An introduction to game theory

The fundamentals of game theory, including Nash equilibrium

Today

- Introduction to game theory
  - We can look at strategic situations with two players
  - Although we will look at situations where each player can make only one of two decisions, theory easily extends to three or more decisions

Who is this?

John Nash, the person portrayed in “A Beautiful Mind”

John Nash

- One of the early researchers in game theory
- His work resulted in a form of equilibrium named after him

Three elements in every game

- Players
  - Two or more (n) for most games
- Strategies available to each player
- Payoffs
  - Based on your decision(s) and the decision(s) of other(s)

Game theory: Payoff matrix

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	12, 4	10, 1

- A payoff matrix shows the payout to each player, given the decision of each player

How do we interpret this box?

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	12, 4	10, 1

- The first number in each box determines the payout for Person 1
- The second number determines the payout for Person 2

How do we interpret this box?

		Person 2	
		Action C	Action D
Person 1	Action A	10, 2	8, 3
	Action B	12, 4	10, 1

- Example
  - If Person 1 chooses Action A and Person 2 chooses Action D, then Person 1 receives a payout of 8 and Person 2 receives a payout of 3

Back to a Core Principle: Equilibrium

- The type of equilibrium we are looking for here is called Nash equilibrium
  - Nash equilibrium: "Any combination of strategies in which each player's strategy is his or her best choice, given the other players' choices"
  - Exactly one person deviating from a NE strategy would result in the same payout or lower payout for that person

How do we find Nash equilibrium (NE)?

- Step 1: Pretend you are one of the players
- Step 2: Assume that your "opponent" picks a particular action
- Step 3: Determine your best strategy (strategies), given your opponent's action
  - Underline any best choice in the payoff matrix
- Step 4: Repeat Steps 2 & 3 for any other opponent strategies
- Step 5: Repeat Steps 1 through 4 for the other player
- Step 6: Any entry with all numbers underlined is NE

Steps 1 and 2

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	12, 4	10, 1

- Assume that you are Person 1
- Given that Person 2 chooses Action C, what is Person 1's best choice?

Step 3:

		Person 2	
		Action C	Action D
Person 1	Action A	10, 2	8, 3
	Action B	12, 4	10, 1

- Underline best payout, given the choice of the other player
- Choose Action B, since 12 > 10 ➔ underline 12

Step 4

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	12, 4	10, 1

- Now assume that Person 2 chooses Action D
- Here, 10 > 8 → Choose and underline 10

Step 5

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	12, 4	10, 1

- Now, assume you are Person 2
- If Person 1 chooses A
  - 3 > 2 ➔ underline 3
- If Person 1 chooses B
  - 4 > 1 ➔ underline 4

Step 6

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	$\boxed{12, 4}$	10, 1

- Which box(es) have underlines under both numbers?
  - Person 1 chooses B and Person 2 chooses C
  - This is the only NE

Double check our NE

Person 2

Person 1		Action C	Action D
	Action A	10, 2	8, 3
	Action B	12, 4	10, 1

- What if Person 1 deviates from NE?
  - Could choose A and get 10
  - Person 1's payout is lower by deviating

Double check our NE

Person 2

Person 1

	Action C	Action D
Action A	10, 2	8, 3
Action B	12, 4	10, 1

- What if Person 2 deviates from NE?
  - Could choose D and get 1
  - Person 2's payout is lower by deviating

Dominant strategy

Person 2

Person 1		Action C	Action D
	Action A	10, 2	8, 3
	Action B	12, 4	10, 1

- A strategy is dominant if that choice is definitely made no matter what the other person chooses
  - Example: Person 1 has a dominant strategy of choosing B

New example

Person 2

Person 1	Yes	No
Yes	20, 20	5, 10
No	10, 5	10, 10

- Suppose in this example that two people are simultaneously going to decide on this game

