# Monoidify! Monoids as a Design Principle for Efficient MapReduce Algorithms

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### 1. INTRODUCTION

The purpose of this short paper is to share a recent observation I made in the context of my introductory graduate course on MapReduce at the University of Maryland. It is well known that since the sort/shuffle stage in MapReduce is costly, local aggregation is one important principle to designing efficient algorithms. This typically involves using combiners or the so-called in-mapper combiner technique [5]. However, can we be more precise in formulating this design principle for pedagogical purposes? Simply saying "use combiners" or "use in-mapper combining" is unsatisfying because it leaves open the obvious question of how? What follows is my attempt to formulate a more precise design principle in terms of monoids—the idea is quite simple, but I haven't seen anyone else make this observation before in the context of MapReduce.

Let me illustrate with a running example I often use to illustrate MapReduce algorithm design, which is detailed in Lin and Dyer [5]. Given a large number of key-value pairs where the keys are strings and the values are integers, we wish to find the average of all the values by key. In SQL, this is accomplished with a simple group-by and Avg. Here is the naïve MapReduce algorithm:

#### Algorithm 1

```
1: class Mapper
       method Map(string t, integer r)
2:
            Emit(t,r)
3:
1: class Reducer
        method Reduce(string t, integers [r_1, r_2, ...])
2:
3:
            sum \leftarrow 0
4:
            cnt \leftarrow 0
            for all r \in [r_1, r_2, ...] do
5:
6:
                sum \leftarrow sum + r
7:
                cnt \leftarrow cnt + 1
            r_{avg} \leftarrow sum/cnt
8:
            EMIT(t, r_{avq})
9:
```

This isn't a particularly efficient algorithm because the mappers do no work and all data are shuffled (across the network) over to the reducers. Furthermore, the reducer cannot be used as a combiner. Consider what would happen if we did: the combiner would compute the mean of an arbitrary subset of values with the same key, and the reducer would compute the mean of those values. As a concrete example, we know that:

```
Avg(1, 2, 3, 4, 5) \neq Avg(Avg(1, 2), Avg(3, 4, 5))
```

In general, the mean of means of arbitrary subsets of a set of values is not the same as the mean of the set of values.

So how might we properly take advantage of combiners? An attempt is shown in Algorithm 2.

# Algorithm 2

```
1: class Mapper
2:
        method Map(string t, integer r)
3:
            Emit(string t, integer r)
1: class Combiner
2:
        method Combine(string t, integers [r_1, r_2, \ldots])
3:
            sum \leftarrow 0
4:
            cnt \leftarrow 0
5:
            for all integer r \in \text{integers } [r_1, r_2, \ldots] do
6:
                sum \leftarrow sum + r
7:
                cnt \leftarrow cnt + 1
8:
            Emit(string t, pair (sum, cnt))
1:
   class Reducer
2:
        method Reduce(string t, pairs [(s_1, c_1), (s_2, c_2)...])
3:
            sum \leftarrow 0
4:
            cnt \leftarrow 0
5:
            for all pair (s, c) \in \text{pairs } [(s_1, c_1), (s_2, c_2) \dots] do
6:
                sum \leftarrow sum + s
7:
                cnt \leftarrow cnt + c
            r_{avg} \leftarrow sum/cnt
8:
9:
            Emit(string t, integer r_{avq})
```

The mapper remains the same, but we have added a combiner that partially aggregates results by separately tracking the numeric components necessary to arrive at the mean. The combiner receives each string and the associated list of integers, from which it computes the sum of those values and the number of integers encountered (i.e., the count). The sum and count are packaged into a pair and emitted as the output of the combiner, with the same string as the key. In the reducer, pairs of partial sums and counts can be aggregated to arrive at the mean.

The problem with this algorithm is that it doesn't actually work. Combiners must have the same input and output key-value type, which also must be the same as the mapper output type and the reducer input type. This is clearly not the case. To understand why this restriction is necessary, remember that combiners are optimizations that cannot change the correctness of the algorithm. So let us remove the combiner and see what happens: the output value type of the mapper is integer, so the reducer should receive a list of integers. But the reducer actually expects a list of pairs! The

correctness of the algorithm is contingent on the combiner running on the output of the mappers, and more specifically, that the combiner is run exactly once. Hadoop, for example, makes no guarantees on how many times combiners are called; it could be zero, one, or multiple times. This algorithm violates the MapReduce programming model.

