GRE®

Graduate Record Examinations®

MATHEMATICS TEST (RESCALED)

PRACTICE BOOK

This practice book contains

- one actual full-length GRE Mathematics Test (Rescaled)

- test-taking strategies

Become familiar with

- test structure and content

- test instructions and answering procedures

Compare your practice test results with the performance of those who took the test at a GRE administration.

ETS
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This practice book and the interpretive information included in it are appropriate for individuals taking the Mathematics Test (Rescaled) after October 1, 2001. The GRE Mathematics Test will be rescaled effective October 2001 and renamed "Mathematics Test (Rescaled)"; **the content of the test will not change**. Although the range of scores for the Mathematics Test (Rescaled) will continue to extend from 200 to 990, **scores earned on the test after October 2001 should not be compared to scores earned earlier.**

The primary reason for rescaling the Mathematics Test is to make the test scores more useful for admissions committees and other score users by spreading out high-ability examinees on the score scale. The scores of the Mathematics Test population have increased substantially since the test was first scaled in 1952, and an increasingly large percentage of individuals each year have been earning 990, the highest possible score on the scale. Based on recent interpretive data (individuals who were tested between October 1996 and October 1999), 18 percent of the examinees received 990 on the test (in other words, the percentage of test takers scoring below 990 was 82). The scale of the new Mathematics Test (Rescaled) will enable admissions committees and other score users to distinguish among high-scoring examinees, something the former scale was no longer able to do.

Note to Test Takers:

Keep this practice book until you receive your score report.

This book contains important information about content specifications and scoring.

Table of Contents

Purpose of the GRE Subject Tests

The GRE Subject Tests are designed to help graduate school admission committees and fellowship sponsors assess the qualifications of applicants in specific fields of study. The tests also provide you with an assessment of your own qualifications.

Scores on the tests are intended to indicate knowledge of the subject matter emphasized in many undergraduate programs as preparation for graduate study. Because past achievement is usually a good indicator of future performance, the scores are helpful in predicting success in graduate study. Because the tests are standardized, the test scores permit comparison of students from different institutions with different undergraduate programs. For some Subject Tests, subscores are provided in addition to the total score; these subscores indicate the strengths and weaknesses of your preparation, and they may help you plan future studies.

The GRE Board recommends that scores on the Subject Tests be considered in conjunction with other relevant information about applicants. Because numerous factors influence success in graduate school, reliance on a single measure to predict success is not advisable. Other indicators of competence typically include undergraduate transcripts showing courses taken and grades earned, letters of recommendation, the GRE Writing Assessment score, and GRE General Test scores. For information about the appropriate use of GRE scores, write to GRE Program, Educational Testing Service, Mail Stop 57-L, Princeton, NJ 08541, or visit our Web site at www.gre.org/codelst.html.

Development of the Subject Tests

Each new edition of a Subject Test is developed by a committee of examiners composed of professors in the subject who are on undergraduate and graduate faculties in different types of institutions and in different regions of the United States and Canada. In selecting members for each committee, the GRE Program seeks the advice of the appropriate professional associations in the subject.

The content and scope of each test are specified and reviewed periodically by the committee of examiners. Test questions are written by the committee and by other faculty who are also subject-matter specialists and by subject-matter specialists at ETS. All questions proposed for the test are reviewed by the committee and revised as necessary. The accepted questions are assembled into a test in accordance with the content specifications developed by the committee to ensure adequate coverage of the various aspects of the field and, at the same time, to prevent overemphasis on any single topic. The entire test is then reviewed and approved by the committee.

Subject-matter and measurement specialists on the ETS staff assist the committee, providing information and advice about methods of test construction and helping to prepare the questions and assemble the test. In addition, each test question is reviewed to eliminate language, symbols, or content considered potentially offensive, inappropriate for major subgroups of the test-taking population, or likely to perpetuate any negative attitude that may be conveyed to these subgroups. The test as a whole is also reviewed to ensure that the test questions, where applicable, include an appropriate balance of people in different groups and different roles.

Because of the diversity of undergraduate curricula, it is not possible for a single test to cover all the material you may have studied. The examiners, therefore, select questions that test the basic knowledge and skills most important for successful graduate study in the particular field. The committee keeps the test up-to-date by regularly developing new editions and revising existing editions. In this way, the test content changes steadily but gradually, much like most curricula. In addition, curriculum surveys are conducted periodically to ensure that the content of a test reflects what is currently being taught in the undergraduate curriculum.

After a new edition of a Subject Test is first administered, examinees' responses to each test question are analyzed in a variety of ways to determine whether each question functioned as expected. These analyses may reveal that a question is ambiguous, requires knowledge beyond the scope of the test, or is inappropriate for the total group or a particular subgroup of examinees taking the test. Answers to such questions are not used in computing scores.

Following this analysis, the new test edition is equated to an existing test edition. In the equating process, statistical methods are used to assess the difficulty of the new test. Then scores are adjusted so that examinees who took a difficult edition of the test are not penalized, and examinees who took an easier edition of the test do not have an advantage. Variations in the number of questions in the different editions of the test are also taken into account in this process.

Scores on the Subject Tests are reported as three-digit scaled scores with the third digit always zero. The maximum possible range for all Subject Test total scores is from 200 to 990. The actual range of scores for a particular Subject Test, however, may be smaller. The maximum possible range of Subject Test subscores is 20 to 99; however, the actual range of subscores for any test or test edition may be smaller. Subject Test score interpretive information is provided in *Interpreting Your GRE Scores*, which you will receive with your GRE score report, and on the GRE Web site at www.gre.org/codelst.html.

Content of the Mathematics Test (Rescaled)

The test consists of 66 multiple-choice questions, drawn from courses commonly offered at the undergraduate level. Although the Mathematics Test has been rescaled, the content of the test has not changed. Approximately 50 percent of the questions involve calculus and its applications—subject matter that can be assumed to be common to the backgrounds of almost all mathematics majors. About 25 percent of the questions in the test are in elementary algebra, linear algebra, abstract algebra, and number theory. The remaining questions deal with other areas of mathematics currently studied by undergraduates in many institutions.

The following content descriptions may assist students in preparing for the test. The percentages given are estimates; actual percentages will vary somewhat from one edition of the test to another.

