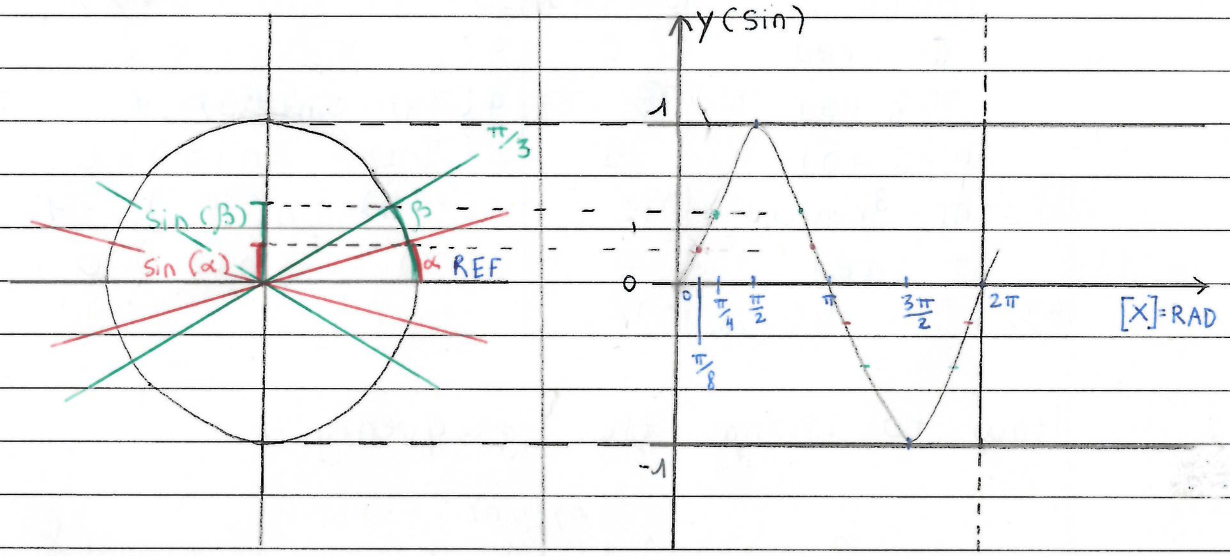
4.4 Trigonometric Functions

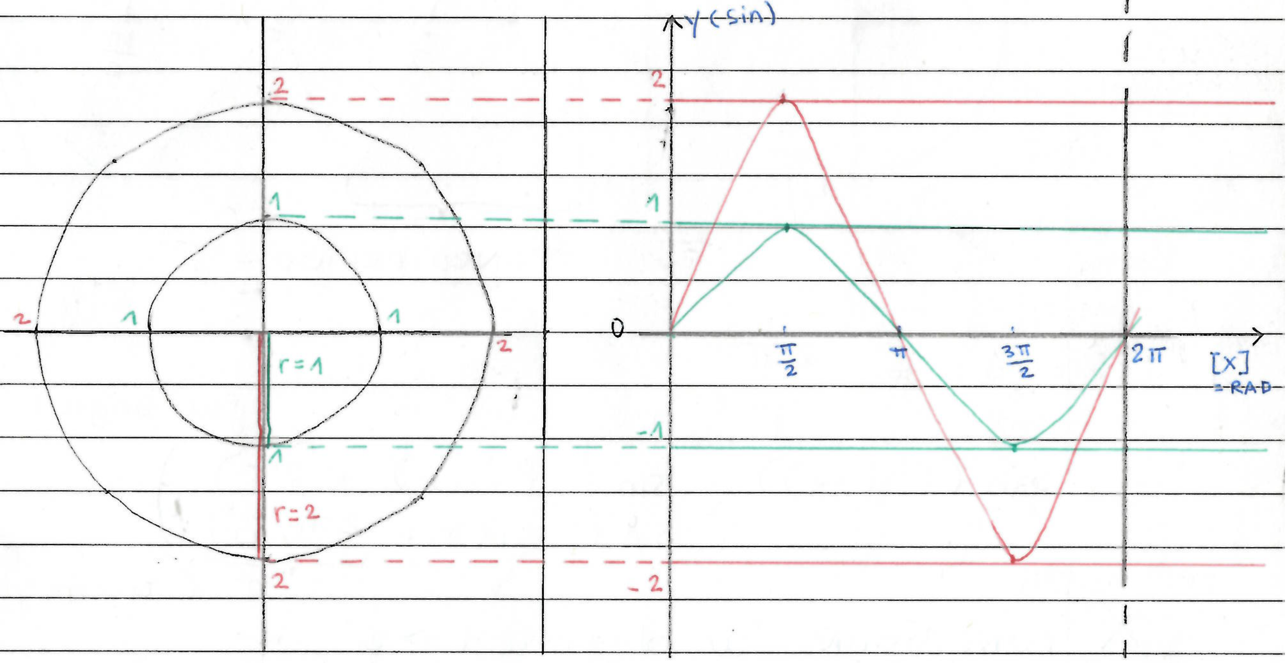
STATIC ANGLES (α) VARIABLE ANGLES (x)

