Prediction Markets.

Alexey Osipov

20.02.2020

Equal probabilities experiment.

- Suppose we vote on an unbiased coin (50%-50%).
  What should be the ratio of votes?

- Suppose we bet instead of voting?
  What should be the ratio of volumes?

Inequal probabilities experiment.

- Suppose we vote on a biased coin (80%-20%).
  What should be the ratio of votes?

- Suppose we bet instead of voting?
  What should be the ratio of volumes?

What is a prediction market?

It is a market of betting on events with a random outcome. The probabilities of the outcome are identified from the bets.
In most cases the outcome probability is almost constant.

Examples of prediction markets:

1. www.betfair.com (betting)
2. www.crowdmed.com (medicine)

Non-examples:

1. stock market
2. betting company

? www.ladbrokes.com ?
Could they be used in insurance?

Bet for and bet against.

Suppose we have an outcome.

- A bet for the outcome by price k is a contract: either get $k-1$ or loose 1 depending on the event outcome.
- A bet against the outcome by price k is a contract: either loose $k-1$ or get 1 depending on the event outcome.

Naturally, they are complementary.
A bet against the outcome can be considered as the bet for the alternative outcome and vice versa (see the next slide).

Simple betting example.

Let k_1, k_2 be the prices:

$$p_1 = \frac{1}{k_1}, \quad p_2 = \frac{1}{k_2}, \qquad \frac{1}{k_1} + \frac{1}{k_2} = 1.$$

Let s_1, s_2 be the volumes. We expect:

$$s_1/p_1 = s_2/p_2$$

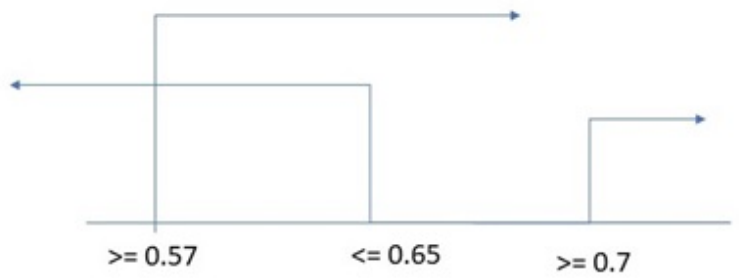
if the prices agree with probabilities.
The prices that agree with probabilities are **equilibrium prices**.
Suppose we are allowed to adjust the prices depending on the volumes. If we do it in a reasonable way, the prices will converge to **equilibrium prices**.
The question is how soon...

Components of prediction market.

- The way of matching orders of different people into bets.
- The way of identifying the outcome probability by orders and bets.
- The policy about cancellation of orders.
- The policy about visibility of orders and bets.
- The market-maker (usually, it is used).

Order book.

Number of voices	Ray	Price	Ray	Number of voices
50	<= 0.8	1.25|5		
31	<= 0.7	1.43|3.33		
85	<= 0.65	1.54|2.85		
		1.76|2.32	>= 0.57	78
		2|2	>= 0.5	96
		2.49|1.67	>= 0.4	32
		5|1.25	>= 0.2	14



Identifying the probability.

If the outcome probability does not change, at some point the prices should stabilize.
Once they are more or less stable, we can say

$$p = 1/k.$$

There are problems with this approach:

1. What to do if there are not enough orders/bets (**cold start**)?
2. What to do if bid-ask spread is too large (**non-liquid market**)?

We try to resolve this problems with a model.

Key model components.

1. Small world assumption: M is the maximal amount of capital.
2. Utility functions:

$$U(v, \lambda) = \frac{1 - e^{-\lambda x/W}}{1 - e^{-\lambda M/W}}.$$

3. Predictions of gamblers come from the same distribution (like $N(\mu, \sigma)$).
4. Maximization of a likelihood function.
5. Iterative optimization for finding μ, σ, λ.
6. Once μ (and λ, σ are found) we may define p and k by:

$$p = \mu, \qquad k = 1/p.$$