Formula sheet: Basic laws about sums

Let a_0, a_1, a_2, ... be a sequence of numbers.
For example if $a_n = n$, we get the sequence 0, 1, 2, 3, ...
And if $a_n = n^2$, we get the sequence 0, 1, 4, 9, ...

1. The sum of all numbers in the sequence in the interval $m \leq k \leq n$ is written as

$$\textstyle\sum_{k=1}^{n} a_k = a_m + a_{m+1} + a_{m+2} + ... + a_{n-1} + a_n$$

2. In particular, the sum of the numbers a_1 through a_n is

$$\textstyle\sum_{k=1}^{n} a_k = a_1 + a_2 + ... + a_n$$

   Moreover, as special cases we get

$$\textstyle\sum_{k=1}^{1} a_k = a_1 \qquad \sum_{k=1}^{0} a_k = 0$$

   The second equality holds because no numbers are being added, so the sum is empty, which yields zero by default.

3. Any sum can be broken down into smaller intervals:

$$\textstyle\sum_{k=1}^{n+m} a_k = (\sum_{k=1}^{n} a_k) + (\sum_{k=1}^{m} a_{n+k})$$

   As a special case, we get

$$\textstyle\sum_{k=1}^{n+1} a_k = (\sum_{k=1}^{n} a_k) + a_{n+1}$$

4. If c is any constant, then

$$\textstyle\sum_{k=m}^{n} c \cdot a_k = c \cdot \sum_{k=m}^{n} a_k$$

5. If b_0, b_1, b_2, ... is another sequence, then

$$\textstyle\sum_{k=m}^{n} (a_k + b_k) = (\sum_{k=m}^{n} a_k) + (\sum_{k=m}^{n} b_k)$$

6. $\sum_{k=0}^{n} = \frac{n(n+1)}{2}$
7. If $x \neq 1$, then $\sum_{k=0}^{n} x^n = \frac{x^{n+1}-1}{x-1}$