2010 GRE sub-math 回忆题

1. The linear equation system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 = b_4 \end{cases}$$

has exactly one solution. Let

$$\mathbf{M} = \left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \\ a_{41} & a_{42} & a_{43} & b_4 \end{array}\right)$$

Then which of the following statement may be false?

A. the rank of M is 3

B. there exists a line which is the combination of other three.

C. the determination of M is zero.

D. the fourth column is the combination of the first three columns.

E. M can become the unity matrix after elementary row matrix transformation.

2. function g and f are continuous, $f \circ g$ is a constant function, then which of the following statement is true

A. f is constant B. g is constant C. $g \circ f$ is constant

3. $C = \{e^{i\theta} | 0 \leq \theta \leq \pi\}$, $f(z) = 1 + z + z^2 + z^3 + z^4$, evaluate $\int_C f(z)dz$

4. six empty seat, three persons A, B, C, one person needs one seat, how many arrangement are there?

5. there is a cube with edge n, and there are balls contained completely in the ball with the center at the integral coordinate and the radius $\frac{1}{2}$, try to get the limit of the ratio of the total volume of the balls to the volume of the cube as n is towards infinity.

6. $|A \cup B \cup C| = 68, |A| = 47, |B| = 25, |C| = 15, |A \cap B \cap C| = 7$, how many elements which are exactly in two of $\{A, B, C\}$?

7. $a \sqcap b$ means $\max\{a, b\}$, then which of the following statements are true?
A. $a \sqcap b = b \sqcap a$ B.$(a \sqcap b) \sqcap c = a \sqcap (b \sqcap c)$ C.$a \sqcap (b + c) = a \sqcap b + a \sqcap c$

8. consider the maps g which maps $\{1, 2, 3, 4, 5\}$ onto $\{11, 12, 13, 14\}$ and $g(1) \neq g(2)$ how many such g are there?

9. calculate the convergence radius of the series

$$\sum_{n=1}^{+\infty} \frac{x^{2n}}{2^n n^2}$$

10. a person wants to get to a boat which is 200 far away from the bank and the person is 500 away from the point, the speed of the person by walk is $\frac{5}{3}$ times the speed in the water, let you to get the minimum distance.

11. $f''(x) < 0, f'(0) = 0$ $T = f(0)+2f(2)+2f(4)+f(6)$ $I = \int_0^6 f(x)dx$ $R = 2f(2) + 2f(4) + 2f(6)$ 从小到大排列T, I, R

12. for which function does the function $f(x) = e^x$ intersect at a positive point? the answer is $tan\frac{1}{1+x}$

13. which function is not uniformly continuous? the answer is $sin(x^2)$.

14. f: X→ Y is continuous bijection,
A. if X is compact then Y is compact
B. if X is Hausdorff space then Y is Hausdorff space
C. if X is compact and Y is Hausdorff space then f^{-1} exist

Which of them are correct?

15. Which is a semi-group but not a group?

A. $\mathbb{R}, x * y = \frac{x}{y}$

B. all irrational numbers, multiplicaiton

C. all continuous functions f on $\mathbb{R}$, composition

D. all 2*2 matrices with determinant=1, multiplication

E. all polynomials whose coefficients are integers and degree no more than 5

16. $C = \{e^{\theta i} : 0 \leq \theta \leq \pi\}, \quad \int_C (1 + 2z + 3z^2 + 4z^3)dz =$

17. f is strictly increasing, then which is necessarily WRONG?

A. $\forall x, f(2x) = 2f(x)$

C. $\int_0^1 f(x)dx = \int_1^2 f(x)dx$

D. $lim_{x\to\infty} f^{'}(x) = 0$

E. $f^{'}(1) = -f^{'}(2)$

18. select 2 elements randomly in 1,2,3,4,5,6,7,8, the probability of that the sum of them is divided by 3 is?

19. $a, b \in$ group G, both have finite orders

A. if $ab = ba$, then ab has finite order

B. if ab has finite order, then ba has finite order

C. if ab has finite order, then $a^{-1}b^{-1}$ has finite order

which of them are correct?

20. $(1 - x^2)y^{''} - xy^{'} + y = 0$ if $x = sint$, then determine the new equation

21. find the minimum value of $x^2 + y^2 + z^2$ on the surface $ax + by + cz = 1$

22. $\frac{dy}{dx} = \frac{3y+1}{x^2}$ solve $y(x)$

23. 给$f^{'}$的图像，问它改变凹凸性的地方，选二阶导数是0的地方就可以了

24. 计算曲面$z = f(x, y)$在某处的切面

25. 一个正方形内切一个圆形，这个圆形又内接一个正方形，问这两个正方形的面积之比?

26. $\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$

27. $ax + by + cz + dw = e, fx + gy + hz + iw = l$ and (1,2,0,0),(0,0,3,4) are the solutions of this system of equation, which one of the following must be the solution to the system of equation $ax+by+cz+dw = 0, fx+gy+hz+iw = 0$?

28. M 是一个4维空间，问M有几个不同构的子空间？

29. there is a uniform distribution in $0 < x < \pi, 0 < y < \pi, 0 < z < \pi$, then what is the probability of $z < cos^2(x)sin^2(y)$

30. R^n中一个紧连通集的补集的连通分支最多几个？

31. p, q are prime numbers, which one of the following can not be the order of a finite domain? The answer is pq