Econometrics - Homework 3

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3.1 Repeat this exercise for sample sizes of 50, 100, and 200. What happens to the bias of β_1 and β_2 as the sample size is increased?

As we can see the bias shrink, thus our parameters estimation are converging on the true parameters.

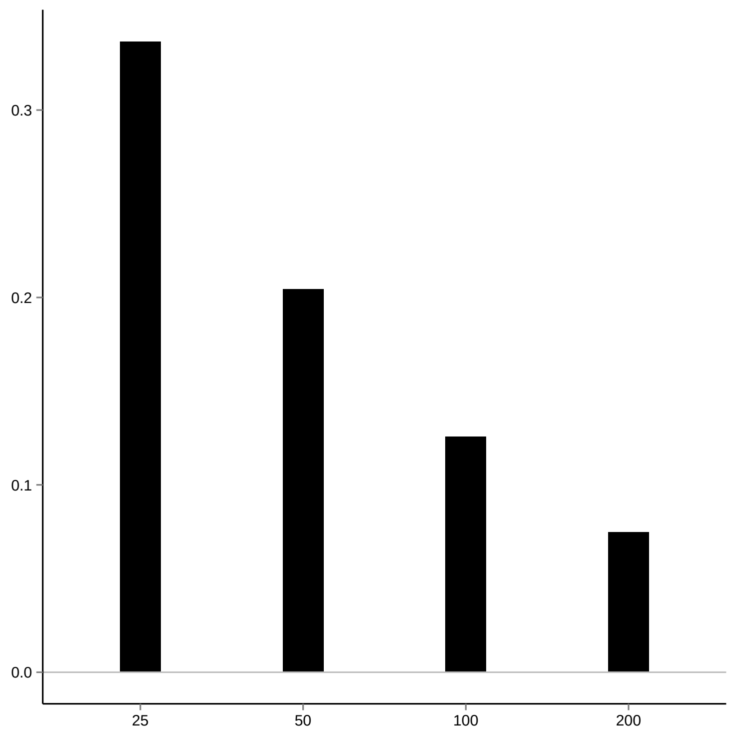

Figure 1: Intercept Bias

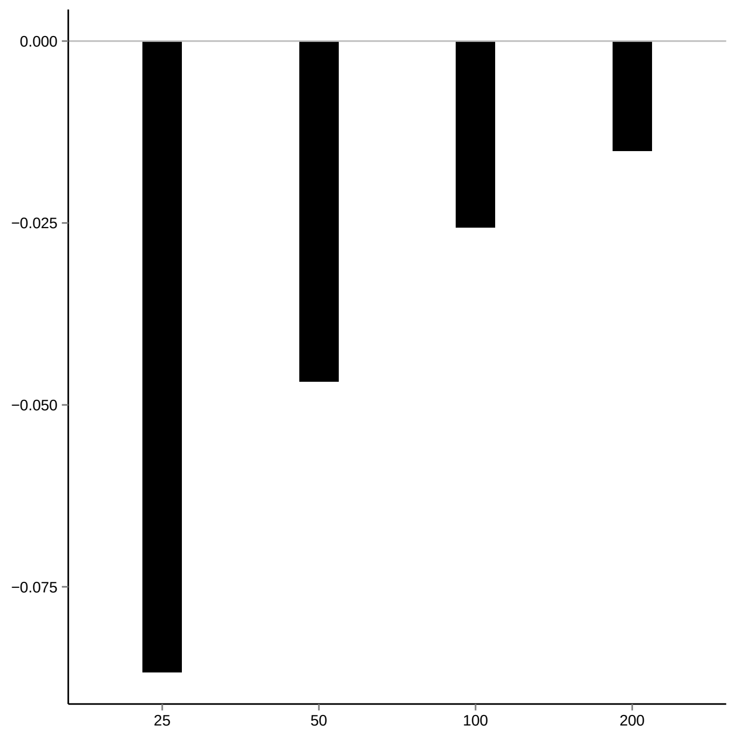

Figure 2: Slope Bias

3.7 If A is a positive definite matrix, show that A^{-1} is also positive definite.

$$x^T A^{-1} x = x^T A^{-1} A A^{-1} x = (A^{-1}x)^T A (A^{-1}x)$$

where the last expression is positive definite since A is positive definite for any vector $x \neq 0$

