1 Trigonometry

1.1 Angles

α = lengte / straal

° to radians	Radians to °
360 = 2π	2π = 360°
1 ° = 2π rad/360 °	1 rad = 360 ° /2π
30° = 2π rad / 360 ° * 30 = π/6 rad	Π / 6 rad = 360 ° /2π * π/6 = 30 °

Acute angle = 0 degrees < α < 90 degrees

Right angle = α = 90 degrees

obtuse angle = 90 degrees < α < 180 degrees

straight angle = α = 180 degrees

1.2 Triangles

hoek A + hoek B + hoek C = 180 degrees = pi rad

Right angle: one of the interior angles is equal to 90 degrees the edge opposite of the right angle is called the **Hypotenuse**

isosceles triangle: a triangle with 2 equal sides through the apex and their corresponding base angles having the same measure.

Equilateral triangle: is a triangle with three equal sides their interior angles measuring 60 degrees

bisector: of a triangle is the straight line through a vertex which cuts the corresponding angle in half

Median or side bisector: of a triangle is the straight line through a vertex and the midpoint of the opposite side

Altitude: of a triangle is the straight line through a vertex and perpendicular to the opposite side this opposite side is called the base of the altitude the altitude and its intersection point is called the foot

perpendicular bisector: of a triangle is a straight line through midpoint of a side and being perpendicular to it

Pythagoras

$$\textbf{(Hypotnuse)}^2 = \textbf{(side1)}^2 + \textbf{(side2)}^2$$

$$\mathbf{a^2 + b^2 = c^2}$$

1.3 Right triangle

sin α = Opposite side / hypotenuse	Csc α = 1 / sin α
cos α = adjacent side / hypotenuse	Sec α = 1 / cos α
tan α = opposite side / adjacent side	Cot α = 1 / tan α
Tan α = sin α / cos α	

SOH CAH TOA / SOS CAS TOA

1.4 Unit circle

$$(\sin\alpha)^2 + (\cos\alpha)^2 = 1$$

$$\frac{\mathbf{a}}{\sin\alpha} = \frac{\mathbf{b}}{\sin\beta} = \frac{\mathbf{c}}{\sin\gamma} \;(\textbf{Law of sines})$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

1.5 Special angle

sin α = [-1, 1]	cos α = [-1, 1]
tan α = [-infinity, infinity]	cot α = [-infinity, infinity]

1.6 Pairs of angles

oppositely signed angles: their measures add up to 0 degrees. α + β = 0 or β = -α

Complementary angles: their measures add up to 90 degrees in. α + β = 90 or β = 90 – α

1.7 Sun Identities

sin(α + β) = sin α * cos β + cos α * sin β	Sin(α – β) = sin α * cos β - cos α * sin β
Cos(α + β) = cos α * cos β – sin α * sin β	Cos(α - β) = cos α * cos β + cos α * cos β
Tan(α + β) = (tan α + tan β) / (1 – tan α * tan β)	Tan(α - β) = (tan α - tan β) / (1 + tan α * tan β)

1.8 summary of trigonometric formulas

Definition of an angle	A = l / r
Conversion from radians to degrees	Degrees * pi / 180
Conversion from degrees to radians	Rad * 180 / pi
The Pythagorean theorem	$a^2 + b^2 = c^2$
The formula of the sine, cosine and tangent of the acute angle in a right triangle	Sinus = Opposite side / Hypotenuse Cosine = Adjacent side / Hypotenuse Tangent = Opposite side / Adjacent side
The law of sines	a / sin(α) = b / sin(β) = c / sin(γ)
The law of cosine	$a^2 = b^2 + c^2 * 2bc * \cos\alpha$ $\alpha = \cos^{-1}((a^2 - b^2 - c^2)/-2bc)$ $b^2 = a^2 + c^2 * 2ac * \cos\beta$ $c^2 = a^2 + b^2 * 2ab * \cos\gamma$
The area of the triangle	Area = (a * b * sin(γ)) / 2

