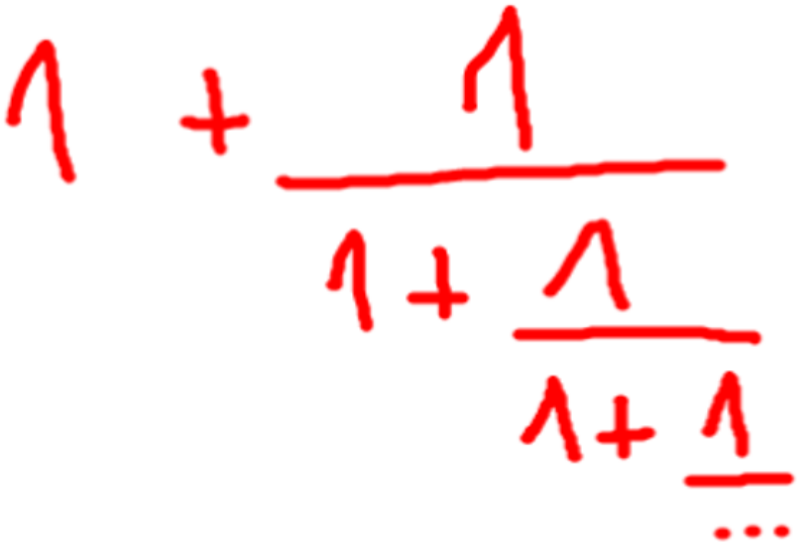
Karlsruhe Institute of Technology

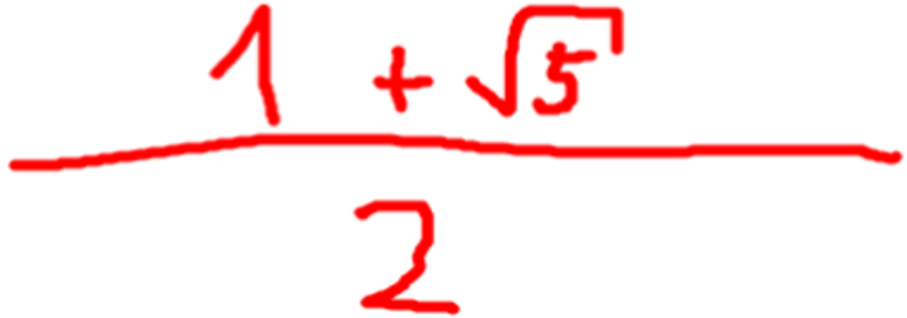
Carnegie Mellon University

On-line Recognition of Handwritten Mathematical Symbols

Bachelor's thesis of Martin Thoma

Martin Thoma | 5th of June, 2014





Contents

What is my Bachelor's thesis about?

- Recognition of handwritten mathematical symbols
- On-line recognition, not OCR!
- Given a series of points $(x(t), y(t), b(t))$
  I want to get the LaTeX command.

Why did I work on this topic?

- LaTeX is easy as soon as you know the \commands.
- It's hard to find the LaTeX command of single symbols.
- It's much harder to find complete formulas.

For now: recognition of isolated symbols.

Preprocessing Algorithms

- Normalizing
  - Scaling
  - Shifting
  - Resampling
- Noise reduction
  - Smoothing (e.g. moving average)
  - Dot reduction
  - Filtering (by distance, speed or angle)
  - Stroke connection

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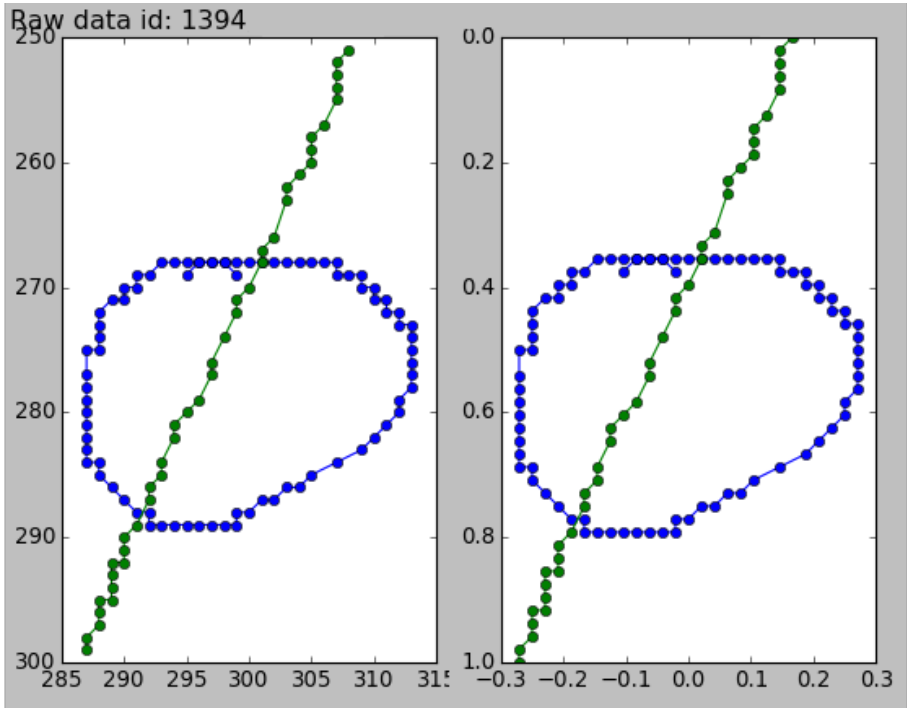

