Robust H_∞ Control Algorithm for Twin Rotor MIMO System

Lidiya John[1], Mija S.J[2]
[1,2]Dept. of Electrical Engineering,National institute of Technology, Calicut
[1]lidiyajohn1990@gmail.com ,[2]mija@nitc.ac.in

***Abstract*-In this paper, H_∞ control strategy of robust control scheme is designed for Twin Rotor MIMO System (TRMS). It is a highly nonlinear, cross coupled, under actuated MIMO laboratory model of actual helicopter having two degree of freedom. The mixed sensitivity robust control method is proposed to improve the tracking performance. The effectiveness of the designed controller is verified by performing simulations; The disturbance rejection capability of the controller is also established.**

***Keywords- Mixed Sensitivity, Robust control, Cross coupling, two Riccati equation methods*.**

I. INTRODUCTION

The twin rotor multi input multi output system (TRMS) is a laboratory setup developed by M/S feedback Instruments limited [1] for control experiments. The system is perceived as a challenging engineering problem owing to its higher nonlinearity, cross coupling between its two axes, and inaccessibility of some of its states for measurements.

Good amount of quality research was found to have taken place in TRMS over the years. The concept of robust controller design for a laboratory twin-rotor system has been developed gradually and manifested in various national and international conferences and journals. The block diagram of TRMS with details of each block is given in [2]. Also [2] provided more information regarding the TRMS dynamics. In [3] a robust control strategy used in TRMS is discussed. The methods involve decoupling the TRMS into two SISO sub systems for applying the control strategy which is basically developed for SISO systems. A similar control strategy is discussed in [4] which discuss a decoupled time optimal robust controller.

The concept of nonlinear H_∞ disturbance rejection procedure is used for the stabilization of helicopter model in [5], analyzed as a nonlinear PID controller with time varying gains. [6] achieves compensation of a physical twin rotor multiple-input and multiple-output system in two steps: (i) input–output decoupling its transfer function model, obtained by linearising its non-linear model around an operating point, using an open-loop, minimal precompensator and (ii) effecting 2-degree of freedom single-input and single-output(SISO) compensations for the resulting SISO-decoupled units. In [7] nonlinear model of TRMS a linearised model is obtained and a controller is designed to regulate the states.

Here H_∞ Mixed Sensitivity design is extended to the TRMS setup. The TRMS nonlinear model obtained from the Feedback systems operation manual [1] is linearized and is used for H_∞ controller design. The rest of the paper is organized as follows: a description of the system along with the mathematical model is exposed in Section II. Section III describes the classical decoupling technique in TRMS. Section IV and V includes H_∞ Mixed sensitivity controller design. Section VI deals with the simulation results and conclusions in section VII

II. SYSTEM MODELING

A. TRMS Description

TRMS is a laboratory size helicopter model built to study the complications involved in the actual helicopter. As shown in figure.1, TRMS mechanical unit consists of a beam pivoted on its base in such a way that it can rotate freely both in its horizontal and vertical planes. At both end of a beam, there are two propellers driven by DC motors. The two motors are placed perpendicular to each other. The aerodynamic force is controlled by varying the speed of the motors. Therefore, the control inputs are the supply voltages of the DC motors. The TRMS system has main and tail rotors for generating vertical and horizontal propeller thrust. There is a counter weight fixed to the beam and it determines a stable equilibrium position. Apart from the mechanical unit, the electrical unit placed under the unit pays an important role for TRMS control. It allows for the measured signal transfer to PC and the control signal application via an I/O card. The measured signals are: position of beam in space, i.e. two position angles. The controls of the system are the motor supply voltages.

B. Dynamics of the TRMS

The momentum equations for vertical plane of motion may be written as:

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$$I_1\ddot{\psi} = M_1 \text{-} M_{FG} \text{-} M_{B\psi} \text{-} M_G \quad (1)$$