New example

Person 1 \ Person 2	Yes	No
Yes	20, 20	5, 10
No	10, 5	10, 10

- We will go through the same steps to determine NE

Two NE possible

Person 2

Person 1

	Yes	No
Yes	20, 20	5, 10
No	10, 5	10, 10

- (Yes, Yes) and (No, No) are both NE
- Although (Yes, Yes) is the more efficient outcome, we have no way to predict which outcome will actually occur

Two NE possible

- When there are multiple NE that are possible, economic theory tells us little about which outcome occurs with certainty

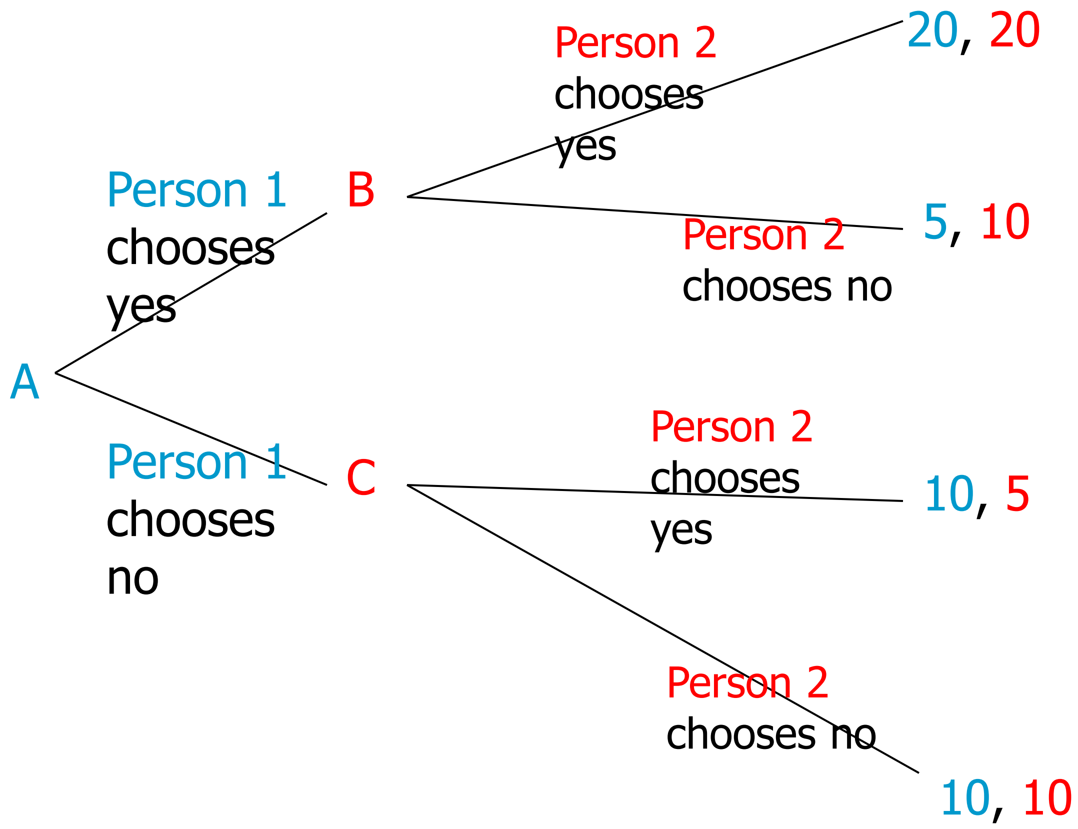
Two NE possible

- Additional information or actions may help to determine outcome
  - If people could act sequentially instead of simultaneously, we could see that 20, 20 would occur in equilibrium

