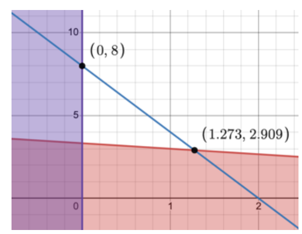
Week 5 Notes
1) Handling non-standard forms (≥, = constraints; minimizations; unrestricted variables), and generalizing the simplex method
2) Interpreting basic sensitivity information graphically and from the solver.

(1) Non-standard forms:

- Minimize Z: Change to maximize $-Z$
- = Constraints: Add an "artificial" variable with a LARGE negative Z-coefficient.
- ≥ Constraints: Add a "surplus" variable AND an "artificial" variable.
- Unrestricted in sign variables:
  - IF there is a lower limit L to the amount allowed, then Replace x_i with $x'_i + L$ to make the new variable meet the non-negativity requirement.
  - If there is no lower limit, replace x_i with $x_i^+ - x_i^-$

First Approach: "BigM" – i.e., Add more variables and "penalize" them

Let's look at Problem 3 from HW2: Serving steak and potatoes with one change: Suppose we want exactly 40 oz of protein.



x_1 = ounces of steak to serve; x_2 = ounces of potatoes to serve

Min cost $Z = 8\,x_1 + 4\,x_2$ needs fixing:
Subject to:

gr of carbs $5x_1 + 15\,x_2 \geq 50$ needs fixing:
gr protein $20x_1 + 5\,x_2 = 40$ needs fixing:
gr fat $15x_1 + 2\,x_2 \leq 60$ OK
Non-negativity $x_1, x_2 \geq 0$

(WRONG) Tableau:

	x1	x2	s1	s2	s3	s4	rhs
z	8	4	0	0	0	0	0
s1	5	15	-1	0	0	0	50
s2	20	5	0	-1	0	0	40
s3	20	5	0	0	1	0	40
s4	15	2	0	0	0	1	60

Fix the first 2 constraints: ADD "artificial" variables with Penalties

$$5\,x_1 + 15\,x_2 - s_1 + a_1 = 50$$
$$20\,x_1 + 5\,x_2 - s_2 + a_2 = 40$$

	x1	x2	s1	s2	s3	s4	a1	a2	rhs
z	8	4	0	0	0	0	1000	1000	0
a1	5	15	-1	0	0	0	1	0	50
a2	20	5	0	-1	0	0	0	1	40
s3	20	5	0	0	1	0	0	0	40
s4	15	2	0	0	0	1	0	0	60

Fix the Z equation to ensure the basic variables all have zero coefficients:

$$Z + 8\,x_1 + 4\,x_2 + 0s_1 + 0s_2 + 0\,s_3 + 0\,s_4 + 1000\,a_1 + 1000\,a_2 = 0$$

$$-1000(\,5x_1 + 15\,x_2 - s_1 + 0s_2 + 0\,s_3 + 0\,s_4 + a_1 = 50)$$
$$-1000(\,20x_1 + 5\,x_2 + 0s_1 - s_2 + 0\,s_3 + 0\,s_4 + a_2 = 40)$$

	x1	x2	s1	s2	s3	s4	a1	a2	rhs
z	-24992	-19996	1000	1000	0	0	0	0	-90000
a1	5	15	-1	0	0	0	1	0	50
a2	20	5	0	-1	0	0	0	1	40
s3	20	5	0	0	1	0	0	0	40
s4	15	2	0	0	0	1	0	0	60

See Excel…..

Alternate approach: Two Phase (Avoid not knowing a good estimate for M)

Phase 1: minimize $Z = a_1 + a_2$ (i.e. maximize $Z = -a_1 - a_2$) until the artificial variables are all non-basic
Phase 2: remove them from the tableau, add in the original objective $Z = -8\,x_1 - 4\,x_2$, fixing the z-row as necessary, and continue.