Calculus — 50%

Material learned in the usual sequence of elementary calculus courses—differential and integral calculus of one and of several variables—including calculus-based applications and connections with coordinate geometry, trigonometry, differential equations, and other branches of mathematics

Algebra — 25%

Elementary algebra: basic algebraic techniques and manipulations acquired in high school and used throughout mathematics

Linear algebra: matrix algebra, systems of linear equations, vector spaces, linear transformations, characteristic polynomials, eigenvalues and eigenvectors

Abstract algebra and number theory: elementary topics from group theory, the theory of rings and modules, field theory, and number theory

Additional Topics — 25%

Introductory real analysis: sequences and series of numbers and functions, continuity, differentiability and integrability, elementary topology of $\mathbb{R}$ and $\mathbb{R}^n$

Discrete mathematics: logic, set theory, combinatorics, graph theory, and algorithms

Other topics: general topology, geometry, complex variables, probability and statistics, and numerical analysis

The above descriptions of topics covered in the test should not be considered exhaustive; it is necessary to understand many other related concepts. Prospective test takers should be aware that questions requiring no more than a good precalculus background may be quite challenging; some of these questions turn out to be among the most difficult questions on the test. In general, the questions are intended not only to test recall of information, but also to assess the test taker's understanding of fundamental concepts and the ability to apply these concepts in various situations.

Preparing for a Subject Test

GRE Subject Test questions are designed to measure skills and knowledge gained over a long period of time. Although you might increase your scores to some extent through preparation a few weeks or months before you take the test, last minute cramming is unlikely to be of further help. The following information may be helpful.

- A general review of your college courses is probably the best preparation for the test. However, the test covers a broad range of subject matter, and no one is expected to be familiar with the content of every question.

- Use this practice book to become familiar with the types of questions in the GRE Mathematics Test (Rescaled), paying special attention to the directions. If you thoroughly understand the directions before you take the test, you will have more time during the test to focus on the questions themselves.

Test-Taking Strategies

The questions in the practice test in this book illustrate the types of multiple-choice questions in the test. When you take the test, you will mark your answers on a separate machine-scorable answer sheet. Total testing time is two hours and fifty minutes; there are no separately timed sections. Following are some general test-taking strategies you may want to consider.

- Read the test directions carefully, and work as rapidly as you can without being careless. For each question, choose the best answer from the available options.

- All questions are of equal value; do not waste time pondering individual questions you find extremely difficult or unfamiliar.

- You may want to work through the test quite rapidly, first answering only the questions about which you feel confident, then going back and answering questions that require more thought, and concluding with the most difficult questions if there is time.

- If you decide to change an answer, make sure you completely erase it and fill in the oval corresponding to your desired answer.

- Questions for which you mark no answer or more than one answer are not counted in scoring.

- As a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly is subtracted from the number of questions you answer correctly. It is improbable that mere guessing will improve your score significantly; it may even lower your score. If, however, you are not certain of the correct answer but have some knowledge of the question and are able to eliminate one or more of the answer choices, your chance of getting the right answer is improved, and it may be to your advantage to answer the question.

- Record all answers on your answer sheet. Answers recorded in your test book will not be counted.

- Do not wait until the last five minutes of a testing session to record answers on your answer sheet.

What Your Scores Mean

Your raw score — that is, the number of questions you answered correctly minus one-fourth of the number you answered incorrectly — is converted to the scaled score that is reported. This conversion ensures that a scaled score reported for any edition of a Subject Test is comparable to the same scaled score earned on any other edition of the same test. Thus, equal scaled scores on a particular Subject Test indicate essentially equal levels of performance regardless of the test edition taken. Test scores should be compared only with other scores on the same Subject Test. (For example, a 680 on the Computer Science Test is not equivalent to a 680 on the Mathematics Test.)

Note that the Mathematics Test has been rescaled effective October 2001 and renamed "Mathematics Test (Rescaled)." Scores earned on the Mathematics Test (Rescaled) after October 2001 should not be compared to Mathematics Test scores earned before October 2001. Additional information about the rescaled test is available on the GRE Web site and will also be included in the score interpretive leaflet that will accompany score reports.

Before taking the test, you may find it useful to know approximately what raw scores would be required to obtain a certain scaled score. Several factors influence the conversion of your raw score to your scaled score, such as the difficulty of the test edition and the number of test questions included in the computation of your raw score. Based on recent editions of the Mathematics Test, the following table gives the range of raw scores associated with selected scaled scores for three different test editions that have been rescaled. (Note that when the number of scored questions for a given test is greater than the range of possible scaled scores, it is likely that two or more raw scores will convert to the same scaled score.) The

three test editions in the table that follows were selected to reflect varying degrees of difficulty. Examinees should note that future test editions may be somewhat more or less difficult than the test editions illustrated in the table.

Range of Raw Scores* Needed to Earn Selected Scaled Scores on Three Mathematics Test (Rescaled) Editions That Differ in Difficulty

Scaled Score	Raw Scores		
	Form A	Form B	Form C
900	66	62	60 – 61
800	55	51	49
700	43	40	38
600	30	29	28
Number of Questions Used to Compute Raw Score			
	66	66	65

*Raw Score = Number of correct answers minus one-fourth the number of incorrect answers, rounded to the nearest integer.

For a particular test edition, there are many ways to earn the same raw score. For example, on the edition listed above as "Form A," a raw score of 30 would earn a scaled score of 600. Below are a few of the possible ways in which a scaled score of 600 could be earned on that edition.

Examples of Ways to Earn a Scaled Score of 600 on the Edition Labeled As "Form A"

Raw Score	Questions Answered Correctly	Questions Answered Incorrectly	Questions Not Answered	Number of Questions Used to Compute Raw Score
30	30	0	36	66
30	33	14	19	66
30	37	27	2	66

MATHEMATICS TEST (RESCALED)
PRACTICE BOOK

PRACTICE TEST

To become familiar with how the administration will be conducted at the test center, first remove the answer sheet (pages 59 and 60). Then go to the back cover of the test book (page 54) and follow the instructions for completing the identification areas of the answer sheet. When you are ready to begin the test, note the time and begin marking your answers on the answer sheet.

THE GRADUATE RECORD
EXAMINATIONS®

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MATHEMATICS TEST (RESCALED)

ETS

*Do not break the seal
until you are told to do so.*

*The contents of this test are confidential.
Disclosure or reproduction of any portion
of it is prohibited.*

THIS TEST BOOK MUST NOT BE TAKEN FROM THE ROOM.

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms with an unspecified base are natural logarithms (that is, with base e).

(2) The set of all real numbers x such that $a \le x \le b$ is denoted by $[a, b]$.

(3) The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of integers, rational numbers, real numbers, and complex numbers, respectively.

---

1. If $F(x) = \int_{e}^{x} \log t \, dt$ for all positive x, then $F'(x) =$

(A) x

(B) $\dfrac{1}{x}$

(C) $\log x$

(D) $x \log x$

(E) $x \log x - 1$

---

2. If $F(1) = 2$ and $F(n) = F(n - 1) + \dfrac{1}{2}$ for all integers $n > 1$, then $F(101) =$

(A) 49
(B) 50
(C) 51
(D) 52
(E) 53

GO ON TO THE NEXT PAGE.