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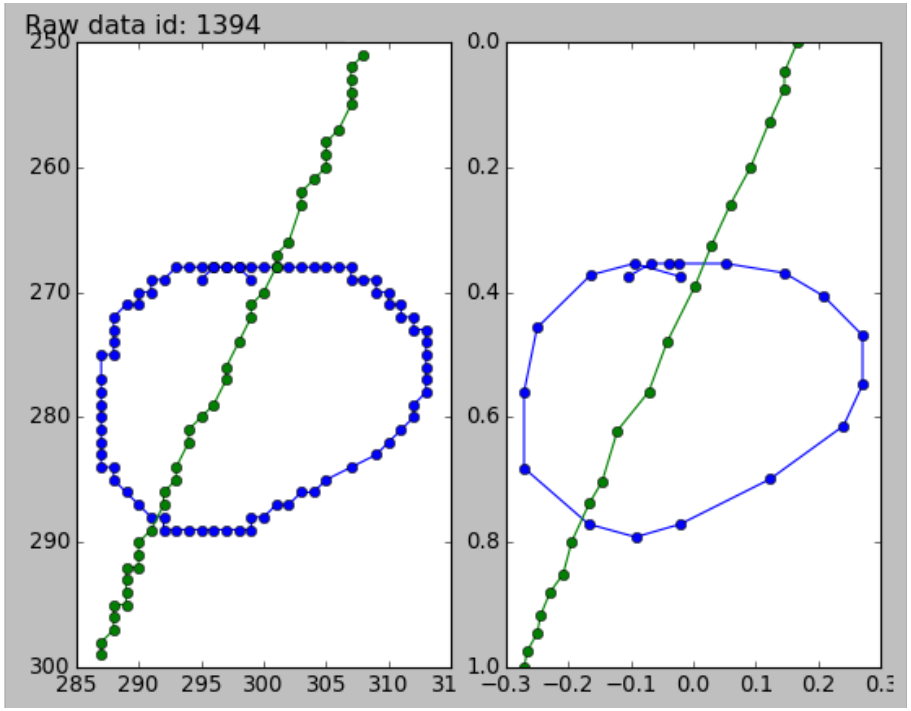

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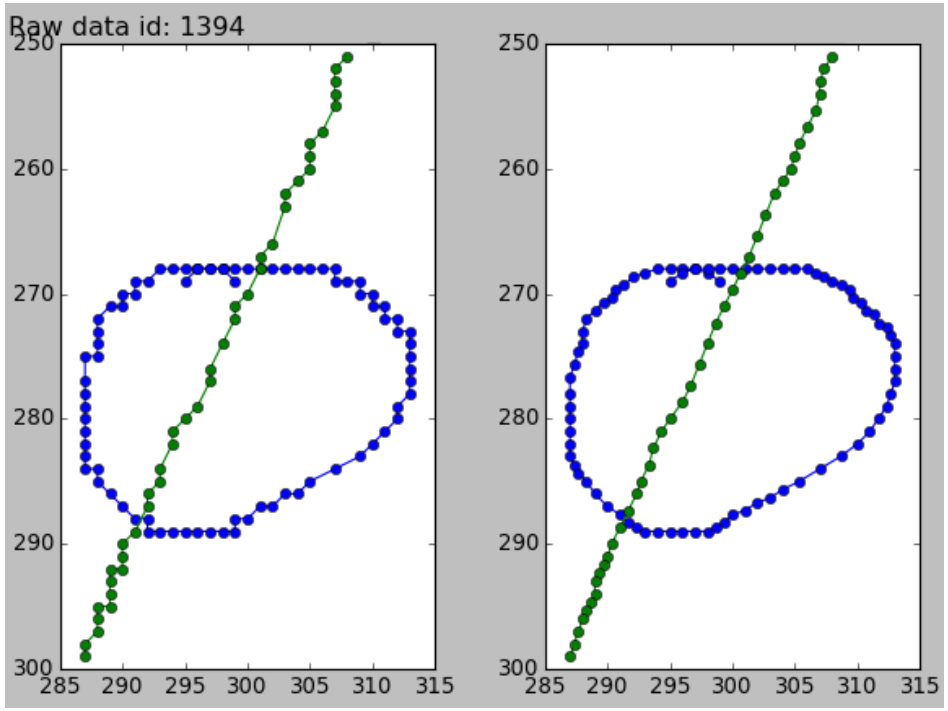

