Modeling Scenarios

Pollution through a Chain of Lakes. - Tables 1, 2, 3

We are monitoring the pollution levels in a chain of three lakes that connect to each other via rivers. If we consider only the flow of water between the lakes, we could model the flow of pollutant between the lakes if we know the average volume of water in each lake and the water flow rates between the lakes.

Suppose we have the following information about flow rate per volume: Preston Lake is spring fed with fresh water; 20% of Preston Lake flows through the river into Henry Lake each month, 18% of Henry Lake flows into Meadow Lake each month, and 16% of Meadow Lake is flushed into the ocean each month.

1. If 600 kg of pollutant is spilled into Preston Lake how long will it take for the pollution to wash out to sea?
2. If 100 kg per month of pollutant is dumped into Preston Lake for 5 months before the leaky pipe is found, will pollution levels ever exceed 200 kg in any of the lakes? How long before the pollution is essentially washed out to sea?
3. The percentages of water that flows between the lakes and out to sea assume a normal rainfall year, with the numbers falling as much as 5% lower in dry years and increasing as much as 3% in rainy years. How sensitive to the amount of rainfall are the answers to the questions?

Death due to Ebola. - Tables 4, 5

You are a public health expert with *Doctors Without Borders* and you have been sent to west Africa to help with treating Ebola patients. If just one copy of the virus enters the body, the virus can replicate rapidly, with essentially a 100% increase (i.e., doubling) each hour. The body's immune system in a healthy person can combat the virus once it detects the virus, but the virus is not detectable until 1 million copies are floating in the body. One of the first actions of the immune system is to raise the body temperature, which lowers the reproductive rate to 50% per hour. Once the immune system reaches its maximum efficiency (after about 12 hours from first detecting the virus), the immune system can kill about 200,000 copies of the virus per hour. Once one billion copies of the virus are present, the virus cannot be stopped and the patient will die. With this information determine the following.

1. When will the immune response kick in? When will it reach maximum efficiency? Will the patient die? If so when?

2. The drug Ebanga is approved to treat the Zaire variant of ebola. The drug does not impact the virus reproductive rate, but does assist the immune system in killing the virus by increasing the kill rate to 500 million copies of the virus per hour. What is the latest time we can administer Ebanga and save the patient's life?

3. The immune system varies in people depending on their state of health at the time of infection. If the person is an infant or elderly or suffering from some level of malnutrition, the immune system may be up to 30% less effective at killing copies of the virus; in addition it may take up to 2 full days to reach maximum effectiveness. How sensitive to the effectiveness of the immune system are the answers to the questions?

The Cooling Cadaver. - Tables 6, 7

You have been called in to investigate a death. Here are the particulars: The body was found at 1:00 AM, and the coroner arrived at 1:15 AM. At 1:30 AM he took a core temperature reading and the body was 32° C. Sixty minutes later, the body had cooled 2 more degrees C; sixty minutes after that, the body had cooled 1.75 more degrees C. The temperature in the room was 25° C at 1:30 AM, and was holding steady. One simple model that we have seen already is Newton's Law of Cooling, which in differential equation form is:

$$T'(t) = k(A - T)$$

where T is the temperature of the body, A is the ambient temperature in the room, and k is the constant of proportionality.

1. Use the data for the body temperature given at the three measurements to determine an estimate for k.
2. Assuming that the person when alive had a "normal" core temperature of 37° C, estimate the time of death within 5 minutes.
3. There is some research that implies that the normal core body temperature of humans can vary from 36.5° to 38.5° C. Given this information, what would you suggest as the most probable time of death?