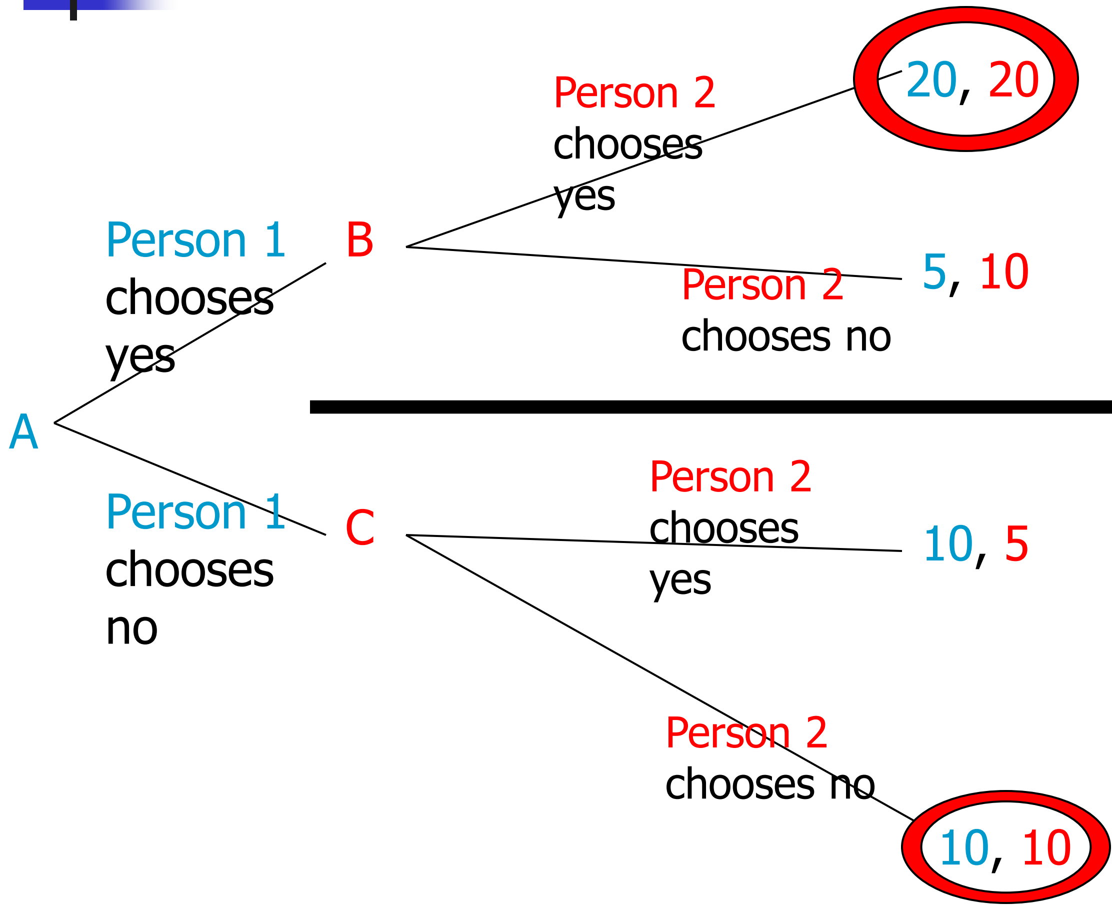
Sequential decisions

- Suppose that decisions can be made sequentially
- We can work backwards to determine how people will behave
  - We will examine the last decision first and then work toward the first decision
- To do this, we will use a decision tree

