Modern Sampling Methods

Class 4: Treatment and Policy Choice

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Outline

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- CES Rules and Minmax Regret
- Local Asymptotic Optimality
- Empirical Welfare Maximization

Basic Setup

Based on Manski (2004) and Dehejia (2005).

$\mathcal{T} = \{0, 1\}$: set of possible treatments.

$Y(0)$, $Y(1)$: potential outcomes

$X \in \mathcal{X}$: background characteristics.

Let θ be parameters associated with potential outcomes:

$$\begin{aligned} X &\sim F_X(\cdot) \\ Y(0)|X = x &\sim F_0(\cdot|x, \theta) \\ Y(1)|X = x &\sim F_1(\cdot|x, \theta) \end{aligned}$$

Treatment Assignment Rules and Social Welfare

A treatment assignment rule selects treatment based on X:

$$\delta : \mathcal{X} \to \{0, 1\}.$$

(Could also allow for randomization.)

Suppose we want to maximize average outcomes

$$W(\theta, \delta, x) = \delta(x) E_\theta[Y(1)|X = x] + (1 - \delta(x)) E_\theta[Y(0)|X = x];$$

$$W(\theta, \delta) = \int W(\theta, \delta, x) dF_X(x).$$

The ideal policy is

$$\delta^*(x) = \mathbf{1}\left\{E_\theta[Y(1)|X = x] \geq E_\theta[Y(0)|X = x]\right\}.$$

Statistical Treatment Rule

This is not feasible in general b/c we do not know θ.

Suppose we have some data that is informative about θ. How to use the past data to inform the future choice of treatment rule?

<u>Statistical Treatment Rule</u>:

Before making our treatment assignment, we observe some data $Z \sim P_\theta$

(Assume Z independent of the future individual to be treated.)

We then choose δ based on the data z.

Note the timing:

1. observe Z (e.g. from a randomized controlled trial);
2. take a <u>new</u> individual, and observe their X;
3. assign this individual to treatment based on her own X as well as the data of others collected in Z.

Notation:

$$\delta(x, z)$$

indicating that the policy depends on data and on any information we have about the new individual.

Ex ante probability of assigning individuals with $X = x$ to treatment:

$$\beta(\delta, x, \theta) = E_\theta[\delta(x, Z)] = \int \delta(x, z) dP_\theta(z).$$

Ex ante expected social welfare for a given rule δ:

$$E_\theta[W(\theta, \delta(\cdot, Z))] =$$

$$\int \int \Big\{ \delta(x, z) \cdot E_\theta[Y(1) | X = x)]$$

$$+ (1 - \delta(x, z)) \cdot E_\theta[Y(0) | X = x] \Big\} dF_X(x) dP_\theta(z).$$

Example: Dehejia (2005)

GAIN experiment, a randomized evaluation of a job training program in California. (Data from Alameda County.)

Tobit model for earnings of individual i in quarter t:

$$Y_{it}^* = x_{it1}'\beta_1 + T_i \cdot x_{it1}'\beta_2 + x_{it2}'\beta_3 + \epsilon_{it}, \quad \epsilon_{it} \overset{iid}{\sim} N(0, \sigma^2),$$

$$Y_{it} = 1(Y_{it}^* > 0) Y_{it}^*.$$

Parameter vector: $\theta = (\beta, \sigma^2)$. The data are:

$$Z = \{(x_{it1}, x_{it2}, T_i, , Y_{it}) : i = 1, \ldots, n, t = 1, \ldots, T\}.$$

Use Bayesian methods to simulate posterior distribution $p(\theta|Z)$.

Hypothetical decision problem: counselor is dealing with a new individual (person $n+1$), whose covariates $x_{n+1,t}$ are observed and whose earnings will follow the same Tobit model.

Can simulate outcomes for person $n+1$:

- Draw θ from posterior $p(\theta|Z)$.
- Simulate $Y_{n+1}(0)$ given $x_{n+1,t}$ and setting $T_{n+1} = 0$.
- Simulate $Y_{n+1}(1)$ given $x_{n+1,t}$ and setting $T_{n+1} = 1$.

