Week 2: A Deeper Dive Into the LP Model Paradigm (3.2-3.4)

The “standard” format for LP

Z = value of overall measure of performance.
x_j = level of activity j (for $j = 1, 2, \ldots, n$).
c_j = increase in Z that would result from each unit increase in level of activity j.
b_i = amount of resource i that is available for allocation to activities (for $i = 1, 2, \ldots, m$).
a_{ij} = amount of resource i consumed by each unit of activity j.

TABLE 3.3 Data needed for a linear programming model involving the allocation of resources to activities

	Resource Usage per Unit of Activity				
	Activity				
Resource	**1**	**2**	**. . .**	***n***	**Amount of Resource Available**
1	a_{11}	a_{12}	. . .	a_{1n}	b_1
2	a_{21}	a_{22}	. . .	a_{2n}	b_2
.					.
.	. . .	. . .	. . .	. . .	.
.					.
m	a_{m1}	a_{m2}	. . .	a_{mn}	b_m
Contribution to Z per unit of activity	c_1	c_2	. . .	c_n	

$$\text{Maximize} \quad Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n,$$

subject to the restrictions

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq b_m, \end{aligned}$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad \ldots, \quad x_n \geq 0.$$

LP/OR Terminology

- Decision Variable (activity)
- Objective
- Constraint
- Feasible Solution
- Infeasible Solution
- Feasible Region
- Most Favorable Value
- Optimal Solution
- Unbounded Objective
- Corner Point Feasible Solution (extreme value, vertex)

LP Underlying Assumptions – Assumed for the Algorithms Used to Solve the Problems

- Proportionality of the Objective Function and the Constrained Totals

- Additivity of the Individual Contributions of the Decision Variables

- Certainty of the Parameter Values

- Divisibility, i.e., Fractional Answers Must be Interpretable

Alternates that arise:

$$\text{Minimize} \quad Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n.$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i \quad \text{for some values of } i.$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i \quad \text{for some values of } i.$$

$$x_j \text{ unrestricted in sign} \quad \text{for some values of } j.$$

Task for the Week: Set up a Bunch!!! Read the sections and consider the examples provided. I have posted two additional worked examples in separate videos.

Example 1: Problem 3.1-11

The Omega Manufacturing Company has discontinued the production of a certain unprofitable product line. This act created considerable excess production capacity. Management is considering devoting this excess capacity to one or more of three products; call them products 1, 2, and 3. The available capacity on the machines that might limit output is summarized in the following table. Also provided was the number of machine hours required for each unit in the second table.

Machine Type	Available Time (Machine Hours per Week)
Milling machine	500
Lathe	350
Grinder	150

Productivity coefficient (in machine hours per unit)

Machine Type	Product 1	Product 2	Product 3
Milling machine	9	3	5
Lathe	5	4	0
Grinder	3	0	2

The sales department indicates that the sales potential for products 1 and 2 exceeds the maximum production rate and that the sales potential for product 3 is 20 units per week. The unit profit would be \$50, \$20, and \$25, respectively, on products 1, 2, and 3. The objective is to determine how much of each product Omega should produce to maximize profit.

Example 2: Problem 3.4-16

Joyce and Marvin run a day care for preschoolers. They are trying to decide what to feed the children for lunches. They would like to keep their costs down, but also need to meet the nutritional requirements of the children. They have already decided to go with peanut butter and jelly sandwiches, and some combination of graham crackers, milk, and orange juice. The nutritional content of each food choice and its cost are given in the table below.

Food Item	Calories from Fat	Total Calories	Vitamin C (mg)	Protein (g)	Cost (¢)
Bread (1 slice)	10	70	0	3	5
Peanut butter (1 tbsp)	75	100	0	4	4
Strawberry jelly (1 tbsp)	0	50	3	0	7
Graham cracker (1 cracker)	20	60	0	1	8
Milk (1 cup)	70	150	2	8	15
Juice (1 cup)	0	100	120	1	35

The nutritional requirements are as follows. Each child should receive between 400 and 600 calories. No more than 30 percent of the total calories should come from fat. Each child should consume at least 60 milligrams (mg) of vitamin C and 12 grams (g) of protein. Furthermore, for practical reasons, each child needs exactly 2 slices of bread (to make the sandwich), at least twice as much peanut butter as jelly, and at least 1 cup of liquid (milk and/or juice). Joyce and Marvin would like to select the food choices for each child which minimize cost while meeting the above requirements.