Cognitive Algorithms - Assignment 1
Part 1 - Linear Algebra

Stefan Cobeli, 5147789
Hau Ke, 337398
Michael Stenzel, 386258

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Task 1

1. 3
2. $\mathbf{v}^\intercal \mathbf{w} = \mathbf{w}^\intercal \mathbf{v}$
3. $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$
4. Commutativity
5. $\mathbf{v} = A^{-1}\mathbf{w}$
6. 1
7. $\text{rank}(A) = n \iff A$ is invertible
8. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Task 2

1. Partial derivatives:

   (a)
   $$\frac{\partial f(\mathbf{x})}{\partial x_j} = \frac{\partial}{\partial x_j}\mathbf{u}^\intercal \mathbf{x} = \frac{\partial}{\partial x_j}\sum_{k=1}^{d} u_k x_k = \sum_{k=1}^{d} u_k \frac{\partial x_k}{\partial x_j} = \sum_{k=1}^{d} u_k \delta_{kj} = u_j$$

   (b)
   $$\frac{\partial g(\mathbf{x})}{\partial x_j} = \frac{\partial}{\partial x_j}\mathbf{x}^\intercal \mathbf{x} = \frac{\partial}{\partial x_j}\sum_{k=1}^{d} x_k^2 = \sum_{k=1}^{d} 2x_k \frac{\partial x_k}{\partial x_j} = \sum_{k=1}^{d} 2x_k \delta_{kj} = 2x_j$$

2. Gradients:

   (a) $\nabla f(\mathbf{x}) = (u_1, ..., u_d)^\intercal = \mathbf{u}$

   (b) $\nabla g(\mathbf{x}) = (2x_1, ..., 2x_d)^\intercal = 2(x_1, ..., x_d)^\intercal = 2\mathbf{x}$

Task 3

Proof.

$$\lambda \mathbf{v}^\intercal \mathbf{w} = (\lambda \mathbf{v})^\intercal \mathbf{w} = (A\mathbf{v})^\intercal \mathbf{w} = \mathbf{v}^\intercal A^\intercal \mathbf{w} \overset{\text{A is symmetric}}{=} \mathbf{v}^\intercal A \mathbf{w} = \mathbf{v}^\intercal(\mu \mathbf{w}) = \mu \mathbf{v}^\intercal \mathbf{w}$$

$\iff \lambda \mathbf{v}^\intercal \mathbf{w} - \mu \mathbf{v}^\intercal \mathbf{w} = 0$
$\iff (\lambda - \mu)\mathbf{v}^\intercal \mathbf{w} = 0$
$\overset{\lambda-\mu\neq 0}{\iff} \mathbf{v}^\intercal \mathbf{w} = 0$ □