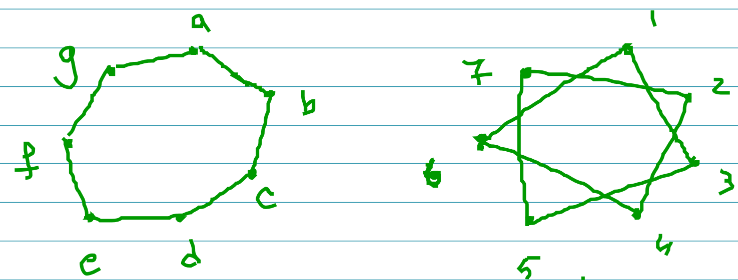
MTH 344 11/7/17

§8.6 Isomorphisms of Graphs.

Def: Graphs G_1 & G_2 are isomorphic if $\exists$ bijections ($f: V_{G_1} \to V_{G_2}$ s.t f is 1-1 (injective) and onto (surjective))

$f: V_{G_1} \to V_{G_2}$ & $g: E_{G_1} \to E_{G_2}$ s.t (such that) an edge e is incident on v & w in G_1 if and only if the edge $g(e)$ is incident on $f(v)$ and $f(w)$ in G_2.
The pair of functions f & g is called an isomorphism of G_1 onto G_2.

#1) Prove that the graphs G_1 & G_2 are isomorphic



G_1

$a \mapsto 1$	$a \mapsto 1$
$b \mapsto 3$	$b \mapsto 3$
$g \mapsto 6$	$c \mapsto 5$
$c \mapsto 5$	$d \mapsto 7$
$d \mapsto 7$	$e \mapsto 2$
$e \mapsto 2$	$f \mapsto 4$
$f \mapsto 4$	$g \mapsto 6$

Thm 8.6.4 Graphs G_1 and G_2 are isomorphic $\Leftrightarrow$ for some ordering of their vertices, their adjacency matrices are equal.

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Def: Graphs G_1 & G_2 are isomorphic if $\exists$ bijections ($f: V_{G_1} \longrightarrow V_{G_2}$ s.t f is 1-1 (injective) and onto (surjective))

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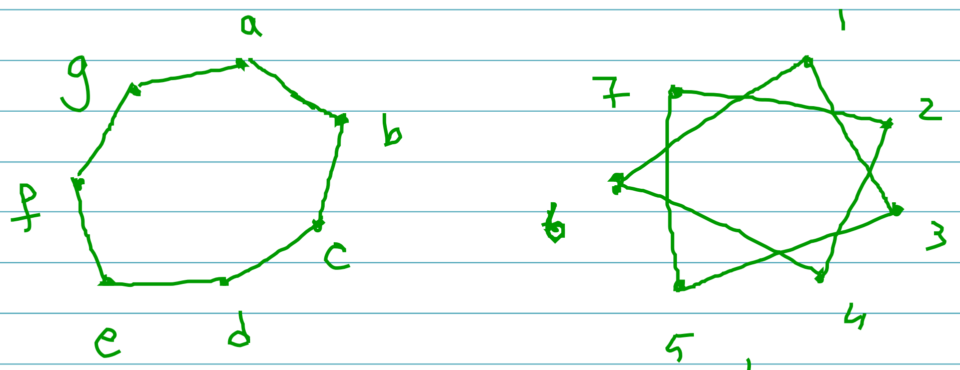
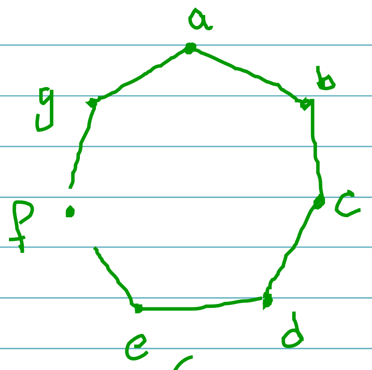
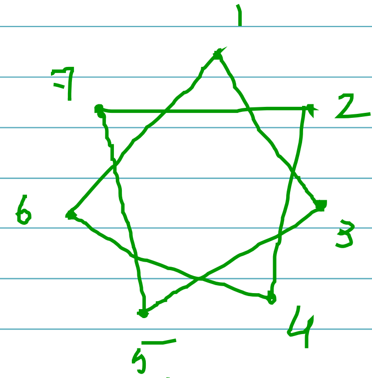


G_1

$a \longmapsto 1$	$a \longmapsto 1$
$b \longmapsto 3$	$b \longmapsto 3$
$g \longmapsto 6$	$c \longmapsto 5$
$c \longmapsto 5$	$d \longmapsto 7$
$d \longmapsto 7$	$e \longmapsto 2$
$e \longmapsto 2$	$f \longmapsto 4$
$f \longmapsto 4$	$g \longmapsto 6$

Thm 8.6.4: Graphs G_1 and G_2 are isomorphic $\Leftrightarrow$ for some ordering of their vertices, their adjacency matrices are equal.





$a \mapsto 1$
$b \mapsto 3$
$c \mapsto 5$
$d \mapsto 7$
$e \mapsto 2$
$f \mapsto 4$
$g \mapsto 6$

	a	b	c	d	e	f	g
a	0	1	0	0	0	0	1
b	1	0	1	0	0	0	0
c	0	1	0	1	0	0	0
d	0	0	1	0	1	0	0
e	0	0	0	1	0	1	0
f	0	0	0	0	1	0	1
g	1	0	0	0	0	1	0

	1	3	5	7	2	4	6
1	0	1	0	0	0	0	1
3	1	0	1	0	0	0	0
5	0	1	0	1	0	0	0
7	0	0	1	0	1	0	0
2	0	0	0	1	0	1	0
4	0	0	0	0	1	0	1
6	1	0	0	0	0	1	0

Corollary 8.6.5. Let G_1 & G_2 be simple graphs. The following are equivalent.

vertex set of G_1 vertex set of G_2

a) G_1 & G_2 are isomorphic.

b) $\exists$ a bijection $f : V_{G_1} \to V_{G_2}$ satisfying the following.

There exists

Vertices v & w are adjacent in $G_1 \Leftrightarrow$ the vertices $f(v)$ and $f(w)$ are adjacent in G_2.

Note: To show that 2 simple graphs G_1 & G_2 are not isomorphic, find a property of G_1 that G_2 does not have, but that G_2 would have if G_1 and G_2 were isomorphic.

Such a property is called <u>invariant</u>.

<u>Def</u>: A property P is an invariant if whenever G_1 & G are isomorphic graphs:

If G_1 has property P, G_2 has property P.