AMP(1) - Lab12 – Scene Graphs

1. Content

2. Learning objectives

2.1. Exam objectives

By the end of this lab you should be able to (pen and paper):

- Design the object tree of a scene graph rooted in 2D World Space, with all of its subsequent (parent-child) local space nodes
- Construct the embeddings which tie up the total scene graph
- Design the object tree of a bone structure rooted in 2D World Space, with all of its subsequent (parent-child) bone space nodes
- Construct the embedding matrices between bones which tie up the structure
- Design the object tree of a solar system rooted a central star, with all of its subsequent (parent-child) planet space nodes
- Construct the embedding transformation matrices between planets

We advise you to **make your own summary of topics** which are new to you.

2.2. Supportive objectives

2.2.1. Self-support by GeoGebra

More specifically related to the above you should in GeoGebra:

- Construct the scene graph in GeoGebra
- Visualize the scene graph in GeoGebra
- Organize angular sliders in GeoGebra to demonstrate the scene graph

3. Exercises

Dependent of the lab session you may work individually or teamed (organized by the lab attendant). In either case make sure that throughout the course of this lab, you backup sufficiently your solution file **on your local machine** as

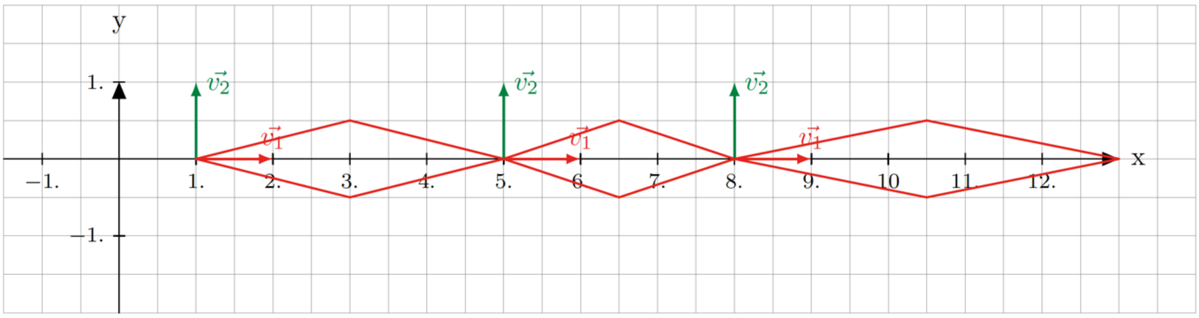
1DAExx-0y-name1(+name2+name3).GGB given **xx**=groupcode, **0y**=labindex

If not already on your machine, get **GeoGebra Classic 5.0 or 6.0** via https://www.geogebra.org/download

3.1. Contextual practice

3.1.1. Bone structure

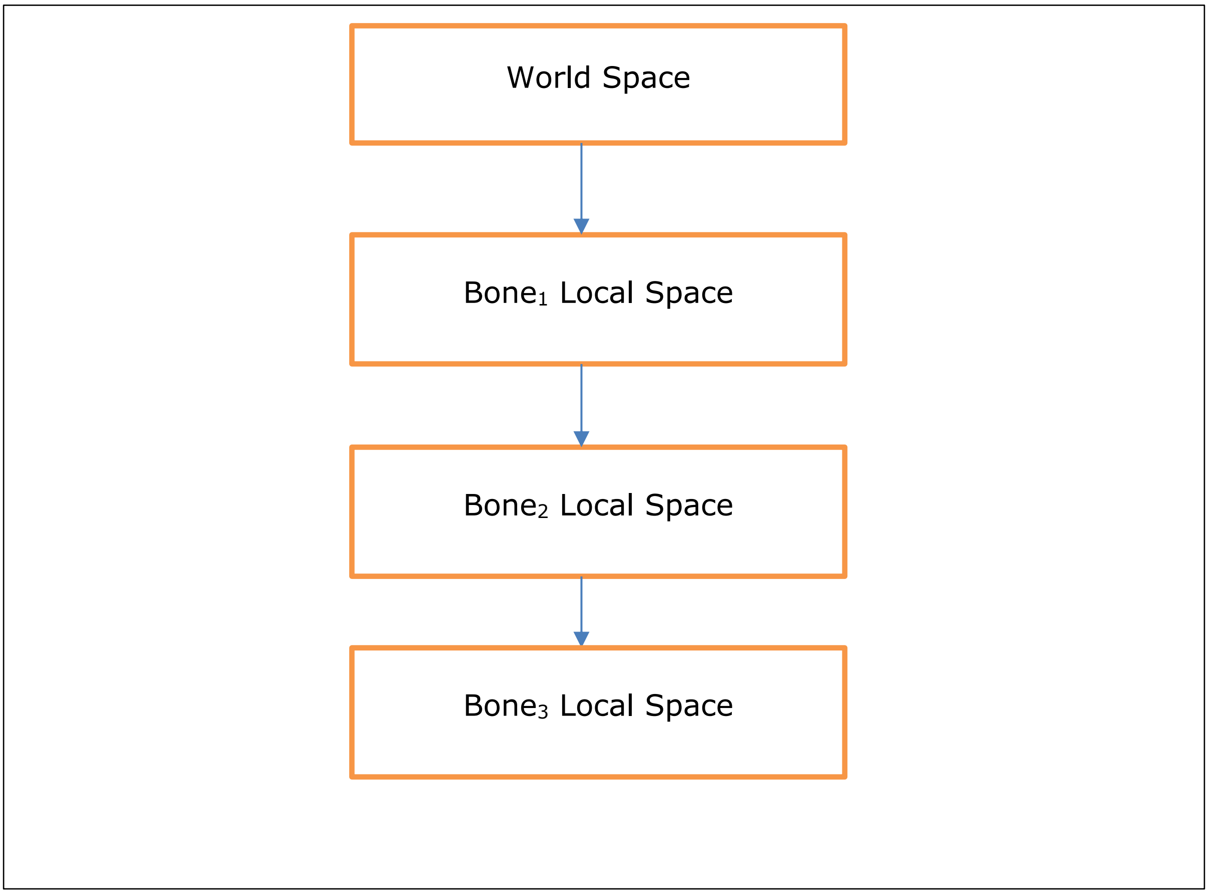
Below you see a representation of three bones B_1, B_2, B_3 that model a robot arm. These bones B_1, B_2, B_3 are respectively 4, 3 and 5 long. The first bone's position is in point (1,0). In this way the (x,y)-World Space contains three embedded Local Spaces:



We model the bones itself by scaling the blueprint diamond B_0 accordingly to their required size:

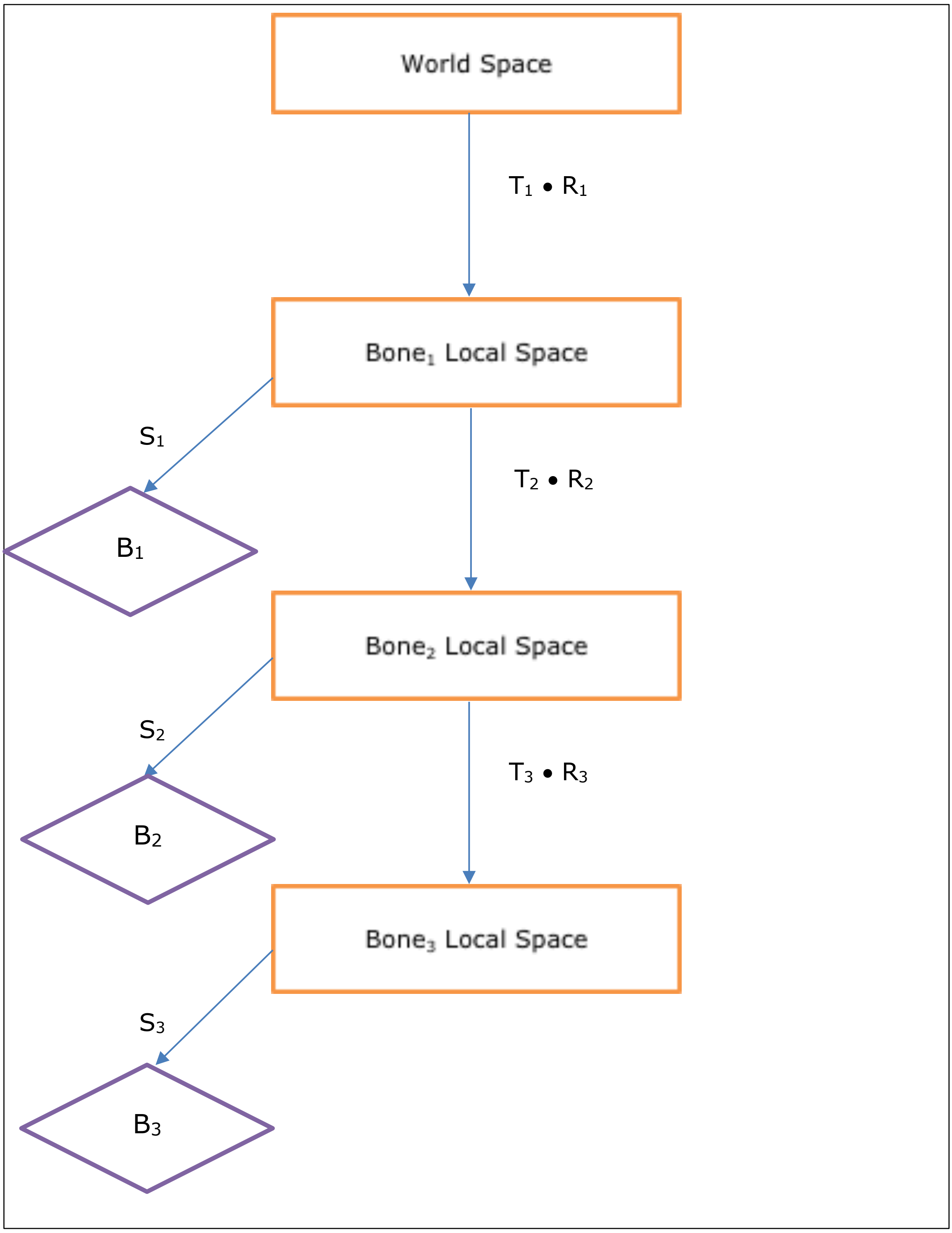
$$B_0 = \begin{pmatrix} 0 & 0.5 & 1 & 0.5 \\ 0 & 0.5 & 0 & -0.5 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Exercise 1: Design the (parent to child) **object tree** for this robot arm