Then choose treatment that has higher expected outcome.

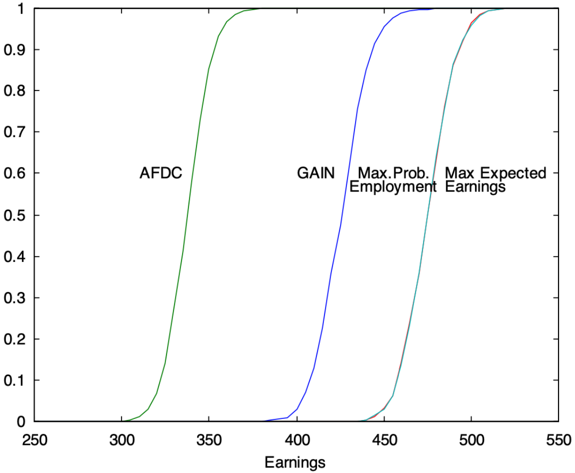

Fig. 7. Predictive distributions for average earnings.

From: Dehejia (2005)

Some other economic applications

- Job Training Programs: JTPA (Kitagawa and Tetenov 2018)
- Environmental Policy (Assuncao et al, 2019)
- Energy Incentives (Knittel and Stolper, 2019)
- Marketing (Rossi et al 1996, Dube et al 2017)

Manski (2004)

Suppose the covariate X is discrete, taking on possible values $\{x_j : j = 1, \ldots, k\}$.

Suppose data Z are obtained from a block-randomized experiment:

N_j units with $X = x_j$, of which
N_j^1 treated and N_j^0 controls.

<u>Conditional Empirical Success (CES) Rule</u>:

$$\hat{\beta}_j := \frac{1}{N_j^1} \sum_{i=1}^{N_j} T_{ji} Y_{ji} - \frac{1}{N_j^0} \sum_{i=1}^{N_j} (1 - T_{ji}) Y_{ji}.$$

Then define

$$\hat{\delta}(x_j, Z) = 1(\hat{\beta}_j > 0).$$

Manksi's CES rule is nonparametric and intuitive, but not immediately clear whether it is in some sense optimal.

Related question: if covariate X takes on many values (or is continuous), will CES work well or are there alternatives?

To analyze further, we need some measure of a statistical treatment rule's performance.

We will consider expected welfare <u>regret</u>:

$$R(\theta, \delta) = E_\theta \left[W(\theta, \delta^*) - W(\theta, \delta) \right],$$

where δ^* is the ideal rule.

Stoye (2009)

Matched Pairs Experiment:

- Y_0 and Y_1 are binary variables
- n is even and we observe exactly $n/2$ treated and $n/2$ controls in our data Z.
- Let $\bar{Y}_1$ and $\bar{Y}_0$ be the averages in the two groups.

Let

$$\hat{\delta}(Z) = \begin{cases} 0 & \text{if } \bar{Y}_1 < \bar{Y}_0 \\ \frac{1}{2} & \text{if } \bar{Y}_1 = \bar{Y}_0 \\ 1 & \text{if } \bar{Y}_1 > \bar{Y}_0 \end{cases}$$

(Essentially the CES rule.)

Stoye (2009) shows that $\hat{\hat{\delta}}$ is minmax with respect to regret:

$$\hat{\hat{\delta}} = \arg\min_{\delta} \max_{\theta} R(\theta, \delta),$$

where $\max_\theta$ takes the maximum over all possible distributions.

This result also extends to bounded outcomes, and (with minor adjustment of rule) to some different RCT randomization schemes.

The result also holds with covariates: then the minmax regret rule conditions *fully* on X, even if this means very few observations per cell, or even some empty cells.

Results as sharp as Stoye's are difficult to obtain in more complex situations with

- More complex outcome distributions
- Structured/parametrized outcome distributions
- Nonexperimental (observational) data, or data from adaptive experiments
- Restrictions on the class of rules (e.g. constraints on complexity of rule)
- etc.