UNIT CIRCLE (RESCALED (ORTHO) PLOT WINDOW



When the r becomes larger ($f(x) = r \cdot \sin(x)$)

1



$f(x) = r \cdot \sin(x)$ $[r > 0]$

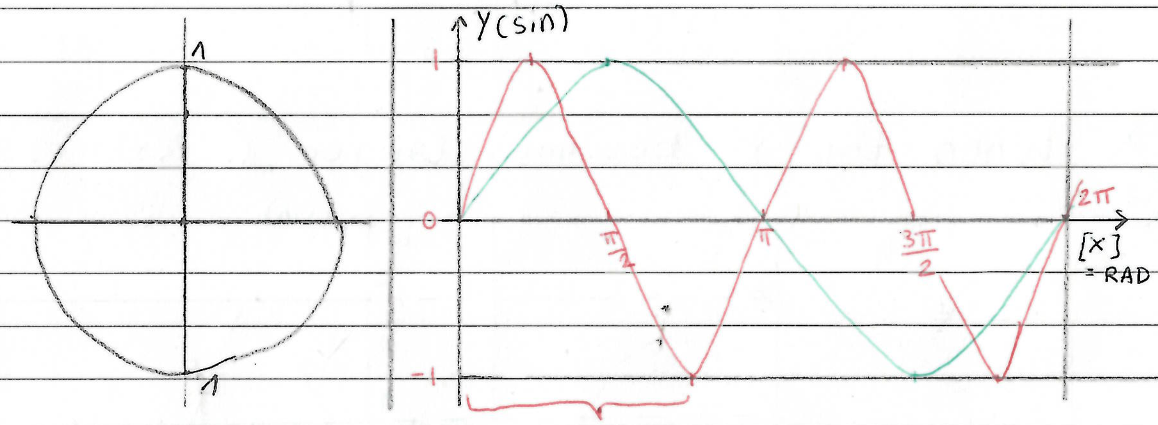
$f(x) = 2 \cdot \sin(x)$

② What if; - $\sin(x) = \sin(1 \cdot x)$
$g(x) = \sin(2 \cdot x)$
$f(x) = \sin(\omega \cdot x)$

* ω = 'omega'

TABLE x	$\sin(x)$	$g(x) = \sin(2 \cdot x)$
0 rad	0	$g(0) = \sin(0) = 0$
$\pi/4$ rad	$\sqrt{2}/2 \approx 0.7$	$g(\pi/4) = \sin(\pi/2) = 1$
$\pi/2$ rad	1	$g(\pi/2) = \sin(\pi) = 0$
$\frac{2\pi}{2 \cdot 2} + \frac{\pi}{4} = 3\pi/4$ rad	$\sqrt{2}/2$	$g(3\pi/4) = \sin(3\pi/2) = -1$
π rad	0	$g(\pi) = -\sin(2\pi) = 0$
$4\pi/4 + \pi/4 = 5\pi/4$ rad	$-\sqrt{2}/2$	

