Economics of Gambling

A side Trip

Choice under Uncertainty:
Lotteries and Risk Preferences

Probability

- The probability of a repetitive event happening is the relative frequency with which it will occur
  - probability of obtaining a head on the fair-flip of a coin is 0.5
  - Probability of Lions winning on Sunday subjective probability
- If a lottery offers n distinct prizes and the probabilities of winning the prizes are π_i (i=1,n) then

$$\sum_{i=1}^{n} \pi_i = 1$$

Expected Value

- For a lottery (X) with prizes $x_1, x_2, \ldots, x_n$ and the probabilities of winning $\pi_1, \pi_2, \ldots \pi_n$, the <u>expected value</u> of the lottery is

$$E(X) = \pi_1 x_1 + \pi_2 x_2 + \ldots + \pi_n x_n$$

$$E(X) = \sum_{i=1}^{n} \pi_i x_i$$

- The <u>expected value</u> is a weighted sum of the outcomes
  - – the weights are the respective probabilities

Expected Value

- Suppose that Smith and Jones decide to flip a coin
  - heads (x_1) $\Rightarrow$ Jones will pay Smith \$1
  - tails (x_2) $\Rightarrow$ Smith will pay Jones \$1
- From Smith's point of view,

$$E(X) = \pi_1 x_1 + \pi_2 x_2$$

$$E(X) = \frac{1}{2}(\$1) + \frac{1}{2}(-\$1) = 0$$

Expected Value

- Games which have an expected value of zero (or cost their expected values) are called actuarially fair games or zero sum games
- Negative sum games expected value less than zero
- Positive sum games expected value greater than zero
  - a common observation is that people often refuse to participate in actuarially fair games
  - A common observation other times people participate in negative sum games

Expected Utility

In 1728, Gabriel Cramer, in a letter to Nicolas Bernoulli, wrote,
"the mathematicians estimate money in proportion to its quantity, and men of good sense in proportion to the usage that they may make of it."

Instead of working with the amount of money (x) *a lottery pays*, we will work with the *utility* ($u(x)$) this money gives you.

This approach was formally proposed and justified by John vonNeumann and Oskar Morgenstern in 1944, and has ben adopted as the main tecnique to analyze decisions under risk ever since

The von Neumann-Morgenstern Theorem

- The von Neumann-Morgenstern method is to define the utility of x_i as the expected utility of the gamble that the individual considers equally desirable to x_i

$$U(x_i) = \pi_i \cdot U(x_n) + (1 - \pi_i) \cdot U(x_1)$$

The Expected Utility Theory

According to the Expected Utility Theory, individuals will prefer that lottery that produces the highest "expected utility" according to their own utility function over money u·

That is, given two lotteries L_1 and certainty,

L_1 is preferred to certainty if $E(u(L_1)) > U(certainty)$

certainty is preferred to L_1 if $U(certainty) > E(u(L_1))$

L1 and certainty are indifferent if $E(u(L_1)) = U(certainty)$

Attitudes towards Risk

Consider the following example,

If we flip a coin 1000 times, and we get $2 with heads and lose $1 with tails . . . How much will we earn at the end ? (approximately)

Most likely, we well get around 500 heads and 500 tails. That is, we will earn $1,000 and lose $500. At the end, we expect to make around $500

What do you prefer, playing the gamble or getting $500 for sure ?

- Some individuals would say:

  “I know that most likely I would get $500 if I play the gamble.
  Nevertheless, I'd better take the $500 because of the risk
  of getting less than $500 if I play the gamble”

- Others might say:

  “I know that most likely I would get $500 if I play the gamble.
  But I prefer to play it rather than getting $500 for sure
  because of the possibility of earning more than $500”

- Finally, some could say:

  “I know that most likely I would get $500 if I play the gamble.
  Hence, for me is the same to play the gamble than to get
  $500 for sure.