3. If $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ is invertible under matrix multiplication, then its inverse is
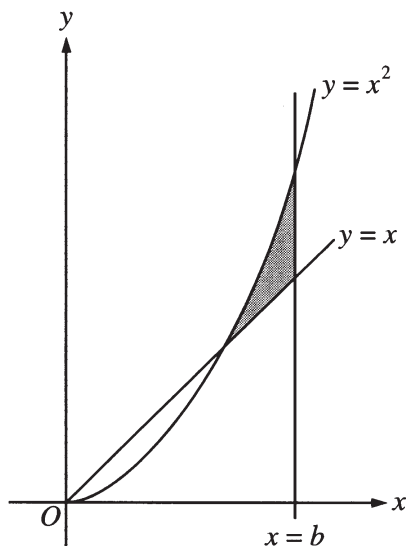
(A) $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(B) $\dfrac{1}{a^2 + b^2} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(C) $\dfrac{1}{a^2 + b^2} \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(D) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

(E) $\dfrac{1}{a^2 - b^2} \begin{pmatrix} -b & a \\ a & b \end{pmatrix}$

4. If $b > 0$ and if $\displaystyle\int_0^b x\, dx = \int_0^b x^2\, dx$, then the area of the shaded region in the figure above is
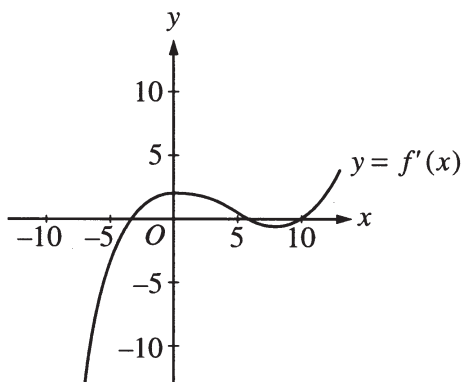
(A) $\dfrac{1}{12}$

(B) $\dfrac{1}{6}$

(C) $\dfrac{1}{4}$
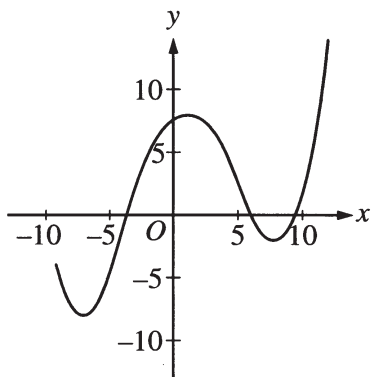
(D) $\dfrac{1}{3}$

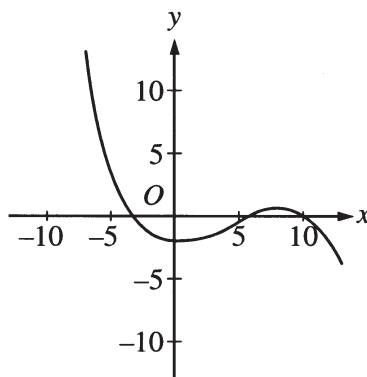(E) $\dfrac{1}{2}$

GO ON TO THE NEXT PAGE.

5. If the figure above is the graph of $y = f'(x)$, which of the following could be the graph of $y = f(x)$?

(A)

(D)

(B)
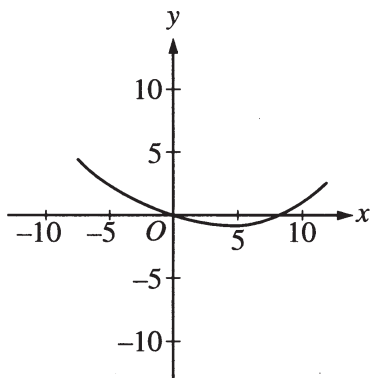
(E)
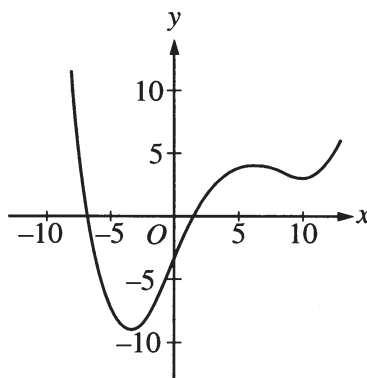
(C)
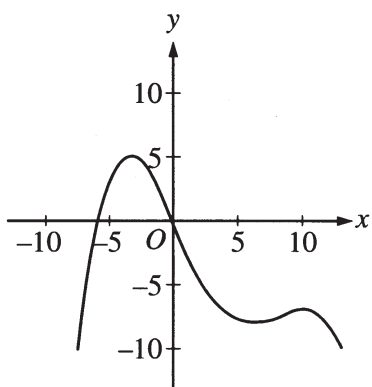
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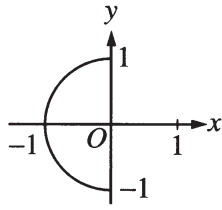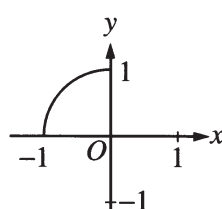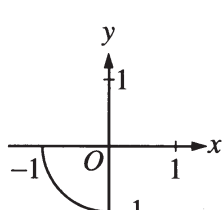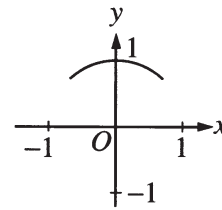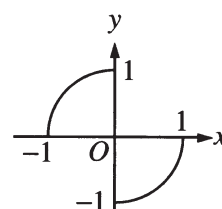
6. Consider the following sequence of instructions.

    1. Set $k = 999$, $i = 1$, and $p = 0$.
    2. If $k > i$, then go to step 3; otherwise go to step 5.
    3. Replace i with $2i$ and replace p with $p + 1$.
    4. Go to step 2.
    5. Print p.

If these instructions are followed, what number will be printed at step 5 ?

(A)   1
(B)   2
(C) 10
(D) 512
(E) 999

---

7. Which of the following indicates the graph of $\left\{(\sin t, \cos t): -\dfrac{\pi}{2} \le t \le 0\right\}$ in the xy-plane?

