

Using induction, show that  $b_n = 4b_{n-1} + 5$   
and  $b_n = \left(\frac{11}{3}\right) 4^{n-1} - \frac{5}{3}$  produce the same sequence.

PF (induction)

$$\text{BC: } b_1 = \frac{11}{3} 4^{1-1} - \frac{5}{3} = \frac{11}{3} 4^0 - \frac{5}{3} = \frac{11}{3} - \frac{5}{3} = \frac{6}{3} = 2 = b_1 \quad \checkmark$$

IC: Assume  $b_k = \left(\frac{11}{3}\right) 4^{k-1} - \frac{5}{3}$  for some  $k \geq 1$ .

We need to show  $b_{k+1} = \left(\frac{11}{3}\right) 4^k - \frac{5}{3}$ .

$$\begin{aligned} \text{We note that } b_{k+1} &= 4b_k + 5 && \text{by RR} \\ &= 4\left(\frac{11}{3} 4^{k-1} - \frac{5}{3}\right) + 5 && \text{by IH} \\ &= \left(\frac{11}{3}\right) 4^k - 4\left(\frac{5}{3}\right) + 5 \\ &= \left(\frac{11}{3}\right) 4^k - \frac{20}{3} + \frac{15}{3} \\ &= \left(\frac{11}{3}\right) 4^k - \frac{5}{3} \end{aligned}$$

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We need to show  $b_{k+1} = \left(\frac{11}{3}\right) 4^k - \frac{5}{3}$ .

We note that  $b_{k+1} = 4b_k + 5$  by RR

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