Mathematical fact

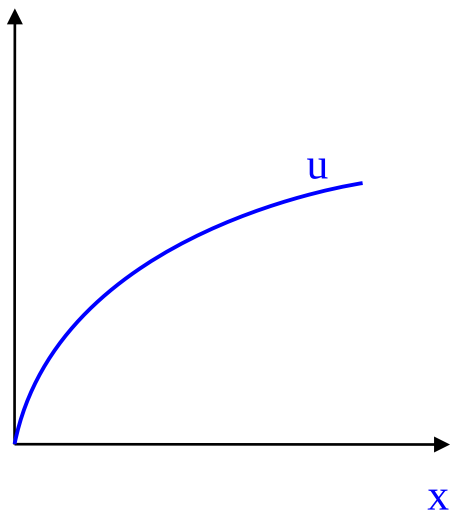
- Risk averse individuals have utility functions u that are "concave"
- Risk lovers individuals have utility functions u that are "convex"
- Risk neutral individuals have utility functions u that are "linear"

Summary

Risk averse

Prefers the expected value of the gamble rather than the gamble

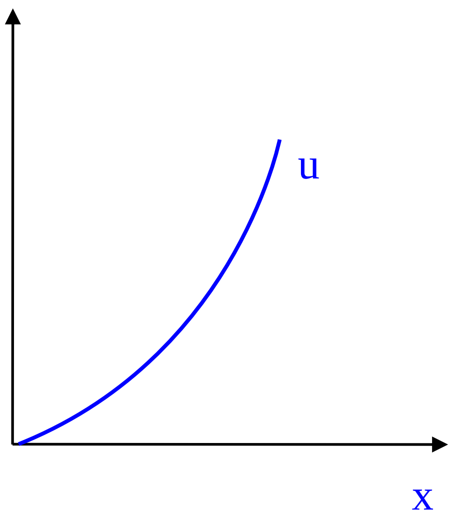
u(E(L)) > E(u(L))



Risk lover

Prefers the gamble better than the expected value of the gamble

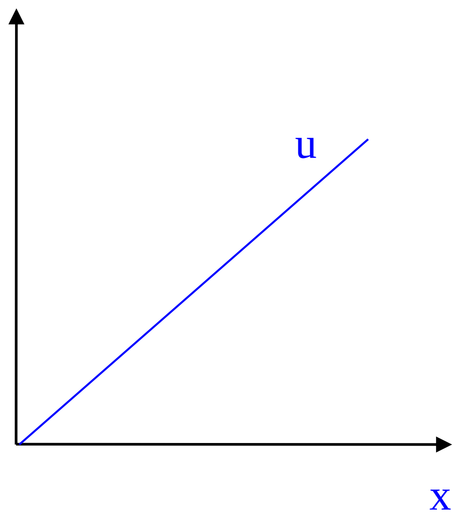
u(E(L)) < E(u(L))