How to change the frequency



$$\omega = \frac{2\pi}{T}$$

$$\sin((\omega x)) = \sin\Big((\omega x) + \underbrace{2\pi}_{\text{(full angle)}}\Big)$$
$$\sin\Big(\omega(x + \underbrace{2\pi/\omega}_{T \text{ (tuned period)}})\Big)$$

CONCLUSION; Tuned period => $T = \frac{2\pi}{\omega}$ solve for $\omega = \frac{2\pi}{T}$

$$\underbrace{\omega = 2\pi \cdot \overbrace{F}^{}}_{\text{pulsation}}$$

angular frequency

Measure = Unit
Lenght = M(eter)
ω = $[2\pi \text{ rad}][1/T]$
= rad 1/2

Interprate like; (ω) = rad/s = angular speed
$(\omega) = (\text{rad})\, s^{-1}$ s = seconds
$= (\text{rad})\, Hz$ Hertz

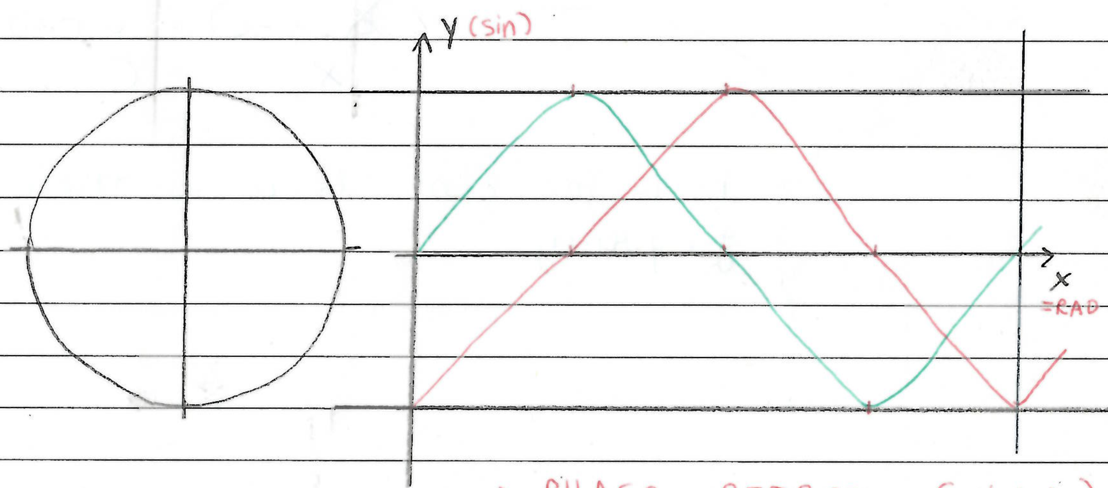
Sound	Base (tone) ↗	,	pitch
Light	IR (colour toning) ↗		UV

How to replace your function on the x-axis

3 $\sin(x) = \sin(x+0)$ ⇝ $h(x) = \sin\left(x + \left(\frac{-\pi}{2}\right)\right)$

X $\sin(x + \theta)$ Theta

TABLE X	Sin (x)	$h(x) = \sin(x(^{-\pi}/_2))$
0 r	0	$h(0) = \sin(^{-\pi}/_2) = -1$
$\pi/4$ r	$\sqrt{2}/2 \approx 0.7$	$h(\pi/4) = \sin(^{-\pi}/_4) = -\sqrt{2}/2$
$\pi/2$ r	1	$h(\pi/2) = \sin(0) = 0$
$3\pi/4$ r	$\sqrt{2}/2$	$h(3\pi/4) = \sin(\pi/4) = \sqrt{2}/2$
π r	0	$h(\pi) = \sin(\pi/2) = 1$



→ PHASE EFFECT (h(x))

There are 4 → 6 appearances of the generalised sign-functions

- $f_1(x) = a \sin(bx+c)+d$
- $f_2(x) = a \sin(b(x+c))+d$
- $f_3(x) = a \sin(bx-c)+d$
- $f_4(x) = a \sin(b(x-c))+d$
- $f_5(x) = -a \sin(b(x-c))+d$

→ odd ⇒ $a \sin(-(b(x-c))+d)$

* $d=0$

⚠ ADVISED!

