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§7.2

#15) $a_n = 6a_{n-1} - 8a_{n-2}$, $a_0 = 1$, $a_1 = 0$

closed form $a_n = 2(2^n) - 4^n = 2^{n+1} - 4^n$

Pf (induction)

BC ✓

IC: Assume $a_r = 2^{r+1} - 4^r$ for all r, $1 \leq r \leq k$ for some $k \geq 1$

Need to show $a_{k+1} = 2^{k+2} - 4^{k+1}$.

Note that $a_{k+1} \overset{RR}{=} 6a_k - 8a_{k-1}$

$$= 6(2^{k+1} - 4^k) - 8(2^k - 4^{k-1})$$

$$= \frac{6}{2} 2^{k+2} - \frac{6}{4} 4^{k+1} - \frac{8}{2^2} 2^{k+2} + \frac{8}{4^2} 4^{k+1}$$

$$= 2^{k+2}\underbrace{(3-2)}_{=1} - 4^{k+1}\underbrace{\left(\frac{3}{2} - \frac{1}{2}\right)}_{=1}$$

$$a_{k+1} = 2^{k+2} - 4^{k+1}$$

Therefore, by the PMI, the two formulas produce the same sequence of values. ∎

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