(A)

(D)

(B)

(E)

(C)

---

8. $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx =$

(A) 1

(B) $\dfrac{\pi}{4}$

(C) $\tan^{-1}\dfrac{\sqrt{2}}{2}$

(D) $\log 2$

(E) $\log \sqrt{2}$

---

9. If S is a nonempty finite set with k elements, then the number of one-to-one functions from S onto S is

(A) $k!$
(B) k^2
(C) k^k
(D) 2^k
(E) 2^{k+1}

---

10. Let g be the function defined on the set of all real numbers by

$$g(x) = \begin{cases} 1 & \text{if } x \text{ is rational,} \\ e^x & \text{if } x \text{ is irrational.} \end{cases}$$

Then the set of numbers at which g is continuous is

(A) the empty set
(B) $\{0\}$
(C) $\{1\}$
(D) the set of rational numbers
(E) the set of irrational numbers

---

11. For all real numbers x and y, the expression $\dfrac{x + y + |x - y|}{2}$ is equal to

(A) the maximum of x and y
(B) the minimum of x and y
(C) $|x + y|$
(D) the average of $|x|$ and $|y|$
(E) the average of $|x + y|$ and $x - y$

GO ON TO THE NEXT PAGE.

12. Let B be a nonempty bounded set of real numbers and let b be the least upper bound of B. If b is <u>not</u> a member of B, which of the following is necessarily true?

(A) B is closed.
(B) B is not open.
(C) b is a limit point of B.
(D) No sequence in B converges to b.
(E) There is an open interval containing b but containing no point of B.

---

13. A drawer contains 2 blue, 4 red, and 2 yellow socks. If 2 socks are to be randomly selected from the drawer, what is the probability that they will be the same color?

(A) $\dfrac{2}{7}$

(B) $\dfrac{2}{5}$

(C) $\dfrac{3}{7}$

(D) $\dfrac{1}{2}$

(E) $\dfrac{3}{5}$

---

14. Let $\mathbb{R}$ be the set of real numbers and let f and g be functions from $\mathbb{R}$ into $\mathbb{R}$. The negation of the statement

"For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$."

is which of the following?

(A) For each s in $\mathbb{R}$, there does not exist an r in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(B) For each s in $\mathbb{R}$, there exists an r in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(C) There exists an s in $\mathbb{R}$ such that for each r in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.
(D) There exists an s in $\mathbb{R}$ and there exists an r in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.
(E) For each r in $\mathbb{R}$, there exists an s in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.

---

15. If g is a function defined on the open interval (a, b) such that $a < g(x) < x$ for all $x \in (a, b)$, then g is

(A) an unbounded function
(B) a nonconstant function
(C) a nonnegative function
(D) a strictly increasing function
(E) a polynomial function of degree 1

GO ON TO THE NEXT PAGE.

16. For what value (or values) of m is the vector $(1, 2, m, 5)$ a linear combination of the vectors $(0, 1, 1, 1)$, $(0, 0, 0, 1)$, and $(1, 1, 2, 0)$?

(A) For no value of m
(B) -1 only
(C) 1 only
(D) 3 only
(E) For infinitely many values of m

---

17. For a function f, the finite differences $\Delta f(x)$ and $\Delta^2 f(x)$ are defined by $\Delta f(x) = f(x + 1) - f(x)$ and $\Delta^2 f(x) = \Delta f(x + 1) - \Delta f(x)$. What is the value of $f(4)$, given the following partially completed finite difference table?

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	-1	4	
2		-2	6
3			
4			

(A) -5
(B) -1
(C) 1
(D) 3
(E) 5

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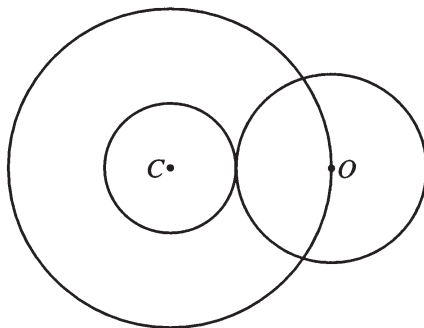
18. In the figure above, the annulus with center C has inner radius r and outer radius 1. As r increases, the circle with center O contracts and remains tangent to the inner circle. If $A(r)$ is the area of the annulus and $a(r)$ is the area of the circular region with center O, then $\lim\limits_{r \to 1^-} \dfrac{A(r)}{a(r)} =$

(A) 0

(B) $\dfrac{2}{\pi}$

(C) 1

(D) $\dfrac{\pi}{2}$

(E) ∞

GO ON TO THE NEXT PAGE.

19. Which of the following are multiplication tables for groups with four elements?

I.

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

II.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	a
d	d	c	a	b

III.

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	c	d
d	d	c	d	c

(A) None
(B) I only
(C) I and II only
(D) II and III only
(E) I, II, and III

20. Which of the following statements are true for every function f, defined on the set of all real numbers, such that $\lim_{x \to 0} \frac{f(x)}{x}$ is a real number L and $f(0) = 0$?

  I. f is differentiable at 0.

  II. $L = 0$

  III. $\lim_{x \to 0} f(x) = 0$

(A) None
(B) I only
(C) III only
(D) I and III only
(E) I, II, and III

21. What is the area of the region bounded by the coordinate axes and the line tangent to the graph of $y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$ at the point $(0, 1)$?

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) 1

(E) 2

GO ON TO THE NEXT PAGE.

22. Let $\mathbb{Z}$ be the group of all integers under the operation of addition. Which of the following subsets of $\mathbb{Z}$ is NOT a subgroup of $\mathbb{Z}$?

(A) $\{0\}$
(B) $\{n \in \mathbb{Z}: n \geq 0\}$
(C) $\{n \in \mathbb{Z}: n \text{ is an even integer}\}$
(D) $\{n \in \mathbb{Z}: n \text{ is divisible by both 6 and 9}\}$
(E) $\mathbb{Z}$

---

23. In the Euclidean plane, point A is on a circle centered at point O, and O is on a circle centered at A. The circles intersect at points B and C. What is the measure of angle BAC ?

(A) 60°
(B) 90°
(C) 120°
(D) 135°
(E) 150°

---

24. Which of the following sets of vectors is a basis for the subspace of Euclidean 4-space consisting of all vectors that are orthogonal to both $(0, 1, 1, 1)$ and $(1, 1, 1, 0)$?

(A) $\{(0, -1, 1, 0)\}$
(B) $\{(1, 0, 0, 0), (0, 0, 0, 1)\}$
(C) $\{(-2, 1, 1, -2), (0, 1, -1, 0)\}$
(D) $\{(1, -1, 0, 1), (-1, 1, 0, -1), (0, 1, -1, 0)\}$
(E) $\{(0, 0, 0, 0), (-1, 1, 0, -1), (0, 1, -1, 0)\}$

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25. Let f be the function defined by $f(x, y) = 5x - 4y$ on the region in the xy-plane satisfying the inequalities $x \leq 2$, $y \geq 0$, $x + y \geq 1$, and $y - x \leq 0$. The maximum value of f on this region is

(A) 1
(B) 2
(C) 5
(D) 10
(E) 15

GO ON TO THE NEXT PAGE.

