AMP(1) - Lab08 – Dot Product

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# Learning objectives

## Exam objectives

By the end of this lab you should be able to (pen and paper):

* Apply the dot product of vectors
* Be mindful of dot product’s properties: commutativity and square of a vector
* Apply the dot product’s geometric formula to retrieve the subtended angle
* Apply the dot product’s criterion for orthogonality

We advise you to **make your own summary of topics** which are new to you.

## Supportive objectives

More specifically related to the above you should in GeoGebra be able to:

* Apply the dot product of vectors
* Apply the dot product to retrieve and visualize the subtended angle in the View/Graphics

# Exercises

Dependent of the lab session you may work individually or teamed (organized by the lab attendant). In either case make sure that throughout the course of this lab, you re-save sufficiently your solution file on your local machine as

**1DAExx-0y-name1**(+name2+name3).GGB given **xx**=groupcode, **0y**=labindex

## Basic exercises

### Recap length of a vector

Calculate the length of the following vectors in 2 different ways:

-By using the Pythagorean Theorem

-By using the dot product

*Tip: the angle between a vector and itself is 0, so*

1) =

Pithagoras

||a|| = sqrt(5\*5 + 4\*4) = 6.4

Dot product

a \* a = ||a|| ||a|| cos alpha = ||a|| ||a|| cos 0 (1) =

||a||2 = a\*a = sqrt(a\*a) = sqrt(25 + 16) = sqrt(41) = 6.4

2) =

Pithagoras

||b|| = sqrt((-1)2 + 32 + 22) = sqrt(1 + 9 +4) = sqrt(14) = 3.74

Dot Product

b \* b = ||b|| ||b|| cos 0

||b|| 2= b\*b = sqrt(b\*b) = sqrt(1 + 9 + 4) = 3.74

### Recap Normalising a vector

Lookup the definition of normalizing a vector and explain this in your own words

to your neighbour.

Normalizing a vector is dividing the vector by its length.

Normalize the following vectors.

1. = normalized(a) = a / || a|| = (3 / 3.61, 2/3.61) = **(0.83, 0.55).**

=

||a|| = sqrt(32 + 22) = sqrt(9 + 4) = sqrt(13) = 3.61

1. = normalized(b) = b / ||b|| = (-2/3.74, 3/3.74, 1/3.74) = **(-0.67, 0.80, 0.27)**

|| b || = sqrt(-22 + 32 + 12) = sqrt(4+9+1) =

= sqrt(14) = 3.74

### Calculate dot product

Draw the following vectors

= = = =

And calculate the dot product for the following combinations:

* = 0 + 18 = 18
* = 36 + 1 = 37
* = 0 + 2 = 2
* = 0 + 0 = 0

What does it mean when the dot product is 0?

That vectors are orthogonal.

## Bridging Practice

### Calculate 2D Angle

Given the two vectors and in this 2D-frame

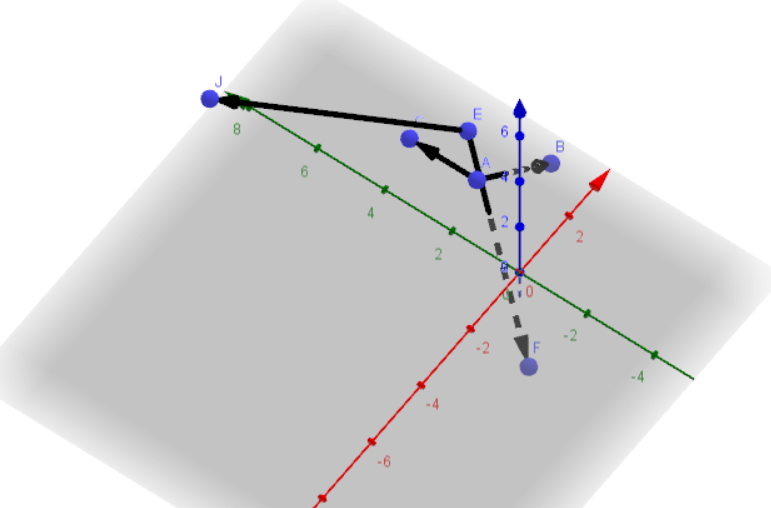
### 

Answer (in positive degrees with 3 decimal places accurate) the angle α

subtended by these vectors using the dot product. You need to type all

intermediate steps (from spanning the free vectors up to the arccosine)!