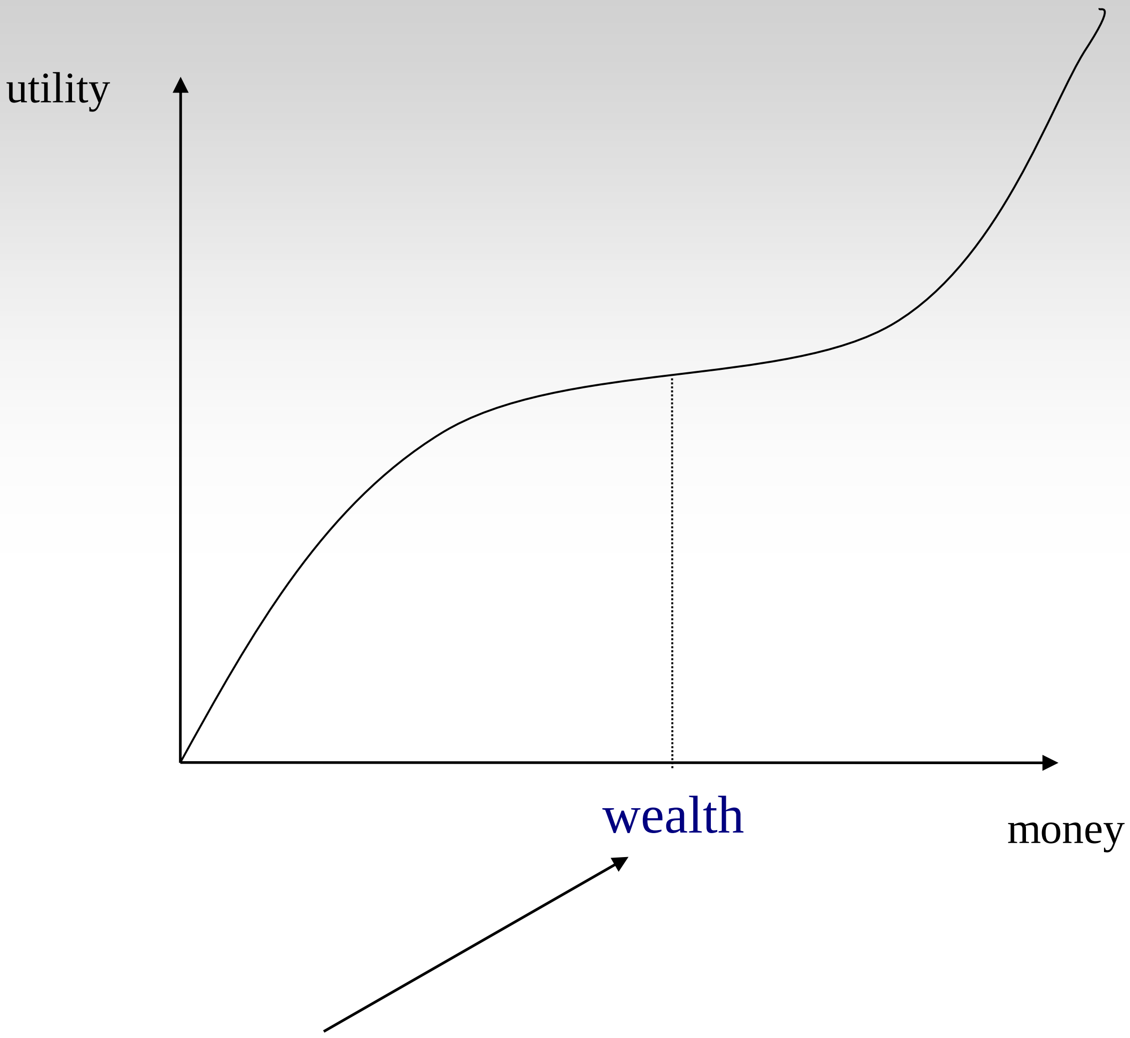


Risk neutral

Indifferent

u(E(L)) = E(u(L))





"Status Quo" Downside and Upside Risk preferences

Risk Aversion

In general, we assume that the marginal utility of wealth falls as wealth gets larger

- a flip of a coin for $1,000 promises a small gain in utility if you win, but a large loss in utility if you lose
- a flip of a coin for $1 is inconsequential as the gain in utility from a win is not much different as the drop in utility from a loss

