Impressions about School on Insurance, Finance, and Risk Management.

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Presentation 2.

Tail risk measures

Risk measures.

Suppose we have data on losses. How to measure risk of this losses?

- **mean** (average loss)
- **variance** (average deviation from the average loss)
- **variance premium** (combination of the 2 measures above)

  $$VP(X) = E(X) + \alpha Var(X),$$

  where α is a smartly chosen constant (analog of loading?).
- **VaR** (how much money is required to cover 95% of losses for sure)

Tail risk measures.

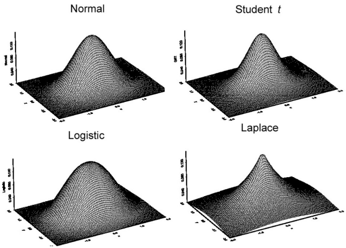
What if we are interested mostly in rare events (5% of worst cases)?

- TE(X), **tail conditional expectation** (shortfall, **TVaR**),
- TV(X), **tail variance**
- TVP(X), **tail variance premium**

$$TVP(X) = TE(X) + \alpha TV(X).$$

Elliptical distributions.

Elliptical (mostly for finance):



Nonelliptical: (multivariate) Pareto (mostly for insurance)

Asymptotic behaviour in one-dimensional case.

Z. Landsman has results for asymptotic behaviour of $TE_q(X)$ and $TV_q(X)$ (when q approaches 1) for:

- one-dimensional normal distribution
- one-dimensional Student distribution

Multivariate tail risk measures.

- MTE (multivariate tail conditional expectation)
- MTCOV (multivariate tail covariance)
- MTCOR (multivariate tail correlation)

Z. Landsman has more or less explicit formulas for this tail risk measures for elliptical distributions.

Risk measures and portfolio management.

We have a number of assets/stocks: $X_1, \ldots, X_n$. Our goal is to construct optimal portfolio out of this assets:

$$X = w_1 X_1 + \ldots + w_n X_n,$$

$$\sum_{i=1}^{n} w_i = 1.$$

so that certain risk measure is optimized:

$$\rho(X) \longrightarrow \min .$$

Z. Landsman has results for portfolio optimization for various risk measures (all the risk measures discussed in the talk).

Presentation 1.

Claims evaluation

Claims: goal and examples.

- We want to estimate the current fair values of the future claims (e.g., in 1 year).
- X is euro-dollar exchange rate in a year
- Y is 1 euro if insured dies within 1 year, otherwise it is 0
- $X \cdot Y$ is the insured contract in dollars.

Evaluation: questions.

1. **Financial quant.** What is a price of claim when traded in an arbitrage-free market?
2. **Traditional actuary.** What is a price for taking over the liability ignoring the financial market except the risk-free bank account?
3. **Modern actuary.** What is a price for taking over the liability taking into account hedging opportunities in the financial market?

Non-arbitrage market.

- we have $n+1$ assets, we want to connect there values at different time moments
- r is the risk free rate, time period is 1 (e.g., year)
- the assets are liquid
- the assets are not-redundant
- the market is *arbitrage-free*: starting from 0 we can not make some profit with some probability with probability 1 of not losing

Financial valuation.

- Suppose that market is arbitrage-free.
- **Hedgeable claim** is a claim that is a result of some trading strategy involving the assets in the market.
- **Hedgeable claim:** price of euro-dollar in a year.
- **Non-hedgeable claim:** price of euro-dollar now, life insurance contract.
- Hedgeable claims can be evaluated using techniques from financial mathematics (EMM-measures).

$$value(S) = e^{-r} E^{Q}(S).$$

- But non hedgeable claims cannot be estimated this way.

Actuarial valuation.

- We have a market of n traded assets/claims.
- **Orthogonal claim** is a claim that is independent of the traded assets.
- **Orthogonal claim:**
  Y is 1 euro if insured dies within 1 year, otherwise it is 0
- **Non-orthogonal claim:** euro-dollar rate in a year.
- Orthogonal claims can be evaluated using techniques from actuarial mathematics (actuarial model, risk margin)

$$value(S) = e^{-r}(E^P(S) + RM(S)).$$

Examples of risk margins.

- r is the risk-free rate.
- Cost-of-capital principle:

$$RM(S) = e^{-r} i(VaR_{\alpha}^{P}(S) - E^{P}(S)),$$

where i is the cost-of-capital rate.

- Standard deviation principle:

$$RM(S) = e^{-r}(E^{P}(S) + \alpha\sigma^{P}(S)).$$

- Or just

$$RM(S) = 2e^{-r}\sigma^{P}(S).$$

Valuation of hybrid claims.

