AMP(1) - Lab08 – Dot Product

1. Content

2. Learning objectives

2.1. Exam objectives

By the end of this lab you should be able to (pen and paper):

- Apply the dot product of vectors
- Be mindful of dot product's properties: commutativity and square of a vector
- Apply the dot product's geometric formula to retrieve the subtended angle
- Apply the dot product's criterion for orthogonality

We advise you to **make your own summary of topics** which are new to you.

2.2. Supportive objectives

More specifically related to the above you should in GeoGebra be able to:

- Apply the dot product of vectors
- Apply the dot product to retrieve and visualize the subtended angle in the View/Graphics

3. Exercises

Dependent of the lab session you may work individually or teamed (organized by the lab attendant). In either case make sure that throughout the course of this lab, you re-save sufficiently your solution file on your local machine as

1DAExx-0y-name1(+name2+name3).GGB given **xx**=groupcode, **0y**=labindex

3.1. Basic exercises

3.1.1. Recap length of a vector

Calculate the length of the following vectors in 2 different ways:

-By using the Pythagorean Theorem

-By using the dot product

Tip: the angle between a vector and itself is 0, so $\cos(0°) = 1$

1) $\vec{a} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ = sqrt(5^2 + 4^2) = sqrt(41)

= sqrt (dot(a, a))

2) $\vec{b} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ = sqrt((-1)^2 + 3^2 + 2^2) = sqrt(14)

= sqrt (dot(b, b))

3.1.2. Recap Normalising a vector

Lookup the definition of normalizing a vector and explain this in your own words to your neighbour.

Normalize the following vectors.

a) $\vec{a} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

=> $\|\vec{a}\| = sqrt(3^2 + 2^2)$ = sqrt(13)

$\vec{n} = \begin{pmatrix} 3/\mathrm{sqrt}(13) \\ 2/\mathrm{sqrt}(13) \end{pmatrix}$ => $\vec{n} = \begin{pmatrix} 0.83 \\ 0.55 \end{pmatrix}$

a) $\vec{b} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

=> $\|\vec{a}\| = sqrt((-2)^2 + 3^2 + 1\text{^}2)$ = sqrt(14)

$\vec{n} = \begin{pmatrix} -2/\mathrm{sqrt}(14) \\ \frac{3}{\mathrm{sqrt}(14)} \\ 1/sqrt(14) \end{pmatrix}$ => $\vec{n} = \begin{pmatrix} -0.53 \\ 0.8 \\ 0.27 \end{pmatrix}$

3.1.3. Calculate dot product

Draw the following vectors

$\vec{a} = \begin{pmatrix} -8 \\ 9 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ $\vec{c} = \begin{pmatrix} -6 \\ 1 \end{pmatrix}$ $\vec{d} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

And calculate the dot product for the following combinations:

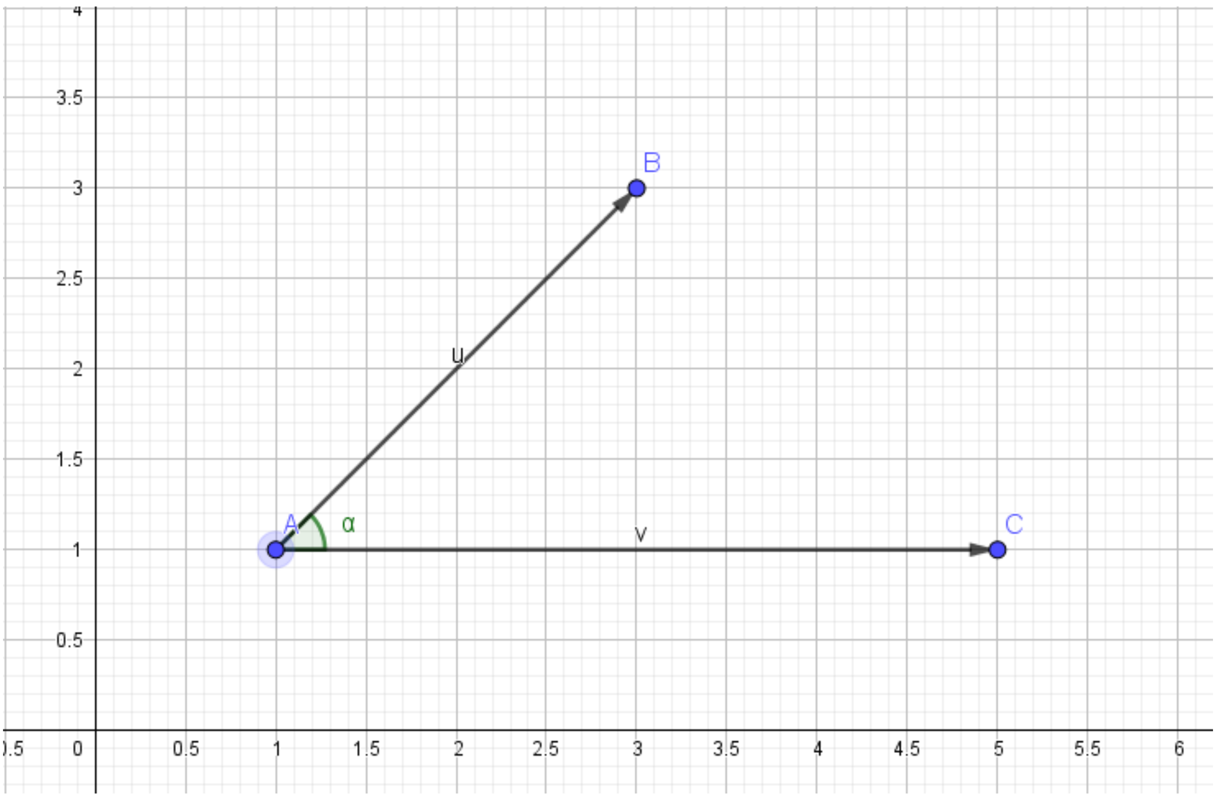
- $\vec{a} \cdot \vec{b}$ = -8* 0 + 9 * 2 = 18
- $\vec{c} \cdot \vec{c}$ = 37
- $\vec{b} \cdot \vec{c}$ = 2
- $\vec{b} \cdot \vec{d}$ = 0

What does it mean when the dot product is 0?

The vectors are orthogonal/perpendicular

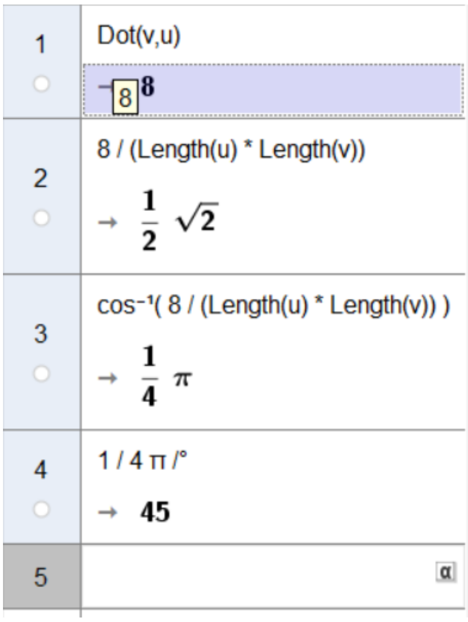
3.2. Bridging Practice

3.2.1. Calculate 2D Angle

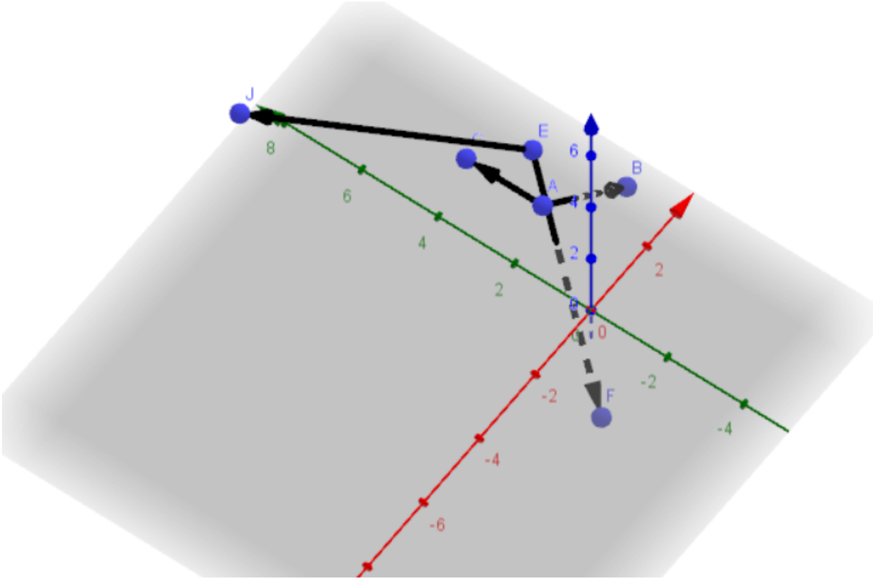


Given the 2 vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$

Calculate the angle α between these 2 vectors using the dot product.

