AMP(1) – Lab11 – Transformation Analysis

1. Content

2. Learning objectives

2.1. Exam objectives

By the end of this lab you should be able to (pen and paper):

- Understand and apply the 3D-translation operator matrix
- Understand and apply the **standard** 3D-transformation matrices for scaling and rotation
- Combine scaling (first), rotation and translation (last) into a composite transformation (in this obliged order to prevent skewing)
- Construct **non-standard** composite transformations as well
- Analyze any given composite transformation, deducting its scaling-, rotation- and translation-parts

We advise you to **make your own summary of topics** which are new to you.

2.2. Supportive objectives

2.2.1. Self-support by GeoGebra

More specifically related to the above you should in GeoGebra:

- Insert and apply the various operator matrices (translation, scaling, rotation) on (the vertices of) a polygon
- Apply composite transformations
- Visualize both the original and the transformed polygon

3. Exercises

Dependent of the lab session you may work individually or teamed (organized by the lab attendant). In either case make sure that throughout the course of this lab, you backup sufficiently your solution file **on your local machine** as

1DAExx-0y-name1(+name2+name3).GGB given **xx**=groupcode, **0y**=labindex

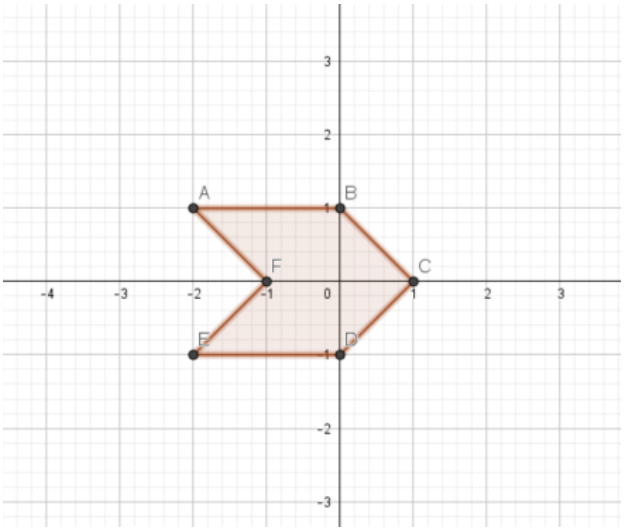
If not already on your machine, get **GeoGebra Classic 5.0 or 6.0** via https://www.geogebra.org/download

3.1. Basic exercises

3.1.1. Translation, rotation and scaling: step by step

In this exercise we will do a step by step transformation of a shape. After every transformation we will do an intermediate calculation on all the different vertices of the shape.

We'll start with a shape whose pivot point is already aligned on the origin of the world.



The shape is represented by this matrix:

$$Arrow = \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Draw the shape in GeoGebra.

a. Scaling matrix

Create a scaling matrix with the following scale factors:

$S\binom{2}{4} =$	$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Calculate the new vertices after scaling and visualize the scaled shape in geogebra:

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

=

$$\begin{pmatrix} -4 & 0 & 2 & 0 & -4 & -2 \\ 4 & 4 & 0 & -4 & -4 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

b. Rotation matrix

We apply the rotation after the scaling to avoid skewing

our orthonormal coordinate system. Create a rotation matrix for the following angle:

$R_O(40°)=$	$\begin{pmatrix} cos(40°) & -sin(40°) & 0 \\ sin(40°) & cos(40°) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ = $\begin{pmatrix} 0.76 & -0.64 & 0 \\ 0.64 & 0.76 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Calculate the new vertices after rotating the scaled vertices and visualize the shape in geogebra:

$$\begin{pmatrix} 0.76 & -0.64 & 0 \\ 0.64 & 0.76 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -4 & 0 & 2 & 0 & -4 & -2 \\ 4 & 4 & 0 & -4 & -4 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

=

$$\begin{pmatrix} -5.64 & -2.57 & 1.53 & 2.57 & -0.49 & -1.53 \\ 0.49 & 3.06 & 1.29 & -3.06 & -5.64 & -1.29 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

c. Translation matrix

If we where to do translations before either scaling or rotations, we would see that our translation gets scaled or rotated too. We will explore this in later exercises.