Decision tree in a sequential game: Person 1 chooses first



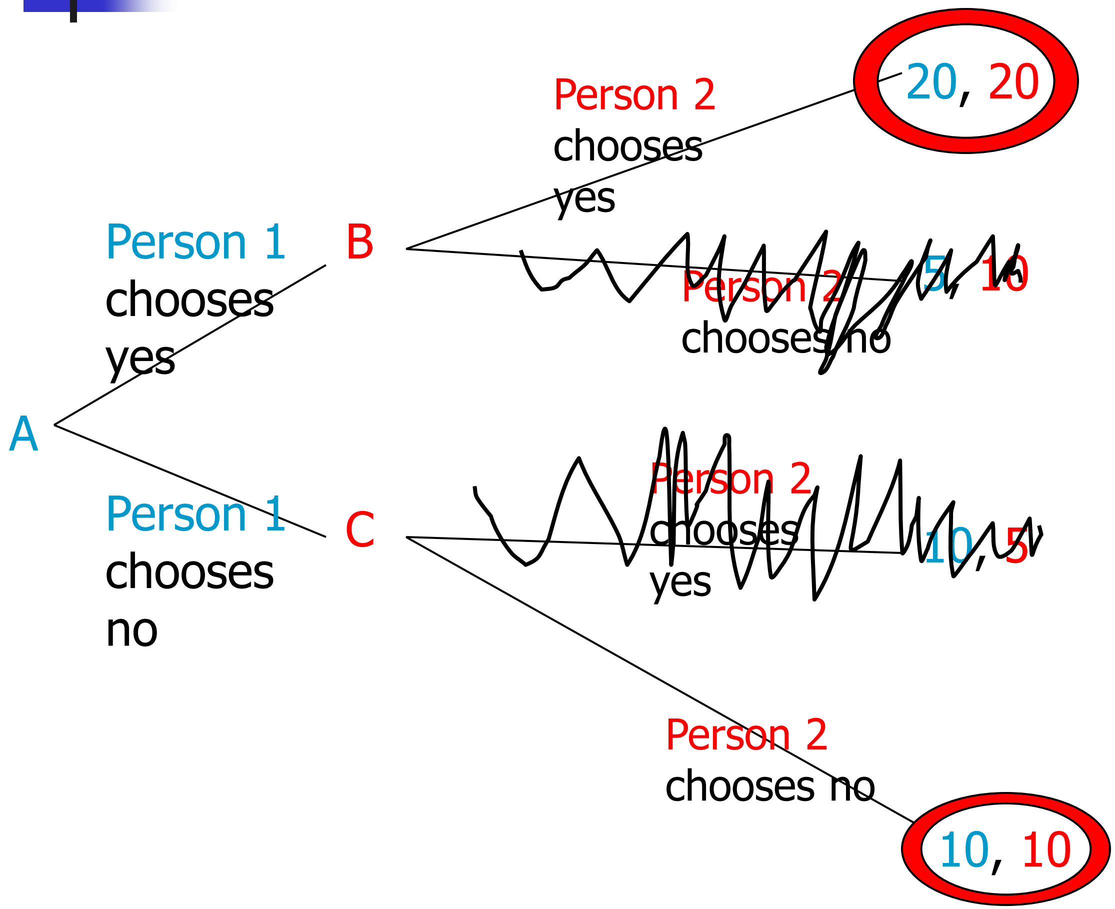
Decision tree in a sequential game: Person 1 chooses first



- Given point B, Person 2 will choose yes (20 > 10)

