Calculo de Integral elíptica

$$T = \sqrt{\frac{2\ell}{g}} \int_{-\theta_0}^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

$$= \underbrace{\sqrt{\frac{2\ell}{g}} \int_{-\theta_0+\varepsilon}^{\theta_0-\varepsilon} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}}_{\text{numérico}} + \underbrace{\sqrt{\frac{2\ell}{g}} \int_{-\theta_0}^{-\theta_0+\varepsilon} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}}_{\text{analítico}} + \sqrt{\frac{2\ell}{g}} \int_{\theta_0-\varepsilon}^{\theta_0} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} = N + 2A$$

A

$\underbrace{\qquad\qquad}_{\text{analítico}}$

B

$\theta \to -\theta \Rightarrow A = B.$

$$A = \sqrt{\frac{2\ell}{g}} \int_{-\theta_0}^{-\theta_0+\varepsilon} \frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

← θ é próximo de $-\theta_0$

$\theta = -\theta_0 + \varphi$ $\quad \varphi$ pequeno

$$\cos\theta - \cos\theta_0 = \cos(-\theta_0 + \varphi) = \cos\theta_0 \underbrace{\cos\varphi}_{\approx 1} + \operatorname{sen}\theta \underbrace{\operatorname{sen}\varphi}_{\approx \varphi}$$

$$-\cos\theta_0 \simeq \operatorname{sen}\theta\, \varphi,$$

$$A = \sqrt{\frac{2\ell}{g}} \int_0^{\varepsilon} \frac{1}{\sqrt{\operatorname{sen}\theta_0}} \frac{1}{\sqrt{\varphi}}\, d\varphi = \underbrace{\sqrt{\frac{2\ell}{g}} \frac{1}{\sqrt{\operatorname{sen}\theta_0}} \frac{\sqrt{\varepsilon}}{2}}_{\text{Adicionar ao numérico}}$$