Another stab at the solution is shown in Algorithm 3:

#### Algorithm 3

```
1: class Mapper
2:
       method Map(string t, integer r)
3:
            \text{Emit}(t,(r,1))
1: class Combiner
2:
        method Combine(string t, pairs [(s_1, c_1), (s_2, c_2)...])
3:
            sum \leftarrow 0
4:
            cnt \leftarrow 0
5:
            for all (s,c) \in [(s_1,c_1),(s_2,c_2)...] do
6:
                sum \leftarrow sum + s
7:
                cnt \leftarrow cnt + c
8:
            Emit(t, (sum, cnt))
1: class Reducer
2:
       method REDUCE(string t, pairs [(s_1, c_1), (s_2, c_2)...])
3:
            sum \leftarrow 0
4:
            cnt \leftarrow 0
5:
            for all (s,c) \in [(s_1,c_1),(s_2,c_2)...] do
                sum \leftarrow sum + s
6:
7:
                cnt \leftarrow cnt + c
8:
            r_{avg} \leftarrow sum/cnt
            \text{EMIT}(t, r_{avg})
9:
```

The algorithm is now correct. In the mapper we emit as the intermediate value a pair consisting of the integer and one—this corresponds to a partial count over one instance. The combiner separately aggregates the partial sums and the partial counts (as before), and emits pairs with updated sums and counts. The reducer is similar to the combiner, except that the mean is computed at the end. In essence, this algorithm transforms a non-associative operation (mean of values) into an associative operation (element-wise sum of a pair of numbers, with a division at the end).

Finally, Algorithm 4 shows an even more efficient algorithm that exploits the in-mapper combining pattern:

## Algorithm 4

```
1: class Mapper
 2:
        method Initialize()
 3:
            S \leftarrow \text{new AssociativeArray}
            C \leftarrow \text{new AssociativeArray}
 4:
        method Map(string t, integer r)
 5:
 6:
            S\{t\} \leftarrow S\{t\} + r
 7:
            C\{t\} \leftarrow C\{t\} + 1
        method Close()
 8:
 9:
            for all term t \in S do
                Emit(term t, pair (S\{t\}, C\{t\}))
10:
```

Inside the mapper, the partial sums and counts associated with each string are held in memory across input key-value pairs. Intermediate key-value pairs are emitted only after the entire input split has been processed; similar to before, the value is a pair consisting of the sum and count. The reducer is exactly the same as in Algorithm 3. Mov-

ing partial aggregation from the combiner into the mapper assumes that the intermediate data structures will fit into memory, which may not be a valid assumption. However, in cases where the assumption holds, the in-mapper combining technique can be substantially faster than using normal combiners, primarily due to the savings in not needing to materialize intermediate key-values pairs.

#### 2. MONOIDIFY!

Okay, what have we done to make this particular algorithm work? The answer is that we've created a monoid out of the intermediate value!

How so? First, a recap on monoids: a monoid is an algebraic structure with a single associative binary operation and an identity element. As a simple example, the natural numbers form a monoid under addition with the identity element 0. Applied to our running example, it's now evident that the intermediate value in Algorithm 3 forms a monoid: the set of all tuples of non-negative integers with the identity element (0,0) and the element-wise sum operation,  $(a,b) \oplus (c,d) = (a+c,b+d)$ .

Thus, one principle for designing efficient MapReduce algorithms can be precisely articulated as follows: create a monoid out of the intermediate value emitted by the mapper. Once we "monoidify" the object, proper use of combiners and the in-mapper combining techniques becomes straightforward. This principle also explains why the reducer in Algorithm 1 cannot be used as a combiner and why Algorithm 2 doesn't work.

#### 3. OTHER EXAMPLES

The "monoidify" principle readily explains another Map-Reduce algorithm I often use for pedagogical purposes: the problem of building word co-occurrence matrices from large natural language corpora, a common task in corpus linguistics and statistical natural language processing. Formally, the co-occurrence matrix of a corpus is a square  $n \times n$  matrix where n is the number of unique words in the corpus (i.e., the vocabulary size). A cell  $m_{ij}$  contains the number of times word  $w_i$  co-occurs with word  $w_i$  within a specific context—a natural unit such as a sentence, paragraph, or a document, or a certain window of m words (where m is an application-dependent parameter). Note that the upper and lower triangles of the matrix are identical since cooccurrence is a symmetric relation, though in the general case relations between words need not be symmetric. For example, a co-occurrence matrix M where  $m_{ij}$  is the count of how many times word i was immediately succeeded by word j (i.e., bigrams) would not be symmetric. Beyond simple co-occurrence counts, the MapReduce algorithm for this task extends readily to computing relative frequencies and forms the basis of more sophisticated algorithms such

<sup>&</sup>lt;sup>1</sup>In many cases the operation is commutative as well, so we actually have a commutative monoid, although in this paper I won't focus on this distinction (i.e., in many places where I refer to a monoid, to be more precise it's actually a commutative monoid). <sup>2</sup>In Algorithm 4, the elements of the tuple have been pulled apart and stored in separate data structures, but that's a specific implementation choice not germane to the design principle.