Where M_1 is the gross momentum of main rotor , which is related to Vertical DC motor momentum τ_1 ,by the non linear static characteristic $M_1 = a_1.\tau_1^2 + b_1.\tau_1$. M_{FG} = Mg.sinψ is the gravity momentum. $M_{B\psi} = B_{1\psi}.\dot{\psi} + B_{2\psi}.\sin(2\psi).\dot{\varphi}^2$ is the frictional force momentum and $M_G = K_{gy}.M_1.\dot{\varphi}.\cos(\psi)$ is the gyroscopic momentum. The motor and the electrical control circuit is approximated by a first order transfer function. Thus in Laplace domain, the motor momentum is given by,

$$\tau_1 = \frac{K_1}{T_{11}s + T_{10}}.u_1 \quad (2)$$

Similar momentum equations refer to the horizontal plane motion

$$I_2\ddot{\varphi} = M_2 \text{-} M_{B\varphi} \text{-} M_R \quad (3)$$

Where M_2 is the gross momentum of tail rotor , which is related to horizontal DC motor momentum τ_2 ,by the non linear static characteristic $M_2 = a_2.\tau_2^2 + b_2.\tau_2$. $M_{B\varphi} = B_{1\varphi}.\dot{\varphi}$ is the friction force momentum. $M_R = k_c.\frac{T_0 s+1}{T_p s+1}.M_1$ is the approximate cross reaction momentum.

Again the DC motor with electrical circuit is given by,

$$\tau_2 = \frac{K_2}{T_{21}s + T_{20}}.u_2 \quad (4)$$

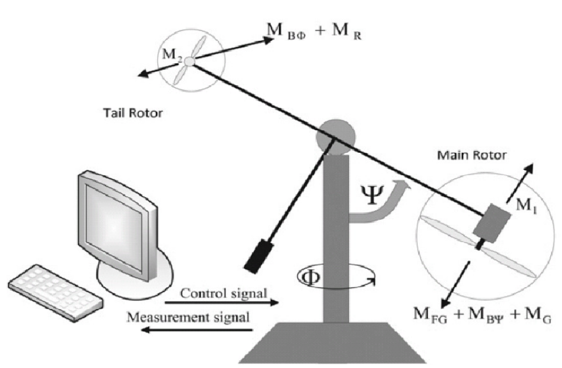

Figure 1. Twin rotor MIMO system

Parameters	Value
I_1 – moment of inertia of vertical motor	0.068 kg m^2
I_2 – moment of inertia of horizontal motor	0.02 kg m^2

a_1 – static characteristic parameter	0.0135
b_1 – static characteristic parameter	0.0924
a_2 – static characteristic parameter	0.02
b_2 – static characteristic parameter	0.09
Mg – gravity momentum	0.32 N m
$B_{1\psi}$ – friction momentum function parameter	0.006 N m s/rad
$B_{2\psi}$ – friction momentum function parameter	0.001 N m s2/rad
$B_{1\square}$ – friction momentum function parameter	0.1 N m s/rad
K_{gy} – gyroscopic momentum parameter	0.05 s / rad
k_1 – motor 1 gain	1.1
k_2 – motor 2 gain	0.8
T_{11} – motor 1 denominator parameter	1.1
T_{10} – motor 2 denominator parameter	1
T_{21} – motor 1 denominator parameter	1
T_{20} – motor 2 denominator parameter	1
T_p – cross reaction momentum parameter	2
T_0 – cross reaction momentum parameter	3.5
k_0 – cross reaction momentum gain	-0.8

TABLE 1. PARAMETERS OF TRMS SETUP

C. Transfer Function Model Of The TRMS

Using the dynamical equations given by operation manual for TRMS, the state space model of linearized plant is given below.

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Where the state vector $x = [\psi\ \dot{\psi}\ \varphi\ \dot{\varphi}\ \tau_1\ \tau_2]^T$,input vector $u = [u_1\ u_2]^T$ and the output vector $y = [\psi\ \varphi]^T$. ψ and φ are the pitch and yaw angles respectively, and u_1 and u_2 are input to motors corresponding to main and tail rotors respectively.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-Mg}{I_1} & \frac{-B_1}{I_1} & 0 & 0 & \frac{b_1}{I_1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{-B_2}{I_2} & \frac{-1.75\,k_c\,b_1}{I_2} & \frac{b_2}{I_2} \\ 0 & 0 & 0 & 0 & \frac{-T_{10}}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-T_{20}}{T_{21}} \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{K_1}{T_{11}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{K_2}{T_{21}} \end{bmatrix}^T$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad D=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The plant transfer function matrix (from u to y) is obtained as:

$$P \triangleq C(sI - A)^{-1}B + D \qquad (5)$$

$$P=\begin{bmatrix} \frac{1.359}{s^3+0.9973s^2+4.786\,s+4.278} & 0 \\ \frac{1.617}{s^3+5.909s^2+4.545\,s} & \frac{3.6}{s^3+6s^2+5\,s} \end{bmatrix}$$

It is obvious from above model that there is a considerable coupling between the main rotor's motor input(u_1) and the yaw angle(φ). The open loop response of yaw angle is unstable.