Create a translation matrix for the following displacement:

$T\binom{3}{7}=$	$\begin{pmatrix}1 & 0 & 3\\ 0 & 1 & 7\\ 0 & 0 & 1\end{pmatrix}$

Calculate the new vertices after translating the scaled and rotated vertices and visualize the final shape in geogebra:

$$\begin{pmatrix}1 & 0 & 3\\ 0 & 1 & 7\\ 0 & 0 & 1\end{pmatrix} \cdot \begin{pmatrix}-5.64 & -2.57 & 1.53 & 2.57 & -0.49 & -1.53\\ 0.49 & 3.06 & 1.29 & -3.06 & -5.64 & -1.29\\ 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix} =$$

$$\begin{pmatrix}-2.64 & 0.43 & 4.53 & 5.57 & 2.51 & 1.47\\ 7.49 & 10.06 & 8.29 & 3.94 & 1.36 & 5.71\\ 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$$

3.1.2. Translation, rotation and scaling: composite matrices

Calculating every vertex at each intermediate step is clearly not the most performant solution. Matrices are especially useful when we combine all different transformation matrices into one transformation matrix that combines all previous ones. We already had a look at how we can create a matrix when given the rotation, translation and scaling, but that is far from the only or most flexible way to combine transformations.

Take the matrixes from above and let's combine them one by one.

Multipy the three different matrices in the TRS-order

$$\begin{pmatrix}1 & 0 & 3\\ 0 & 1 & 7\\ 0 & 0 & 1\end{pmatrix} \cdot \begin{pmatrix}0.76 & -0.64 & 0\\ 0.64 & 0.76 & 0\\ 0 & 0 & 1\end{pmatrix} \cdot \begin{pmatrix}2 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix} 1.53 & -2.57 & 3 \\ 1.29 & 3.06 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

Retrieve the original translation, rotation and scaling from this composite matrix:

Translation: $T\binom{3}{7}$

Rotation: atan2(1.29, 1.53) = 40° and atan2(3.06, -2.57) = 130°

Scaling: $\sqrt{1.53^2 + 1.29^2} \approx 2$ and $\sqrt{(2.57)^2 + 3.06^2} \approx 4$

Apply this matrix to the original vertices, compare your results and draw the shape in GeoGebra:

$$\begin{pmatrix} 1.53 & -2.57 & 3 \\ 1.29 & 3.06 & 7 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -2.64 & 0.43 & 4.53 & 5.57 & 2.51 & 1.47 \\ 7.49 & 10.06 & 8.29 & 3.94 & 1.36 & 5.71 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

3.2. Bridging exercises

3.2.1. Off center vertices

When our pivot point is not on the origin, strange effects will occur. In the next few exercises we explore these effects. We start with the following matrix which represents the same shape but now with off centered vertices:

$$Vessel = \begin{pmatrix} 3 & 5 & 6 & 5 & 3 & 4 & 3 \\ 7 & 7 & 6 & 5 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

a. Off center scaling

Before you start this exercise, draw the image in GeoGebra and think about what will happen with the image when we scale it. Use the previously calculated scaling matrix and the off center vertices.

-Calculate the new vertices:

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 & 6 & 5 & 3 & 4 & 3 \\ 7 & 7 & 6 & 5 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 6 & 10 & 12 & 10 & 6 & 8 \\ 28 & 28 & 24 & 20 & 20 & 24 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape in GeoGebra

b. Off center rotation

Again, try to formulate what will happen when we rotate the off center shape before doing the actual math. Use the previously calculated rotation matrix and the original (non-scaled) off center vertices

-Calculate the new vertices:

$$\begin{pmatrix} 0.76 & -0.64 & 0 \\ 0.64 & 0.76 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 & 6 & 5 & 3 & 4 & 3 \\ 7 & 7 & 6 & 5 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -2.2 & -0.67 & 0.74 & 0.62 & -0.92 & -0.79 \\ 7.29 & 8.58 & 8.45 & 7.04 & 5.76 & 7.17 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape in GeoGebra

c. Translation

Before you start this exercise, think about what will happen with the image above when we apply a translation. Use the previously calculated translation matrix and the original (non-scaled, non-rotated) off center vertices

-Calculate the new vertices:

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 & 6 & 5 & 3 & 4 & 3 \\ 7 & 7 & 6 & 5 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 6 & 8 & 9 & 8 & 6 & 7 \\ 14 & 14 & 13 & 12 & 12 & 13 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape in GeoGebra

d. Composite Matrix

Combine the previously calculated transformation matrices and apply them to the original off center vertices.