<sup>&</sup>lt;sup>3</sup>This exposition glosses over the fact that at the end of the computation, we break apart the pair to arrive at the mean, which destroys the monoid, but this is a one-time termination operation that can be treated as "post-processing".

as those for expectation-maximization (where we're keeping track of pseudo-counts rather than actual observed counts).

The so-called "stripes" algorithm [5] for accomplishing the co-occurrence computation is as follows:

## Algorithm 5

```
1: class Mapper
       method MAP(docid a, doc d)
2:
           for all term w \in \text{doc } d do
3:
4:
               H \leftarrow \text{new AssociativeArray}
               for all term u \in \text{Neighbors}(w) do
5:
6:
                   H\{u\} \leftarrow H\{u\} + 1
7:
               Emit(Term w, Stripe H)
1: class Reducer
2:
       method Reduce(term w, stripes [H_1, H_2, H_3, \ldots])
3:
           H_f \leftarrow \text{new AssociativeArray}
           for all stripe H \in \text{stripes } [H_1, H_2, H_3, \ldots] do
4:
5:
               Sum(H_f, H)
6:
           Emit(term w, stripe H_f)
```

In this case, the reducer can also be used as a combiner because associative arrays form a monoid under the operation of element-wise sum with the empty associative array as the identity element.

Here's another non-trivial example: Lin and Kolck [6] advocate the use of stochastic gradient descent (SGD) for scaling out the training of classifiers. Viewed from this perspective, SGD "works" because the model parameter (i.e., a weight vector for linear models) comprise a monoid under incremental training.

Other examples of interesting monoids that are useful for large-scale data processing are found in Twitter's Algebird package.<sup>4</sup> These include Bloom filters [1], count-min sketches [2], hyperloglog counters [4].

Finally, it is interesting to note that regular languages form a monoid under intersection, union, subtraction, and concatenation. Since finite-state techniques are widely used in computational linguistics and natural language processing, this observation might hold implications for scaling out text processing applications.

## 4. OPTIMIZATIONS AND BEYOND

In the context of MapReduce, it may be possible to elevate "monoidification" from a design principle (that requires manual effort by a developer) to an automatic optimization that can be mechanistically applied. For example, in Hadoop, one can imagine declaring Java objects as monoids (for example, via an interface). When these objects are used as intermediate values in a MapReduce algorithm, some optimization layer can automatically create combiners (or apply in-mapper combining) as appropriate.

The observation that monoids represent a design principle for efficient MapReduce algorithms extends more broadly to large-scale data processing in general. One concrete example is Twitter's Summingbird project, which takes advantage of associativity to integrate real-time and batch processing. The same monoid (from Algebird, mentioned above) can be used to hold state in a low-latency online application (i.e., operating on an infinite stream) as well as in a scale-out batch processing job (e.g., on Hadoop).

## 5. CONCLUSIONS

None of the ideas in this paper are completely novel: the property of associativity and commutativity in enabling combiners to work properly was pointed out in the original MapReduce paper [3]. Independently, there has been a recent resurgence of interest in functional programming and its theoretical underpinnings in category theory. However, I haven't seen anyone draw the connection between MapReduce algorithm design and monoids in the way that I have articulated here—and therein lies the small contribution of this piece: identifying a phenomenon and giving it a name.

However, it remains to be seen whether this observation is actually useful. Perhaps I am gratuitously introducing monoids just because category theory is "hip" and in vogue. In a way, a monoid is simply a convenient shorthand for saying: associative operations give an execution framework great flexibility in sequencing computations, thus allow opportunities for much more efficient execution. Thus, another way to phrase the takeaway lesson is: take advantage of associativity (and commutativity) to the greatest possible extent. This rephrasing conveys the gist without needing to invoke references to algebraic structures.

Finally, there remains the question of whether this observation is actually useful as a pedagogical tool for teaching students how to think in MapReduce (which was the original motivation for this paper). It is often the case that introducing additional layers of abstraction actually confuses students more than it clarifies (especially in light of the previous paragraph). This remains an empirical question I hope to explore in future offerings of my MapReduce course.

To conclude, the point of this paper can be summed up in a pithy directive: Go forth and monoidify!

# 6. ACKNOWLEDGMENTS

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# 7. REFERENCES

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 $<sup>^4 \</sup>verb|github.com/twitter/algebird|$