III. DECOUPLING CONTROLLER FOR TRMS

The decoupling controller D(s) is employed to achieve input-output decoupling in open loop. Given a MIMO plant P, a method to design D such that the transfer function G=D*P has only diagonal element.

$$D(s) = \begin{bmatrix} 1 & D_1(s) \\ D_2(s) & 1 \end{bmatrix}$$

$D_1(s)$ and $D_2(s)$ are introduced in order to eliminate the cross coupling effect in the TRMS subsystem.

$$D_1(s) = -\left[\frac{1.617\,s^3+1.613\,s^2+7.739\,s+6.918}{1.359s^3+8.029\,s^2+6.176\,s}\right]$$

$$D_2(s) = 0$$

So that

$$G=\begin{bmatrix} \frac{1.359}{s^3+0.9973s^2+4.786\,s+4.278} & 0 \\ 0 & \frac{3.6}{s^3+6s^2+5\,s} \end{bmatrix}$$ has only diagonal element.

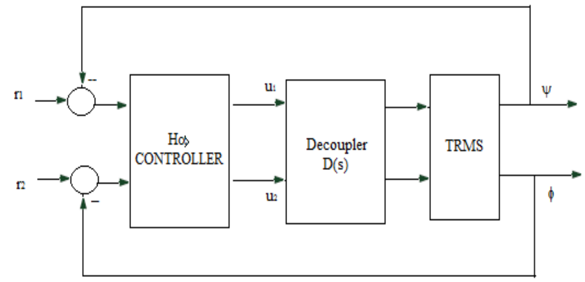

Figure 2. TRMS with H_∞ controller

IV. H_∞ CONTROLLER DESIGN PROBLEM

Consider the system shown in block diagram shown in figure 3. The plant P and controller K are assumed to be real rational and proper. The generalized plant P contains the plant in the control problem plus all weighing functions. Plant P has two inputs, the exogenous input w including disturbances, sensor noise and commands and the control input u. There are two outputs, controlled output z that we want to minimize and measured variable v [8].

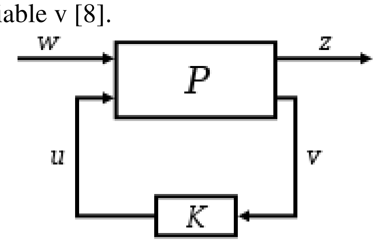

Figure 3.Generalized plant and controller configuration

The transfer functions will be denoted as

$$P(s)=\begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

The linear fractional transformation and is the transfer matrix from exogenous input w to output z is denoted by $F_l(P,K)$ is given by:

$$F_l(P,K) = (P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}) \qquad (6)$$

The infinity norm of transfer function matrix $F_l(P,K)$ is defined as

$$||F_l\,(P,K)\,||_\infty = \sup_{\omega} \bar{\sigma}(F_l(P,K)(j\omega)) \qquad (7)$$

The objective of H_∞ control design is to minimize the closed loop transfer function $F_l(P,K)$ according to H_∞ norm.

V. THE MIXED SENSITIVITY OPTIMIZATION PROBLEM

The sensitivity and complementary sensitivity function can characterize the robustness of the system. A general feedback system is shown in figure 4. The sensitivity and complementary sensitivity function are defined as follows [9]:

$$S = \frac{e}{r} = \frac{1}{(1+GK)} \qquad (8)$$

$$T = \frac{y}{r} = \frac{GK}{(1+GK)} \qquad (9)$$

The relationship between S and T is S+T=I

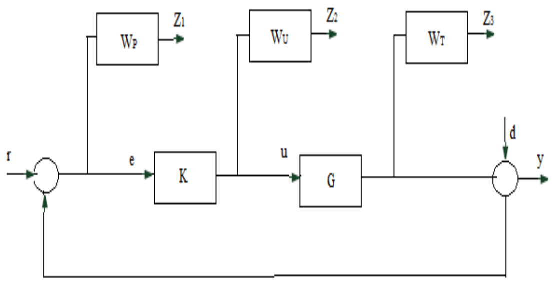

Figure 4.Closed loop system structure for mixed sensitivity minimization

Considering the standard frame of the mixed sensitivity problem the generalized plant P(s) can be expressed:

$$P(s) = \begin{bmatrix} W_P & -w_P G \\ 0 & W_u \\ 0 & W_T G \\ I & -G \end{bmatrix} \qquad (10)$$

u=Ke

The mixed sensitivity problem is looking for a real rational function controller K, and it can make the closed loop system stable and meet,

$$\left\| \begin{matrix} w_p S \\ w_u KS \\ w_T T \end{matrix} \right\|_\infty = 1$$

It is called H_∞ optimization problem. Two Riccati equation method is introduced to find out the controller.

