#15 in section 5.3 **Worksheet 4**

Name: **Date:**

To prove #15, first try the following example.

Let a = 198 and b = 462. Let CD(a,b) represent the set of the common divisors of a and b. Now find the set of common divisors of a and a+b; CD(a, a+b).

CD(a,b) = {2, 3, 11, 66, 22, 33, 6}

CD=(a, a+b) = {2, 3, 6, 11, 22, 33, 66}.

198, 660

Are they the same? What is the GCD(a,b)? What is the GCD(a, a+b)

GCD(a,b) = 66 GCD(a, a+b) = 66

#15) If a and b are positive integers, show that gcd(a,b) = gcd(a,a+b).

Proof: Let a and b be positive integers.

To show that gcd(a,b) = gcd(a,a+b), we will show that the CD(a,b) $\subseteq$ = $\supseteq$ CD(a, a+b).

($\subseteq$) Let c be an element of CD(a,b). Then $c \mid a$ and $c \mid b$. To show $c \in CD(a, a+b)$, we need to have $c \mid a$ and $c \mid a+b$. Since $c \mid a$ and $c \mid b$, $c \mid a+b$ by 5.1.3(a). Therefore $c \in CD(a, a+b)$.

($\supseteq$) Let $d \in CD(a, a+b)$. Therefore, $d \mid a$ and $d \mid a+b$. To show $d \in CD(a,b)$, we need to show $d \mid a$ and $d \mid b$. Since $d \mid a+b$ and $d \mid a$, $d \mid (a+b) - a = b$ by #27 §5.1. Thus, $d \mid a$ and $d \mid b$. Therefore $d \in CD(a,b)$.

$\boxtimes$