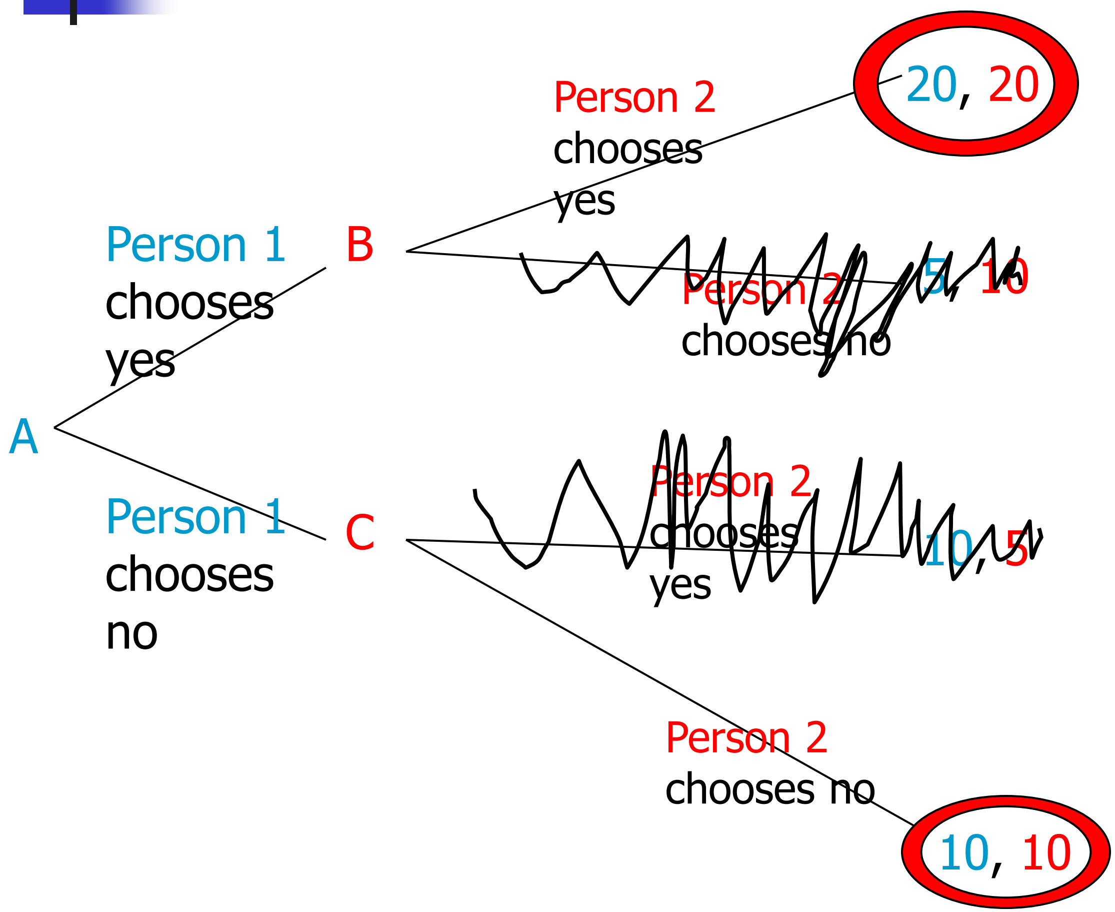
- Given point C, Person 2 will choose no (10 > 5)

Decision tree in a sequential game: Person 1 chooses first



- If Person 1 is rational, she will ignore potential choices that Person 2 will not make
- Example: Person 2 will not choose yes after Person 1 chooses no

Decision tree in a sequential game: Person 1 chooses first



- If Person 1 knows that Person 2 is rational, then she will choose yes, since 20 > 10
- Person 2 makes a decision from point B, and he will choose yes also
- Payout: (20, 20)

The Prisoners' Dilemma Game

- Two players, prisoners 1, 2.
- Each prisoner has two possible actions.
  - Prisoner 1: Silent, Talk
  - Prisoner 2: Silent, Talk
- Players choose actions simultaneously without knowing the action chosen by the other.
- Payoff consequences quantified in prison years.
  - If both silent, each gets 3 year
  - If both talks, each gets 7 years
  - If 1 talks, he gets 1 year and other gets 8 years
- Fewer years=greater satisfaction=>higher payoff.
  - Prisoner 1 payoff first, followed by prisoner 2 payoff.