X Phase-effect = 1st incoming root to find

example; 1 $h(x_0) = \sin(x_0 - \pi/2) = 0$

$$x_0 - \pi/2 = 0 \text{ rad}$$
$$x_0 = \pi/2 \text{ rad}$$

1ST ROOT of $h(x)$, so; the x-shift to the right hand side

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example 2; $f_4(x) = a\sin(b(x-c))+d = 0 \quad (a \neq 0)$

$$\cancel{b}(x-c) = 0 \text{ rad}$$
$$x_0 = c = \text{PHASE}$$

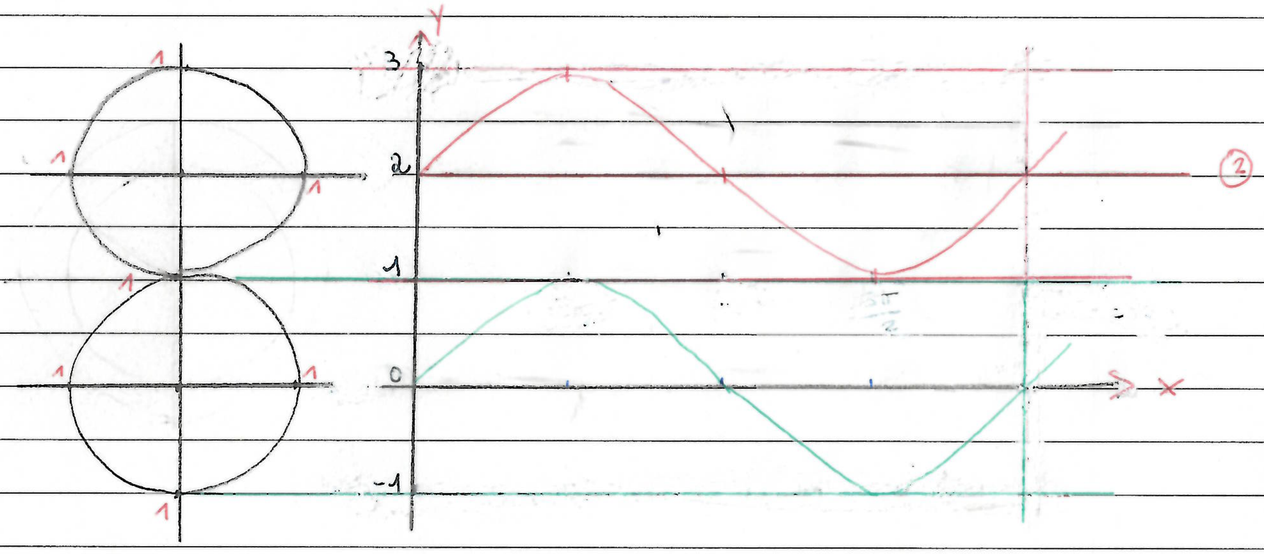
b is innocent because there is no frequency.

④ Constant term in recipe (placing the function higher on the y-axis)

$\sin(x) = 1 \cdot \sin(1x + 0) + 0$

$k(x) = \sin(x) + 2 \rightsquigarrow k(0) = 0 + 2 = 2$

$\ell(x) = \sin(x) + c \rightsquigarrow \ell(0) = 0 + c = c$



I.D. Trigonometric Sin-function

dom $_{\sin} = \mathbb{R} =]-\infty, +\infty[$

range $_{\sin} = [-1, +1] \in \mathbb{R}$

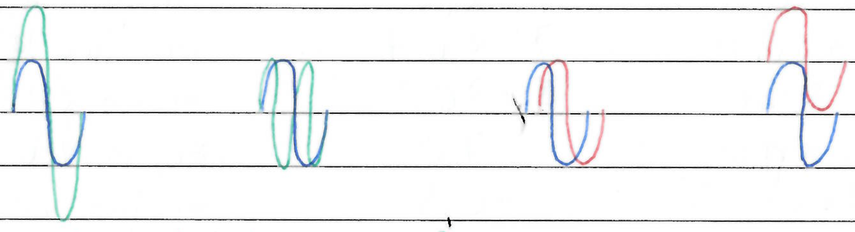
X root(s) $_{\sin} = \ldots, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots = X_0 = k\pi$
$(k \in \mathbb{Z}$

feature(s) $_{\sin}$ = HAS PERIODICITY BY NATURAL PERIODICITY
= 2π rad
= full angle

Transforming a sine-function.

