

- 1 $f(x)$ is function, for $x > 0$ $\ln f(x) = \ln 2 - \ln(x)$, find $f'(x)$
- 2 T_1 is a equilateral triangle with side length 1. T_2 is a triangle whose vertices are the midpoints of T_1 's sides. Find the area of T_2 . inter
- 3 A, B, C are three sets. $|A|=17, |B|=15, |C|=20, |A \cap B|=10, |A \cap C|=4, |C \cap B|=2, |A \cap B \cap C|=1$, find $|A \cup B \cup C|$
- 4 Given that $\int_0^\infty \exp(-x^2) dx = \sqrt{\pi}/2$, $a > 0$, find $\int_0^\infty \exp(-x^2/a^2) dx$
- 5 A bag contains 4 green balls and 5 black balls. One picks two balls randomly from it without putting back. Find the probability of picking two green balls.
- 6 $f(x,y)$ is continuous, $\int_0^2 (\int_0^{x^2} f(x,y) dy) dx =$ what (exchange the order of the two integrals)
- 7 $f(x) = \int_0^x \frac{(t^2-1)}{(t^2+1)} dt$, what is the set of the critical points of $f(x)$?
- 8 In a vector space with inner product $\langle a, b \rangle$ and norm $\|a\|$, a, b, c are vectors, $\langle a, b \rangle = 2, \langle b, c \rangle = 5, \langle a, c \rangle = -3, \|a\| = 1, \|b\| = 2, \|c\| = 7$. Find $\langle a+b, b+c \rangle$
- 9 What the graph of $y = 3x^4 - 4x^3 + 1$ like?
- 10 Which function is not even?
(a) $f(x) = \sin(x \sin(x))$ (b) $f(x) = \sin(\cos(x))$ (c) $f(x) = \cos(\sin(x))$ (d) $f(x) = \exp(\cos(x)) - \exp(\cos(-x))$ (e) $f(x) = \cos(\exp(2x))$
- 11 M_2 is the vector space of all 2×2 matrices with real entries. For what real number r the set $\{(a, b, c, d) | a+b+c+d=r\}$ is a subspace of M_2 ?
- 12 Find $\int_0^1 \sqrt{\exp(x) + \exp(-x) + 2} dx$
- 13 Find the congruence of $4^{578} \pmod{7}$
- 14 ABCD is a quadrilateral, $AB=BC=3, AD=DC=4, BD=5$. find AC
- 15 f and g are bijections from interval $[0,1]$ to itself. which map must be an one-to-one map?
(a) $f+g$ (b) $f-g$ (c) $f \circ g$ (d) $(f^2+g^2)/2$ (e) $f(g)$
- 16 一张边长 24 的正方形纸, 四个角各剪掉一个边长为 h 的正方形, 剩下的部分可以折成一个无盖的长方体, 求 h 使长方体体积最大。
- 17 A and B are two invertible 5×5 matrices, B^T is the transposition of B . X is a 5×1 vector. Which statement must be true?

I. $(B^T)A$ is invertible

II. the equation $BX=AX$ has only the trivial solution $X=0$.

III. the dimension of the column space of A equals to the dimension of the column space of B

18 g is a twice-differentiable function, $g(0)=g'(0)=g''(0)=1$, $f(x)=g(x^2)+g(x)+1$, find $f'(0)$

19 f is a function that is differentiable at 0, find $\lim_{x \rightarrow 0} [f(x^5)-f(0)]/2x^3$

20 the graph of the derivative $g'(x)$ of function $y=g(x)$ has the graph as follows (题目给了图, 我没法画, 用解析式表示大概是): 当 x 不大于 2, $g'(x)=x-1$; 当 $2 \leq x \leq 3$, $g'(x)=3-x$; 当 $3 \leq x \leq 4$, $g'(x)=x-3$; 当 x 不小于 4, $g'(x)=5-x$. Which one is not true

(a) g has a local minimum at $x=1$ (b) g has a local maximum at $x=2$ (c) g has an inflection point at $x=3$ (d) g has an inflection point at $x=4$ (e) g has an absolute maximum at $x=5$

21 Given that the 3×3 matrix: (three row vectors are $(0,0,-2), (1,2,1), (1,0,3)$

, respectively) has an eigenvalue of 2, find a base of the subspace of eigenvectors corresponding to 2.

22 On a curve $r=r(t)$ (r 是向量), a point is regular if dr/dt is not zero vector. Then how many points on the curve $x(t)=[\sin(t)]^3$, $y(t)=[\cos(t)]^3$ are not regular?

ditons

23 Let $z=x+iy$, x and y are real numbers. $f(z)=6x-4y+i(ax+by)$, a and b are real numbers. If $f(z)$ is differentiable, find a and b .

24 a, b, c belongs to interval $[0,1]$ and $a < b < c$. Which function's graph satisfy the following conditions: (1) f is nonnegative; (2) $\int_a^b f(x) dx = \int_a^c f(x) dx$; (3) $f(a)=f(c)$ 图像就不画了, 反正很简单。

25 $z=x+iy$, $x > 0$ and $y > 0$. If $z^2 = -1/2 + i\sqrt{3}/2$, then $z^3 + 2z + 1 = ?$

26 $\{a_n\}$ is a sequence, $a_0=0, a_1=1$, $a_{n+1}=a_n+2a_{n-1}$ for $n \geq 1$. Find $\lim_{n \rightarrow \infty} a_{n+1}/a_n$

27 Let A be the area of the triangle formed by x -axle, y -axle and the tangent line of $y=x^p$ ($p < 0$) at $x=c$. Find p such that A is independent of c .