Prisoners' Dilemma in "Normal" or "Strategic" Form

	Silent	Talk
Silent	-3,-3	-8,-1
Talk	-1,-8	-7,-7

Talk

Talk

Talk Talk

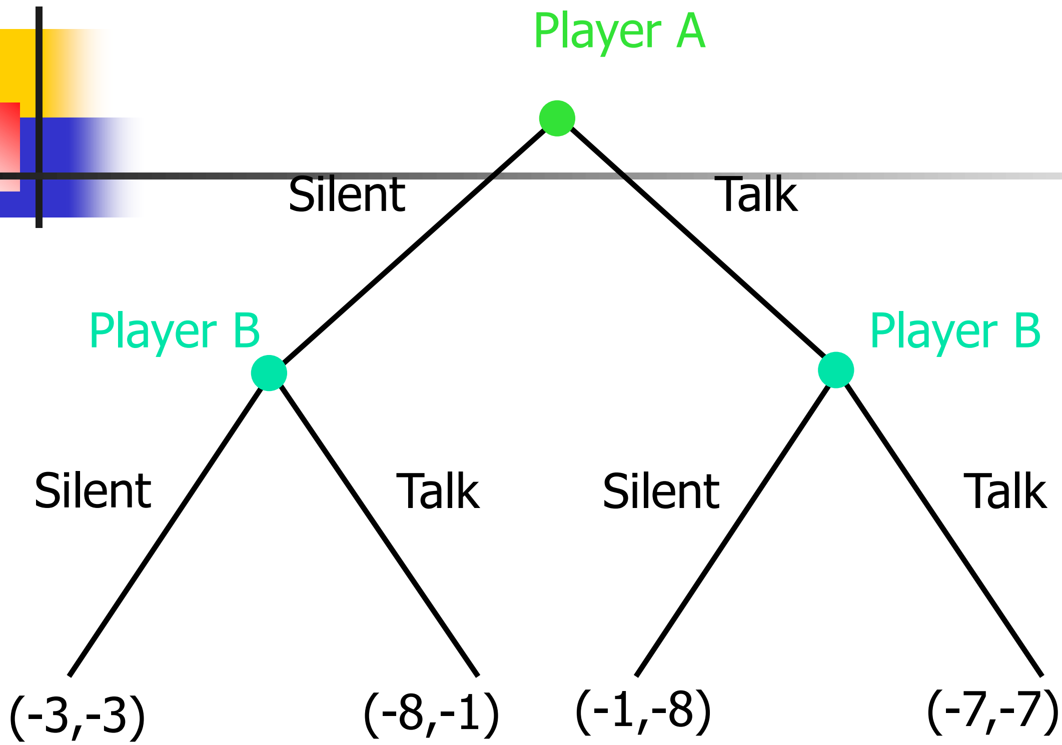
Prisoner’s Dilemma

- Players acting in their own best interest end up in worst possible joint outcome
- Dominant strategy equilibrium
- Talk,Talk
- Incentive to cheat +2
- External Cost of cheating -5
- (Value of reputation +4) Repeated Game

Prisoner’s Dilemma Sports

- Arms race (NCAA)
- Arms race (Superstar wages)
- Brand advertising
- Doping in Sports
- Other?

Prisoner's Dilemma Sequential Game



A plays first.
B plays second.

Player A knows Player B is rationally self interested

How do you solve the dilemma?

- Trust
  - Stag Hunt Game
  - Norms
- Punishment
  - Repeated game
  - Tit for tat strategy

Stag Hunt

	Stag	Hare	
Stag	10,10	0,5	Stag
Hare	5,0	5,5	Hare
	Stag	Hare	

Stag hunt drafting in NASCAR

- In aerodynamically intense stock-car races like the Daytona 500, the drivers form into multi-car draft lines to gain extra speed. A driver who does not enter a draft line (slipstream) will lose. Once in a line, a driver must attract a drafting partner in order to break out and try to get further ahead. Thus the effort to win leads to ever-shifting patterns of cooperation and competition among rivals. This provides a curious laboratory for several social science theories: (1) complexity theory, since the racers self-organize into structures that oscillate between order and chaos; (2) social network analysis, since draft lines are line networks whose organization depends on a driver's social capital as well as his human capital; and (3) game theory, since racers face a "prisoner's dilemma" in seeking drafting partners who will not defect and leave them stranded. Perhaps draft lines and related "bump and run" tactics amount to a little-recognized dynamic of everyday life, including in structures evolving on the Internet.

Driving Game (norms)

	Left	Right
Left	10,10	-5,-5
Right	-5,-5	10,10

Left

Right

Left Right

One-Shot versus Repeated Games

- One-shot: play of the game occurs once. Players likely to not know much about one another. Example - tipping on your vacation
- Repeated: play of the game is repeated with the same players.
  - Indefinitely versus finitely repeated games
  - Reputational concerns matter; opportunities for cooperative behavior may arise.
- Advise: If you plan to pursue an aggressive strategy, ask yourself whether you are in a one-shot or in a repeated game. If a repeated game, think again

Repeated Game Strategies

- In repeated games, the sequential nature of the relationship allows for the adoption of strategies that are contingent on the actions chosen in previous plays of the game.
- Most contingent strategies are of the type known as "trigger" strategies.
  - Example trigger strategies
    - In prisoners' dilemma: Initially play Silent. If your opponent plays Talk, then play Talk from then on.