Exercise 2: Determine the **embedding tranformations E** that link each successive Local Space of the scenegraph. Firstly, do this pen and paper for

B_1 constantly 20° inclined (instead of horizontally in the World Space), with
B_2 variably α° inclined within its B_1-Local Space, and finally
B_3 variably β° inclined within its B_2-Local Space.



Hint: given diamond B_0 and the above pictured representation of the robot arm, retrieve the respective scale transformations creating the bones B_1, B_2, B_3.
Moreover, sufficiently allow each bone to take its required space via an appropriate translation for that.

Firstly - given the previous hint - we tackle the required scale transformations:

S_1 = $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ creates $Bone_1$ as image $B_1 = S_1 \bullet B_0$

S_2 = $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ creates $Bone_2$ as image $B_2 = S_2 \bullet B_0$

S_1 = $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ creates $Bone_3$ as image $B_3 = S_3 \bullet B_0$

Secondly, we determine all subsequent Local Space - linking transformations:

To adequately position **$Bone_1$ Local Space** within World Space, we orient it with

R_1 = $\begin{pmatrix} \cos 20° & -\sin 20° & 0 \\ \sin 20° & \cos 20° & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and then we position it with

T_1 = $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ because the first bone's position should be in point (1,0).

To adequately position **$Bone_2$ Local Space** within $Bone_1$ Local Space, we orient

R_2 = $\begin{pmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and then we position it with

T_2 = $\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ because the $Bone_1$ runs up to the point (4,0) within its space.

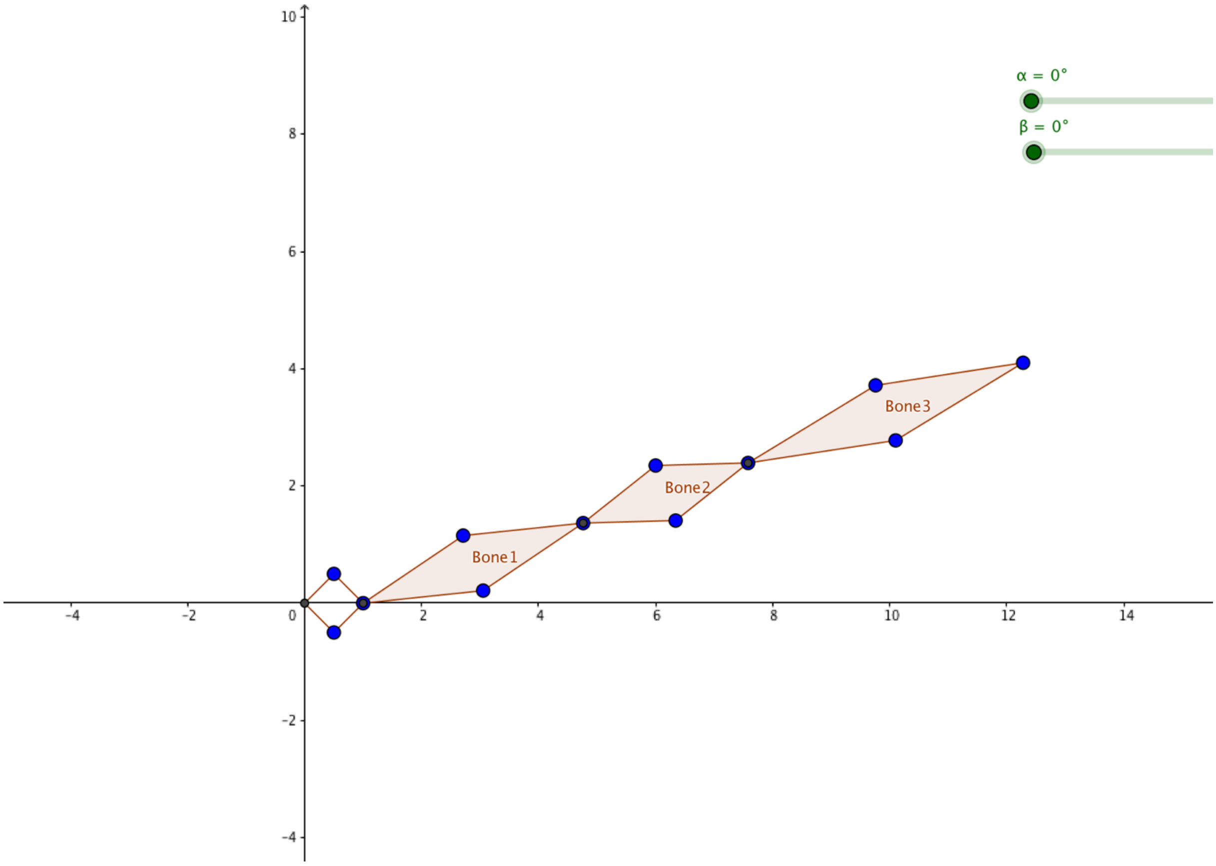
To adequately position **$Bone_3$ Local Space** within $Bone_2$ Local Space, we orient