A. Selection of Weighing Function

For proper tracking and disturbance rejection the sensitivity transfer function must be small in low frequency ranges. This can be formulated as specifying that the sensitivity is below a frequency dependent weight, W_P which is connected between e(s) and plant

$$|S| \le |W_P{}^{-1}|$$

Noise rejection requires that the complementary sensitivity transfer function must be small in high frequency ranges. This can be formulated as specifying the complementary sensitivity is below a frequency dependent weight W_T., ie

$$|T| \le |W_T{}^{-1}|$$

Control transmission weight W_U is incorporated with u(s) to limit the control input with in specified values.

The weights W_P and W_u are your tuning parameters, and it typically requires some iterations to obtain weights which will yield a good controller. That being said, a good starting point is to choose

$$W_P = \frac{s/M + w_0}{s + w_0 A} \qquad (11)$$

; $W_u = constant,$

Where A < 1 is the maximum allowed steady state offset, w_0 is the desired bandwidth and M is the sensitivity peak. For the controller synthesis,

the inverse of W_P is an upper bound on the desired sensitivity loop shape, and W_u^{-1} will effectively limit the controller output u. Shape the complementary sensitivity function T = GK(I +GK)$^{-1}$. A starting point is to choose

$$W_T = \frac{s + w_{0/M}}{As + w_0} \qquad (12)$$

Which is symmetric to W_P around the line $\omega = \omega_0$

By combining all these functions it is possible to influence both low-frequency and high frequency properties of the closed loop system. In practice, it is desirable to find the controller K(s) which minimizes a weighed norm which combines these three transfer functions.

$$\left\| \begin{matrix} W_P(j\omega)S(j\omega) \\ W_T(j\omega)T(j\omega) \\ W_U(j\omega)KS(j\omega) \end{matrix} \right\|_\infty$$

VI. RESULTS AND DISCUSSION

As explained in section V for pitch and yaw angle controls the weighing matrices are computed as:

Wp1=(0.6667 s + 0.23)/(s + 0.00023)

Wu1=1: is taken as a constant

W_T1= (s + 0.1533)/(0.001 s + 0.23)

Wp2= (0.6667 s + 0.9/(s + 0.0009)

Wu2=0.088: is taken as a constant;

W_T2 is not assigned

The controller for pitch angle control u_1=K1*e_1

Where,

K1=

$$\begin{bmatrix} -0.00023 & 0 & 0 & 0 & -1.11e-016 & 23.75 \\ 0 & -230 & 0 & 0 & 347.9 & 0 \\ 65.6 & 0.01127 & -23.59 & -17.53 & -46.11 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2.762 & 0.0004745 & -0.9512 & -0.6879 & -1.896 & 0 \end{bmatrix}$$

Similarly for yaw angle control, u_2=K2*e_2

$$K2=\begin{bmatrix} -0.0008929 & 0 & 0 & -2.812e-008 & 0.4298 \\ 2.121e+005 & -2.516e+004 & -1.225e+005 & -4.235e+005 & 6.173e-027 \\ -3.194e-031 & 2 & 5.821e-011 & -4.039e-026 & -9.645e-027 \\ -9.207e-016 & 0 & 1 & -5.821e-011 & -2.78e-011 \\ 4.934e+005 & -5.853e+004 & -2.849e+005 & -9.853e+005 & 0 \end{bmatrix}$$

