

Exercises 3

Which of the following inferences are sound?

For those which are not, give a concrete example — an assignment of variables to “True” or “False” — where the hypotheses are all true but the conclusion is not.

$$\frac{X \rightarrow Y \quad Z \rightarrow Y}{\neg Y}$$

$$\frac{X \rightarrow Y \quad \neg X}{Y}$$

$$\frac{X \rightarrow Y \quad Z \rightarrow X}{Z \rightarrow Y}$$

$$\frac{X \rightarrow Y \quad \neg Z \rightarrow \neg Y}{X \rightarrow Z}$$

$$\frac{X \rightarrow Y \quad \neg X \rightarrow Y}{Y}$$

$$\frac{X \rightarrow Y \quad X \rightarrow \neg Y}{\neg X}$$

$$\frac{(X \vee Y) \rightarrow Z \quad \neg X}{Y \rightarrow Z}$$

$$\frac{(X \vee Y) \rightarrow Z \quad \neg Z}{\neg X}$$

$$\frac{X \rightarrow (Y \vee Z) \quad \neg Y}{\neg X}$$

$$\frac{(X \vee \neg Z) \rightarrow \neg Y}{Y \rightarrow Z}$$

$$\frac{(X \wedge Y) \rightarrow Z \quad \neg Z}{\neg X}$$

$$\frac{(X \wedge Y) \rightarrow Z \quad \neg Z \quad Y}{\neg X}$$

$$\frac{X \rightarrow (Y \vee Z) \quad \neg Y \rightarrow Z}{X \rightarrow Z}$$

$$\frac{X \rightarrow (Y \vee Z) \quad Y \rightarrow Z}{X \rightarrow Z}$$

$$\frac{(X \vee Z) \rightarrow Y \quad Z \vee \neg Y}{\neg X}$$

$$\frac{(X \vee Z) \rightarrow Y \quad Z \wedge \neg Y}{\neg X}$$

$$\frac{(X \vee Y) \rightarrow (\neg X \wedge Z)}{Y \rightarrow Z}$$

$$\frac{(X \wedge Y) \rightarrow Z \quad \neg X}{Z}$$

Sound inference rules

Modus Ponens

$$\frac{X \rightarrow Y \quad X}{Y}$$

Modus Tollens

$$\frac{X \rightarrow Y \quad \neg Y}{\neg X}$$

Syllogism

$$\frac{X \rightarrow Y \quad Y \rightarrow Z}{X \rightarrow Z}$$

Conjunction introduction

$$\frac{X \quad Y}{X \wedge Y}$$

Conjunction elimination

$$\frac{X \wedge Y}{X} \quad \frac{X \wedge Y}{Y}$$

Disjunction introduction

$$\frac{X}{X \vee Y} \quad \frac{Y}{X \vee Y}$$

Disjunction elimination

$$\frac{X \vee Y \quad X \rightarrow Z \quad Y \rightarrow Z}{Z}$$

Ex Falso ¹

$$\frac{\perp}{X}$$

Excluded middle

$$\overline{X \vee \neg X}$$

Double Negation Elimination

$$\frac{\neg \neg X}{X} \quad \frac{\neg X \rightarrow \perp}{X}$$

¹ \perp is the symbol for “False”, the absurd proposition.

Whenever you’ve proved \perp , you have derived a contradiction.