AMP(1)-Lab03–Trigonometric Formulas

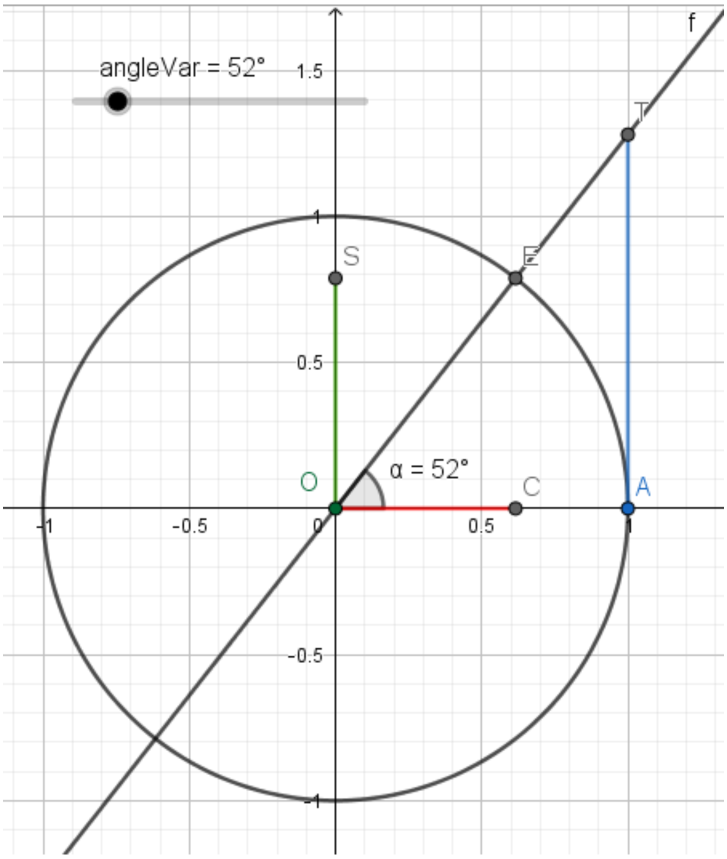
1. Content

2. Learning objectives

3. Exercises

3.1. Basic exercises

3.1.1. Exploring trigonometric ratios in the unit circle



- For which angle values is the tangent undetermined?
  90° and 270°
- Give the angle interval(s) that results in negative cosine values?
  ] 90°, 270°[
- Give the angle interval(s) that results in negative sine values?
  ] 180°, 360°[
- Give the angle interval(s) that results in negative tangent values?
  ] 90°, 180°[
  ] 270°, 360°[
- What is the maximum and minimum value of the cosine and sine?
  max is 1
  min is -1
- Which angles have the same cosine and sine value?
  45°, 225°

3.1.2. Angles

a. Special angles

a) Prove solely by pen and paper the three trigonometric ratios (sine, cosine and tangent) for **at least one** of these special angles: 30° or 45° or 60°

b) Scaffold each geometric reasoning step by a situation sketch within the unit circle.

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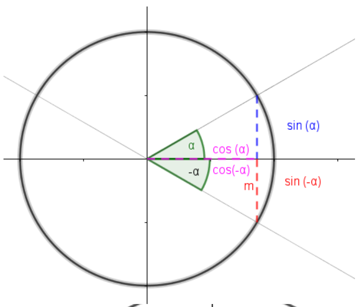
c) Prove pure calculations by providing each intermediate calculation step.

See ISBN 9789401474955 (Animation Maths NE/2021), par 3.5 (special angles)

b. Coterminal and complementary angles

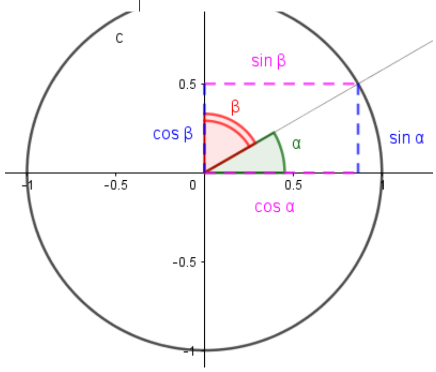
a) What is the result of sin(α) + sin(β) when α and β are coterminal angles?



The result is 0, because when these are coterminal angles, then the drawing shows that sin(-α) is the same as -sin(α) so we can rewrite the expression as follows:

sin(α) + sin(β) = sin(α) - sin(α) = 0

b) What is the result of $\sin^2(\alpha)+\sin^2(\beta)$ when α and β are complementary angles?