2 Functions

2.1 basics

functions: as a mapping f that for each argument x returns at most one image f(x)

domain: the set of arguments x which have exactly one image f(x)

range: the of all images f(x) returned by the function f

root: each argument x_0 that maps $f(x_0) = 0$\

2.1.1 Linear functions

y = mx + c   c < 0 = descending   c = 0 = horizontal   c > 0 ascending

2.1.2 Quadratic functions

$y = ax^2 + bx + c$ a > 0 = U   a < 0 = ∩

$y^2 = x$

2.2 Trigonometric functions

amplitude or elongation: r > 0 is the maximum position from the equilibrium

angular speed or pulsation: ω > 0 is the inner coëfficiënt we use to stretch the sine function

Phase: first incoming crossing

y-intercept: offset from x axis from equilibrium

$f(x) = r\sin(\omega x + \Theta_0) + c$

$r = y_{max} - y_{min}$

$c = y_{min} + y_{max} / 2$

ω = first incoming crossing

$\Theta_0 = - \omega$

3 Vector

a **vector** is a arrow uniquely determined by its **length** (or **normal** or **magnitude**) and its **direction** (holding an orientation and a sense)

4 Kinematics

4.1 Measures

Measure: an aspect from reality that records a directly observable or computable value

Measure	symbol	SI-unit
length	l	[l] = m meter
Mass	m	[m] = kg kilogram
Time	t	[t] = s second

4.2 Delta time

delta time: the time elapsed between 2 successive frames [△t] = s

frame rate: frt the refresh rate of our runtime screen [frt] = fps

△t * frt = 1 frame

s_n -> location vector

v_n -> velocity

4.3 Transitional motion

Velocity	displacement / delta time	v	$\triangle$s /$\triangle$t
Acceleration	Change of velocity / delta time	a	$\triangle$v /$\triangle$t
Deceleration	Decrease of velocity / delta time	$\mathrm{a\,.\,v} < 0 = \mathrm{Deceleration}$ $\mathrm{a\,.\,v} > 0 = \mathrm{Acceleration}$	

$$v = v_0 + at$$

$$s = s_0 + v_0 t + \frac{1}{2}at$$

Free fall example (p 165)

$$s(t) = s_0 + v_0 t + \frac{1}{2}at^2$$

applied to Free Fall yields

$$s(t) = 100 + 0t + \frac{1}{2}(-9.81)^2$$

retrieving its roots at

$$t_{1,2} = \sqrt{\frac{2 * (-1000)}{-9.81}}$$

$$t_1 = 14 \; or \; t_2 = -14$$

Determining its vertex at

$t_{vertex} = -v_0/ 2g = -0/2(-9.81) = 0$

4.4 Circular motion

Angular speed	Angular displacement / delta time	ω	$\triangle\Theta$ / $\triangle$t (rad / s)

y(t) = r * sin(ω*t)

f(t) = r sin(ωt + Θ_0)

v = rω in m/s

4.5 summary of formulas

$v = v_0 + at$

$s = s_0 + v_0t + ½at^2$

5 Dot product

$$\theta \; = \; arcoss(\frac{ab}{\sqrt{a^2} + \sqrt{b^2}})$$

a is perpendicular to b when a.b = 0

6 Cross Product

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \; \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} a_2 * b_3 - b_2 * a_3 \\ -a_1 * b_3 + b_1 * a_3 \\ a_1 * b_2 - b_1 * a_2 \end{pmatrix}$$

Example:

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \; and \; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \; and \; \vec{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\vec{e_1} \, x \, \vec{e_2} = \begin{pmatrix} 0 * 0 - 1 * 0 \\ -0 * 0 + 0 * 0 \\ 1 * 1 - 0 * 0 \end{pmatrix} = \vec{e_3}$$

7 Matrices

Matrix: a matrix is a rectangle of numbers

Identity Matrix: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7.1 Determinant

Determinant 2x2 matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11} * a_{22} - a_{21} * a_{12}$$

Determinant 3x3 matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$
$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

Example:

$$\begin{bmatrix} 3 & 2 & 1 \\ 7 & 4 & 2 \\ -2 & 0 & 5 \end{bmatrix} = 3 \begin{bmatrix} 4 & 2 \\ 0 & 5 \end{bmatrix} - 2 \begin{bmatrix} 7 & 2 \\ -2 & 5 \end{bmatrix} + 1 \begin{bmatrix} 7 & 4 \\ -2 & 0 \end{bmatrix}$$
$$= 3(4 * 5 - 0 * 2) - 2(7 * 5 - (-2) * 2) + 1(7 * 0 - (-2) * 4) = -10$$