3.8 If A is a symmetric positive definite $k \times k$ matrix, then $I - A$ is positive definite if and only if $A^{-1} - I$ is positive definite, where I is the $k \times k$ identity matrix. Prove this result by considering the quadratic form $x^T(I - A)x$ and expressing x as $R^{-1}z$, where R is a symmetric matrix such that $A = R^2$.

$$x^T(I - A)x = z^T R^{-1}(I - A)R^{-1} z = z^T(R^{-1}R^{-1} - R^{-1}AR^{-1})z$$

where $(R^{-1})^2 = A^{-1}$ and $R^{-1}AR^{-1} = I$

Thus $x^T(I - A)x = z^T(A^{-1} - I)z$

If $x^T(I - A)x$ is positive definite $\forall x \neq 0$, $z^T(A^{-1} - I)z$ must also be positive definite $\forall z \neq 0$. Therefore $I - A$ is positive definite if and only if $A^{-1} - I$ is positive definite.

Extend the above result to show that, if A and B are symmetric positive definite matrices of the same dimensions, then $A - B$ is positive definite if and only if $B^{-1} - A^{-1}$ is positive definite.

$R^{-1}(A - B)R^{-1} = I - R^{-1}BR^{-1}$
Using what we proved above if the preceding matrix is positive definite the following matrix should also be positive definite.
$RB^{-1}R - I$
Multipliying on both sides by R^{-1} gives us
$R^{-1}RBRR^{-1} - R^{-1}R^{-1} = B^{-1} - A^{-1}$
If $A - B$ is positive definite $B^{-1} - A^{-1}$ must also be positive definite.
Therefore $A - B$ is positive definite if and only if $B^{-1} - A^{-1}$ is positive definite.

3.23 Use them to estimate, for the period 1953:1 to 1996:4, the following autoregressive distributed lag model:

$$c_t = \alpha + \beta c_{t-1} + \gamma_0 y_t + \gamma_1 y_{t-1} + u_t \ (1)$$

Such models are often expressed in first-difference form, that is, as

$$\Delta c_t = \delta + \phi c_{t-1} + \theta y_t + \psi y_{t-1} + u_t \ (2)$$

Estimate the first-difference model (2), and then, without using the results of (1), rederive the estimates of α, β, γ_0, and $gamma_1$ solely on the basis of your results from (2)

Call:
lm(formula = diffc   logc1 + diffy + logy1, data = cons)
##
Residuals:
Min 1Q Median 3Q Max
-0.03388 -0.00636 0.00097 0.00611 0.02521
##
Coefficients:
Estimate Std. Error t value Pr(> $\|t\|$)
(Intercept) 0.0639 0.0217 2.95 0.0036
logc1 -0.0308 0.0223 -1.38 0.1696
diffy 0.2910 0.0551 5.28 3.9e-07
logy1 0.0258 0.0210 1.23 0.2213
##
Residual standard error: 0.0096 on 172 degrees of freedom
Multiple R-squared: 0.191, Adjusted R-squared: 0.177
F-statistic: 13.6 on 3 and 172 DF, p-value: 5.5e-08

We can re-write the first equation as

$$c_t - c_{t-1} = \alpha + \beta c_{t-1} + \gamma_0 y_t + \gamma_1 y_{t-1} + u_t - c_{t-1}$$
$$= \alpha + (\beta - 1)c_{t-1} + \gamma_0 y_t + \gamma_1 y_{t-1} + u_t - \gamma_0(y_{t-1} - y_{t-1})$$
$$= \alpha + \beta c_{t-1} + \gamma_0(y_t - y_{t-1}) + (\gamma_0 + \gamma_1)y_{t-1}.$$

