GRE SUB Test problems*

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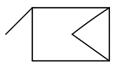
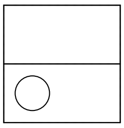
1. Check whether two vectors are linear independent or not.
2. Given two points on the Euclid Plane and determine whether they lie in a square with unit area.
3. Find out all the maximum points of the following function:
$$e^{-x}\sin^2\pi x,\ 0<x<10.$$
4. What is the minimum value of
$$a+b+c+d$$
when the following identity satisfies:
$$4a=3b=5c=15d.$$
5. If the mean of $a<b<c<d$ is 100, what is the minimum value of $a+d$.
6. It is known that $f''>0$ and $f(x)=f(-x)$, which of the following statements are correct?
$$f(0)<f(1),\ f(4)-f(3)<f(6)-f(5),\ f(-2)<\frac{f(-3)+f(-1)}{2}.$$
7. Suppose there is a field of q elements. Determine the number of invertible matrix on this field.
8. f_n and f are continuous functions. $f_n(x)\to f(x)$ pointwise. Which of the following is/are correct?
$$\int_0^x F_n(t)\,\mathrm{d}t\to\int_0^x F(t)\,\mathrm{d}t,\ F_n'(x)\to f(x),\ \int_0^x f_n(t)\,\mathrm{d}t\to\int_0^x f(t)\,\mathrm{d}t,$$
where $F(x)=\int f(x)\,\mathrm{d}x$ and $F_n(x)=\int f_n(x)\,\mathrm{d}x$
9. Which person made a great contribution to the modern analysis?
10. Which letter is not homomorphic to letter **C**? **J, N, S, O, U.**
11. Determine the number of the trees of the following graph. 
12. Ten questions in all. If four of the first five must be answer at least, then how many way are there to answer seven of all the ten questions?
13. Fair coins are tossed and when either four consecutive heads or tails appear the process will be stopped. What is the probability of two consecutive head or tail or any one of them in one row?
14. T is a one-to-one and onto mapping defined as:
$$\mathrm{T}:\ \mathbb{R}^3\to\mathbb{R}^3.$$
Which of the following is/are correct?
$$T\circ(S\circ N)=(T\circ S)\circ N,\ T\circ N=N\circ T,\ T\circ A=T\circ B\Longrightarrow A=B.$$

*Based on the GRE SUB Test in Year 2003-2004

15. A 10×10 cm square card is folded as. A circle with radius 1 cm is placed so that it is entirely on the square. The upper part of the square is blue and the lower part of the square is red. What is the probability that the circle lies entirely in the red part? 

16. Choose the graph of
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin y.$$

17. Check the consistence of the linear equations (2 equations in all).

18. In a group $\mathbf{G}$, a, $b \in \mathbf{G}$ such that $ab \neq ba$. Choose the correct relation. (Easy, direct check)

19. Function f is a linear function composed of two linear pieces and continuous on $[0, 2]$. $f(0) = f(2) = 0$ and $\max f = 1$. What is the length of the graph of f?

20. Two particles move along x and y directions separately. v_x and v_y are constant velocity. Which of the following can be used to calculate the distance between particles on axis x and y at time $t = 1$? When $t = 0$, the particles are at the origin.
    - $t = 3$ the distance between x and y;
    - $t = 10$ the travelled distance of v_x and v_y

21. z on the unit circle $|z| = 1$. How many ?? part ?? does the transformed region $z \to e^z$ has?

22. Change the coordinates of the function.

23. $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Calculate
$$I + \sum_{n=0}^{\infty} \frac{t^n}{n!} A^n.$$

24. Calculate the volume of the solid generated by rotate the region $\{x = 1,\ y = 2,\ y = x\}$ along the the y axis.

25. Write the formula to calculate the volume of the intersection of $x^2 + y^2 + z^2 = 4$ and $r = 2\cos\theta$.

26.
$$\int_c \Delta \vec{F} \cdot \vec{t}\ \mathrm{d}s = ?$$
where $F = x - y$ and c is the following circle (counterclockwise rotation).