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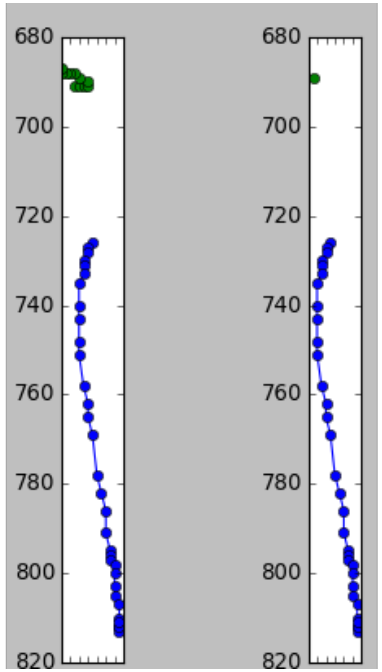

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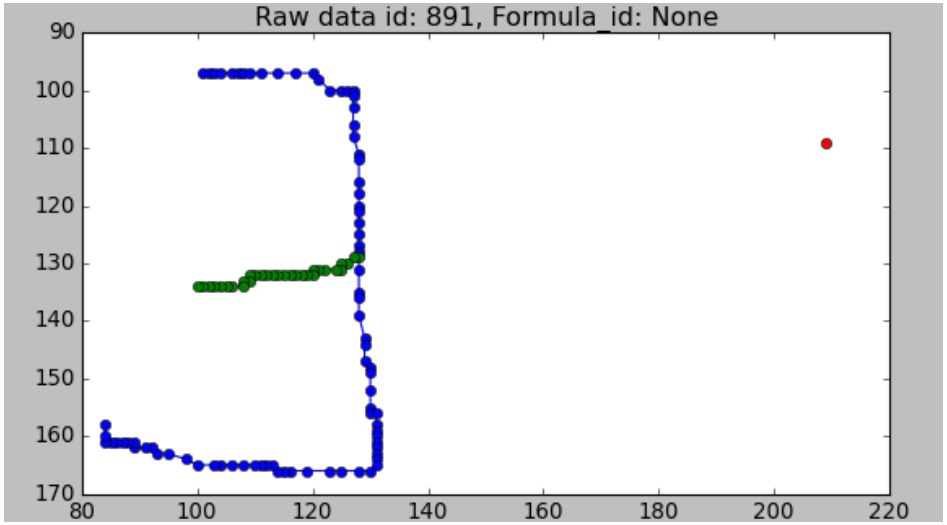

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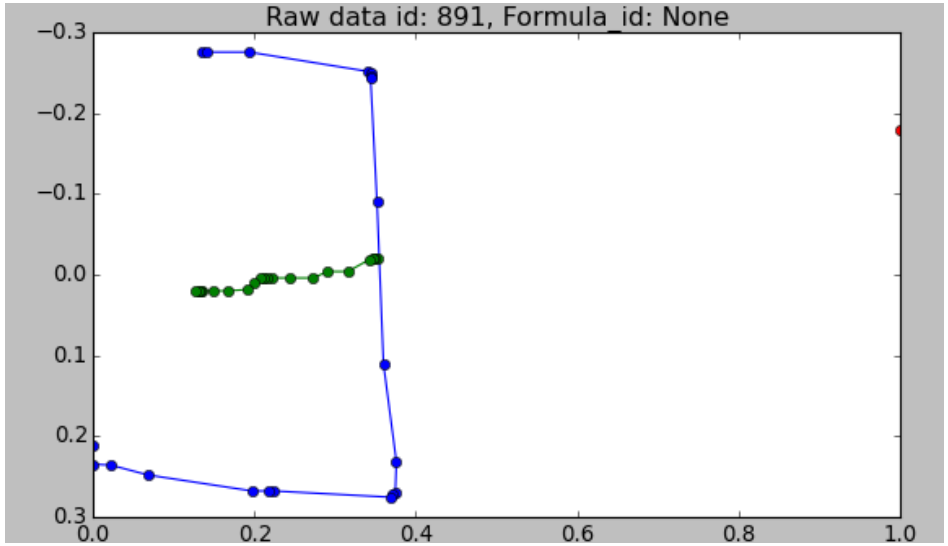