Prisoners' Dilemma in "Normal" or "Strategic" Form

	cooperate	compete	
cooperate	10,10	13,3	compete
compete	13,3	6,6	compete
	compete	compete	

End of the period problem

- Value of reputation (+4)
- If there is an end the solution unravels
- Unravelling problem

Observation about Nash

- Fairness Norm
- Tend to be nicer than expected
  - Tipping on a trip
- Tend to be meaner than expected
  - Small Claims court
  - Ultimatum Game

Ultimatum Game

- Player 1 chooses ($10-x, x)
- Player 2 faced with ultimatum
  - Accept (x, 10-x)
  - Reject (0,0)

Prisoners’ Dilemma Norms

Franks—always share
Bens—always fight
Clint—Tit for Tat

	Share	Fight
Share	5,5	0,7
Fight	7,0	2,2

Prisoners' Dilemma Norms

Franks—always share
Bens—always fight
Clint—Tit for Tat

	Frank	Ben	Clint	Total
Frank	5	0	5	10/3
Ben	7	2	2	11/3
Clint	5	2	5	12/3

EVOLUTION OF COMMUNITY DETERRENCE: EVIDENCE FROM THE NATIONAL HOCKEY LEAGUE

Craig A. Depken II Peter A. Groothuis Mark C. Strazicich

- Community and specialized enforcement are recognized as important components of deterring antisocial behavior. To provide insights on the interplay between deterrence methods, we examine the empirical evolution of fighting and scoring in the National Hockey League using time series data. We identify structural changes that correlate with changes in player behavior and rules. In particular, we find that player behavior related to fighting changed 4 or 5 years prior to most rule changes aimed at reducing fighting. We conclude that the decline in fighting in hockey was more closely associated with a change in community rather than specialized deterrence methods.