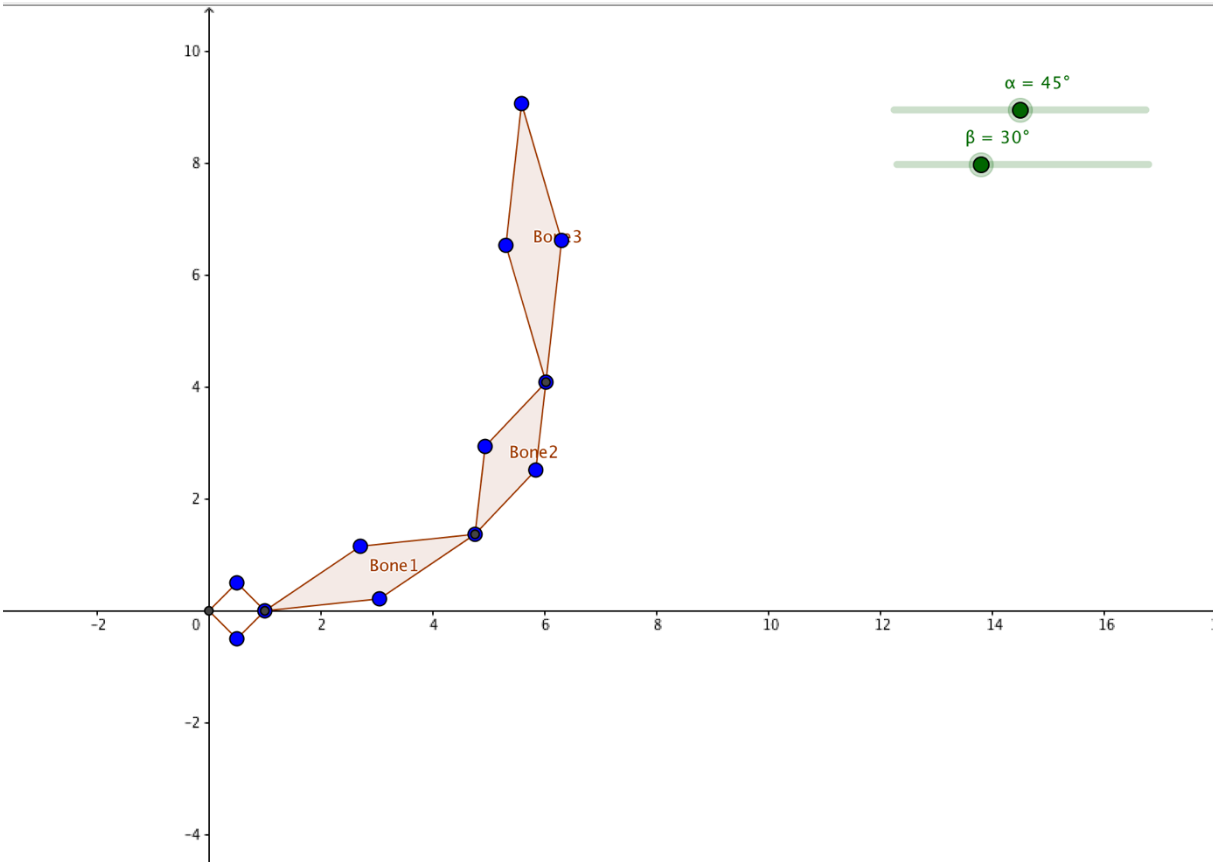
R_3 = $\begin{pmatrix} \cos \beta & -\sin \beta & 0 \\ \sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and then we position it with

T_3 = $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ because the $Bone_2$ runs up to the point (3,0) within its space.