26. Let f be the function defined by

$$f(x) = \begin{cases} -x^2 + 4x - 2 & \text{if } x < 1, \\ -x^2 + 2 & \text{if } x \geq 1. \end{cases}$$

Which of the following statements about f is true?

(A) f has an absolute maximum at $x = 0$.
(B) f has an absolute maximum at $x = 1$.
(C) f has an absolute maximum at $x = 2$.
(D) f has no absolute maximum.
(E) f has local maxima at both $x = 0$ and $x = 2$.

---

27. Let f be a function such that $f(x) = f(1 - x)$ for all real numbers x. If f is differentiable everywhere, then $f'(0) =$

(A) $f(0)$
(B) $f(1)$
(C) $-f(0)$
(D) $f'(1)$
(E) $-f'(1)$

---

28. If V_1 and V_2 are 6-dimensional subspaces of a 10-dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 6

---

29. Assume that p is a polynomial function on the set of real numbers. If $p(0) = p(2) = 3$ and

$p'(0) = p'(2) = -1$, then $\int_0^2 xp''(x)\, dx =$

(A) -3
(B) -2
(C) -1
(D) 1
(E) 2

GO ON TO THE NEXT PAGE.

28

30. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?

(A) The zero vector of V is an element of B.
(B) B has a proper subset that spans V.
(C) B is a proper subset of a linearly independent subset of V.
(D) There is a basis for V that is disjoint from B.
(E) One of the vectors in B is a linear combination of the other vectors in B.

---

31. Which of the following CANNOT be a root of a polynomial in x of the form $9x^5 + ax^3 + b$, where a and b are integers?

(A) -9

(B) -5

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) 9

---

32. When 20 children in a classroom line up for lunch, Pat insists on being somewhere ahead of Lynn. If Pat's demand is to be satisfied, in how many ways can the children line up?

(A) 20!

(B) 19!

(C) 18!

(D) $\dfrac{20!}{2}$

(E) $20 \cdot 19$

GO ON TO THE NEXT PAGE.

33. How many integers from 1 to 1,000 are divisible by 30 but not by 16 ?

(A) 29
(B) 31
(C) 32
(D) 33
(E) 38

34. Suppose f is a differentiable function for which $\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f'(x)$ both exist and are finite. Which of the following must be true?

(A) $\lim_{x \to \infty} f'(x) = 0$

(B) $\lim_{x \to \infty} f''(x) = 0$

(C) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} f'(x)$

(D) f is a constant function.

(E) f' is a constant function.

35. In xyz-space, an equation of the plane tangent to the surface $z = e^{-x} \sin y$ at the point where $x = 0$ and $y = \dfrac{\pi}{2}$ is

(A) $x + y = 1$
(B) $x + z = 1$
(C) $x - z = 1$
(D) $y + z = 1$
(E) $y - z = 1$

36. For each real number x, let $\mu(x)$ be the mean of the numbers 4, 9, 7, 5, and x; and let $\eta(x)$ be the median of these five numbers. For how many values of x is $\mu(x) = \eta(x)$?

(A) None
(B) One
(C) Two
(D) Three
(E) Infinitely many

GO ON TO THE NEXT PAGE.

37. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k!} =$

(A) e

(B) $2e$

(C) $(e + 1)(e - 1)$

(D) e^2

(E) ∞

---

38. Which of the following integrals on the interval $\left[0, \dfrac{\pi}{4}\right]$ has the greatest value?

(A) $\displaystyle\int_0^{\frac{\pi}{4}} \sin t \; dt$

(B) $\displaystyle\int_0^{\frac{\pi}{4}} \cos t \; dt$

(C) $\displaystyle\int_0^{\frac{\pi}{4}} \cos^2 t \; dt$

(D) $\displaystyle\int_0^{\frac{\pi}{4}} \cos 2t \; dt$

(E) $\displaystyle\int_0^{\frac{\pi}{4}} \sin t \cos t \; dt$

GO ON TO THE NEXT PAGE.

39. Consider the function f defined by $f(x) = e^{-x}$ on the interval $[0, 10]$. Let $n > 1$ and let $x_0, x_1, \ldots, x_n$ be numbers such that $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = 10$. Which of the following is greatest?

(A) $\displaystyle\sum_{j=1}^{n} f(x_j)(x_j - x_{j-1})$

(B) $\displaystyle\sum_{j=1}^{n} f(x_{j-1})(x_j - x_{j-1})$

(C) $\displaystyle\sum_{j=1}^{n} f\left(\frac{x_j + x_{j-1}}{2}\right)(x_j - x_{j-1})$

(D) $\displaystyle\int_0^{10} f(x)\, dx$

(E) 0

---

40. A fair coin is to be tossed 8 times. What is the probability that more of the tosses will result in heads than will result in tails?

(A) $\dfrac{1}{4}$

(B) $\dfrac{1}{3}$

(C) $\dfrac{87}{256}$

(D) $\dfrac{23}{64}$

(E) $\dfrac{93}{256}$

---

41. The function $f(x, y) = xy - x^3 - y^3$ has a relative maximum at the point

(A) $(0, 0)$

(B) $(1, 1)$

(C) $(-1, -1)$

(D) $(1, 3)$

(E) $\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$

GO ON TO THE NEXT PAGE.

42. Consider the points $A = (-1, 2)$, $B = (6, 4)$, and $C = (1, -20)$ in the plane. For how many different points D in the plane are A, B, C, and D the vertices of a parallelogram?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

---

43. If A is a 3×3 matrix such that $A\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $A\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then the product $A\begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}$ is

(A) $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

(D) $\begin{pmatrix} 9 \\ 10 \\ 11 \end{pmatrix}$

(E) not uniquely determined by the information given

---

44. Let f denote the function defined for all $x > 0$ by $f(x) = (\sqrt{x})^x$. Which of the following statements is FALSE?

(A) $\lim_{x \to 0^+} f(x) = 1$

(B) $\lim_{x \to \infty} f(x) = \infty$

(C) $f(x) = x^{x/2}$ for all $x > 0$.

(D) The derivative $f'(x)$ is positive for all $x > 0$.

(E) The derivative $f'(x)$ is increasing for all $x > 0$.

GO ON TO THE NEXT PAGE.

38

45. An experimental car is found to have a fuel efficiency $E(v)$, in miles per gallon of fuel, where v is the speed of the car, in miles per hour. For a certain 4-hour trip, if $v = v(t)$ is the speed of the car t hours after the trip started, which of the following integrals represents the number of gallons of fuel that the car used on the trip?