The result is 1.

Having a look at the drawing. It tells us that sin(β) has the same value as the cos (α). So we can transform the expression as follows:

$\sin^2(\alpha)+\sin^2(\beta) = \sin^2(\alpha)+ \mathbf{\cos^2(\alpha)} = 1$

c) What is the result of $\cos^2(\alpha)+\cos^2(\beta)$ when α and β are complementary angles?

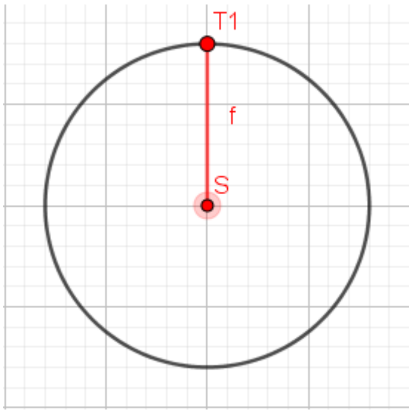
Again it is 1, because the cos (β) has the same value as the sin(α). Thus we can transform the expression as follows: $\cos^2(\alpha)+\cos^2(\beta) =$

$\mathbf{\sin^2(\beta)}+\cos^2(\beta) = 1$

3.2. Contextual practice

Now solving following real world examples should be a piece of cake.

3.2.1. Clock

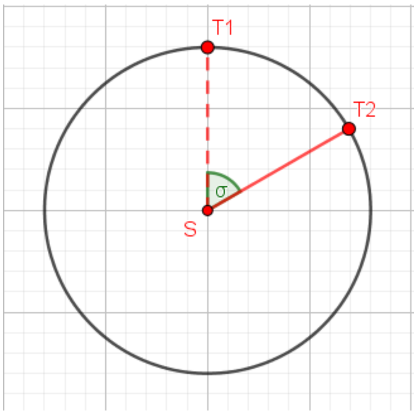


A game shows a chronometer that measures the time an attack takes.

The arm starts upwardly as indicated by [ST1].

And it makes a whole tour in 1 minute.

It has a diameter of 16 units.



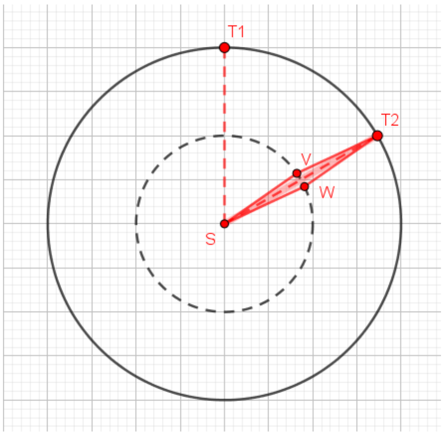
T2 indicates the position of the arm after 10 seconds. Answer the questions mentioned in the table below.

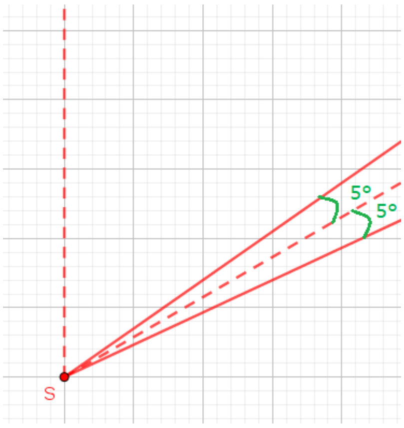
	Formula	Result
a) Size in degrees of angle σ	360° * 10 sec / 60 sec	60°
b) Horizontal distance between S and T2	16 / 2 * sin(60°)	6.93
c) Vertical distance between S and T2	16 / 2 * cos(60°)	4

Check your results in Geogebra.

3.2.2. Advanced clock

The project leader wants you to draw a more advanced arm.





