

Impressions about School on Insurance, Finance, and Risk Management.

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Presentation 2.

Tail risk measures

Risk measures.

Suppose we have data on losses. How to measure risk of this losses?

- **mean** (average loss)
- **variance** (average deviation from the average loss)
- **variance premium** (combination of the 2 measures above)

$$VP(X) = E(X) + \alpha Var(X),$$

where α is a smartly chosen constant (analog of loading?).

- **VaR** (how much money is required to cover 95% of losses for sure)

Tail risk measures.

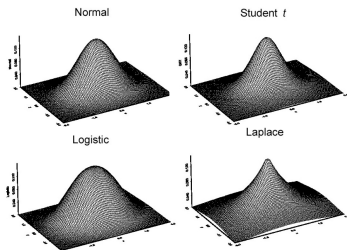
What if we are interested mostly in rare events (5% of worst cases)?

- $TE(X)$, **tail conditional expectation** (shortfall, **TVaR**),
- $TV(X)$, **tail variance**
- $TVP(X)$, **tail variance premium**

$$TVP(X) = TE(X) + \alpha TV(X).$$

Elliptical distributions.

Elliptical (mostly for finance):



Nonelliptical: (multivariate) Pareto (mostly for insurance)

Asymptotic behaviour in one-dimensional case.

Z. Landsman has results for asymptotic behaviour of $TE_q(X)$ and $TV_q(X)$ (when q approaches 1) for:

- one-dimensional normal distribution
- one-dimensional Student distribution

Multivariate tail risk measures.

- MTE (multivariate tail conditional expectation)
- MTCOV (multivariate tail covariance)
- MTCOR (multivariate tail correlation)

Z. Landsman has more or less explicit formulas for this tail risk measures for elliptical distributions.

Risk measures and portfolio management.

We have a number of assets/stocks: X_1, \dots, X_n . Our goal is to construct optimal portfolio out of this assets:

$$X = w_1 X_1 + \dots + w_n X_n,$$

$$\sum_{i=1}^n w_i = 1.$$

so that certain risk measure is optimized:

$$\rho(X) \longrightarrow \min.$$

Z. Landsman has results for portfolio optimization for various risk measures (all the risk measures discussed in the talk).

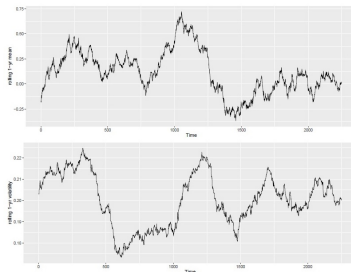
Presentation 3.

Options for dependent assets.

Problems from financial mathematics

- 1 **Portfolio selection.** Which stocks will go up, which will go down?
 - Individual stocks are studied separately.
 - Volatility and average return.
 - Dynamics of the stock price process.
- 2 **Systemic risk measurement.** How likely are the stocks to move (down) together?
 - Dependence between stocks is studied.
 - Multivariate modeling.
 - Copulas.

Estimating mean and variance.



Estimating mean is much more difficult than estimating variance in the world of finance.

Options.

- 1 A derivative is an asset which value depends on values of other more basic/underlying assets.
- 2 **European call option.** The **right** to buy asset S by strike price K at date T .
- 3 **European put option.** The **right** to sell asset S by strike price K at date T .
- 4 **American options** are almost the same. Not at date T but no later than date T .
- 5 What if S is a basket? That is a combination of stocks, like an index...

Black-Scholes model.

- **Put-call parity.**

$$C(K) = P(K) + S - e^{-rT} K.$$

- **Black-Scholes equation.**

The option price is a solution of PDE (with boundary conditions depending on the option type)

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

- **Black-Scholes model.**

$$C(K) = N(d_1)S - N(d_2)e^{-rT} K.$$

- Only volatility/variance in formulas, no means. We are **lucky!**



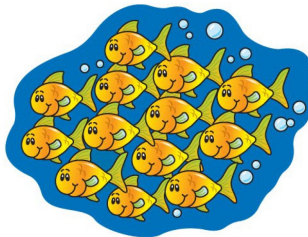
Options on indices.

- Index is a weighted sum of stocks:

$$I = w_1 S_1 + \dots + w_n S_n.$$

- Index shows the power of economy/industry.
- Indices are not traded.
- Options on indices are traded.
- Stocks are dependent.
- Not all strikes are traded for all stocks.
- Choice 1. Approximate the option price.
- Choice 2. Derive the upper bound for it.