-Calculate the new vertices:

$$\begin{pmatrix} 1.53 & -2.57 & 3 \\ 1.29 & 3.06 & 7 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 & 6 & 5 & 3 & 4 & 3 \\ 7 & 7 & 6 & 5 & 5 & 6 & 7 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -10.4 & -7.34 & -3.23 & -2.2 & -5.26 & -6.3 \\ 32.31 & 34.88 & 33.1 & 28.75 & 26.18 & 30.53 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape in GeoGebra

3.2.2. Off center vertices: improved approach

As you noticed, the problem is in the scaling and in the rotation. To overcome this, we need to align the pivot point of the shape to the world's origin before doing those.

a. Centering

The displacement between the world's origin and the pivot of our shape is $\binom{5}{6}$. So we will need the opposite displacement to bring the shape to the center.

Create a translation matrix for the following displacement:

$T\binom{-5}{-6}$	$\begin{pmatrix}1 & 0 & -5\\0 & 1 & -6\\0 & 0 & 1\end{pmatrix}$

-Calculate the new vertices after translation:

$$\begin{pmatrix}1 & 0 & -5\\0 & 1 & -6\\0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}3 & 5 & 6 & 5 & 3 & 4 & 3\\7 & 7 & 6 & 5 & 5 & 6 & 7\\1 & 1 & 1 & 1 & 1 & 1 & 1\end{pmatrix} =$$

$$\begin{pmatrix}-2 & 0 & 1 & 0 & -2 & -1\\1 & 1 & 0 & -1 & -1 & 0\\1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

b. Rotate

Use the previous rotation matrix and rotate the vertices.

-Calculate the new vertices after rotation:

$$\begin{pmatrix}0.76 & -0.64 & 0\\0.64 & 0.76 & 0\\0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}-2 & 0 & 1 & 0 & -2 & -1\\1 & 1 & 0 & -1 & -1 & 0\\1 & 1 & 1 & 1 & 1 & 1\end{pmatrix} =$$

$$\begin{pmatrix}-2.17 & -0.64 & 0.77 & 0.64 & -0.89 & -0.77\\-0.52 & 0.77 & 0.64 & -0.77 & -2.05 & -0.64\\1 & 1 & 1 & 1 & 1 & 1\end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

c. Repositioning

The last step is to put the pivot point back in its original position.

Create a translation matrix for the following displacement:

$T\binom{5}{6}$	$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix}$

-Calculate the new vertices after translation:

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2.17 & -0.64 & 0.77 & 0.64 & -0.89 & -0.77 \\ -0.52 & 0.77 & 0.64 & -0.77 & -2.05 & -0.64 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 2.83 & 4.36 & 5.77 & 5.64 & 4.11 & 4.23 \\ 5.48 & 6.77 & 6.64 & 5.23 & 3.95 & 5.36 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

3.2.3. Off center vertices: composite matrices

Multiply the center (inverse translation) matrix with the composite matrix from the previous exercises:

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1.53 & -2.57 & 3 \\ 1.29 & 3.06 & 7 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1.53 & -2.57 & 8 \\ 1.29 & 3.06 & 13 \\ 0 & 0 & 1 \end{pmatrix}$$

Multiply the previous matrix with the repositioning (translation) matrix:

$$\begin{pmatrix} 1.53 & -2.57 & 8 \\ 1.29 & 3.06 & 13 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1.53 & -2.57 & 15.77 \\ 1.29 & 3.06 & -11.81 \\ 0 & 0 & 1 \end{pmatrix}$$

Retrieve the original translation, rotation and scaling from this composite matrix:

Translation: $T\binom{15}{-11.81}$

Rotation: atan2(1.29, 1.53) = 40° and atan2(3.06, -2.57) = 130°

Scaling: $\sqrt{1.53^2 + 1.29^2} \approx 2$ and $\sqrt{(2.57)^2 + 3.06^2} \approx 4$