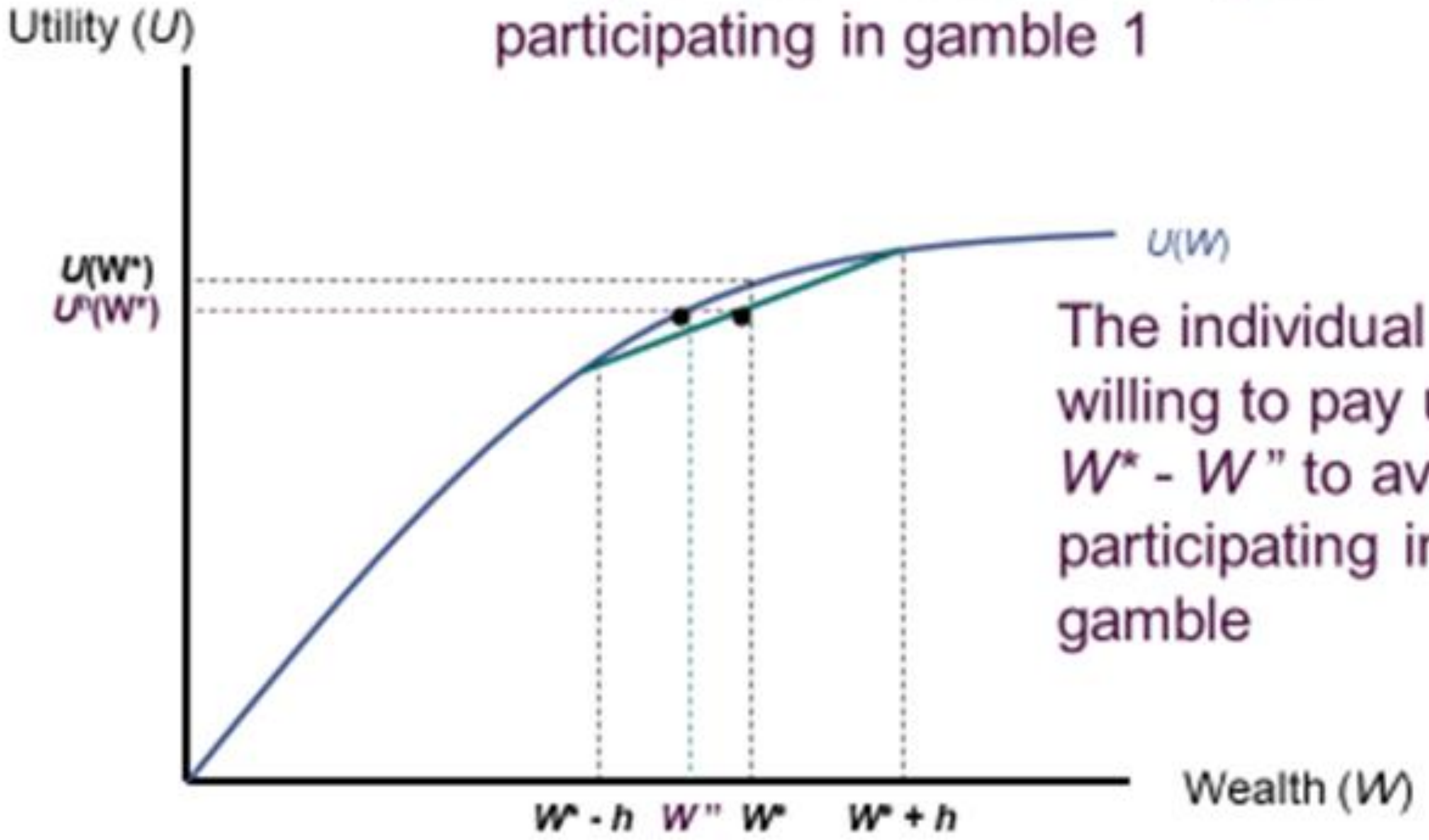
Risk Aversion and Insurance

The person might be willing to pay some amount to avoid participating in a gamble

This helps to explain why some individuals purchase insurance

Risk Aversion and insurance

W " provides the same utility as participating in gamble 1



The individual will be willing to pay up to *W** - *W* " to avoid participating in the gamble

Willingness to Pay for Insurance

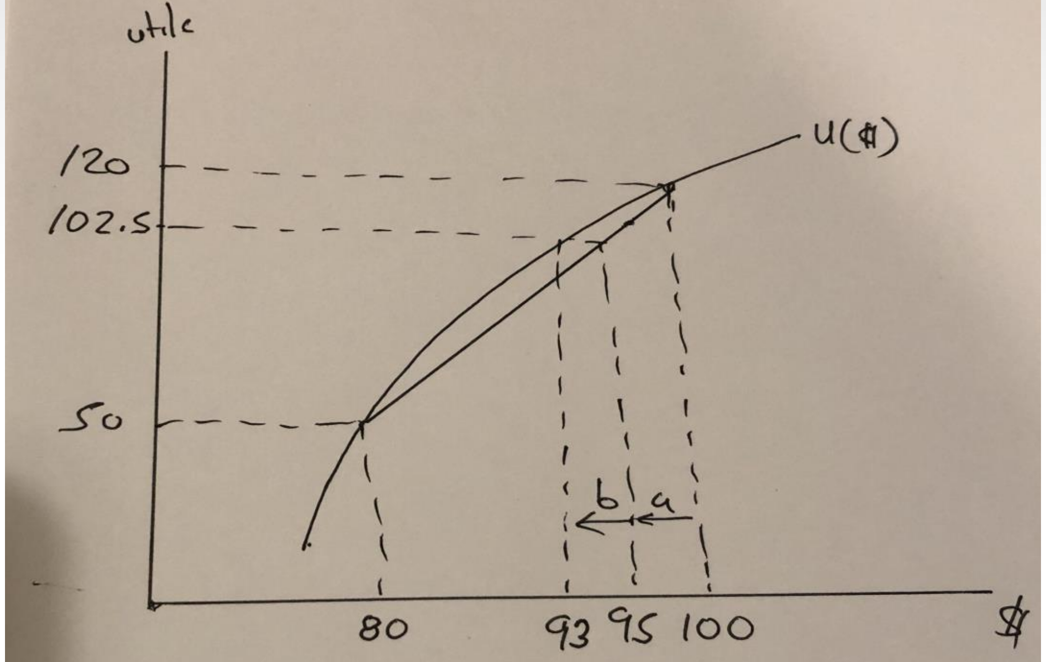
- Consider a person with a current wealth of $100,000 who faces a 25% chance of losing his automobile worth $20,000
- The person's expected loss will be