Comonotonicity.



- All stocks are the same as in the original market situation.
- But they are moving in the same direction.
- All stocks depend on the same random source U .
 $F_{X_i}^{-1}(U)$ are values for stock S_i .

Use of comonotonicity.

- 1 We can calculate easily options on comonotonic stocks.
- 2 The price of an option for real (non-comonotonic stocks) is less than the price in the comonotonic situation.

$$C(K) \leq \sum_{i=1}^n w_i C_i(K_i^*)$$

(result by Daniel Linders)

- 3 Stress test.

Measuring herd behavior.

- How well do the stocks move together?
- Size of the gap between the real world and the comonotonic world.
- **The ATM comonotonicity gap.**

$$ATMGAP = \frac{C_{real}(K)}{C_{comon}(K)}$$

- **Herd behavior index, HIX, Comonotonicity index, CIX.**
The same but for $u\left(\frac{S}{ES}\right)$ for index S for u chosen in a smart way, like:

$$u(x) = (x - 1)^2, \quad \text{or} \quad u(x) = -2 \ln(x).$$

- **DHIX, downside herd behavior index.**
How well do the stocks move down together?

Presentation 1.

Claims evaluation

Claims: goal and examples.

- We want to estimate the current fair values of the future claims (e.g., in 1 year).
- X is euro-dollar exchange rate in a year
- Y is 1 euro if insured dies within 1 year, otherwise it is 0
- $X \cdot Y$ is the insured contract in dollars.

Evaluation: questions.

- 1 **Financial quant.** What is a price of claim when traded in an arbitrage-free market?
- 2 **Traditional actuary.** What is a price for taking over the liability ignoring the financial market except the risk-free bank account?
- 3 **Modern actuary.** What is a price for taking over the liability taking into account hedging opportunities in the financial market?

Non-arbitrage market.

- we have $n + 1$ assets, we want to connect there values at different time moments
- r is the risk free rate, time period is 1 (e.g., year)
- the assets are liquid
- the assets are not-redundant
- the market is *arbitrage-free*: starting from 0 we can not make some profit with some probability with probability 1 of not losing

Financial valuation.

- Suppose that market is arbitrage-free.
- **Hedgeable claim** is a claim that is a result of some trading strategy involving the assets in the market.
- **Hedgeable claim:** price of euro-dollar in a year.
- **Non-hedgeable claim:** price of euro-dollar now, life insurance contract.
- Hedgeable claims can be evaluated using techniques from financial mathematics (EMM-measures).

$$\text{value}(S) = e^{-r} E^Q(S).$$

- But non hedgeable claims cannot be estimated this way.

Actuarial valuation.

- We have a market of n traded assets/claims.
- **Orthogonal claim** is a claim that is independent of the traded assets.
- **Orthogonal claim:**
Y is 1 euro if insured dies within 1 year, otherwise it is 0
- **Non-orthogonal claim:** euro-dollar rate in a year.
- Orthogonal claims can be evaluated using techniques from actuarial mathematics (actuarial model, risk margin)

$$value(S) = e^{-r}(E^P(S) + RM(S)).$$

Examples of risk margins.

- r is the risk-free rate.
- Cost-of-capital principle:

$$RM(S) = i(VaR_{\alpha}^P(S) - E^P(S)),$$

where i is the cost-of-capital rate.

- Standard deviation principle:

$$RM(S) = \alpha \sigma^P(S).$$

Valuation of hybrid claims.

Karim Barigou and Jan Dhaene offer a way of evaluating some types of **hybrid claims**. These claims have both hedgeable and orthogonal parts. It is the combination of financial and actuarial evaluations.

Typical examples are:

- **Hedgeable claim:** X is euro-dollar exchange rate in a year
- **Orthogonal claim:** Y is 1 euro if insured dies within 1 year, otherwise it is 0
- **Hybrid claim:** $X \cdot Y$ is the insured contract in dollars.

$$S = S^h \times S^o,$$

$$\text{value}(S) = e^{-r} E^Q(S^h) \times E^P(S^o).$$

Case of several years is also considered.

Brief summary.

- 1 Tail risk measures are important.
- 2 We can calculate them in the multivariate case as well.
Can it be used in Actus?
- 3 In financial and actuarial worlds claims are valued differently. It is possible to combine two approaches.
- 4 If we have a lot of random variables like stocks that are dependent on each other, we can try to assume some artificial dependency, like comonotonicity to get an upper bound.
- 5 We can measure the gap between the real world and the comonotonic world.