Thus $\alpha = \delta = 0.0639$, $\beta = 1 + \phi = 1 - 0.0308 = .9692$, $\gamma_0 = \theta = .2910$ and $\gamma_1 = \psi - \theta = 0.0258 - .2910 = -.2652$. Which are corraborate by the following regression output:

```
##
## Call:
## lm(formula = logc   logc1 + logy + logy1, data = cons)
##
## Residuals:
## Min 1Q Median 3Q Max
## -0.03388 -0.00636 0.00097 0.00611 0.02521
##
## Coefficients:
## Estimate Std. Error t value Pr(> ||t||)
## (Intercept) 0.0639 0.0217 2.95 0.0036
## logc1 0.9692 0.0223 43.44 <2e-16
## logy 0.2910 0.0551 5.28 3.9e-07
## logy1 -0.2652 0.0564 -4.70 5.4e-06
##
## Residual standard error: 0.0096 on 172 degrees of freedom
## Multiple R-squared: 1, Adjusted R-squared: 1
## F-statistic: 1.63e+05 on 3 and 172 DF, p-value: <2e-16
##
```

3.24 Plot the residuals from running (1) on the simulated data, and compare the plot with that of the residuals from the real data. Comment

The residuals look pretty similar and I.I.D. therefore we cannot reject that our model is correctly specified.

```
##
## Call:
## lm(formula = logc   logc1 + logy + logy1, data = conssim)
##
## Residuals:
## Min 1Q Median 3Q Max
## -0.025009 -0.007116 -0.000406 0.006820 0.027631
##
## Coefficients:
## Estimate Std. Error t value Pr(> ||t||)
## (Intercept) 0.1024 0.0266 3.85 0.00017
## logc1 0.9506 0.0217 43.81 < 2e-16
## logy 0.3803 0.0566 6.71 2.6e-10
```

logy1 -0.3392 0.0569 -5.96 1.4e-08
##
Residual standard error: 0.00982 on 172 degrees of freedom
Multiple R-squared: 1, Adjusted R-squared: 1
F-statistic: 1.47e+05 on 3 and 172 DF, p-value: <2e-16

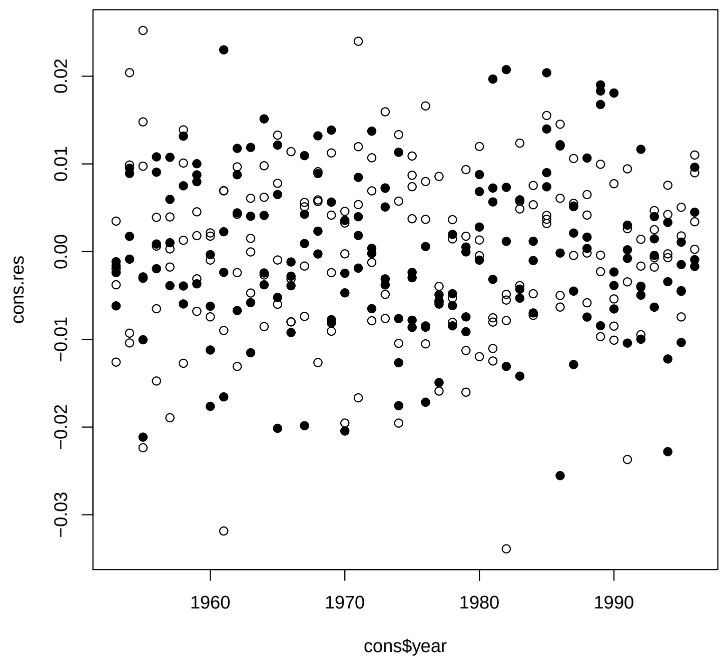

Figure 3: Residuals for the simulated model(filled points) and the model (1) (empty points)