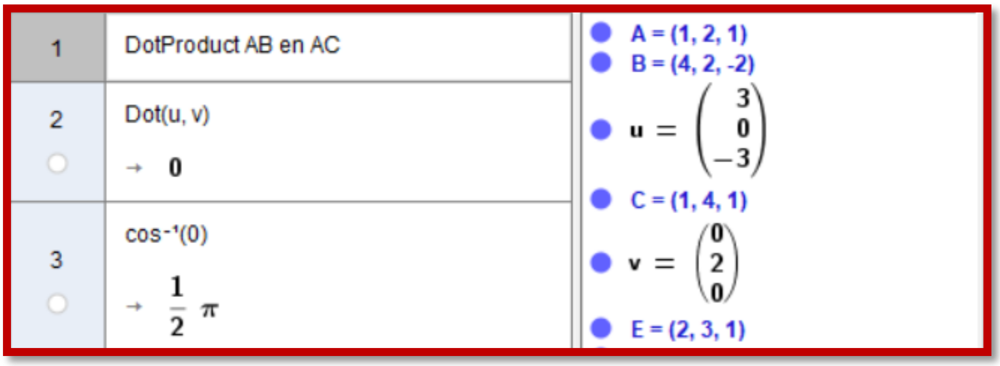


3.2.2. Calculate 3D dot product



Given the points $A(1, 2, 1), \mathrm{B}\,(4, 2\,, -2), \mathrm{C}\,(1, 4, 1)$

a) What is the dot product of $\overrightarrow{AB}$ and $\overrightarrow{AC}$
b) Is the vector $\overrightarrow{AB}$ perpendicular to the vector $\overrightarrow{BC}$? Why/Why Not?
c) Calculate the angle between these 2 vectors in radians



Given the points $\mathrm{E}\,(2, 3, 1), \mathrm{F}\,(-1, -1\,, -2), \mathrm{J}\,(\,-3, 7, 5\,)$

a) What is the dot product of $\overrightarrow{EF}$ and $\overrightarrow{EJ}$
b) Is the vector $\overrightarrow{EF}$ perpendicular to the vector $\overrightarrow{EJ}$? Why?
c) Calculate the angle between these 2 vectors in degrees

5	DotProduct EF en EJ
6	Dot(w, a) → -13
7	cos⁻¹(-13 / (Length(w) * Length(a))) ≈ **1.87057**
8	1.870567981928 / ° ≈ **107.17565**
9	

$$\mathbf{w} = \begin{pmatrix} -3 \\ -4 \\ -3 \end{pmatrix}$$

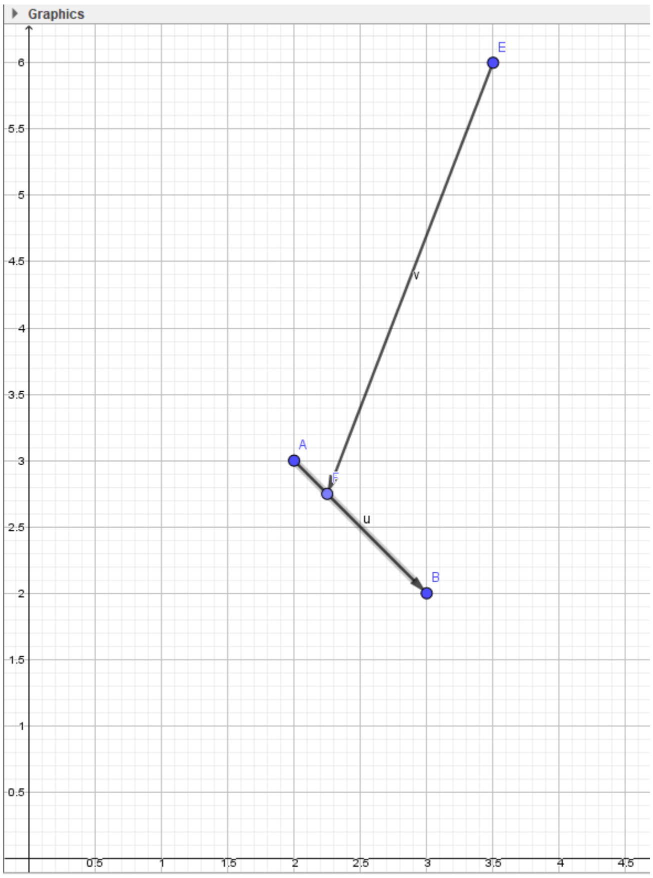
$$\mathbf{a} = \begin{pmatrix} -5 \\ 4 \\ 4 \end{pmatrix}$$