7.2 Addition

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} + \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \vdots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & \cdots & a_{1n} + b_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} + b_{n1} & \cdots & a_{mn} + b_{mn} \end{pmatrix}$$

Example:

$$\begin{pmatrix} -1 & 5 & \sqrt{2} \\ 4 & -7 & \sqrt{3} \end{pmatrix} + \begin{pmatrix} 3 & 2 & -1 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} -1 + 3 & 5 + 2 & \sqrt{2} - 1 \\ 4 + 0 & -7 + -1 & \sqrt{3} - 2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 5 & \sqrt{2} \\ 4 & -7 & \sqrt{3} \end{pmatrix} - \begin{pmatrix} 3 & 2 & -1 \\ 0 & -1 & -2 \end{pmatrix} = \begin{pmatrix} -1 - 3 & 5 - 2 & \sqrt{2} - (-1) \\ 4 - 0 & -7 - (-1) & \sqrt{3} - (-2) \end{pmatrix}$$

7.3 scalar multiplication

$$\lambda\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} = \begin{pmatrix} \lambda a_{11} & \cdots & \lambda a_{1n} \\ \vdots & \vdots & \vdots \\ \lambda a_{m1} & \cdots & \lambda a_{mn} \end{pmatrix}$$

Example:

$$2\begin{pmatrix} -1 & 5 & \sqrt{2} \\ 4 & -7 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} 2*-1 & 2*5 & 2*\sqrt{2} \\ 2*4 & 2*-7 & 2*\sqrt{3} \end{pmatrix}$$

7.4 Transpose a matrix

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}^t = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}$$

7.5 Dot product

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} * \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \end{pmatrix}$$

$$A_{\mathbf{3}x\mathbf{2}} * B_{\mathbf{2}x\mathbf{5}} = C_{\mathbf{3}x\mathbf{5}}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} * \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$$

$$= \begin{pmatrix} a_{11}*b_{11}+a_{12}*b_{21} & a_{11}*b_{12}+a_{12}*b_{22} & a_{11}*b_{13}+a_{12}*b_{23} \\ a_{21}*b_{11}+a_{22}*b_{21} & a_{21}*b_{12}+a_{22}*b_{22} & a_{21}*b_{13}+a_{22}*b_{23} \\ a_{31}*b_{11}+a_{32}*b_{21} & a_{31}*b_{12}+a_{32}*b_{22} & a_{31}*b_{13}+a_{32}*b_{23} \end{pmatrix}$$

8 Transformation analysis

8.1 Translation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 5 \\ 1 \end{pmatrix}$$

8.2 Scaling

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

8.3 Rotation

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

z-axis rotation only affects x- and y-labels

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

x-axis rotation only affects y- and z-labels

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

y-axis rotation only affects x- and z-labels

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

8.4 Reflection

2d reflection over x-axis

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

2d reflection over y-axis

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

2d reflection over origin

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

8.5 Shearing

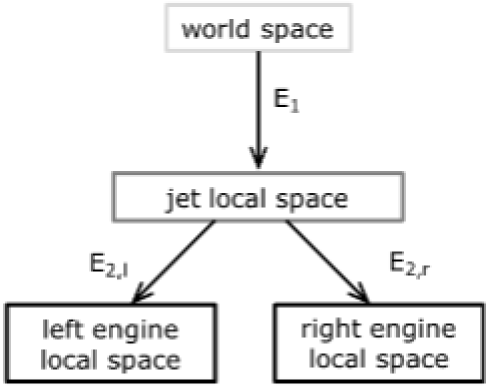
$$tan\ \sigma_x :\ only\ affects\ x - axis$$

$$tan\ \sigma_y :\ blah\ blah\ y - axis$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & \tan\sigma_x & 0 \\ \tan\sigma_y & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

9 Scene graphs

Scene Graph: the parent child tree structure of a composite graphical object



Example:

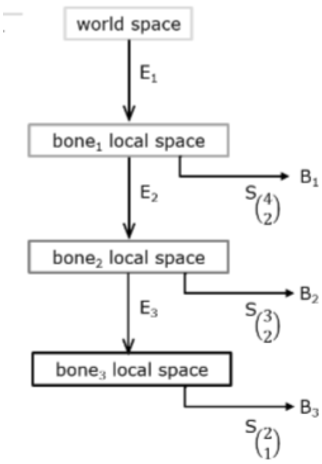
$$E_1 = L_{\overrightarrow{BT}} = \begin{pmatrix} 0.93 & -0.37 & 5 \\ 0.37 & 0.93 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

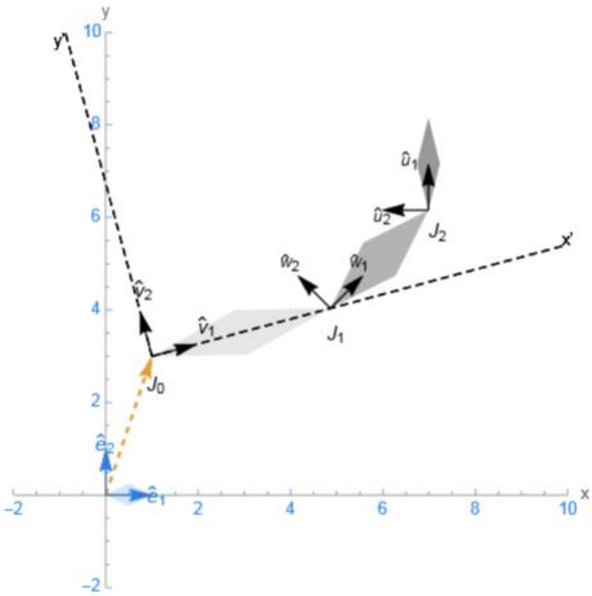
$$E_{2,l} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.3 \\ 0 & 0 & 1 \end{pmatrix} E_{2,r} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -0.3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_r^{(w)} = E_1 * E_{2,r} = \begin{pmatrix} 0.93 & -0.37 & 5 \\ 0.37 & 0.93 & 4 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -0.3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.93 & -0.37 & 5.11 \\ 0.37 & 0.93 & 3.72 \\ 0 & 0 & 1 \end{pmatrix}$$

9.1 Bone structure

$$B_0 = \begin{pmatrix} 0 & 0.5 & 1 & 0.5 \\ 0 & -0.25 & 0 & 0.25 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$





$$E_1 = P_{(1,3)}(15°) = T_{\binom{1}{3}} * R_\theta(15°)$$

$$E_2 = P_{(4,0)}(30°) = T_{\binom{4}{0}} * R_\theta(30°)$$

$$E_3 = P_{(3,0)}(45°) = T_{\binom{3}{0}} * R_\theta(45°)$$

$$B_2^{(W)} = E_1 * E_2 * S_{\binom{3}{2}} * B_0$$

$$= (T_{\binom{1}{3}} * R_\theta(15°)) * (T_{\binom{4}{0}} * R_\theta(30°)) * S_{\binom{3}{2}} * B_0$$

$$J_2^{(W)} = (T_{\binom{1}{3}} * R_\theta(15°)) * (T_{\binom{4}{0}} * R_\theta(30°)) * S_{\binom{3}{2}} * \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= (T_{\binom{1}{3}} * R_\theta(15°)) * (T_{\binom{4}{0}} * R_\theta(30°)) * \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$= (T_{\binom{1}{3}} * R_\theta(15°)) * (\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} cos\ 30° & -sin\ 30° & 0 \\ sin\ 30° & cos\ 30° & 0 \\ 0 & 0 & 1 \end{pmatrix}) * \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

$$= (T_{\binom{1}{3}} * R_\theta(15°)) * \begin{pmatrix} 0.87 & -0.5 & 4 \\ 0.5 & 0.87 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$$

$$= (\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} cos\ 15° & -sin\ 15° & 0 \\ sin\ 15° & cos\ 15° & 0 \\ 0 & 0 & 1 \end{pmatrix}) * \begin{pmatrix} 0.87 & -0.5 & 4 \\ 0.5 & 0.87 & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 6.61 \\ 1.50 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.97 & -0.26 & 1 \\ 0.26 & 0.97 & 3 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 7.02 \\ 6.17 \\ 1 \end{pmatrix} = \begin{pmatrix} 7.02 \\ 6.17 \\ 1 \end{pmatrix}$$