Phase 1:

Fix the Z equation!

$$Z + a_1 + a_2 = 0$$
$$-(\,5x_1 + 15\,x_2 - s_1 + 0s_2 + 0\,s_3 + 0\,s_4 + a_1 = 50)$$
$$-(\,20x_1 + 5\,x_2 + 0s_1 - s_2 + 0\,s_3 + 0\,s_4 + a_2 = 40)$$

See Excel…

2) Sensitivity

Barbie and Ken Sensitivity Report from the Solver:

Variable Cells

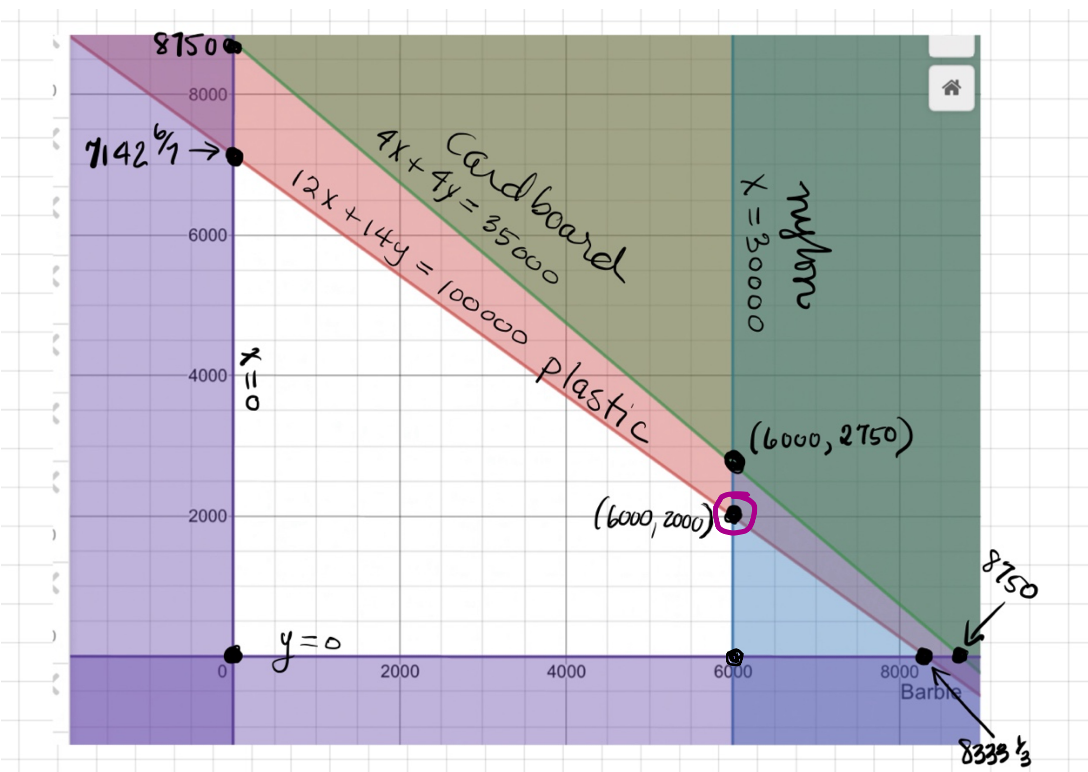
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B1	barbies =	6000	0	6	1E+30	0.428571429
B2	kens =	2000	0	6.5	0.5	6.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B10	cardboard	32000	0	35000	1E+30	3000
B8	plastic	100000	0.464285714	100000	10500	28000
B9	nylon	30000	0.085714286	30000	11666.66667	30000

Barbie and Ken Sensitivity Final Tableau:

	barbie	ken	plastic slack	nylon slack	cardboard slack	right hand side
Z	0	0	0.4642857	0.0857143	0	49000
ken	0	1	0.0714286	-0.171429	0	2000
barbie	1	0	0	0.2	0	6000
Cardboard slack	0	0	-0.285714	-0.114286	1	3000



Maximize Z = 6 B + 6.5 K

Objective Coefficient Ranges: *How much can I change an objective coefficient by and still have the same constraints define the optimal corner (and hence have the decision value variables stay the same)?*

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Shadow Prices: *How much is one unit of the limited resource worth if I could get a little more or less? How big a change can I make and have this value stay the same?*

Right-Hand Side Ranges: *How much can I change the right-hand side of a constraint and still have the same constraints define the optimal corner (with decision variable values possibly changing)?*