Calculate the new vertices after translation:

$$\begin{pmatrix} 1.53 & -2.57 & 15.77 \\ 1.29 & 3.06 & -11.81 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 2.36 & 5.43 & 9.53 & 10.57 & 7.51 & 6.47 \\ 13.49 & 16.06 & 14.29 & 9.94 & 7.36 & 11.71 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

3.2.4. Wrong reordering (to better understand composites)

To help you progress in your understanding of matrices we will be having a look at what happens when you get the order wrong. We will be using the centred shape for these exercises so as to clearly see what goes wrong.

a. Scaling After Rotating

Multiply the scaling matrix with the rotation matrix from the first exercise:

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0.76 & -0.64 & 0 \\ 0.64 & 0.76 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1.53 & -1.29 & 0 \\ 2.57 & 3.06 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Retrieve the original translation, rotation and scaling from this SR-productmatrix:

Translation: $T\binom{0}{0}$

Rotation: atan2(2.57, 1.53) = 60° and atan2(3.06, 1.29) = 112°

Scaling: $\sqrt{1.53^2 + 2.59^2} \approx 3$ and $\sqrt{(1.29)^2 + 3.06^2} \approx 3.3$

Calculate the new vertices after transformation:

$$\begin{pmatrix} 1.53 & -1.29 & 0 \\ 2.57 & 3.06 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -4.35 & -1.29 & 1.53 & 1.29 & -1.78 & -1.53 \\ -2.08 & 3.06 & 2.57 & -3.06 & -8.21 & -2.57 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

b. Scaling after translating

Multiply the scaling matrix with the translation matrix from the first exercise:

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 6 \\ 0 & 4 & 28 \\ 0 & 0 & 1 \end{pmatrix}$$

Retrieve the original translation, rotation and scaling from this ST-productmatrix:

Translation: $T\binom{6}{28}$

Rotation: $0°$ and $90°$

Scaling: $S\binom{2}{4}$

Calculate the new vertices after transformation:

$$\begin{pmatrix} 2 & 0 & 6 \\ 0 & 4 & 28 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 2 & 6 & 8 & 6 & 2 & 4 \\ 32 & 32 & 28 & 24 & 24 & 28 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

c. Rotation after translating

Multiply the rotation matrix with the translation matrix from the first exercise:

$$\begin{pmatrix} 0.76 & -0.64 & 0 \\ 0.64 & 0.76 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.77 & -0.64 & -2.2 \\ 0.64 & 0.77 & 7.29 \\ 0 & 0 & 1 \end{pmatrix}$$

Retrieve the original translation, rotation and scaling from this RT-productmatrix:

Translation: $T\binom{-2.2}{7.29}$

Rotation: atan2(0.64, 0.77) = 40° and atan2(0.77, -0.64) = 130°

Scaling: $S\binom{1}{1}$

Calculate the new vertices after transformation:

$$\begin{pmatrix} 0.77 & -0.64 & -2.2 \\ 0.64 & 0.77 & 7.29 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 0 & 1 & 0 & -2 & -1 \\ 1 & 1 & 0 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} -4.38 & -2.84 & -1.44 & -1.56 & -3.09 & -2.97 \\ 6.77 & 8.06 & 7.93 & 6.52 & 5.24 & 6.65 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

-Draw the shape on the grid in GeoGebra.

3.3. Contextual practice

3.3.1. Pivoting

Pivot the shape $Arrow$ (from the first exercise 4.1.1) around the local center $B(5{,}4)$ by a $30°$ angle.