Exercise 3: Implement all the previous in GeoGebra, realizing a **GeoGebra-simulation** of the robot arm, by means of the two (between 0° and 90° clipped) angular sliders (for respectively α and β)**.**



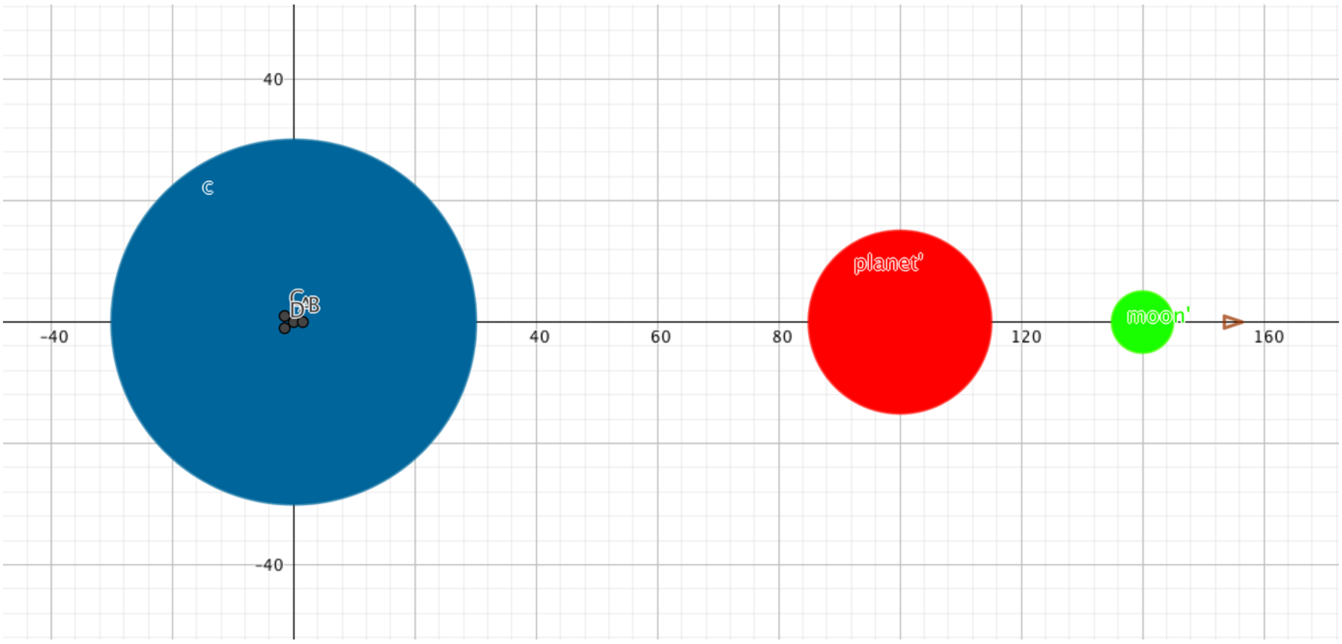


α = 45°
β = 30°
Bone3
Bone2
Bone1

3.1.2. Solar system

Below you see a representation of a solar system featuring a central star with a radius of 30. A planet with a radius of 15 is circling the star at a distance of 100 between the star center and the planet center. The entire planetary orbit counts 350 days. A moon with radius 5 is circling the planet at a distance of 40 and the lunar orbit counts 50 days. Around the moon an isosceles space craft (with its modeled coordinates below) is circling the moon at a distance of 15 and completes its orbit in only 5 days.