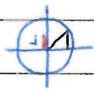
$f(x) = a\sin(bx - c) + d$

Change: a | b | c | d

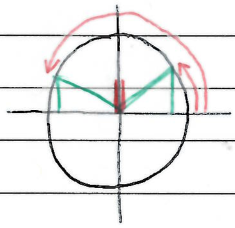


a	b	c	d
(higher/lower) (tops) amplitude	(frequency) (of the 'waves')	move on x-axis	move on y-axis

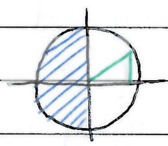
4.5 Inverse Trigonometric Functions

RECALL;  the sin of an angle was given a tangent in order to calculate the sin
$\sin(\alpha) = s$

Now; We want to return to the original arc by;
$\arcsin(\sin(\alpha)) = \arcsin(s)$

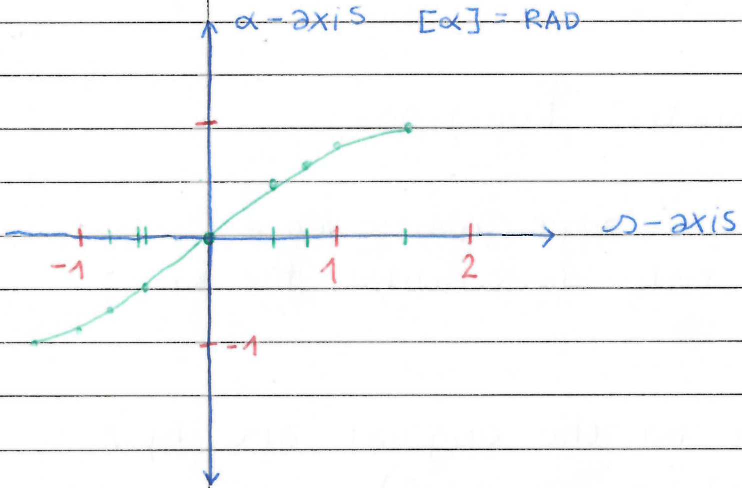


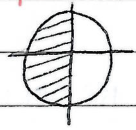
⚠ but we don't know which side of the unit circle the angle is on, because; the sin is the same.

So;  parts get crossed out.

TABLE

$[\alpha] = RAD$	$\sin(\alpha) = s$	$\arcsin(s)$
$0{,}00$	$0{,}0$	$\arcsin(0) = 0$
$\pi/6 \approx 0{,}50$	$0{,}5$	$\arcsin(0{,}5) = \pi/6 \approx 0{,}50$
$\pi/4 \approx 0{,}75$	$\sqrt{2}/2 \approx 0{,}7$	$\arcsin(0{,}7) = \pi/4 \approx 0{,}75$
$\pi/3 \approx 1{,}00$	$\sqrt{3}/2 \approx 0{,}9$	$\arcsin(0{,}9) = \pi/3 \approx 1{,}00$
$\pi/2 \approx 1{,}50$	$1{,}0$	$\arcsin(1{,}0) = \pi/2 \approx 1{,}50$
$-\pi/3 \approx -1{,}00$	$-0{,}9$	$\arcsin(-0{,}9) = -\pi/3 \approx -1{,}00$
$-\pi/4 \approx -0{,}75$	$-0{,}7$	$\arcsin(-0{,}7) = -\pi/4 \approx -0{,}75$
$-\pi/6 \approx -0{,}50$	$-0{,}5$	$\arcsin(-0{,}5) = -\pi/6 \approx -0{,}50$



⚠ There is no periodicity because of 

X ID

Dom (arcsin) = $[-1, +1]$
Range (arcsin) = $[-\pi/2, \pi/2]$
Root(s) (arcsin) = unique root, always 0
Feature (arcsin) = point-symmetric
(example; see graph before this ID)