Karim Barigou and Jan Dhaene offer a way of evaluating some types of **hybrid claims**. This claims have both hedgeable and orthogonal parts. It is the combination of financial and actuarial evaluations.

Typical examples are:

- **Hedgeable claim:** X is euro-dollar exchange rate in a year
- **Orthogonal claim:** Y is 1 euro if insured dies within 1 year, otherwise it is 0
- **Hybrid claim:** $X \cdot Y$ is the insured contract in dollars.

$$S = S^h \times S^o,$$

$$value(S) = e^{-r} E^Q(S^h) \times E^P(S^o).$$

Case of several years is also considered.

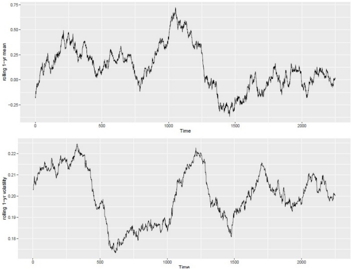
Presentation 3.

Options for dependent assets.

Problems from financial mathematics

1. **Portfolio selection.** Which stocks will go up, which will go down?
   - Individual stocks are studied separately.
   - Volatility and average return.
   - Dynamics of the stock price process.
2. **Systemic risk measurement.** How likely are the stocks to move (down) together?
   - Dependence between stocks is studied.
   - Multivariate modeling.
   - Copulas.

Estimating mean and variance.



Estimating mean is much more difficult than estimating variance in the world of finance.

Options.

1. A derivative is an asset which value depends on values of other more basic/underlying assets.
2. **European call option.** The **right** to buy asset S by strike price K at date T.
3. **European put option.** The **right** to sell asset S by strike price K at date T.
4. **American options** are almost the same. Not at date T but no later than date T.
5. What if S is a basket? That is a combination of stocks, like an index...

Black-Scholes model.

- **Put-call parity.**

$$C(K) = P(K) + S - e^{-rT}K.$$

- **Black-Scholes equation.**
  The option price is a solution of PDE (with boundary conditions depending on the option type)

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

- **Black-Scholes model.**

$$C(K) = N(d_1)S - N(d_2)e^{-rT}K.$$

- Only volatility/variance in formulas, no means. We are **lucky**!

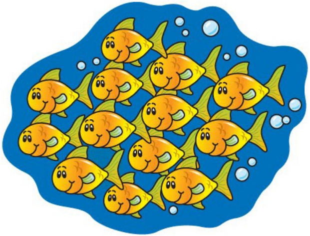
Options on indices.

- Index is a weighted sum of stocks:

$$I = w_1 S_1 + \ldots + w_n S_n.$$

- Index shows the power of economy/industry.
- Indices are not traded.
- Options on indices are traded.
- Stocks are dependent.
- Not all strikes are traded for all stocks.
- Choice 1. Approximate the option price.
- Choice 2. Derive the upper bound for it.

Comonotonicity.



- All stocks are the same as in the original market situation.
- But they are moving in the same direction.
- All stocks depend on the same random source U. $F_{X_i}^{-1}(U)$ are values for stock S_i.

Use of comonotonicity.

1. We can calculate easily options on comonotonic stocks.
2. The price of an option for real (non-comonotonic stocks) is less than the price in the comonotonic situation.

$$C(K) \leq \sum_{i=1}^{n} w_i C_i(K_i^*)$$

   (result by Daniel Linders)
3. Stress test.

Measuring herd behavior.

- How well do the stocks move together?
- Size of the gap between the real world and the comonotonic world.
- **The ATM comonotonicity gap.**

$$ATMGAP = \frac{C_{real}(K)}{C_{comon}(K)}$$

- **Herd behavior index, HIX, Comonotonicity index, CIX.** The same but for $u\left(\frac{S}{ES}\right)$ for index S for u chosen in a smart way, like:

$$u(x) = (x-1)^2, \quad \text{or} \quad u(x) = -2\ln(x).$$

- **DHIX, downside herd behavior index.** How well do the stocks move down together?

Brief summary.

1. Tail risk measures are important.
2. We can calculate them in the multivariate case as well. Can it be used in Actus?
3. In financial and actuarial worlds claims are valuated differently. It is possible to combine two approaches.
4. If we have a lot of random variables like stocks that are dependent on each other, we can try to assume some artifical dependency, like comonotonicity to get an upper bound.
5. We can measure the gap between the real world and the comonotonic world.