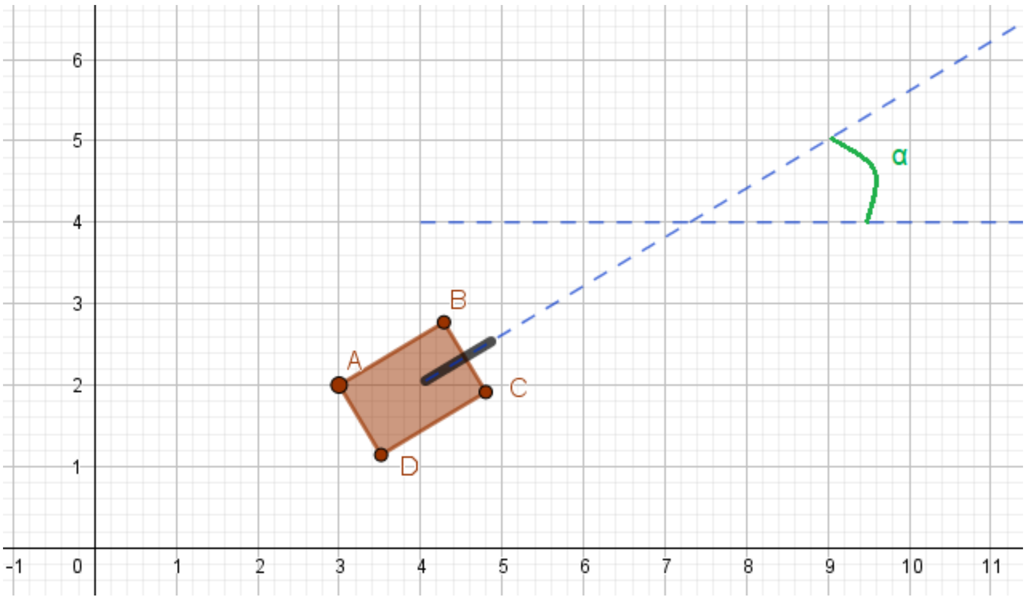
The vertices V and W of the arm are on a circle with diameter half the value (8 cm) of the outer circle (16 cm) and the line segments [SV] and [SW] form an angle of 5° to ST2.

What is the horizontal and vertical distance between S and V and S and W after 10 seconds?

	Formula	Result
a) Horizontal distance between S and V	8 / 2 * sin(60°-5°)	3.28
b) Vertical distance between S and V	8 / 2 * cos(60°-5°)	2.29
c) Horizontal distance between S and W	8 / 2 * sin(60°+5°)	3.63
d) Vertical distance between S and W	8 / 2 * cos(60°+5°)	1.69

3.2.3. Tank wars

We are developing a tank wars game.
The top view of one of the tanks is represented by the image below. It has a rectangular form of which the diagonal [AC] lies horizontal. The coordinates of vertex A are (3,2), segments [AD] and [BC] measure 1 unit each and [AB] and [DC] both measure 1.5 units. The tank its gun is aligned with the tank and pointing in the direction determined by the dashed line with angle α. The slope (=tan(α)) of this direction is +0.6.
Calculate the coordinates of the vertices B, C and D of the tank.



	Formula	Result
a) x of B	α = arctan(0.6) => α = 30.96° x(A) + 1.5 * cos α	4.29
b) y of B	y(A) + 1.5 * sin α	2.77
c) x of D	x(A) + 1 * sin α	3.51
d) y of D	y(A) – 1 * cos α	1.14
e) x of C	x(A) + 1.5 * cos α + 1 * sin α	4.8
f) y of C	y(A) + 1.5 * sin α – 1 * cos α	2.00

3.2.4. Rotation

Position a plane point P(x,y) at a fixed distance *r* to the origin O. Now, rotate this point around the origin O over an angle β along a circular arc (i.e.: keeping its distance *r* to the origin).

Determine the rotated point P’(x’,y’) **coordinates** in terms of the initial coordinates x and y.

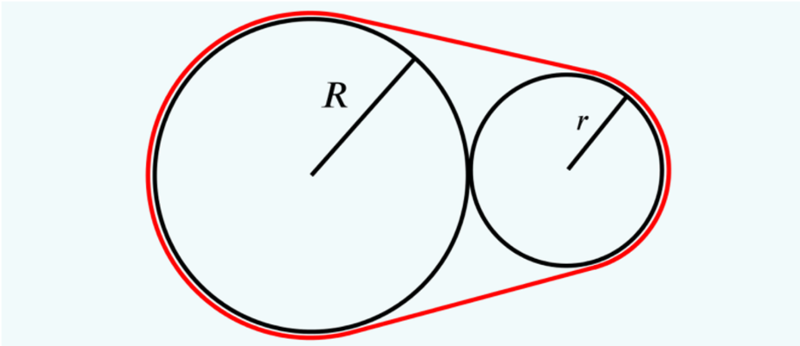
Hint: consider the initial point P(x,y) subtending an angle α to the horizontal x-axis.