3.3. Contextual exercises

3.3.1. Bullet

A Bullet E is shot at a window between points A and B under direction

$\vec{v} = \begin{pmatrix} -1.25 \\ -3.25 \end{pmatrix}$



The window will break if the magnitude of the perpendicular force to the window is greater than 3.

1) Calculate the magnitude of the vector perpendicular to the window?

2) Will the window break or reflect the bullet?

Tip: In a 2D world you can find the clockwise perpendicular from a vector $\vec{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ as $\vec{p} = \begin{pmatrix} y \\ -x \end{pmatrix}$

AB = (1, -1)

N = (1, 1) (dit is in tegenovergestelde richting als de v vector dus omdraaien(*-1)

N = (-1, -1)

nNormalized (0.71, 0.71)

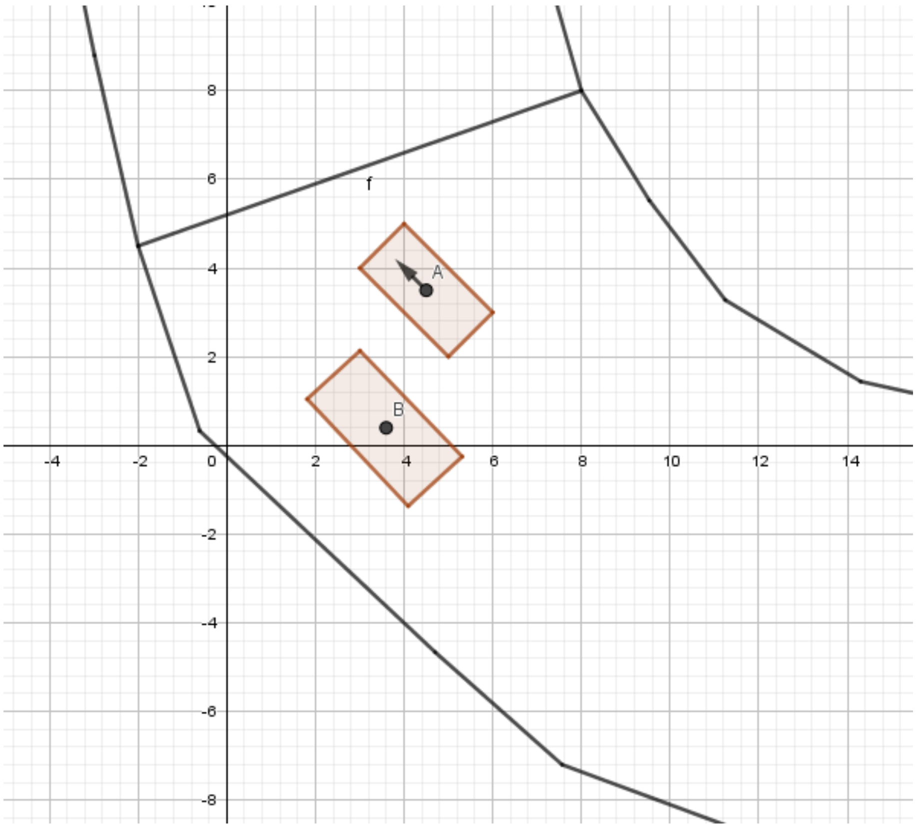
Dot(v, nNormalized) = 3.18 => Projection of v onto normalized perpendicular force on the window, so magnitude is 3.18

3.3.2. Race

You have 2 cars in a race :

-Car 1 with centroid $A(4.5, 3.5)$ and normalized forward vector $\vec{v} = \begin{pmatrix} -0.71 \\ 0.71 \end{pmatrix}$

-Car 2 with centroid $B(3.43,\ 0.3)$



Determine if the car A is in front or behind of car B relative to car A’s forward direction?

