Convergence of Greedy Algorithm

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Independence Gene Model:

$g \in \hat{M}, a_{ij} \sim B(1, p_g), p_g \in [p_L, p_U]$.

$P(\hat{M}) = rm . 0 \leqslant r \leqslant nt, \min p(o), p(2)/2$

$P(M) \geqslant \frac{|M|-d}{|\hat{M}|} P(\hat{M}) . M \subset \hat{M}, 0 \leqslant d \leqslant 1$.

where $P(M) = \sum_{i=1}^{m} p(\sum_{j=1}^{n} a_{ij} x_j)$, objective: $\arg\min_{|M|=k} P(M)$.

$p(1) = 0 \leqslant p(0) \leqslant p(2) < p(3) < \cdots$, $p(\cdot)$ should be convex and $p(a) + p(b) \leqslant p(a+b), \forall a, b \in \mathbb{Z}$.

Lemma 1. $M \subset \hat{M}$, $P(M) \leqslant \frac{|M|}{|\hat{M}|}$, $\exists g \in \hat{M}$, s.t. $P(M \cup \{g\}) \leqslant \frac{|M|+1}{|\hat{M}|} P(\hat{M})$.

By contradiction: assume $P(M \cup \{g\}) > \frac{|M|+1}{|\hat{M}|} P(\hat{M})$.

$M_i \triangleq M \cup \{g_1, \cdots, g_i\}$, s.t.

$P(M_{i*}) > \frac{|M|+i^*}{|\hat{M}|} P(\hat{M})$, $P(M_{i^*+1}) \leqslant \frac{|M|+i^*+1}{|\hat{M}|} P(\hat{M})$.

$\Rightarrow P(M_{i^*+1}) - P(M_{i^*}) < \frac{P(\hat{M})}{|\hat{M}|} \leqslant P(M \cup \{g_{i^*+1}\}) - P(M)$.