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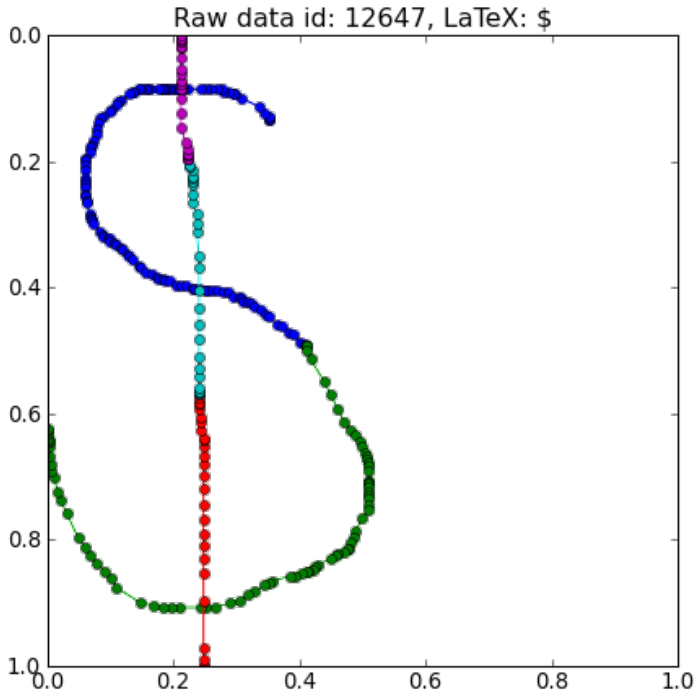

Features

Local Features

- Coordinates
- Speed
- Binary pen pressure
- Direction
- Curvature
- Bitmap-environment
- Hat-Feature

Global Features

- # of dots (i, j, $\therefore$, $\because$, ...)
- # of strokes
- Center point coordinates
- Bitmap
- Bounding box (width, height, time)
- Re-curvature per stroke s $\left(\frac{\mathsf{height}(s)}{d(s[0],s[-1])}\right)$
- Ink

Baseline Systems

Preprocessing: Scaling, shifting and linear interpolation
Features: Coordinates of 80 points (4 strokes with 20 points each)
Learning: MLP, 1000 epochs, LR $\eta = 0.1$, Momentum $\alpha = 0.1$

System	Topology	Classification error		
		TOP1	TOP3	MER
B_1	160:500:369	23.34 %	6.80 %	6.64 %
B_2	160:500:500:369	21.51 %	5.75 %	5.67 %
B_3	160:500:500:500:369	21.93 %	5.74 %	5.64 %

Table: Baseline systems with three different classification error measures. All errors were measured on the test set.

Merged symbols (MER error)

Base symbol		equivalent symbols	
LaTeX	Rendered	LaTeX	Rendered
\sum	$\sum$	Σ	Σ
\prod	$\prod$	Π	Π
		$\sqcap$	$\sqcap$
\coprod	$\coprod$	$\amalg$	$\amalg$
		$\sqcup$	$\sqcup$
\perp	$\perp$	$\bot$	$\bot$
\models	$\models$	$\vDash$	$\vDash$
|	$\|$	\mid	$\mid$
\Delta	Δ	$\triangle$	$\triangle$
		$\vartriangle$	$\vartriangle$

Merged symbols (MER error)

Base symbol		equivalent symbols	
LaTeX	Rendered	LaTeX	Rendered
`\\|`	$\|$	`$\parallel$`	$\parallel$
`\ohm`	$\ohm$	`$\Omega$`	Ω
`\setminus`	$\setminus$	`$\backslash$`	$\backslash$
`\checked`	$\checked$	`$\checkmark$`	$\checkmark$
`\&`	$\&$	`$\with$`	$\with$
`\#`	$\#$	`$\sharp$`	$\sharp$
`\S`	$\S$	`$\mathsection$`	$\mathsection$
`\nabla`	∇	`\triangledown`	$\triangledown$
`\lhd`	$\lhd$	`$\triangleleft$`	$\triangleleft$
		`$\vartriangleleft$`	$\vartriangleleft$
`\oiint`	$\oiint$	`$\varoiint$`	$\varoiint$