$x' = x \cos\beta - y \sin\beta$

$y' = x \sin\beta + y \cos\beta$

3.2.5. Belt

The red belt keeps together two wheels of radii R and r respectively, looping all the way around them both. We logically assume R > r >0.

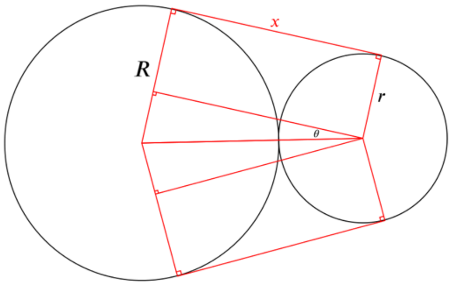


Determine the entire **length** of this looped belt.

Hint 1. Distinguish the four parts of this belt: two external tangent stretches, the left circular arc and the right circular arc. Calculating each of these partial lengths relies on properties of the tangent to a circle, right triangles and (inverse) trigonometry.

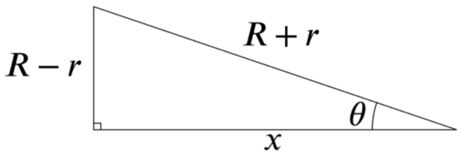
Hint 2. You will need to solve a certain right triangle, and will need to apply the definition of an angle in radian.

The straight part of the wire runs along two lines that are tangent to both circles. Drawing in radii of the circles that are perpendicular to this tangent line and the straight line joining the centres of the two circles, we have the following picture:



The two cylindrical welding rods with part of the wire and some other helpful lines drawn in

We have also drawn in an extra line of the same length of those we want to find, but further down, creating a triangle.



A close-up of the triangle we found

We can use this to find the length of x, as well as the value of θ. Notice that the bottom triangle is a reflection of the top triangle, so that both line segments have the same length. (And we are assuming that $R > r$.)

Using Pythagoras's Theorem, we find that

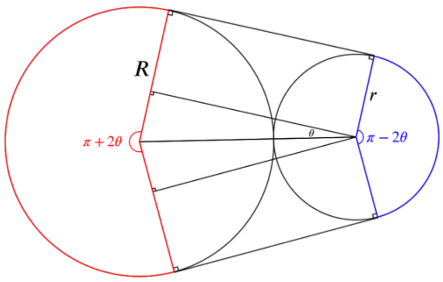
$$x = \sqrt{(R+r)^2 - (R-r)^2} = \sqrt{4Rr} = 2\sqrt{Rr},$$

and using trigonometry we have that

$$\sin\theta = \frac{R-r}{R+r},$$

and so

$$\theta = \sin^{-1}\left(\frac{R-r}{R+r}\right).$$



The two cylindrical welding rods with the arced part of the wire marked

We know the angles of the arcs from our earlier drawing. The red angle (on the left) is the whole circle without two times the third (unmarked) angle in our triangle (which we know must be $\pi/2 - \theta$), and so the red angle must be

$$2\pi - 2\left(\frac{\pi}{2} - \theta\right) = \pi + 2\theta.$$

The blue angle (on the right) is the whole circle minus two times θ minus two right angles, and so must be

$$2\pi - 2\theta - 2\frac{\pi}{2} = \pi - 2\theta.$$

So the length of the red (left) arc is

$$\frac{\pi + 2\theta}{2\pi} 2\pi R = \pi R + 2\theta R = \pi R + 2R\sin^{-1}\left(\frac{R - r}{R + r}\right),$$

and the blue (right) arc has length

$$\frac{\pi - 2\theta}{2\pi} 2\pi r = \pi r - 2\theta r = \pi r - 2r\sin^{-1}\left(\frac{R - r}{R + r}\right).$$

So the total length of the wire is

$$2\{\text{length of straight line}\} + \{\text{length of blue arc}\} + \{\text{length of red arc}\},$$

which is

$$4\sqrt{Rr} + \pi(R + r) + 2(R - r)\sin^{-1}\left(\frac{R - r}{R + r}\right).$$

$$\text{length} = 4\sqrt{R \cdot r} + \pi(R + r) + 2(R - r)\arcsin\left(\frac{R-r}{R+r}\right)$$