28 Given that $x-2y+3z=1$, find the minimum value of $x^2+y^2+z^2$

29 $P(x)$ is a quadratic polynomial of x that has the same value as $\sin(x)$ at $x=0, x=\pi/4$ and $x=\pi/2$. Find $P(\pi)$

30 For real x , let $f(x)=\lim_{n \rightarrow \infty} [\cos(x)]^{2n}$. Then what about the continuity of $f(x)$. (选项不列了, 就是问 f 在哪些点连续)

31 For real x , let $f(x)=x-[x]$, i.e., f is the fractional part of x . If a and b are real numbers and $b=a+1$, find the image of f on interval $[a,b]$.

32 $G(x)=\int_0^x \exp(-t^2)dt$, find $G'(x)$

33 老师给 12 个孩子分 12 个玩具，每人 1 个。只有 4 种不同的玩具，每种分别有 4，5，2，1 个，问有多少种分配的方法。

34 Find the intersection point of plane $3x+2y-z=5$ and line $x=1+2t, y=-1-t, z=-3t$.

35 Find the mean value of function $f(x)=xe^{3x}$ on interval $[0,2]$

36 S and T are nonzero 3×3 matrices. Which of the following cannot be true?

(A) $ST=0$ (B) $ST \neq TS$ (C) $ST=ST^2$ (D) $T=T^4$ (E) $S^4=0$ but $S^3 \neq 0$ (" \neq " means "not equal to")

37 $\{a_n\}$ is a strictly increasing sequence of positive numbers, which of the following sum must converge?

I. $\sum_{i=1}^{\infty} 1/a_i^2$

II. $\sum_{i=1}^{\infty} (a_i/a_{i+1})$

III. $\sum_{i=1}^{\infty} \exp(-a_i)$

38 $a=(a_1, a_2, a_3)$ is a nonzero vector. S is the linear transformation from \mathbb{R}^3 to \mathbb{R}^3 such that $S(x)=a \times x$ (叉乘) for all vector x . Find the determinant of S .

39 P 和 Q 是命题，则“非 (P 且 Q)”等价于以下哪个？ 答案： P 蕴含非 Q

40 Which set is not the union of countably many open sets?

(a) \mathbb{Z} (b) \mathbb{Q} (c) interval $[0, \infty)$ (d) the complement of \mathbb{Q} (e) the complement of \mathbb{Z}

42 h is a continuous function on interval $[a,b]$ and $h(x)$ belongs in \mathbb{Q} for all x . Which statement is true?

(a) h is constant on the interval 后面的选项忘了

43 直线 $y=1000x$ 与 $y=999x$ 的夹角用弧度表示最接近哪个选项？ 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6}

44 Find $\sum_{n=0}^{\infty} ((-1)^n)^*(n+1)/n!$

46 $n \geq 3, k$ is real number, $\sum_{i=1}^n (a_i)^3 \leq k \sqrt{\sum_{i=1}^n (a_i)^6}$ for all real numbers $\{a_i\}$. Find the minimum value of k .

47 C is the circle $x^2+y^2=9$, oriented counterclockwise, find \int_C 的线积分

$$(\exp(\pi i z)/(z-4i)^2/(z-i) dz) .$$

48 For a commutative ring M and ideal A , let $N(A)=\{x \text{ in } M | \text{there exists a nonnegative integer } n \text{ such that } x^n \text{ in } A\}$. Which of following is true for $N(A)=A$?

I. $M=\mathbb{Z}, A=(2)$

II. $M=\mathbb{Z}[x], A=(x^2+2)$

III. $M=\mathbb{Z}/27\mathbb{Z}, A=(18+27\mathbb{Z})$

49 X 和 Y 两个正态分布, 前者期望是 52, 标准差是 6; 后者期望是 40, 标准差是 8。求 $Z=X+Y$ 的标准差

50 对不小于 3 的正整数 n , 求单位圆的外切正 n 边形与内接正 n 边形的周长之比。

How many distinct real root does the equation $x^4-x^3\sin(x)-x^2\cos(x)=0$ have? 0 1 2

3 4

有多少个互不同构的 $3^2 \cdot 11^4 \cdot 17$ 阶 Abel 群?

T_a 是以点 $(0,0), (1,0), (0,a)$ 为顶点的三角形, $F(a)$ 是函数 $f(x,y)=y^2$ 在 T_a 上的积分, 求 $F'(1)$

55 The solution $y(x)$ of $y''+6y'+cy=0$ satisfies that $\lim_{x \rightarrow \text{positive infinity}} y(x)=0$, find all valid c :

(a) $c \leq 0$ (b) $c > 0$ (c) $0 < c \leq 9$ (d) $c \geq 9$ (e) \mathbb{R}

56 Line $y=x$ is tangent to a circle at $(3,3)$ and line $y=2x$ passes the center of that circle. Which circle is possible?

Answer: (a) $(x-2)^2+(y-4)^2=2$

57 X is a compact metric space, f is a continuous function $X \rightarrow X$. Which of the following must be true?

I. f has fixed point

II. f is uniformly continuous

III. f is a closed map

59 G is a group, a and b are non-unit elements of G , $ab=bba$. If the subgroup of G generated by a has order 3, what about the order of the subgroup of G generated by b ?

(a) 3 (b) 5 (c) 7 (d) 9 (e) cannot determined by given information

66 Find $\int_0^{\pi/2} \frac{[\sin(t)]^3}{[\sin(t)]^3 + [\cos(t)]^3} dt$