Merged symbols (MER error)

Base symbol		equivalent symbols	
LaTeX	Rendered	LaTeX	Rendered
`\mathbb{R}`	$\mathbb{R}$	`$\mathds{R}$`	$\mathds{R}$
`\mathbb{Q}`	$\mathbb{Q}$	`\mathds{Q}`	$\mathds{Q}$
`\mathbb{Z}`	$\mathbb{Z}$	`\mathds{Z}`	$\mathds{Z}$
`\mathcal{A}`	$\mathcal{A}$	`\mathscr{A}`	$\mathscr{A}$
`\mathcal{D}`	$\mathcal{D}$	`\mathscr{D}`	$\mathscr{D}$
`\mathcal{N}`	$\mathcal{N}$	`\mathscr{N}`	$\mathscr{N}$
`\mathcal{R}`	$\mathcal{R}$	`\mathscr{R}`	$\mathscr{R}$
`\propto`	$\propto$	`$\varpropto$`	$\varpropto$

Preprocessing: Connect strokes

System	Classification error					
	TOP1	change	TOP3	change	MER	change
$B_{1,\theta=5\,\text{px}}$	23.27 %	−0.07 %	6.50 %	−0.30 %	6.37 %	−0.27 %
$B_{2,\theta=5\,\text{px}}$	21.20 %	−0.31 %	5.59 %	−0.16 %	5.50 %	−0.17 %
$B_{3,\theta=5\,\text{px}}$	21.80 %	−0.13 %	5.54 %	−0.20 %	5.47 %	−0.17 %
$B_{1,\theta=10\,\text{px}}$	23.17 %	−0.17 %	6.61 %	−0.19 %	6.47 %	−0.17 %
$B_{2,\theta=10\,\text{px}}$	20.97 %	−0.54 %	5.43 %	−0.32 %	5.34 %	−0.33 %
$B_{3,\theta=10\,\text{px}}$	21.34 %	−0.59 %	5.42 %	−0.32 %	5.33 %	−0.31 %
$B_{1,\theta=20\,\text{px}}$	22.81 %	−0.53 %	6.28 %	−0.52 %	6.19 %	−0.45 %
$B_{2,\theta=20\,\text{px}}$	21.61 %	0.10 %	5.79 %	0.04 %	5.69 %	0.02 %
$B_{3,\theta=20\,\text{px}}$	21.71 %	−0.22 %	5.55 %	−0.19 %	5.45 %	−0.19 %

Table: Models B_1–B_4 with additionally applied stroke connect algorithm.

Learning: Supervised layer-wise pretraining

System	Classification error					
	TOP1	change	TOP3	change	MER	change
B_1	23.34 %		6.80 %		6.64 %	
$B_{2,p}$	19.89 %	−1.62 %	4.76 %	−0.99 %	4.68 %	−0.99 %
$B_{3,p}$	19.43 %	−2.50 %	4.64 %	−1.10 %	4.54 %	−1.10 %

Table: Supervised layer-wise pretraining, 1000 epochs per layer

Optimized classifier

Preprocessing: Connect strokes, scale, shift and linear interpolation
Features: Coordinates of 80 points (4 strokes with 20 points each), re-curvature per stroke, ink, stroke count, aspect ratio
Learning: MLP, 1000 epochs, LR $\eta = 0.1$, Momentum $\alpha = 0.1$, supervised layer-wise pretraining

System	Classification error					
	TOP1	change	TOP3	change	MER	change
$B_{1,c}$	20.96 %	−2.38 %	5.24 %	−1.56 %	5.13 %	−1.51 %
$B_{2,c}$	18.26 %	−3.25 %	4.07 %	−1.68 %	3.98 %	−1.69 %
$B_{3,c}$	18.19 %	−3.74 %	4.06 %	−1.68 %	3.99 %	−1.65 %

Table: Error rates of the complex recognizer systems.

HWRT and write-math.com

Two software projects were created:

- write-math.com: A website where on-line handwritten data gets collected and classified
- hwrt: The *handwriting recognition toolkit* is a Python project for handwriting recognition

This presentation and the bachelor's thesis will be at martin-thoma.com/write-math.

Image Sources

- Server by RRZEicons
- Desktop Computer by Ed g2s, Ironbrother, Kierancassel and Msgj
- Server by Mimooh

Thanks for Your Attention!