Mathematics Module 1
(Calculus and Statistics)

A Comprehensive Review of Calculus and Statistics Concepts

Part 1: Calculus

A) Formula of differentiation

1.

a.

$$\frac{d}{dx}x^n = nx^{n-1}$$

b.

$$\frac{d}{dx}k = 0, where\ k\ is\ a\ constant$$

2.

a.

$$\frac{d}{dx}e^x = e^x$$

b.

$$\frac{d}{dx}a^x = a^x \cdot \ln a$$

3.

a.

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

b.

$$\frac{d}{dx}\log_a x = \frac{1}{x} \cdot \ln a$$

B) Rules of differentiation

Assume u and v are differentiable functions.

1. Sum and Difference Rule:

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

2. Product Rule:

$$\frac{d}{dx}u \cdot v = u\frac{dv}{dx} + v\frac{du}{dx}$$

3. Quotient Rule:

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

4. Chain Rule:

$$y = f(u), u = g(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{df(u)}{du} \cdot \frac{dg(x)}{dx}$$

C) Rules of integration

There are NO product rule and quotient rule in integration!

1. Change the differential variable dx and simplify

2. Modify the equation by adding or subtracting some terms in order to match the question

3. U-Substitution, remember to change the bounds in Definite Integral $\int_a^b f(x)\,dx$ to match the new variable u

D) Application

1. To find the equation of the tangent or normal line to a curve at a given point

2. To find extreme values and optimize a function

   a. First-order derivative
      i. $\frac{dy}{dx} = 0$

      ii. Draw table

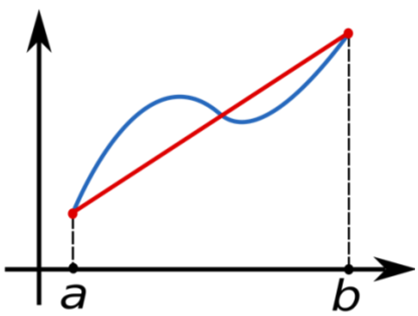
   b. Second- order derivative
      i. $\frac{dy}{dx} = 0$

      ii. $\frac{d^2y}{dx^2}|_{x=a} > 0 \text{ (min) } or < 0\ (max)$

3. Change of rate
   e.g. For a cone:

$$if \quad \frac{dv}{dt} = 3 \quad \rightarrow \quad \frac{dh}{dt} \quad increase.$$

4. Trapezoidal rule



If estimated value > actual, overestimate and $f''(x) > 0$.

If estimated value < actual, underestimate and $f^{\wedge''}(x) < 0$.

Part 1: Statistics

NO NEED to remember all formula, draw a Venn diagram instead

Complementary events:

1. $P(A) + P(A') = 1$

Mutually exclusive events:

1. $P(A \cap B) = 0$
2. $P(A \cup B) = P(A) + P(B)$
3. $P(A|B) = 0$

Independent events:

1. $P(A \cap B) = P(A) \times P(B)$
2. $P(A|B) = P(A)$

Dependent events:

1. $P(A \cap B) = P(A) \times P(B|A)$

Exhaustive events:

1. $P(A \cup B \cup C \cup D) = 1$

Bayes' Theorem:

1. $P(A|B) = \frac{P(A \cap B)}{P(B)}$
2. $P(A|B) \neq P(B|A)$

Others:

1. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
2. $P(A \cap B) + P(A' \cap B) = P(B)$
3. $P(A) + P(B) - P(A \cup B) + P(A' \cup B') = 1$
4. $P(A) + P(B) - P(A \cap B) + P(A' \cap B') = 1$
5. $P(A \cap B)' = P(A' \cup B')$
6. $P(A' \cup B') + P(A \cap B) = 1$
7. $P(A' \cap B') + P(A \cup B) = 1$

Distribution	Parameter	Mean	Variance	$P(X=x)$
Bernoulli	p	p	$p(1-p)$	p
Binomial	n,p	np	$np(1-p)$	$C_x^n(1-p)^{n-x}p^x$
Geometric	p	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$(1-p)^{x-1}p$
Poisson	λ	λ	λ	$\frac{e^{-\lambda}\lambda^x}{x!}$

Normal distribution:

1. Symmetry
2. Mean = Mode = Median
3. Tend to infinity and negative infinity
4. Bell-shape curve
5. $\int_{-\infty}^{\infty} f(x)dx = 1$
6. $x \to \pm\infty,\ f(x) \to 0$
7. $X \sim N(\mu, \sigma^2)$, μ and σ^2 are the parameters

	Population	Sample
Mean	$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Variance	$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i-\mu)^2$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x-\bar{x})^2$

As for the sample variance, Why dividing by $n-1$ instead of n?

1. When sampling, extreme data is not necessarily likely to be included. Therefore, the obtained variance is likely to be smaller.
2. The formula for calculating the sample variance involves dividing by $n-1$ instead of n , as this provides an unbiased estimate of the population variance and it provides a more accurate estimate of the population variance based on the sample data

$$Var(x) = E(x^2) - E(x)^2$$

$$Var(\bar{x}) = [E(\bar{x}^2) - E(\bar{x})^2] \times \frac{n}{n-1}$$

$$E(aX + b) = aE(X) + b$$

$$Var(aX + b) = a^2 Var(X)$$

Remarks:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots, \qquad -\infty < x < \infty$$

$$(a + b)^n = C_0^n a^n + C_1^n a^{n-1} b + \cdots + C_n^n b^n$$

$$\sum_{k=1}^{n} f(k) = f(1) + f(2) + \cdots + f(n)$$

$$\sum_{n=0}^{3} 3 = 3 + 3 + 3 + 3$$

$$\prod_{k=1}^{3} 3k = \prod_{k=1}^{3} 3 \times \prod_{k=1}^{3} k$$

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

$$C_r^n + C_{r+1}^n = C_{r+1}^{n+1}$$