### Calculate 3D dot product



Given the points

1. What is the dot product of and

AB = B – A = (3, 0, -3)

AC = C – A = (0, 2, 0)

AB \* AC = 0 + 0 + 0 = 0

1. Is the vector perpendicular to the vector ? Why/Why Not?

Is perpendicular because the dot product is 0.

1. Calculate the angle between these 2 vectors in radians

angle = arccos(AB\*AC / ||AB|| ||AC||) = arccos(0) = 90º (pi / 2 rad).

Given the points

1. What is the dot product of and

EF = F – E = (-3, -4, -3)

EJ = J – E = (-5, 4, 4)

EF \* EJ = -3 \* -5 + -4 \* 4 + -3 \* 4 = 15 -16 – 12 = - 13

1. Is the vector perpendicular to the vector ? Why?

Is not perpendicular because the dot product != 0

1. Calculate the angle between these 2 vectors in degrees

angle = arccos( EF \* EJ / ||EF|| ||EJ||) = arccos(-13 / 7.55 \* 5.83) =

|| EF ||= sqrt(9 + 16 + 9) = sqrt(34) = 5.83

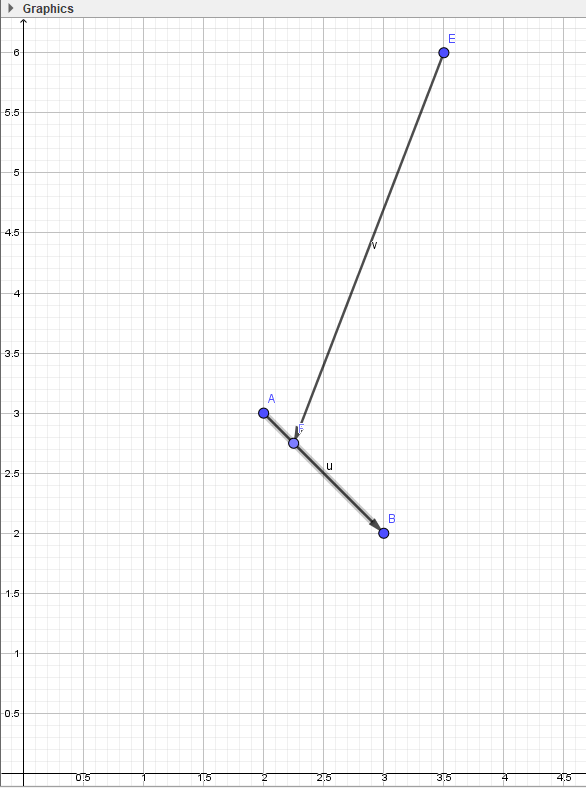
|| EJ ||= sqrt(25 + 16 + 16) = sqrt(57) = 7.55

## Contextual exercises

### Bullet

A Bullet is shot at a window between points and under direction

=



The window will break if the magnitude of the perpendicular force to the window is greater than 3.

1) Calculate the magnitude of the vector perpendicular to the window?

The magnitude is determined by dot(v, normalized vector (n))

Calculate vector → AB = B – A = (3 – 2, 2 – 3) = (1, -1). → We flip the vector so is perpendicular. (1, 1)

||AB|| = sqrt(12 + (1)2 ) = sqrt(2) = 1.412

2) Will the window break or reflect the bullet?