(A) $\displaystyle\int_0^4 \frac{v(t)}{E(v(t))}\, dt$

(B) $\displaystyle\int_0^4 \frac{E(v(t))}{v(t)}\, dt$

(C) $\displaystyle\int_0^4 \frac{tv(t)}{E(v(t))}\, dt$

(D) $\displaystyle\int_0^4 \frac{tE(v(t))}{v(t)}\, dt$

(E) $\displaystyle\int_0^4 v(t)E(v(t))\, dt$

---

46. For $0 < t < \pi$, the matrix $\begin{pmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{pmatrix}$ has distinct complex eigenvalues λ_1 and λ_2. For what value of t, $0 < t < \pi$, is $\lambda_1 + \lambda_2 = 1$?

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{\pi}{2}$

(E) $\dfrac{2\pi}{3}$

GO ON TO THE NEXT PAGE.

47. Let x and y be uniformly distributed, independent random variables on $[0, 1]$. The probability that the distance between x and y is less than $\frac{1}{2}$ is

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) $\frac{3}{4}$

---

48. Consider the change of variables from the xy-plane to the uv-plane given by the equations

$$u = x^{1/3} + y$$
$$v = 1 + y.$$

Under this transformation, the image of the region $\{(x, y): 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$ is which of the following shaded regions?

(A)

$u = v$

(C)

$u = v$

(E)

$v = u^3$

(B)

$v = u^{1/3}$

(D)

$v = 1 + u^{1/3}$
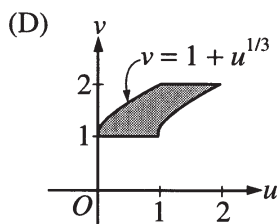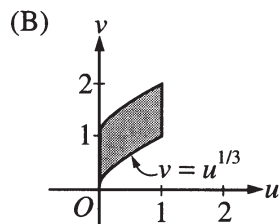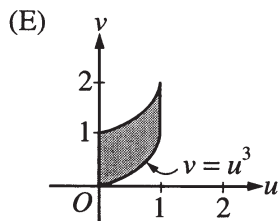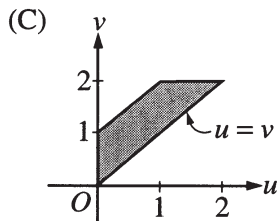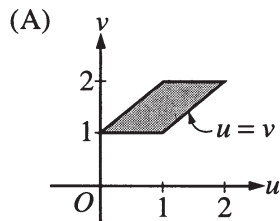
GO ON TO THE NEXT PAGE.

49. If f is a continuous function on the set of real numbers and if a and b are real numbers, which of the following must be true?

I. $\displaystyle\int_a^b f(x)\,dx = \int_{a+3}^{b+3} f(x-3)\,dx$

II. $\displaystyle\int_a^b f(x)\,dx = \int_a^3 f(x)\,dx - \int_b^3 f(x)\,dx$

III. $\displaystyle\int_{3a}^{3b} f(x)\,dx = 3\int_a^b f(3x)\,dx$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

50. How many continuous real-valued functions f are there with domain $[-1, 1]$ such that $(f(x))^2 = x^2$ for each x in $[-1, 1]$?

(A) One
(B) Two
(C) Three
(D) Four
(E) Infinitely many

---

51. Let D be the region in the xy-plane in which the series $\displaystyle\sum_{k=1}^{\infty} \frac{(x+2y)^k}{k}$ converges.
Then the interior of D is

(A) an open disk
(B) the open region bounded by an ellipse
(C) the open region bounded by a quadrilateral
(D) the open region between two parallel lines
(E) an open half plane

GO ON TO THE NEXT PAGE.

52. Consider the following system of linear equations over the real numbers, where x, y, and z are variables and b is a real constant.

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ x + 3y + bz &= 0 \end{aligned}$$

Which of the following statements are true?

   I.   There exists a value of b for which the system has no solution.
   II. There exists a value of b for which the system has exactly one solution.
   III. There exists a value of b for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

53. In the complex plane, let C be the circle $|z| = 2$ with positive (counterclockwise) orientation. Then

$$\int_C \frac{dz}{(z-1)(z+3)^2} =$$

(A) 0

(B) $2\pi i$

(C) $\dfrac{\pi i}{2}$

(D) $\dfrac{\pi i}{8}$

(E) $\dfrac{\pi i}{16}$

---

54. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let $h(t)$ denote the water level, in feet, above the floor of the tank at time t seconds. Starting at time $t = 0$, water pours into the tank at a constant rate of 1 cubic foot per second, and simultaneously, water is removed from the tank at a rate of $0.25h(t)$ cubic feet per second. As $t \to \infty$, what is the limit of the volume of the water in the tank?

(A)   400 cubic feet
(B)   600 cubic feet
(C) 1,000 cubic feet
(D) The limit does not exist.
(E) The limit exists, but it cannot be determined without knowing $h(0)$.

GO ON TO THE NEXT PAGE.

55. Suppose that f is a twice-differentiable function on the set of real numbers and that $f(0)$, $f'(0)$, and $f''(0)$ are all negative. Suppose f'' has all three of the following properties.

   I.   It is increasing on the interval $[0, \infty)$.
   II. It has a unique zero in the interval $[0, \infty)$.
   III. It is unbounded on the interval $[0, \infty)$.

Which of the same three properties does f necessarily have?

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II, and III

---

56. For every set S and every metric d on S, which of the following is a metric on S ?

(A) $4 + d$
(B) $e^d - 1$
(C) $d - |d|$
(D) d^2
(E) $\sqrt{d}$

---

57. Let $\mathbb{R}$ be the field of real numbers and $\mathbb{R}[x]$ the ring of polynomials in x with coefficients in $\mathbb{R}$. Which of the following subsets of $\mathbb{R}[x]$ is a subring of $\mathbb{R}[x]$?

   I.   All polynomials whose coefficient of x is zero
   II. All polynomials whose degree is an even integer, together with the zero polynomial
   III. All polynomials whose coefficients are rational numbers

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II, and III

---

58. Let f be a real-valued function defined and continuous on the set of real numbers $\mathbb{R}$. Which of the following must be true of the set $S = \{f(c): 0 < c < 1\}$?

   I. S is a connected subset of $\mathbb{R}$.
   II. S is an open subset of $\mathbb{R}$.
   III. S is a bounded subset of $\mathbb{R}$.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE.

59. A cyclic group of order 15 has an element x such that the set $\{x^3, x^5, x^9\}$ has exactly two elements. The number of elements in the set $\{x^{13n}: n \text{ is a positive integer}\}$ is

(A) 3
(B) 5
(C) 8
(D) 15
(E) infinite

60. If S is a ring with the property that $s = s^2$ for each $s \in S$, which of the following must be true?

   I. $s + s = 0$ for each $s \in S$.
   II. $(s + t)^2 = s^2 + t^2$ for each $s, t \in S$.
   III. S is commutative.

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

61. What is the greatest integer that divides $p^4 - 1$ for every prime number p greater than 5 ?

(A) 12
(B) 30
(C) 48
(D) 120
(E) 240

62. The coefficient of x^3 in the expansion of $(1 + x)^3(2 + x^2)^{10}$ is

(A) 2^{14}

(B) 31

(C) $\binom{3}{3} + \binom{10}{1}$

(D) $\binom{3}{3} + 2\binom{10}{1}$

(E) $\binom{3}{3}\binom{10}{1}2^9$

GO ON TO THE NEXT PAGE.

