Collective risk model with **actuar**.

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Statement of the problem.

- N wildfires per year. Damage from a wildfire is X. What is aggregate annual damage from wildfires?

$$Y = X_1 + \ldots + X_N, \qquad \text{where } N \text{ is random.}$$

- N is frequency (Poisson). X is severity (Pareto). Result is Compound Poisson distribution or collective risk model.
- We want to find:

$$mean(Y) = ?, \quad var(Y) = ?,$$

$$VaR(Y, 0.99) = ?, \quad TVaR(Y, 0.99) = ?.$$

Pause 1.

Let us look into the code!

Methods from **actuar::aggregateDist**.

1. Monte Carlo.
2. FFT.
3. Panjer recursion.
4. Normal approximation based methods.

actuar::aggregateDist returns us **ecdf**.

Monte Carlo methods: description.

- Fix large number, like $K = 1000000$.
- Generate K random years. For each year generate random number of wildfires. For each wildfire generate damage.
- Complexity is: $O(K\lambda)$.

Pause 2.

Let us look into the code!

Monte Carlo methods: critics.

- How to choose K?
- Should K be connected with mean frequency?
- The method is **slow**, but **flexible**.
- What can be done further? Stratified sampling, Sobol sequence, Latin hypercube, Iman Conover, ...

FFT.

- Discretize frequency.
- Fix n, calculate p_n, probability to have exactly n wildfires.
- $Y_n = X_1 + \ldots + X_n$ is a convolution. It is calculated via FFT.

$$FFT(Y_n) = FFT(X_1) \cdot \ldots \cdot FFT(X_n)$$

$$Y_n = FFT^{-1}(FFT(X_1) \cdot \ldots \cdot FFT(X_n))$$

- complexity is $O(N \ln N)$ (for direct convolution calculation it is $O(N^3)$).

Pause 3.

Let us look into the code!

FFT, critics.

- How to choose discretization step?
- How to choose ranges?
- Is uniform discretization obligatory?
- Is it really FFT in actuar?

Panjer recursion.

- frequency from Panjer class.

$$P(N = k) = (a + b/k)P(N = k - 1).$$

- discretize severity with step h
- calculate recursively $P(S = hk)$
- frequency reduction trick: reduce frequency m times by calculating $\ln(m)/\ln(2)$ of convolutions.
- Complexity is $O(N^2)$.

Pause 4.

Let us look into the code!

Panjer recursion, critics.

- How to choose discretization range?
- How to choose discretization step?
- Should we do frequency reduction trick?
- Is uniform discretization obligatory?

Normal approximation.

Method 1.

- Calculate 2 first moments.
- Approximate by normal distribution.

Method 2.

- Calculate 3 first moments (skewness).
- Normal series approximation.

Pause 5.

Let us look into the code!

Normal approximation, critics.

- What to do if one of the moments is infinite?
- Is it OK to approximate a model with 3 parameters by 2 parameters?
- We underestimate the right tail.

Test description.

- frequency is Poisson, severity is Pareto.
- mean frequency is from 0.1 to 100.
- α from Pareto is from 2.5 to 10.
- x_m from Pareto is from 100000 to 100000000.
- 90 test cases.

Criteria:

- compare means
- compare VaR(0.95)
- compare performance

Results.

	Monte Carlo	FFT	Panjer recursion	Normal approximation
Performance	0.23 sec/0.46 sec	250/413 times slower than MC	1.3/1.6 times slower than MC	109 times faster than MC
Share of cases with correct mean	97%	20%	100%	100%
Share of cases with correct VaR.	92%	23%	100%	60%

What is missing?

- nonuniform discretizations
- algorithms for defining the default parameters
- convolution method is too tricky

What else is in **actuar**?

- Many probability distributions.
- Risk and ruin theory.
- Credibility theory.
- Support of mixture probability distributions.