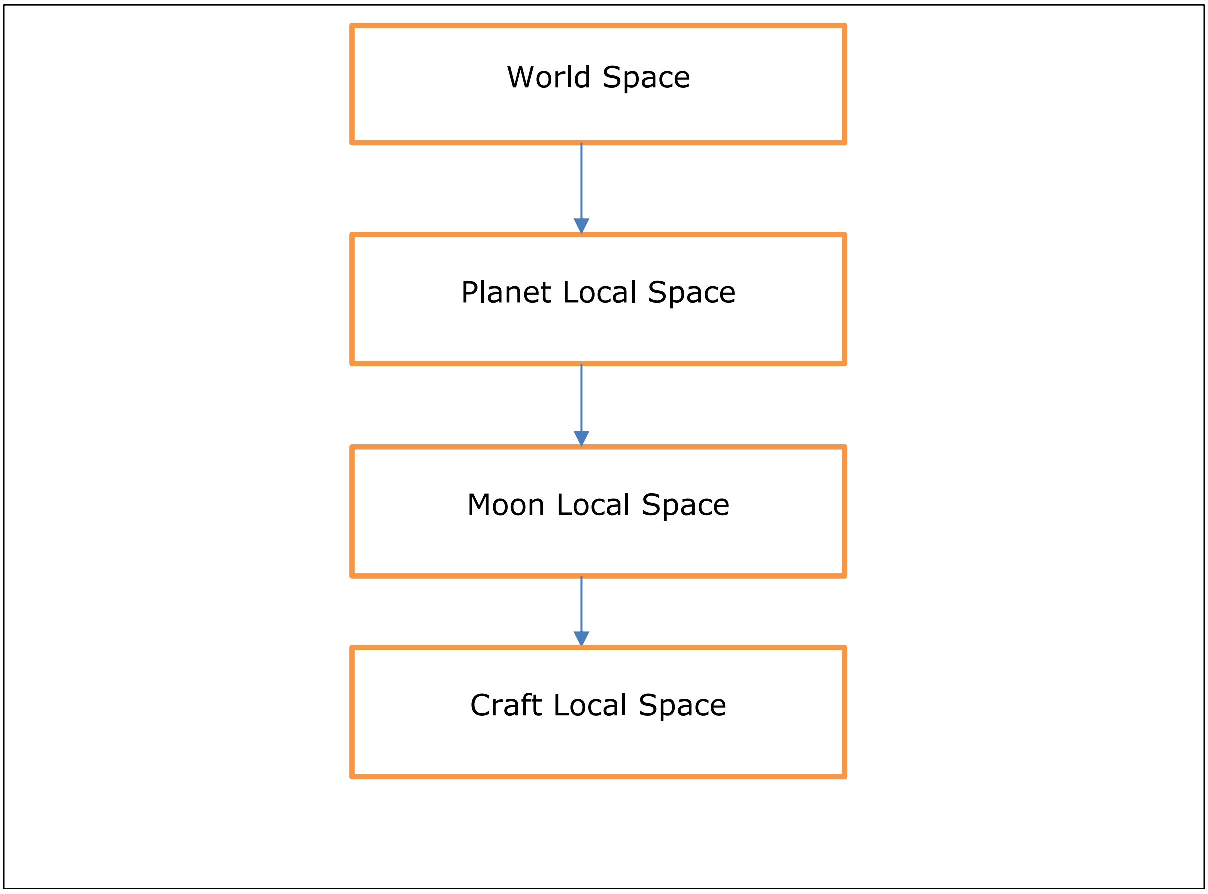
In this way the (x,y)-World Space contains three subsequent Local Spaces:



