 2^{k+2} + \frac{8}{4^2} 4^{k+2}$$

$$= 2^{k+2} \underbrace{(3-2)}_{=1} - 4^{k+1} \underbrace{\left(\frac{3}{2} - \frac{1}{2} \right)}_{=1}$$

$$a_{k+1} = 2^{k+2} - 4^{k+1}$$

Therefore, by the PMI, the two formulas produce the same sequence of values. ■