Tax-sheltered Annuity **Worksheet 3**

Name: **Date:**

In a tax-sheltered annuity, the money invested, as well as the interest earned, is not subject to taxation until withdrawn from the account. In the following, assume that a person invests $~~2~~3000 each year in a tax-sheltered annuity at 10 percent interest compounded annually. Let A_n represent the amount at the end of *n* years

1. Find a recurrence relation for the sequence $A_0, A_1, \ldots$

$$A_0 = \$2000$$
$$A_1 = (2000 + 2000(.10)) + 2000 = \overset{A_0}{2000}(1.10) + 2000$$
$$A_2 = A_1(1.\cancel{0}0) + 2000$$
$$A_3 = A_2(1.10) + 2000$$
$$\vdots$$
$$A_n = A_{n-1}(1.10) + 2000$$

2. Find an initial condition for the sequence $A_0, A_1, \ldots$

$$A_0 = \$2000$$

3. Find A_1, A_2, A_3

$$A_1 = 4200, \quad A_2 = 6620, \quad A_3 = 9282$$

$$A_{n-1} = A_{n-2}(1.1) + 2000$$

$$A_{n-2} = A_{n-3}(1.1) + 2000$$

4. Find an explicit Formula for A_n.

$$A_n = A_{n-1}(1.10) + 2000$$

$$= (1.1)((1.1)A_{n-2} + 2000) + 2000$$

$$= (1.1)^2 A_{n-2} + (1.1)2000 + 2000$$

$$= (1.1)^2((1.1)A_{n-3} + 2000) + (1.1)2000 + 2000$$

$$= (1.1)^{3} A_{n-3} + (1.1)^2 2000 + (1.1)2000 + 2000$$

$$\vdots$$

$$= (1.1)^n \underbrace{\boxed{A_{n-n}}}_{A_0 = 2000} + (1.1)^{n-1} 2000 + (1.1)^{n-2} 2000 + \cdots + (1.1)2000 + 2000$$

$$= 2000 \underbrace{\left((1.1)^n + (1.1)^{n-1} + (1.1)^{n-2} + \cdots + (1.1) + 1\right)}$$

$$= 2000 \left(\frac{1.1^{n+1} - 1}{\underbrace{1.1 - 1}_{.1}}\right)$$

$$= 20{,}000 \left(1.1^{n+1} - 1\right) \leftarrow \text{explicit formula}$$