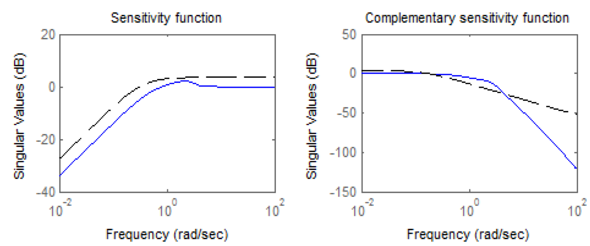

Figure 5.Singular value plot for Pitch angle control

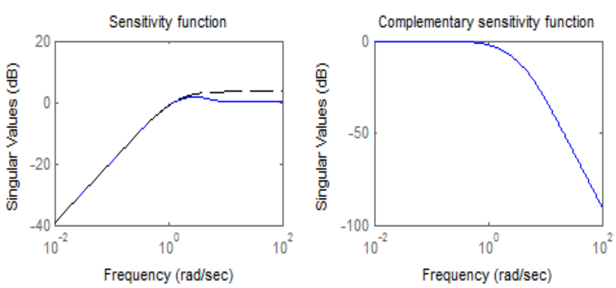

Figure 6. Singular value plot for yaw angle control

Figure 5,6 shows the singular value plot showing the sensitivity and complementary sensitivity functions for both pitch and yaw angle control respectively. The figures confirm that the design specifications given by the equations are satisfied.

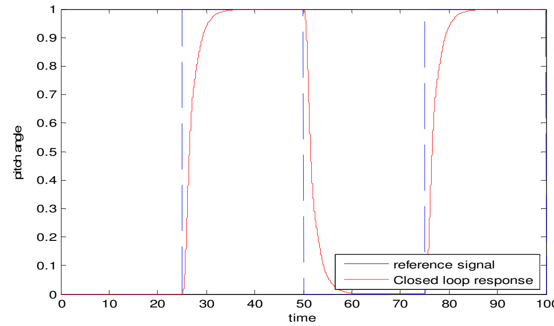

Figure 7.Pitch angle response with H_∞ Controller for time varying reference signal

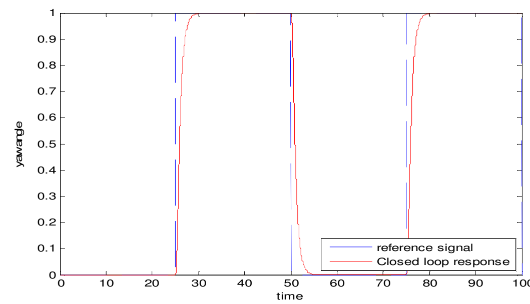

Figure 8.Yaw angle response with H_∞ Controller for time varying reference signal

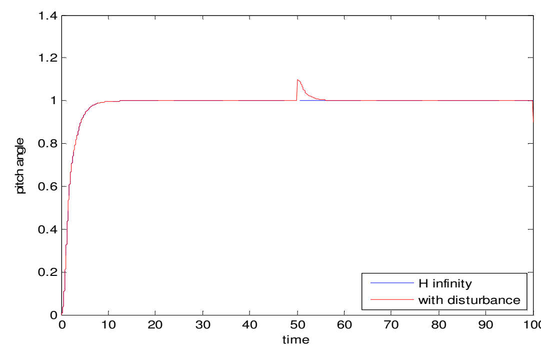

Figure 9.Pitch angle output with H_∞ Controller due to disturbance at output

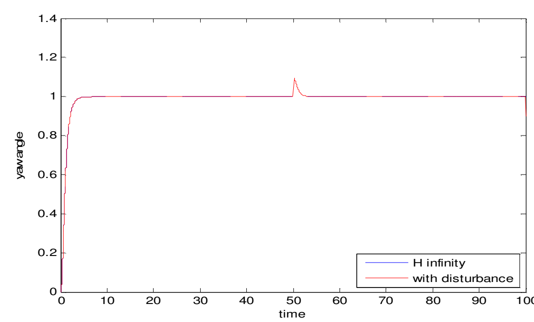

Figure 10. yaw angle output with H_∞ Controller due to disturbance at output

The pitch and yaw angle tracking responses with the designed H_∞ controller for a varying reference input are shown in figure 7 and 8. From the plots it is clear that the controller is capable of tracking the input.

To test the disturbance rejection capability of the controller, a constant disturbance of 0.1 magnitude is applied after the system settled to the reference value. The responses are shown in figure 9 and 10. As expected the H_∞ controller rejects the effect of the disturbance.

VII. CONCLUSION

This paper addresses the mathematical model of twin rotor MIMO system, and designs the H_∞ mixed sensitivity controller. The H_∞ mixed sensitivity controller can meet the design requirements of the tracking performance index and the robust performance index. From the results obtained it is found out that with a better choice of sensitivity function-weights H_∞ controller be the optimal one to stabilize TRMS through feedback.

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