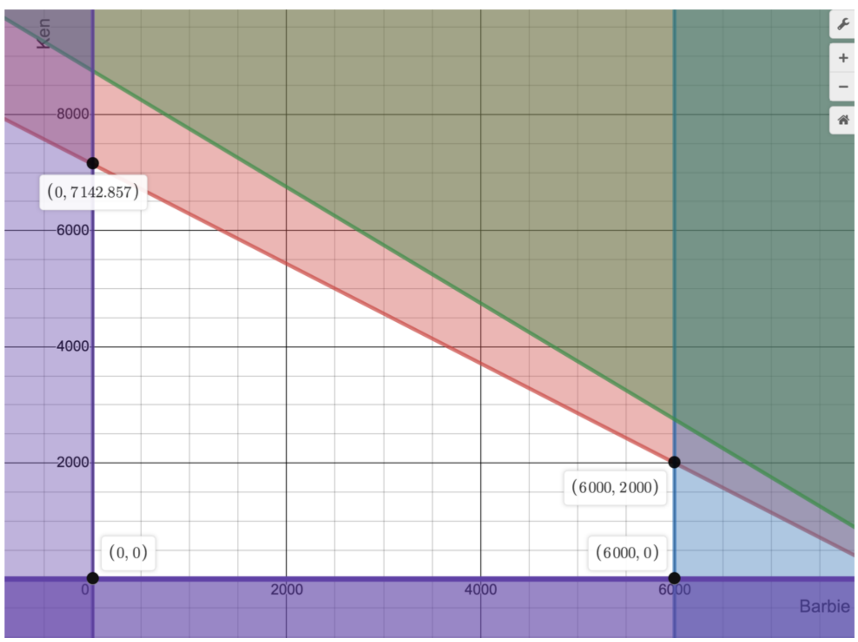
Week 4: Chapter 4 – Simplex method for standard problems

Barbie and Ken (again!)

```
B = the number of Barbies to make per week
K = the number of Kens to make per week
Maximize Z =     6B + 6.5K                (profit $)
Subject to:     12B +  14K ≤ 100,000  (plastic oz)
                 5B        ≤ 30,000   (nylon oz)
                 4B +   4K ≤ 35,000   (cardboard oz)
                  B ≥ 0 and K ≥ 0     (non-negativity)
```

The idea geometrically

(1) Start at a corner (namely the origin).
(2) Pick the direction (edge) giving the largest increase in the objective
(3) Find the nearest intersection (corner) in that direction
(4) Check to see if we've found the best objective value



Translating to algebra, i.e., make it programmable

```
Z -   6B - 6.5K                  = 0
-------------------------------------------
     12B +  14K + s1             = 100,000
      5B              + s2       = 30,000
      4B +   4K            + s3 = 35,000
```

Non-Basic Variables	Basic Variables (AKA Basis)	Feasible?	Objective? (Z=?)
B = 0, K = 0	s1 = 100000, s2 = 30000, s3 = 35000	Yes	0
B = 0, s1 = 0	K = 7142.86, s2 = 30000, s3 = 6428.57	Yes	46428.57
B = 0, s2 = 0	not possible (see constraint 2)	No	xxx
B = 0, s3 = 0	K = 8750, s1 = -22500, s2 = 30000	No (s1 < 0)	xxx
K = 0, s1 = 0	B = 8333.33, s2 = -11666.67, s3 = 1666.67	No (s2 < 0)	xxx
K = 0, s2 = 0	B = 6000, s1= 28000, s3 = 11000	Yes	36000
K = 0, s3 = 0	B = 8750, s1 = -5000, s2 = -13750	No (s2 < 0)	xxx
s1 = 0, s2 = 0	B = 6000, K= 2000, s3 = 3000	Yes	49000
s1 = 0, s3 = 0	B = 11250, K= -2500, s2 = -26250	No (s2< 0)	xxx
s2 = 0, s3 = 0	B = 6000, K = 2750, s1 = -10500	No (s1 < 0)	xxx

Mirroring the Geometry

```
    Z -   6B - 6.5K                  = 0         (profit)
        12B +  14K + s1              = 100,000   (plastic)
          5B            + s2         = 30,000    (nylon)
          4B +   4K          + s3    = 35,000    (cardboard)
```

Start at the origin adding slack variables (Initialize):

	B	K	s1	s2	s3	rhs
Z	-6	-6.5	0	0	0	0
s1	12	14	1	0	0	100000
s2	5	0	0	1	0	30000
s3	4	4	0	0	1	35000

(2) Optimality? If not, pick the direction (edge) giving the largest increase in the objective (Entering Variable)

	B	K	s1	s2	s3	rhs
Z	-6	-6.5	0	0	0	0

(3) Find the nearest intersection (corner) in that direction (Leaving Variable)

s1	12	14	1	0	0	100000
s2	5	0	0	1	0	30000
s3	4	4	0	0	1	35000

(4) Check to see if we've found the best objective value (Pivot and check)

	B	K	s1	s2	s3	rhs
Z	-6	-6.5	0	0	0	0
s1	12	14	1	0	0	100000
s2	5	0	0	1	0	30000
s3	4	4	0	0	1	35000

	B	K	s1	s2	s3	rhs
Z						
K						
s2						
s3						

	B	K	s1	s2	s3	rhs
Z						
K						
s2						
s3						

Summary of Simplex Method
(1) Build the initial tableau, adding slack variables for each constraint and rearranging the objective.
(2) Choose the entering variable by finding the largest negative coefficient in the objective. Circle that column.
(3) Choose the leaving variable by finding the smallest non-negative $\frac{rhs}{entering\ variable\ coefficient}$. Circle that row.
(4) Pivot the tableau by changing the leaving variable to the entering variable in the first column and then unitizing the entering variable column.
(5) If there is still a negative in the objective row, go to step 2, if not the optimal corner has been found.

Another Example: Maximize $3x + 2y$ such that $x + y \leq 10$ and $2x + y \leq 16$

What weirdness might happen:

1) Tie for entering variable

	X	Y	S1	S2	S3	rhs
Z	-2	-2	0	0	0	0
S1	4	3	1	0	0	62
S2	1	3	0	1	0	50
S3	0	1	0	0	1	20

2) Tie for leaving variable

	X	Y	S1	S2	S3	rhs
Z	0	-3	2	0	0	4
X	1	2	-1	0	0	20
S2	0	3	2	1	0	30
S3	0	6	-2	0	1	120

3) No leaving variable

	X	Y	S1	S2	S3	rhs
Z	0	-3	2	0	2	4
X	1	-2	-1	0	-2	11
S2	0	-1	2	1	1	12

4) Multiple optimal solutions

	X	Y	S1	S2	S3	rhs
Z	0	3	0	0	2	4
X	1	2	-1	0	-2	11
S2	0	1	2	1	1	12

5) Other forms of the problem (minimize objective; = or ≥ constraints… Next time!