Mixed Strategy Game
Rock, Paper, Scissors

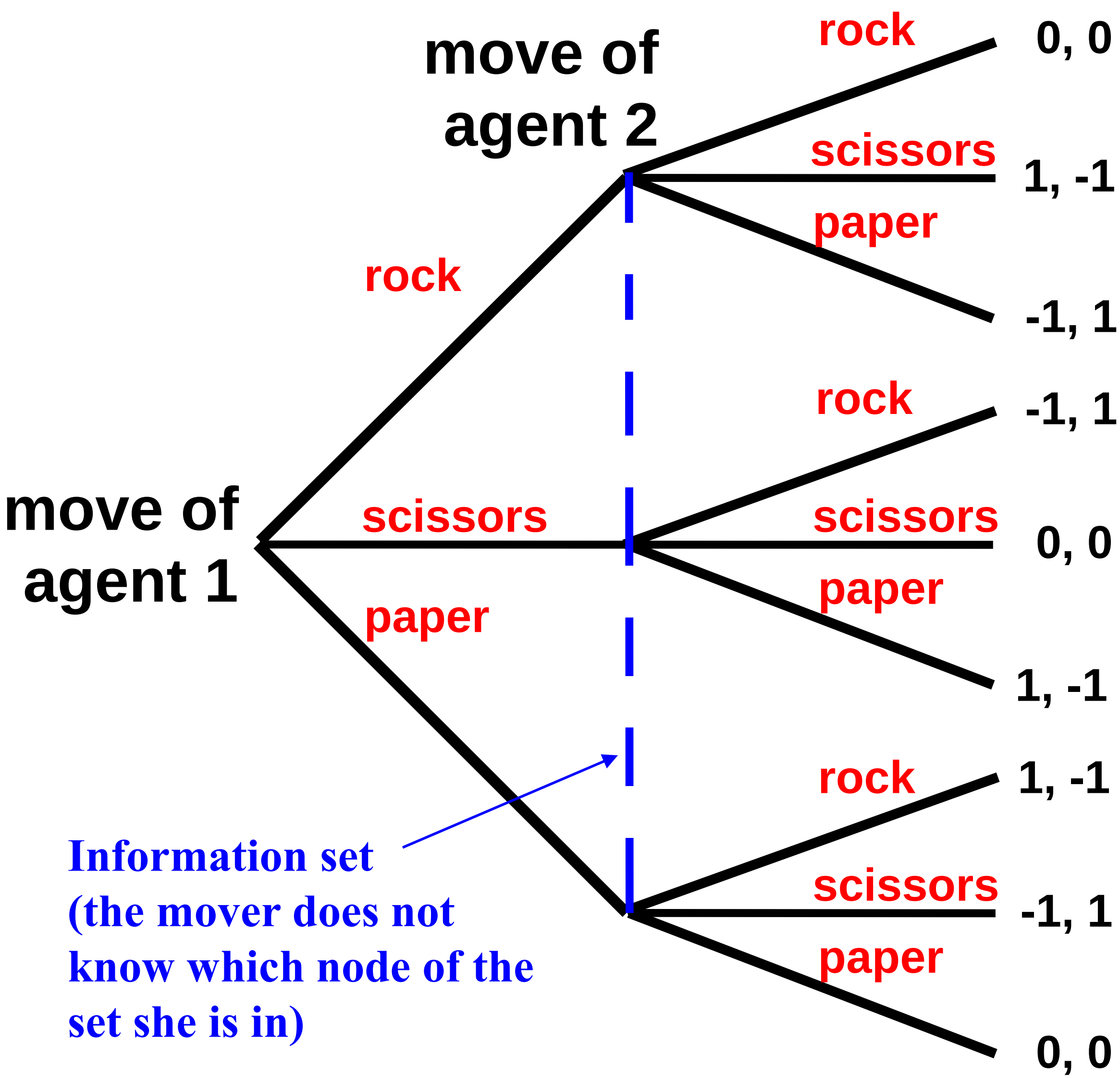
- Simultaneous move game
- Normal-form representation:

		Player 2			
		Rock	Paper	Scissors	
	ROCK	0,0	-1,1	1,-1	Paper
Player 1	PAPER	1,-1	0,0	-1,1	Sissors
	SCISSORS	-1,1	1,-1	0,0	Rock
		Paper	Sissors	Rock	

Diagram courtesy: Dr. Tayfun Sönmez

Mixed strategy Nash equilibrium

Mixed strategy **= agent's chosen probability distribution over pure strategies from its strategy set**



Each agent has a best response strategy and beliefs (consistent with each other)

Symmetric mixed strategy Nash eq: Each player plays each pure strategy with probability 1/3

In mixed strategy equilibrium, each strategy that occurs in the mix of agent i has equal expected utility to i

Example 5: Mixed Strategy and Tennis What about the Real World?

Minimax Play at Wimbleton
Walker and Wooders (AER 2001)

"We use data from classic professional tennis matches to provide an empirical test of the theory of mixed strategy equilibrium. We find that the serve-and-return play of John McEnroe, Bjorn Borg, Boris Becker, Pete Sampras and others is consistent with equilibrium play."

Results: Probability Server wins is the same whether serve right or left. Which side server serves is not "serially independent".

Other examples

- Penalty kicks in soccer
- Pitches in baseball
- Run vs Pass in football
- Fold or Bluff in poker

Game of Chicken (Brinksmanship)

	Swerve	Straight
Swerve	0,0	-1,+1
Straight	+1,-1	-50,-50

Straight

Swerve

Straight Swerve

Game of Life (Advantage of moving first)

Mr. A

Mrs. B

	Boxing	Opera
Boxing	50,20	0,10
Opera	10,0	20,50

Boxing

Opera

Boxing

Opera