- Calculate all of its vertex images

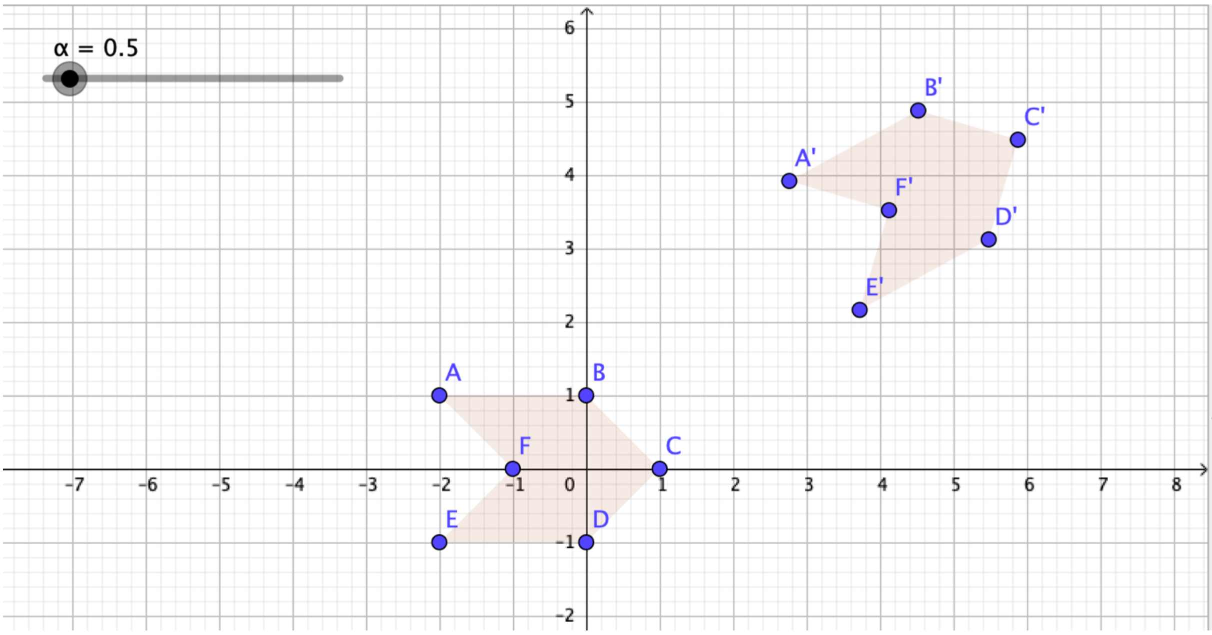
$$P_B(30°) = \begin{pmatrix} cos30° & -sin30° & 5 \\ sin30° & cos30° & 4 \\ 0 & 0 & 1 \end{pmatrix} \approx \begin{pmatrix} 0.88 & -0.48 & 5 \\ 0.48 & 0.88 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

- Draw both the shape and its image in different colours (in GeoGebra)
- Organize a slider on the $30°$ angle to vary it which shows the runtime pivoting
- Retrieve the original translation, rotation and scaling from this pivot matrix:

Translation: $T\binom{5}{4}$

Rotation: atan2(0.48, 0.88) = 30° and atan2(0.88, -0.48) = 120°

Scaling: $S\binom{1}{1}$



3.3.2. Orbiting

Orbit the shape $Arrow$ (from the first exercise 4.1.1) anchored in $B(5,4)$ around the world's origin by a $30°$ angle.

- Calculate all of its vertex images

$$O_B(30°) = \begin{pmatrix} cos30° & -sin30° & 0 \\ sin30° & cos30° & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix} \approx \begin{pmatrix} 0.88 & -0.48 & 2.47 \\ 0.48 & 0.88 & 5.91 \\ 0 & 0 & 1 \end{pmatrix}$$