Then it can be useful to turn to large-sample approximations to study alternative rules.

Local Asymptotics for Treatment Assignment

Consider a setting without covariates, but where data are not necessarily from a RCT: there is just some general statistical model

$$Z^n \sim P_\theta, \quad \theta \in \Theta$$

(where n indicates sample size).

The model parameter θ is informative about average treatment effect through:

$$\text{ATE} = W(\theta, 1) - W(\theta, 0) = g(\theta)$$

for some known function g.

As $n \to \infty$, we will often be able to estimate θ consistently:

$$\hat{\theta} \xrightarrow{p} \theta \quad \Rightarrow \quad g(\hat{\theta}) \xrightarrow{p} g(\theta),$$

and therefore we can learn the optimal rule in the limit.

However, this type of analysis does not capture the finite-sample risk arising from estimation error in $\hat{\theta}$.

One useful way to better reflect finite-sample properties is to consider the behavior of rules when $g(\theta) \approx 0$: let θ_0 satisfy

$$g(\theta_0) = 0,$$

and consider parameters local to θ_0:

$$\theta = \theta_0 + \frac{h}{\sqrt{n}}.$$

Under this local parametrization:

- uncertainly about whether $g(\theta) \lesseqgtr 0$ does not vanish;
- but classic asymptotic normality theory for parametric and semiparametric statistical models largely carries through.

Hirano and Porter (2009): if P_θ is a regular parametric model, and if $\hat{\theta}$ is an asymptotically efficient estimator (e.g. MLE), then the "plug-in" rule

$$\hat{\delta} = 1(g(\hat{\theta} > 0))$$

is locally asymptotically minmax regret.

In semiparametric settings, if $\hat{g}$ is a semiparametrically efficient estimator of the ATE, then $\hat{\delta} = 1(\hat{g} > 0)$ is locally asymptotically minmax regret.

Empirical Welfare Maximization

This suggests to replace unknown welfare with a “good” estimate.

Next consider the problem with covariates: $\delta(\cdot)$ is a function of X.

Let

$$W(\delta) = E_X\left[\delta(X)E[Y(1)|X] + (1-\delta(X))E[Y(0)|X]\right].$$

Suppose

$$\widehat{W}(\delta) = \text{estimate of } W(\delta),$$

and we set

$$\hat{\delta} = \arg\max_{\delta} \widehat{W}(\delta).$$

This is the general empirical welfare maximization principle.

Suppose we have (conditionally) randomized experimental data and X has finite support.

Then we can estimate $E[Y|T=1,X]$ and $E[Y|T=0,X]$ by empirical conditional averages $\hat{\mu}_1(X)$ and $\hat{\mu}_0(X)$.

Then set

$$\widehat{W}(\delta) = \frac{1}{n}\sum_{i=1}^{n}\big[\delta(X_i)\hat{\mu}_1(X_i) + (1-\delta(X_i))\hat{\mu}_0(X_i)\big].$$

This leads to Manski's CES rule.

Next suppose X is continuous.

Then the space of possible functions $\delta(X)$ is very large, and it is not generally possible to estimate $W(\delta)$ uniformly well.

Kitagawa and Tetenov (2018) propose to restrict the class of possible rules δ.

For example, consider only rules of the form

$$\delta(X) = 1(\alpha + \beta X > 0).$$

In applications, it may be more practical to consider simple classes of rules such as this.

For $\delta \in \mathcal{A}$ where $\mathcal{A}$ is sufficiently "small," it may be possible to construct welfare estimators $\widehat{W}(\delta)$ s.t.

$$\widehat{W}(\delta) \xrightarrow{p} W(\delta),$$

and

$$\sqrt{n}\left(\widehat{W}(\delta) - W(\delta)\right) \xrightarrow{d} N(0, V_\delta),$$

uniformly in $\delta \in \mathcal{A}$.

Then

$$\hat{\delta}(X) = \arg\max_{\delta \in \mathcal{A}} \widehat{W}(\delta)$$

will typically have good properties.