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§8.3

Goal: Creating a "ring" model for connecting processors in an n-cube to each other, i.e., find a Hamiltonian cycle for the graph of an n-cube.

Gray code

Theorem 8.3.6 Let G_1 denote the sequence 0, 1. We define G_n in terms of G_{n-1} by the following rules:

a) Let G^R_{n-1} denote the sequence G_{n-1} written in reverse.

b) Let G'_{n-1} denote the sequence obtained by prefixing each member of G_{n-1} with 0.

c) Let G''_{n-1} denote the sequence obtained by prefixing each member of G^R_{n-1} with 1.

d) Let G_n be the sequence consisting of G'_{n-1} followed by G''_{n-1}.

Then G_n is a Gray code for every positive integer n.

Def. A sequence is called a Gray code, if it has the form $s_1, s_2, \ldots, s_{2^n}$ where each s_i is a string of n bits satisfying

- Every n-bit string appears somewhere in the sequence
- s_i and s_{i+1} differ in exactly one bit. $i=1,\ldots,2^n-1$.
- s_{2^n} and s_1 differ in exactly 1 bit.

Note: when $n \geq 2$, a Gray code corresponds to the HC $s_1, s_2, \ldots, s_{2^n}, s_1$, since every vertex appears and the edges (s_i, s_{i+1}), $i=1,\ldots,2^n-1$ and (s_{2^n}, s_1) are distinct.

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§8.3

Goal: Creating a "ring" model for connecting processors in an n-cube to each other, i.e., find a Hamiltonian cycle for the graph of an n-cube.

Gray code

Theorem 8.3.6 Let G_1 denote the sequence 0, 1. We define G_n in terms of G_{n-1} by the following rules:

a) let G_{n-1}^R denote the sequence G_{n-1} written in reverse.

b) let G_{n-1}' denote the sequence obtained by prefixing each member of G_{n-1} with 0.

c) let G_{n-1}'' denote the sequence obtained by prefixing each member of G_{n-1}^R with 1.

d) let G_n be the sequence consisting of G_{n-1}' followed by G_{n-1}''.

Then G_n is a Gray code for every positive integer n.

Def. A sequence is called a Gray code, if it has the form $s_1, s_2, \ldots, s_{2^n}$ where each s_i is a string of n bits satisfying

- Every n-bit string appears somewhere in the sequence
- s_i and s_{i+1} differ in exactly one bit, $i = 1, \ldots, 2^n - 1$.
- s_{2^n} and s_1 differ in exactly 1 bit.

Note: when $n \geq 2$, a Gray code corresponds to the HC $s_1, s_2, \ldots, s_{2^n}, s_1$, since every vertex appears and the edges (s_i, s_{i+1}), $i = 1, \ldots, 2^n - 1$ and (s_{2^n}, s_1) are distinct.

G_1: 0 1

G_1^R: 1 0

G_1': 00 01

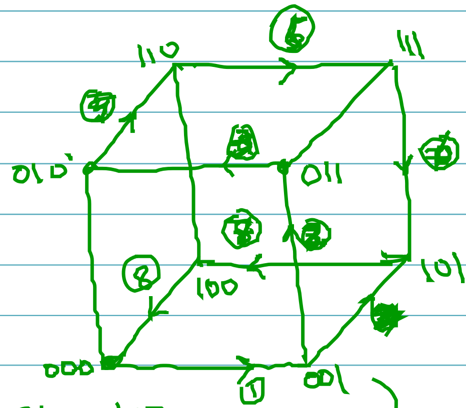
G_1'': 11 10

G_2: 00 01 11 10

G_2^R: 10 11 01 00

G_2': 000 001 011 010

G_2'': 110 111 101 100

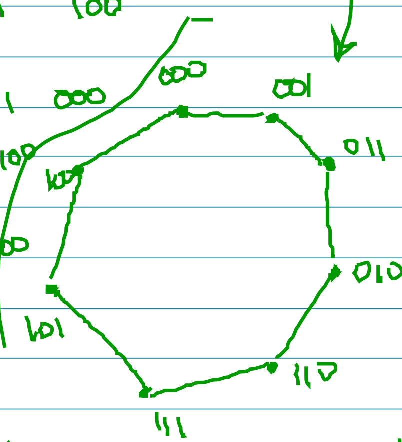


G_3: 000 001 011 010 110 111 101 100

G_3^R: 100 101 111 110 010 011 001 000

G_3': 0000 0001 0011 0010 0110 0111 0101 0100

G_3'': 1100 1101 1111 1110 1010 1011 1001 1000



#19a) G_4

0000 0001 0011 0010 0110 0111 0101 0100 1100 1101 1111 1110 1010 1011 1001 1000

§ 8.4 A shortest-path algorithm