63. At how many points in the xy-plane do the graphs of $y = x^{12}$ and $y = 2^x$ intersect?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

64. Suppose that f is a continuous real-valued function defined on the closed interval $[0, 1]$. Which of the following must be true?

   I. There is a constant $C > 0$ such that $|f(x) - f(y)| \leq C$ for all x and y in $[0, 1]$.

   II. There is a constant $D > 0$ such that $|f(x) - f(y)| \leq 1$ for all x and y in $[0, 1]$ that satisfy $|x - y| \leq D$.

   III. There is a constant $E > 0$ such that $|f(x) - f(y)| \leq E|x - y|$ for all x and y in $[0, 1]$.

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

65. Let $p(x)$ be the polynomial $x^3 + ax^2 + bx + c$, where a, b, and c are real constants. If $p(-3) = p(2) = 0$ and $p'(-3) < 0$, which of the following is a possible value of c ?

(A) -27

(B) -18

(C) -6

(D) -3

(E) $-\dfrac{1}{2}$

66. In the xy-plane, if C is the circle $x^2 + y^2 = 9$, oriented counterclockwise, then $\oint_C -2y\, dx + x^2\, dy =$

(A) 0
(B) 6π
(C) 9π
(D) 12π
(E) 18π

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

I

NOTE: To ensure prompt processing of test results, it is important that you fill in the blanks exactly as directed.

SUBJECT TEST

A. Print and sign your full name in this box:

PRINT: _____
        (LAST)   (FIRST)   (MIDDLE)

SIGN: _____

Copy this code in box 6 on your answer sheet. Then fill in the corresponding ovals exactly as shown.

6. TITLE CODE
6 8 7 5 0

Copy the Test Name and Form Code in box 7 on your answer sheet.

TEST NAME _____*Mathematics*_____

FORM CODE _____*GR9768*_____

GRADUATE RECORD EXAMINATIONS SUBJECT TEST

B. The Subject Tests are intended to measure your achievement in a specialized field of study. Most of the questions are concerned with subject matter that is probably familiar to you, but some of the questions may refer to areas that you have not studied.

Your score will be determined by subtracting one-fourth the number of incorrect answers from the number of correct answers. Questions for which you mark no answer or more than one answer are not counted in scoring. If you have some knowledge of a question and are able to rule out one or more of the answer choices as incorrect, your chances of selecting the correct answer are improved, and answering such questions will likely improve your score. It is unlikely that pure guessing will raise your score; it may lower your score.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult for you. Go on to the other questions and come back to the difficult ones later if you can.

YOU MUST INDICATE ALL YOUR ANSWERS ON THE SEPARATE ANSWER SHEET. No credit will be given for anything written in this examination book, but you may write in the book as much as you wish to work out your answers. After you have decided on your response to a question, fill in the corresponding oval on the answer sheet. BE SURE THAT EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL. Mark only one answer to each question. No credit will be given for multiple answers. Erase all stray marks. If you change an answer, be sure that all previous marks are erased completely. Incomplete erasures may be read as intended answers. Do not be concerned that the answer sheet provides spaces for more answers than there are questions in the test.

Example:

What city is the capital of France?
(A) Rome
(B) Paris
(C) London
(D) Cairo
(E) Oslo

Sample Answer

Ⓐ ● Ⓒ Ⓓ Ⓔ   CORRECT ANSWER PROPERLY MARKED

Ⓐ ⊗ Ⓒ Ⓓ Ⓔ
Ⓐ ☑ Ⓒ Ⓓ Ⓔ
Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   } IMPROPER MARKS
Ⓐ ◑ Ⓒ Ⓓ Ⓔ

DO NOT OPEN YOUR TEST BOOK UNTIL YOU ARE TOLD TO DO SO.

Scoring Your Subject Test

The GRE Mathematics Test (Rescaled) scores typically range from 440 to 810. The range for different editions of a given test may vary because different editions are not of precisely the same difficulty. The differences in ranges among different editions of a given test, however, usually are small. This should be taken into account, especially when comparing two very high scores. In general, differences between scores at the 99th percentile should be ignored. **The score conversion table on page 57 shows the score range for this edition of the test only.**

The worksheet on page 56 lists the correct answers to the questions. Columns are provided for you to mark whether you chose the correct (C) answer or an incorrect (I) answer to each question. Draw a line across any question you omitted, because it is not counted in the scoring. At the bottom of the page, enter the total number correct and the total number incorrect. Divide the total incorrect by 4 and subtract the resulting number from the total correct. This is the adjustment made for guessing. Then round the result to the nearest whole number. This will give you your raw total score. Use the total score conversion table to find the scaled total score that corresponds to your raw total score.

Example: Suppose you chose the correct answers to 48 questions and incorrect answers to 15. Dividing 15 by 4 yields 3.75. Subtracting 3.75 from 48 equals 44.25, which is rounded to 44. The raw score of 44 corresponds to a scaled score of 710 on the new GRE Mathematics scale, effective October 2001. The scaled scores in this book, and those received by individuals taking the GRE Mathematics Test (Rescaled) after October 2001, are *NOT* comparable to Mathematics Test scores reported prior to October 2001.

Worksheet for the GRE Mathematics Test (Rescaled), Form GR9768
Answer Key and Percentages* of Examinees
Answering Each Question Correctly

Question Number	Answer	P +	TOTAL C	TOTAL I	Question Number	Answer	P +	TOTAL C	TOTAL I
1	C	73			36	D	38		
2	D	86			37	B	32		
3	C	88			38	B	54		
4	B	87			39	B	53		
5	E	83			40	E	54		
6	C	87			41	E	49		
7	B	81			42	D	60		
8	E	76			43	B	64		
9	A	77			44	D	39		
10	B	55			45	A	47		
11	A	80			46	C	50		
12	C	75			47	E	32		
13	A	72			48	A	36		
14	C	63			49	E	41		
15	B	61			50	D	34		
16	D	83			51	D	35		
17	E	80			52	D	52		
18	E	63			53	D	35		
19	B	55			54	A	31		
20	D	52			55	D	28		
21	D	71			56	E	19		
22	B	72			57	C	20		
23	C	72			58	C	34		
24	C	50			59	A	28		
25	D	70			60	E	23		
26	B	67			61	E	25		
27	E	65			62	A	36		
28	C	62			63	D	17		
29	B	63			64	C	35		
30	D	55			65	A	44		
31	C	65			66	E	58		
32	D	59							
33	A	72							
34	A	52							
35	B	50							

Correct (**C**) _____

Incorrect (**I**) _____

Total Score:

C – **I**/4 = _____

Scaled Score (SS) = _____

* The P+ column indicates the percentage of Mathematics Test examinees that answered each question correctly; it is based on a sample of December 1997 examinees selected to represent all Mathematics Test examinees tested between October 1, 1997, and September 30, 2000.