$$E(\$) = 0.75\text{x}100{,}000 + 0.25\text{x}80{,}000$$

$$\mathrm{E}(\$) = \$95{,}000$$

- In this situation, the expected loss or actuarially fair insurance premium would be $5,000 (25% of $20,000)

Example Graph



a = $5000
b = $2000

Max WTP = $7000

Examples of insurance in sports

MLB $100 million arm

NCAA football insurance

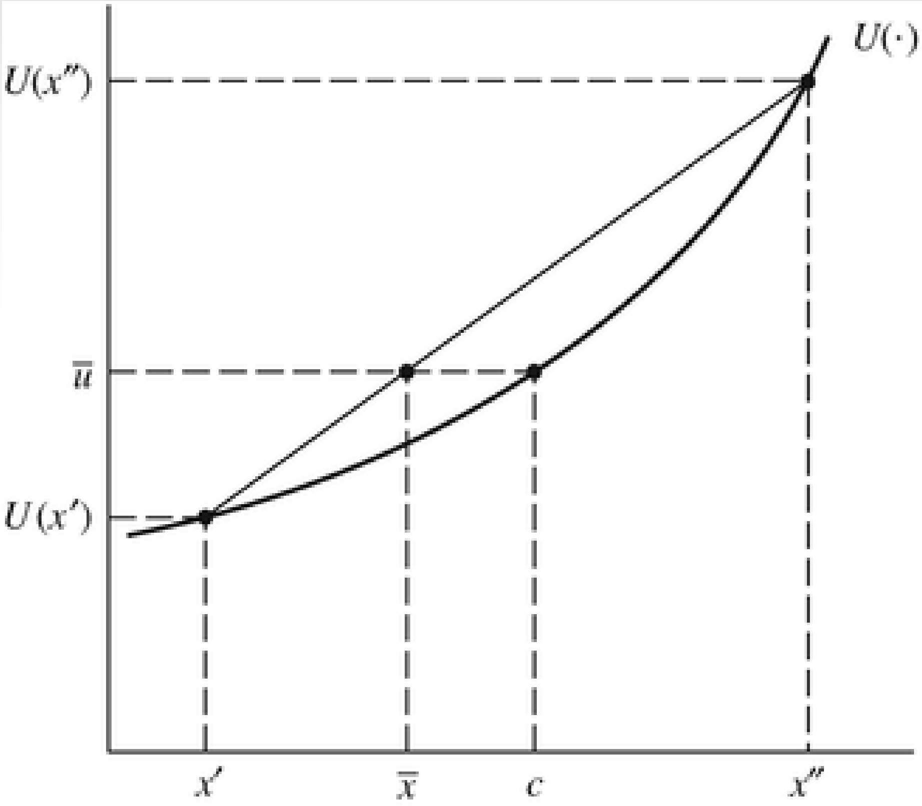
NCAA basketball insurance

Risk Loving and Gambling

The person might be willing to pay some amount to participate in a gamble in a negative sum game

