The problems for the 3rd set of HWK are §7.2: #9, 11–14. I have also asked them to include an induction proof to show their two equations produce the same sequence of values.

please grade #13 for 6 pts; 3 pts for the derivation and 3 pts for the proof. the rest may have 1 pt each without grading. Total = 10. Thank you.

$$a_n = a_{n-1} + n \qquad a_0 = 0$$
$$= (a_{n-2} + (n-1)) + n$$
$$= a_{n-2} + (n-1) + n$$
$$= (a_{n-3} + (n-2)) + (n-1) + n$$
$$= a_{n-3} + (n-2) + (n-1) + n$$
$$\vdots$$
$$= \underbrace{a_{n-n}}_{=0} + 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$
$$= 1 + 2 + \cdots + n = \frac{n(n+1)}{2} = a_n$$

To show that $a_n = a_{n-1} + n$ and $a_n = \frac{n(n+1)}{2}$ produce the same values note that

pf (induction)

Base Case: $a_1 = 0 + 1 \overset{?}{=} \frac{1(1+1)}{2} = 1$ ✓ or $a_0 = 0 = \frac{0(1)}{2} = 0$ ✓

Induction Case: Assume $a_k = \frac{k(k+1)}{2}$ for some $k \geq 1$.
we need to show that $a_{k+1} = \frac{(k+1)(k+2)}{2}$.

Note that $a_{k+1} = a_k + (k+1)$ by the recurrence relation

$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$ by the induction hypothesis

$= \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$.

Principle of Mathematical Induction

Therefore, by the PMI, the two equations produce the same sequence.