Score Conversions* and Percents Below** for GRE Mathematics Test (Rescaled), Form GR9768

TOTAL SCORE					
Raw Score	Scaled Score	%	Raw Score	Scaled Score	%
66	890	99	29	590	43
65	880	98	28	580	41
64	870	97	26-27	570	37
62-63	860	97	25	560	35
61	850	95	24	550	33
60	840	94	23	540	31
59	830	93	21-22	530	29
57-58	820	92	20	520	27
56	810	90	19	510	24
55	800	88	18	500	22
54	790	87	17	490	20
52-53	780	85	15-16	480	17
51	770	83	14	470	15
50	760	82	13	460	13
49	750	80	12	450	11
47-48	740	78	10-11	440	9
46	730	76	9	430	8
45	720	74	8	420	6
44	710	72	7	410	5
43	700	70	5-6	400	3
41-42	690	68	4	390	2
40	680	66	3	380	1
39	670	63	2	370	1
38	660	61	0-1	360	1
36-37	650	58			
35	640	56			
34	630	53			
33	620	51			
31-32	610	48			
30	600	46			

*Score conversions presented in this table are based on the rescaled Mathematics Test.

**Percentage scoring below the scaled score is based on the performance of 6,753 examinees who took the Mathematics Test between October 1, 1997, and September 30, 2000.

Evaluating Your Performance

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test. Both the worksheet on page 56 and the table on page 57 use performance data from GRE Mathematics Test (Rescaled) examinees.

The data in the worksheet on page 56 are based on the performance of a sample of the examinees who took this test in December 2000. This sample was selected to represent the total population of GRE Mathematics Test (Rescaled) examinees tested between October 1997 and September 2000. The numbers in the column labeled "P+" on the worksheet indicate the percentages of examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

The table on page 57 contains, for each scaled score, the percentage of examinees tested between October 1997 and September 2000 who received lower scores. Interpretive data based on the scores earned by examinees tested in this three-year period will be used by admissions officers in the 2001–02 testing year. These percentages appear in the score conversion table in a column to the right of the scaled scores, when their raw scores were placed on the new scale. For example, in the percentage column opposite the scaled score of 620 is the number 51. This means that 51 percent of the GRE Mathematics Test (Rescaled) examinees tested between October 1997 and September 2000 scored lower than 620 when their raw scores are placed on the new scale. To compare yourself with this population, look at the percentage next to the scaled score you earned on the practice test.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other GRE Mathematics Test (Rescaled) examinees, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test (Rescaled) under standard conditions.

DO NOT USE INK

Use only a pencil with soft, black lead (No. 2 or HB) to complete this answer sheet.
Be sure to fill in completely the space that corresponds to your answer choice.
Completely erase any errors or stray marks.

1. NAME

Enter your last name, first name initial (given name), and middle initial if you have one.
Omit spaces, apostrophes, Jr., II., etc.

Last Name only (Family or Surname) - first 15 letters

First Name Initial | Middle Initial

2.

YOUR NAME: (Print)
Last Name (Family or Surname)   First Name (Given)   M.I.

MAILING ADDRESS: (Print)
P.O. Box or Street Address

City   State or Province

Country   Zip or Postal Code

CENTER:
City   State or Province

Country

Center Number   Room Number

SIGNATURE:

· 3. DATE OF BIRTH

Month	Day	Year
Jan.		
Feb.		
Mar.		
April		
May		
June		
July		
Aug.		
Sept.		
Oct.		
Nov.		
Dec.		

4. SOCIAL SECURITY NUMBER (U.S.A. only)

5. REGISTRATION NUMBER (from your admission ticket)

6. TITLE CODE (on back cover of your test book)

7. TEST CODE (on back cover of your test book)

FORM CODE (on back cover of your test book)

7. TEST NAME (on back cover of your test book)

8. TEST BOOK SERIAL NUMBER (red number in upper right corner of front cover of your test book)

SHADED AREA FOR ETS USE ONLY

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

1–38	39–76	77–114
A B C D E	A B C D E	A B C D E

Item responses continued on reverse side.

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2

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SIDE 2

SUBJECT TEST

COMPLETE THE CERTIFICATION STATEMENT, THEN TURN ANSWER SHEET OVER TO SIDE 1.

BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE INTENDED SPACE AS ILLUSTRATED HERE: ●.
YOU MAY FIND MORE RESPONSE SPACES THAN YOU NEED. IF SO, PLEASE LEAVE THEM BLANK.

#	A B C D E	#	A B C D E	#	A B C D E	#	A B C D E
115	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	147	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	179	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	211	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
116	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	148	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	180	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	212	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
117	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	149	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	181	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	213	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
118	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	150	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	182	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	214	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
119	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	151	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	183	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	215	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
120	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	152	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	184	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	216	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
121	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	153	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	185	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	217	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
122	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	154	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	186	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	218	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
123	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	155	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	187	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	219	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
124	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	156	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	188	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	220	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
125	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	157	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	189	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	221	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
126	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	158	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	190	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	222	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
127	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	159	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	191	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	223	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
128	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	160	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	192	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	224	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
129	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	161	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	193	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	225	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
130	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	162	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	194	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	226	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
131	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	163	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	195	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	227	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
132	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	164	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	196	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	228	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
133	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	165	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	197	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	229	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
134	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	166	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	198	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	230	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
135	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	167	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	199	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	231	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
136	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	168	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	200	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	232	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
137	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	169	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	201	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	233	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
138	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	170	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	202	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	234	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
139	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	171	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	203	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	235	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
140	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	172	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	204	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	236	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
141	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	173	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	205	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	237	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
142	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	174	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	206	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	238	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
143	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	175	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	207	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	239	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
144	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	176	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	208	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	240	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
145	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	177	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	209	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	241	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ
146	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	178	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	210	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ	242	Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

TR	TW	TFS	TCS	1R	1W	1FS	1CS	2R	2W	2FS	2CS
	FOR ETS USE ONLY			3R	3W	3FS	3CS	4R	4W	4FS	4CS
				5R	5W	5FS	5CS	6R	6W	6FS	6CS