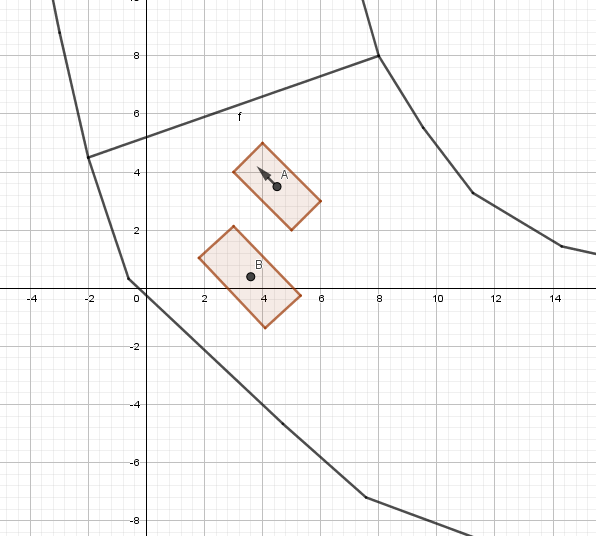
*Tip: In a 2D world you can find the clockwise perpendicular from a vector*  = as =

### Race

You have 2 cars in a race :

-Car 1 with centroid and normalized forward vector =

-Car 2 with centroid

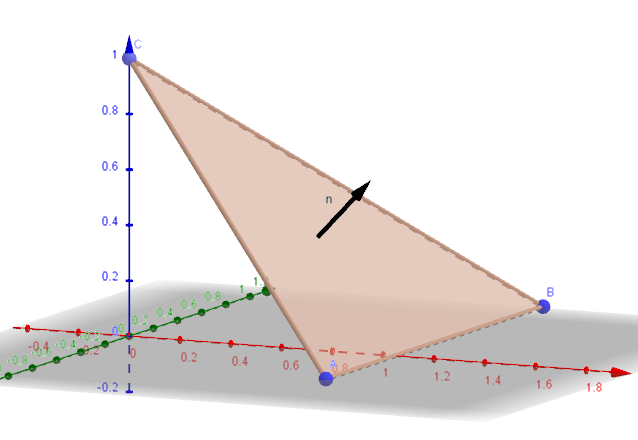
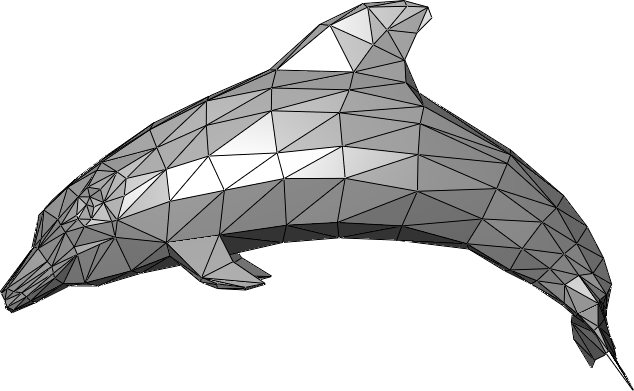


Determine if the car A is in front or behind of car B relative to car A’s forward direction?

How can you know when the cars are next to each other? And on which side?

### Backface culling

A 3D object in a game is composed of triangles that have a front and back face.

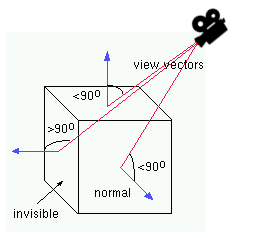


Back

Front

Normally when we are rendering, we only want to show the fronts of our triangles. Backfaces should not be rendered. The front of a triangle (face) is indicated by the direction of the normal.

Assume we have a camera at point that is looking at a point . Below you ﬁnd some normals for faces which are within the view of the camera. Determine which are valid normals for rendering, and which normals are indicating a backface.



Face with perpendicular vector =

Face with perpendicular vector =

Face with perpendicular vector =

Think about how the camera vector should be oriented!

# References

## Basics Explained

Wikipedia Dot Product

<https://en.wikipedia.org/wiki/Dot_product>

Khan Academy Video

<https://www.khanacademy.org/math/linear-algebra/vectors-and-spaces/dot-cross-products/v/vector-dot-product-and-vector-length>

## Applications

Backface culling

<https://en.wikipedia.org/wiki/Back-face_culling>

Reflection

<http://immersivemath.com/ila/ch03_dotproduct/ch03.html>