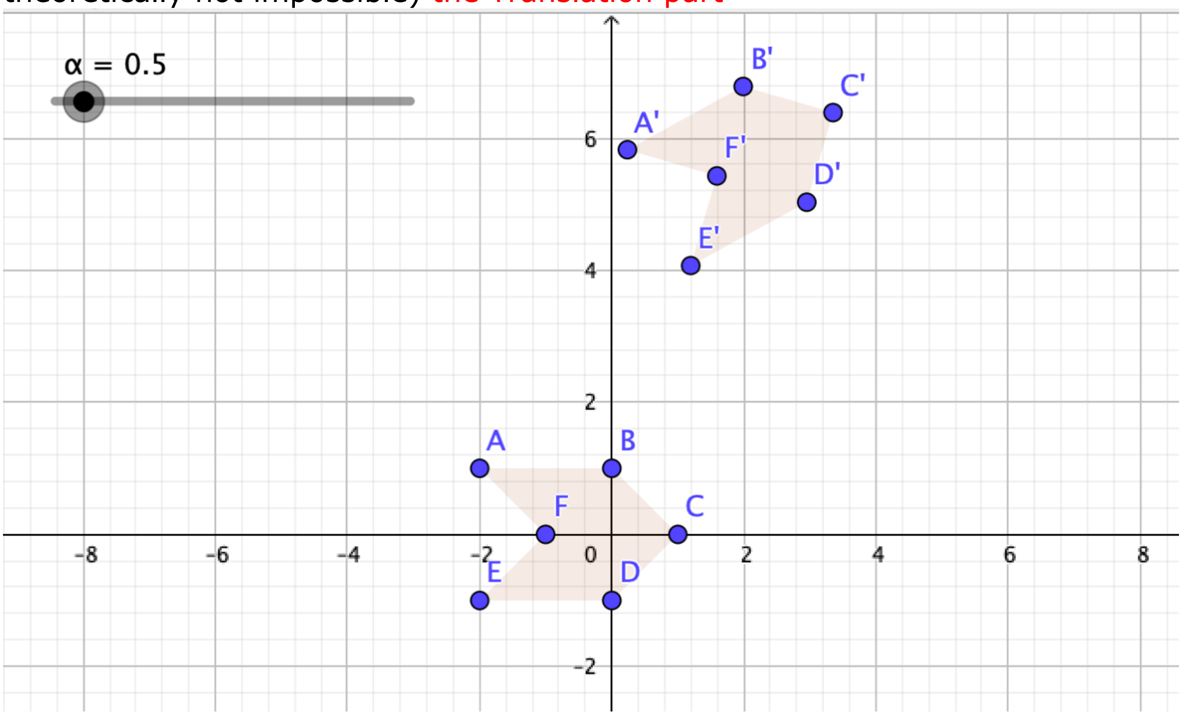
- Draw both the original and its image in different colours (in GeoGebra)
- Organize a slider on the $30°$ angle to vary it which shows the runtime orbiting
- Retrieve the original translation, rotation and scaling from this orbit matrix:

Translation: **?**

Rotation: atan2(0.48, 0.88) = 30° and atan2(0.88, -0.48) = 120°

Scaling: $S\binom{1}{1}$

- Which of the above transformation parts is not to be retrieved? (However theoretically not impossible) the Translation part



3.3.3. A composite 2D rotation

Given the hexagon defined by the vertices $A(2,2)$, $B(4,2)$, $C(5,3)$, $D(4,4)$, $E(2,4)$, $F(1,3)$, determine the transformation matrix to rotate it clockwise around its center Z over an angle of $270°$.

- Calculate all of its vertex images

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 6 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 6 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 4 & 5 & 4 & 2 & 1 \\ 2 & 2 & 3 & 4 & 4 & 3 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & 3 & 2 & 2 & 3 \\ 2 & 4 & 5 & 4 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

- Draw both the original and its image in different colours (in GeoGebra)

3.3.4. A composite 3D scaling

We want to resize the polyhedron defined by the vertices $A(2,2,1)$, $B(5,1,2)$, $C(5,1,-1)$, $D(2,2,-1)$, $E(2,5,1)$, $F(5,4,2)$, $G(5,1,4)$ and $H(2,5,4)$ into a polyhedron by applying scale factor 2 along the x-axis, factor 4 along the y-axis and factor 3 along the z-axis, with respect to its corner vertex A.

Hint: the center Z of the hexagon is the midpoint of the line segment $[AD]$

- Calculate all of its vertex images

$$\begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 & -2 \\ 0 & 4 & 0 & -6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 & -2 \\ 0 & 4 & 0 & -6 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 5 & 5 & 2 & 2 & 5 & 5 & 2 \\ 2 & 1 & 1 & 2 & 5 & 4 & 1 & 5 \\ 1 & 2 & -1 & -1 & 1 & 2 & 4 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 8 & 8 & 2 & 2 & 8 & 8 & 2 \\ 2 & -2 & -2 & 2 & 14 & 10 & -2 & 14 \\ 1 & 4 & -5 & -5 & 1 & 4 & 10 & 10 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

4. References

4.1. Outline of standard and composite matrix transformations

https://www.alanzucconi.com/2016/02/10/tranfsormation-matrix/

4.2. Matrix transformations in games - GDC Vault

https://www.gdcvault.com/play/1017652/Math-for-Game-Programmers-Matrix