

Cognitive Algorithms - Assignment 1

Part 1 - Linear Algebra

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Task 1

1. 3
2. $\mathbf{v}^\top \mathbf{w} = \mathbf{w}^\top \mathbf{v}$
3. $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$
4. Commutativity
5. $\mathbf{v} = A^{-1} \mathbf{w}$
6. 1
7. $\text{rank}(A) = n \iff A$ is invertible
8. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Task 2

1. Partial derivatives:

(a)

$$\frac{\partial f(\mathbf{x})}{\partial x_j} = \frac{\partial}{\partial x_j} \mathbf{u}^\top \mathbf{x} = \frac{\partial}{\partial x_j} \sum_{k=1}^d u_k x_k = \sum_{k=1}^d u_k \frac{\partial x_k}{\partial x_j} = \sum_{k=1}^d u_k \delta_{kj} = u_j$$

(b)

$$\frac{\partial g(\mathbf{x})}{\partial x_j} = \frac{\partial}{\partial x_j} \mathbf{x}^\top \mathbf{x} = \frac{\partial}{\partial x_j} \sum_{k=1}^d x_k^2 = \sum_{k=1}^d 2x_k \frac{\partial x_k}{\partial x_j} = \sum_{k=1}^d 2x_k \delta_{kj} = 2x_j$$

2. Gradients:

$$(a) \quad \nabla f(\mathbf{x}) = (u_1, \dots, u_d)^\top = \mathbf{u}$$

$$(b) \quad \nabla g(\mathbf{x}) = (2x_1, \dots, 2x_d)^\top = 2(x_1, \dots, x_d)^\top = 2\mathbf{x}$$

Task 3

Proof.

$$\lambda \mathbf{v}^\top \mathbf{w} = (\lambda \mathbf{v})^\top \mathbf{w} = (A\mathbf{v})^\top \mathbf{w} = \mathbf{v}^\top A^\top \mathbf{w} \stackrel{A \text{ is symmetric}}{=} \mathbf{v}^\top A \mathbf{w} = \mathbf{v}^\top (\mu \mathbf{w}) = \mu \mathbf{v}^\top \mathbf{w}$$

$$\iff \lambda \mathbf{v}^\top \mathbf{w} - \mu \mathbf{v}^\top \mathbf{w} = 0$$

$$\iff (\lambda - \mu) \mathbf{v}^\top \mathbf{w} = 0$$

$$\stackrel{\lambda - \mu \neq 0}{\iff} \mathbf{v}^\top \mathbf{w} = 0$$

□