Vector AB = (-1.07,-3.2)
Dot(v, AB) **< 0**

Negatief dot product means car B is behind the car A

Positive dot product means the car B is in front of car A

0 dot product means car A and B are right next to eachother

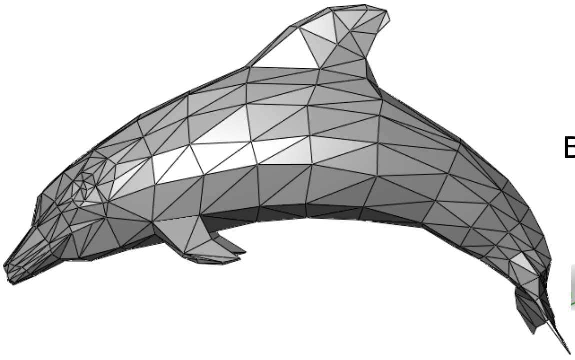
How can you know when the cars are next to each other? And on which side?

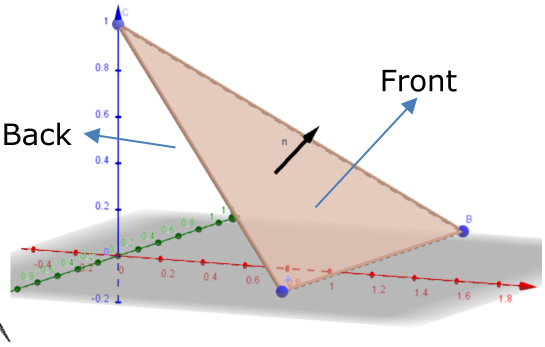
Perpendicular vector on v => n = (-0.71, -0.71) (points to the left under)

Dot(n, AB) **>0** = > Positive so car B is on the left side of car A

3.3.3. Backface culling

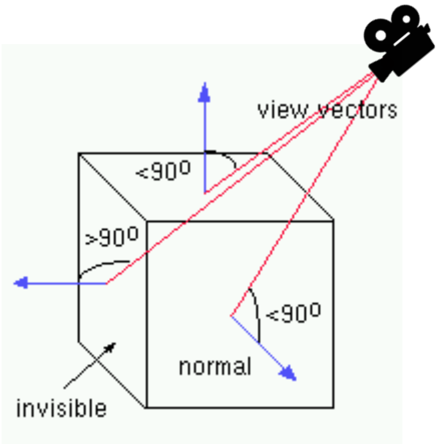
A 3D object in a game is composed of triangles that have a front and back face.





Normally when we are rendering, we only want to show the fronts of our triangles. Backfaces should not be rendered. The front of a triangle (face) is indicated by the direction of the normal.

Assume we have a camera at point $C\,(1, 4, 3)$ that is looking at a point $L\,(5, 2, 1)$. Below you find some normals for faces which are within the view of the camera. Determine which are valid normals for rendering, and which normals are indicating a backface.



Face $A\,(1.5, 0, 2)\,B(0, 0, 3)\,D(0, 2, 0)$ with normal $\vec{n} = \begin{pmatrix} 2 \\ 4.5 \\ 3 \end{pmatrix}$

Face $E\,(1.5, 3, 1)\,F(1.5, 4.5, 0)\,G(0, 3.5, 0)$ with normal $\vec{n} = \begin{pmatrix} 1 \\ -1.2 \\ -2.25 \end{pmatrix}$

Face $H\,(1, 2, 2)\,I(1, 1, 1)\,J(3, 1, 1)$ with normal $\vec{n} = \begin{pmatrix} 2 \\ 4.5 \\ 3 \end{pmatrix}$

Think about how the camera vector should be oriented!