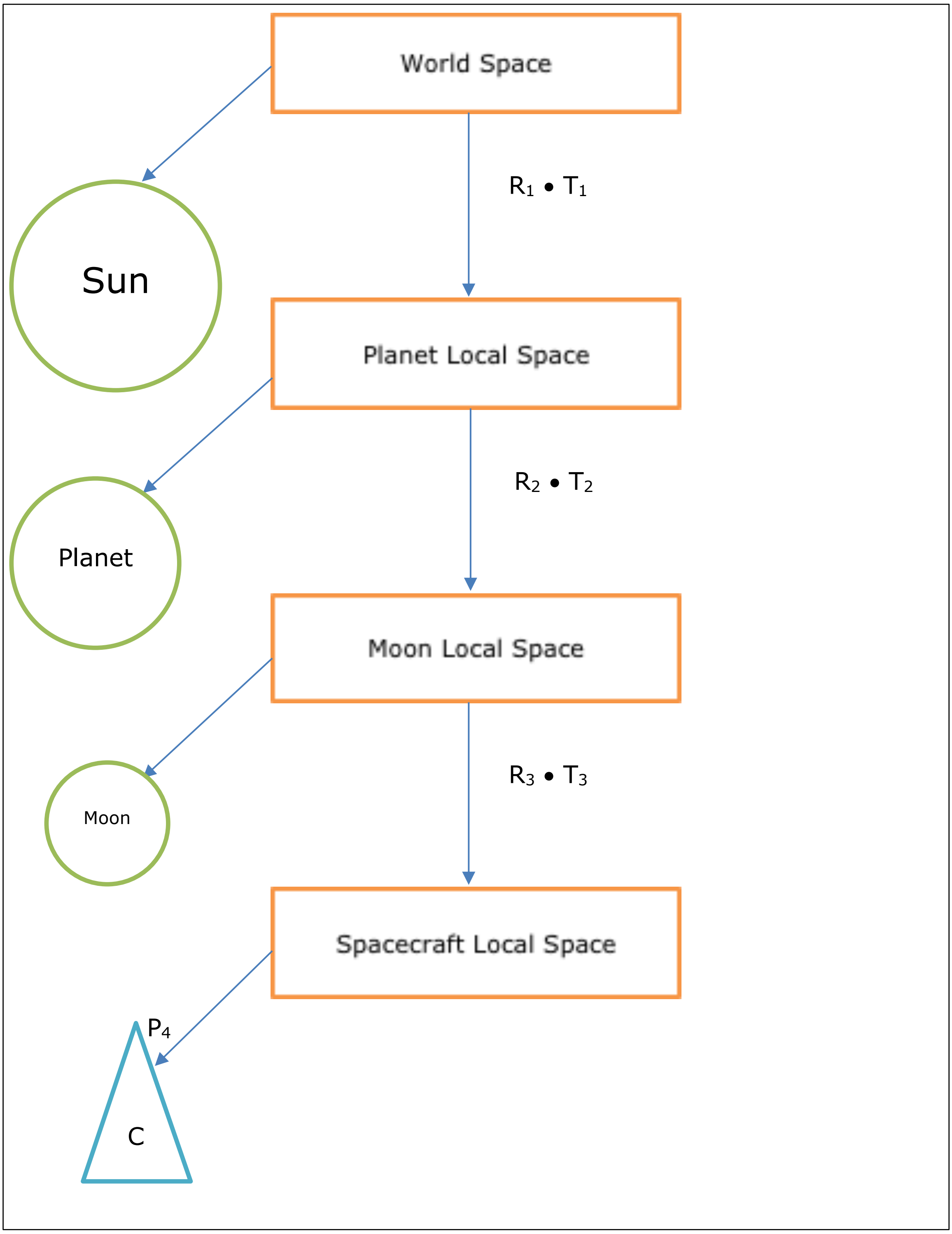
We model the said isosceles space craft by its vertices:

$$\text{Craft} = \begin{pmatrix} 1.5 & -1.5 & -1.5 \\ 0.0 & 1.0 & -1.0 \\ 1 & 1 & 1 \end{pmatrix}$$

Exercise 4: Design the (parent to child) **object tree** for this solar system.



Exercise 5: Determine the **embedding tranformations E** that link each successive Local Space of the scenegraph making use of the variable '**day**' where appropriate.



Let us now specify all subsequent Local Space - linking transformations:

To adequately orbit the **Planet (Local Space)** within World Space, we first position it with

$T_1 = \begin{pmatrix} 1 & 0 & 100 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ because the planet's initial position should be in point (100,0).

and then we orbit it around the World Space's origin (Sun) by applying

$$R_1 = \begin{pmatrix} \cos\left(\frac{2\pi}{350}days\right) & -\sin\left(\frac{2\pi}{350}days\right) & 0 \\ \sin\left(\frac{2\pi}{350}days\right) & \cos\left(\frac{2\pi}{350}days\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

To adequately position **Moon Local Space** within Planet Local Space, we first position it with

$T_2 = \begin{pmatrix} 1 & 0 & 40 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ because the moon's initial position should be in point (40,0)

within the Planet Local Space and then we orbit it around the Local Space's origin (Planet) by means of

$$R_2 = \begin{pmatrix} \cos\left(\frac{2\pi}{50}days\right) & -\sin\left(\frac{2\pi}{50}days\right) & 0 \\ \sin\left(\frac{2\pi}{50}days\right) & \cos\left(\frac{2\pi}{50}days\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