9.2 Solar system

10 View transform

10.1 Camera transformation

Example:

$$F_{\binom{400}{100}}(21.8°) = T_{\binom{400}{100}} * R_\theta(21.8°) * S_{\binom{1}{1}} = \begin{pmatrix} cos\ 21.8° & -sin\ 21.8° & 400 \\ sin\ 21.8° & cos\ 21.8° & 100 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0.93 & -0.37 & 400 \\ 0.37 & 0.93 & 100 \\ 0 & 0 & 1 \end{pmatrix}$$

Generalized:

$$F_c(\theta) = T_{\binom{c_1}{c_2}} * R_\theta(\theta) * S_{\binom{s_x}{s_y}} = \begin{pmatrix} 1 & 0 & c_1 \\ 0 & 1 & c_2 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} cos\ \theta & -sin\ \theta & 0 \\ sin\ \theta & cos\ \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} s_x\ cos\ \theta & -s_y\ sin\ \theta & c_1 \\ s_x\ sin\ \theta & s_y\ cos\ \theta & c_2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} v1_x & v2_x & c_1 \\ v1_y & v2_y & c_2 \\ 0 & 0 & 1 \end{pmatrix}$$

10.2 View transformation

inverse of camera transformation

$$V_{\vec{c}}(\theta) = V_{\vec{c}}(\theta)^{-1} = \left(T_{\binom{c_1}{c_2}} * R_\theta(\theta) * S_{\binom{s_x}{s_y}}\right)^{-1} = S^{-1}_{\binom{s_x}{s_y}} * R^{-1}_\theta(\theta) * T^{-1}_{\binom{c_1}{c_2}} = S_{\binom{1/s_x}{1/s_y}} * R_\theta(-\theta) * T_{\binom{-c_1}{-c_2}}$$

$$= \begin{pmatrix} 1/s_x & 0 & 0 \\ 0 & 1/s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} cos(-\theta) & -sin(-\theta) & 0 \\ sin(-\theta) & cos(-\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & -c_1 \\ 0 & 1 & -c_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/s_x & 0 & 0 \\ 0 & 1/s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} cos\,\theta & sin\,\theta & 0 \\ -sin\,\theta & cos\,\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & -c_1 \\ 0 & 1 & -c_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{s_x} cos\,\theta & \frac{1}{s_x} sin\,\theta & 0 \\ -\frac{1}{s_y} sin\,\theta & \frac{1}{s_y} cos\,\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & -c_1 \\ 0 & 1 & -c_2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{s_x} cos\,\theta & \frac{1}{s_x} sin\,\theta & -\frac{1}{s_x}(c_1 cos\theta \; + c_2 sin\theta) \\ -\frac{1}{s_y} sin\,\theta & \frac{1}{s_y} cos\,\theta & -\frac{1}{s_y}(c_1(-sin\theta) \; + c_2 cos\theta) \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{cos\,\theta}{s_x} & \frac{sin\,\theta}{s_x} & \frac{-v_1 * \vec{c}}{s_x} \\ -\frac{sin\,\theta}{s_y} & \frac{cos\,\theta}{s_y} & \frac{-v_2 * \vec{c}}{s_y} \\ 0 & 0 & 1 \end{pmatrix}$$

Vectors($\overrightarrow{v_1}$, $\overrightarrow{v_2}$)

$$V_{\vec{c}}(\theta) = \begin{pmatrix} \frac{v1_x}{||\overrightarrow{v_1}||} & \frac{v1_y}{||\overrightarrow{v_1}||} & \frac{v_1 * \vec{c}}{||\overrightarrow{v_1}||} \\ \frac{v2_x}{||\overrightarrow{v_2}||} & \frac{v2_y}{||\overrightarrow{v_2}||} & \frac{v_2 * \vec{c}}{||\overrightarrow{v_2}||} \\ 0 & 0 & 1 \end{pmatrix}$$

11 Parameters

12 Extra

$D = b^2 - 4ac$

$$x_{1,2} = \frac{-b \mp \sqrt{D}}{2a}$$