Camera Vector CL = (4, -2, -2)

Camera vector omdraaien om deze vanuit het raakpunt aan de face te richten naar de camera

C = -1 * CL = (-4, 2, 2)

ABC = Dot(n, C) = 7 >0 => Visible

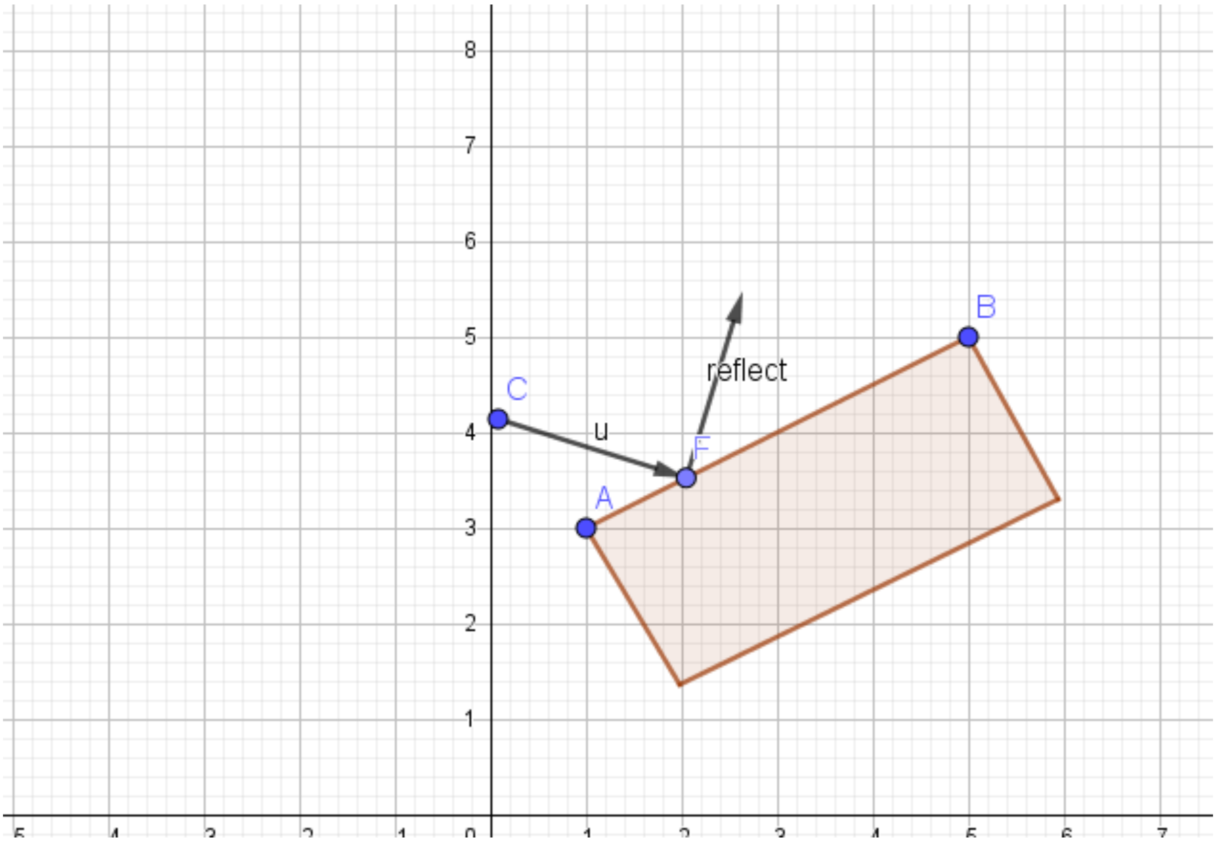
EFG = Dot(n, C) = -11.5 < 0 => Invisible

HIJ = Dot(n, C) = 0 => Perpendicular to the camera, so invisible

3.3.4. Reflection

Calculate the reflection vector using the dot product.

Have a look at the following link(all the way at the bottom 3.23) to find and understand the formula to calculate the reflection vector.



AB = (4, 2) n = (-2, 4), nNormalized = (-0.45, 0.89)

ReflectionVector = u – 2 * Dot(u, nNormalized) * nNormalized

4. References

4.1. Basics Explained

Wikipedia Dot Product

https://en.wikipedia.org/wiki/Dot_product

Khan Academy Video

https://www.khanacademy.org/math/linear-algebra/vectors-and-spaces/dot-cross-products/v/vector-dot-product-and-vector-length

4.2. Applications

Backface culling explained

https://en.wikipedia.org/wiki/Back-face_culling

Reflection Explained

http://immersivemath.com/ila/ch03_dotproduct/ch03.html