This helps to explain why some individuals purchase lottery tickets

Risk acceptance premium



c = certainty
c – xbar = max WTP
Risk acceptance premium

Willingness to Pay for lottery ticket

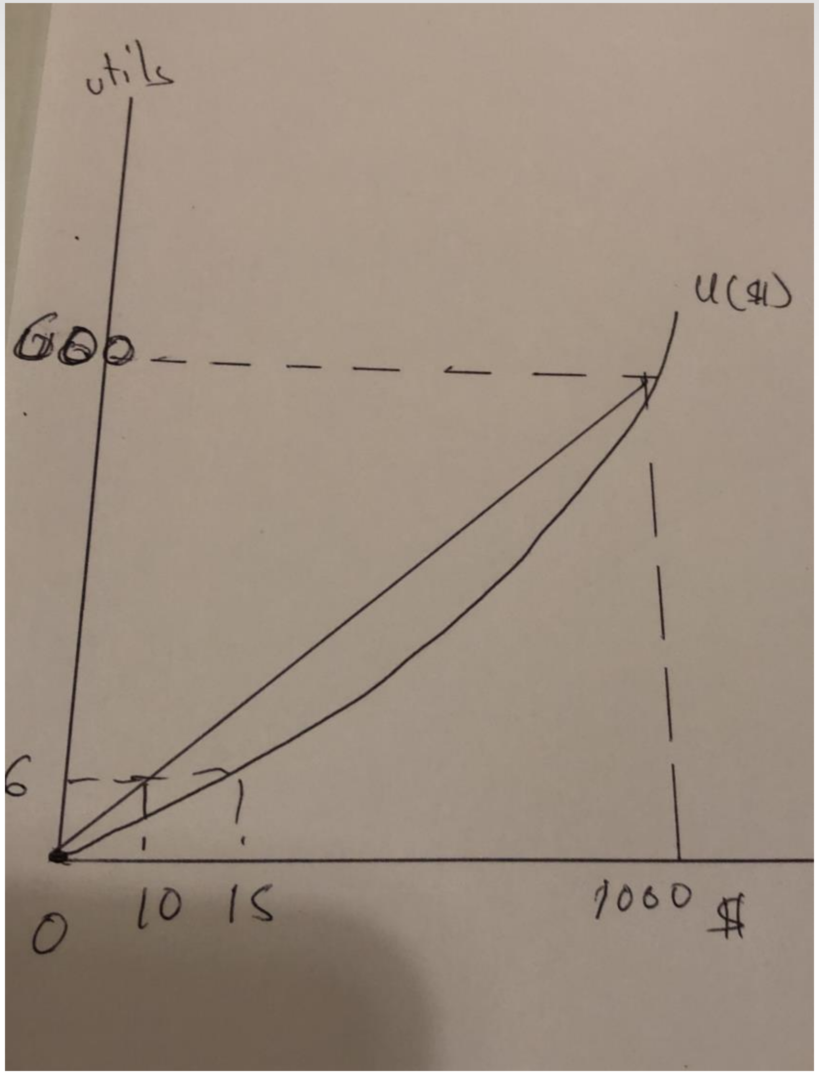
- Consider a person with a 1% chance of winning the lottery worth $1,000 and a 99% of winning zero dollars
- The person's expected gain will be

$$E(\$) = 0.01x1{,}000 + 0.99x0$$

$$\mathrm{E}(\$) = \$10$$

- In this situation, the expected gain or actuarially fair price would be $10
- *How much would a risk lover be willing to pay for a ticket?*

Example Graph



Expected Value

Sports gambling –house take 5%   Expected value $.95

Lottery tickets—government take 45% Expected value $.55

Horse racing—house take 12% Expected value $.88

Sports is subjective probability

Wisdom of crowds-market efficiency

Stocks vs gambling (positive sum game vs negative sum game)

Long shot bias (Over bet low probability high payoff events)

Textbook Version Market Efficiency

"Security prices accurately reflect available information, and respond rapidly to new information as soon as it becomes available" Richard Brealey & Stewart Myers, *Principles of Corporate Finance*, 1996

Harry Roberts, 1967

Weak form efficiency: prices incorporate information about past prices

Semi-strong form: incorporate all publicly available information

Strong form: all information, including inside information

Reasons to Think Gambling Markets Ought to Be Efficient

Marginal gambler determines prices

Smart money dominates trading

Survival of fittest

Gambling and Risk Loving

Long shot bias (Over bet low probability high payoff events)

So can you make money sports gambling?

Difficult need to do better than average to cover take.

Market efficiency

One bet that pays even after the take betting a favorite to place in horse racing