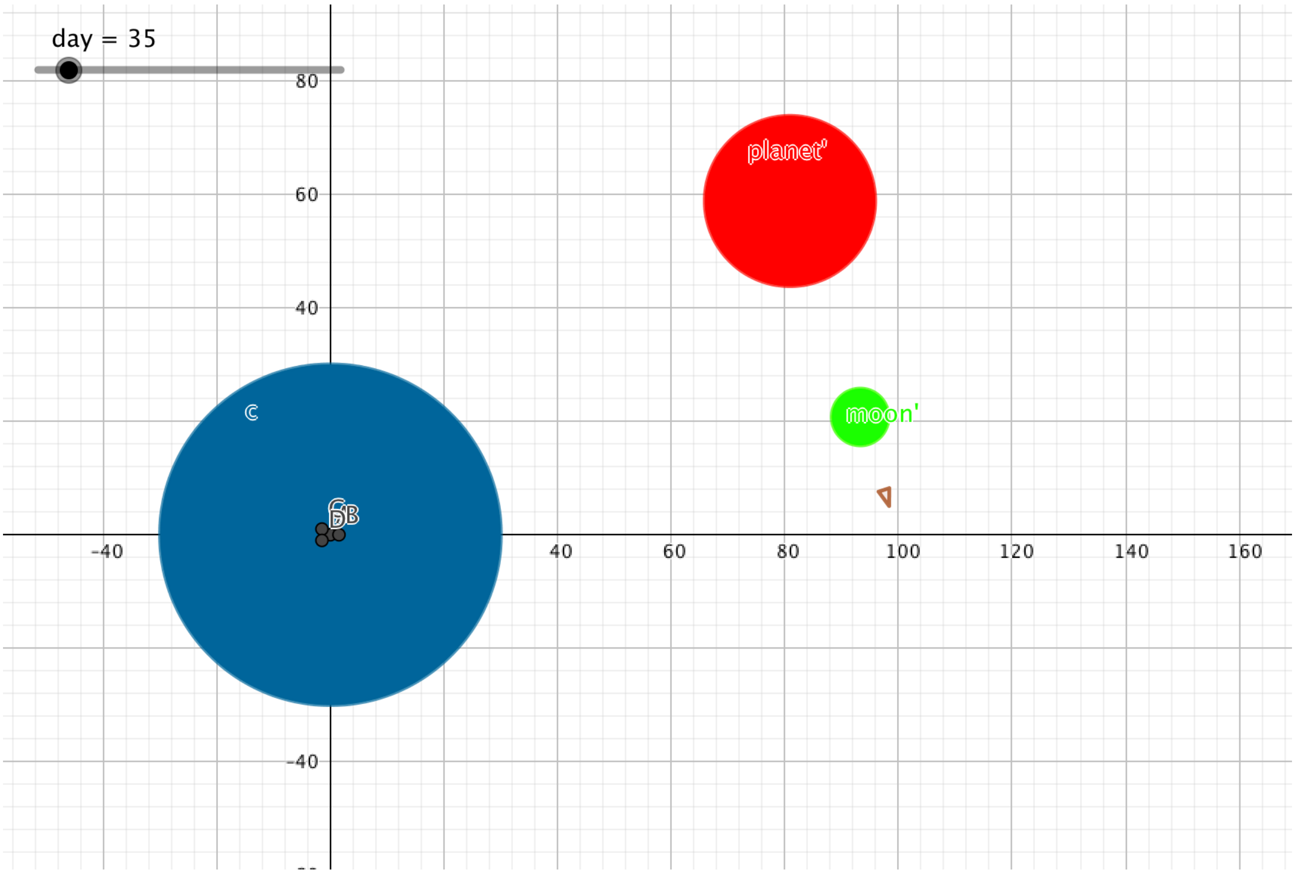
To adequately position **Craft Local Space** within the Moon Local Space, we position it with

$T_3 = \begin{pmatrix} 1 & 0 & 15 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ because the craft's initial position should be in (15,0)

within the Moon Local Space and then we orbit it around the Local Space's origin (Moon) by the rotator

$$R_3 = \begin{pmatrix} \cos\left(\frac{2\pi}{5}days\right) & -\sin\left(\frac{2\pi}{5}days\right) & 0 \\ \sin\left(\frac{2\pi}{5}days\right) & \cos\left(\frac{2\pi}{5}days\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Exercise 6: Implement all the previous in GeoGebra, realizing a **GeoGebra-simulation** of the solar system, by means of a (between 0 and 350 clipped) slider for the variable '**day**'.



Exercise 7 (EXTRA): Extend the previous exercise 6, by making the space craft finally spinning around its centroid C every 4 days**.**

We already had to position the Craft in its own Local Space origin.

So, to finally pivot the **Craft** within its own Local Space around its centroid, we insert this pivot transform as the first transformation acting on it:

$$P_4 = \begin{pmatrix} \cos\left(\frac{2\pi}{4}days\right) & -\sin\left(\frac{2\pi}{4}days\right) & 0 \\ \sin\left(\frac{2\pi}{4}days\right) & \cos\left(\frac{2\pi}{4}days\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Referecences

4.1. Basics

4.1.1. English maths dictionary

http://www.mathwords.com

4.2. Demos in art and programming

4.2.1. Scenegraphs in game engines

https://www.haroldserrano.com/blog/the-purpose-of-a-scenograph-in-a-game-engine