



Chapter 5: Probability: What are the Chances?

Section 5.3

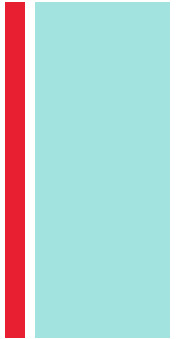
Conditional Probability and Independence

The Practice of Statistics, 4th edition – For AP*
STARNES, YATES, MOORE



Chapter 5

Probability: What Are the Chances?



- 5.1 Randomness, Probability, and Simulation
- 5.2 Probability Rules
- **5.3 Conditional Probability and Independence**



Section 5.3

Conditional Probability and Independence



Learning Objectives

After this section, you should be able to...

- ✓ DEFINE conditional probability
- ✓ COMPUTE conditional probabilities
- ✓ DESCRIBE chance behavior with a tree diagram
- ✓ DEFINE independent events
- ✓ DETERMINE whether two events are independent
- ✓ APPLY the general multiplication rule to solve probability questions

■ What is Conditional Probability?

The probability we assign to an event can change if we know that some other event has occurred. This idea is the key to many applications of probability.

When we are trying to find the probability that one event will happen under the *condition* that some other event is already known to have occurred, we are trying to determine a **conditional probability**.

Definition:

The probability that one event happens given that another event is already known to have happened is called a **conditional probability**. Suppose we know that event A has happened. Then the probability that event B happens *given* that event A has happened is denoted by $P(B | A)$.

Read $|$ as “given that”
or “under the
condition that”

■ Example: Grade Distributions

Consider the two-way table on page 314. Define events

E : the grade comes from an EPS course, and

L : the grade is lower than a B.

School	Grade Level			Total
	A	B	Below B	
Liberal Arts	2,142	1,890	2,268	6300
Engineering and Physical Sciences	368	432	800	1600
Health and Human Services	882	630	588	2100
Total	3392	2952	3656	10000

Find $P(L)$

$$P(L) = 3656 / 10000 = 0.3656$$

Find $P(E | L)$

$$P(E | L) = 800 / 3656 = 0.2188$$

Find $P(L | E)$

$$P(L | E) = 800 / 1600 = 0.5000$$

■ Conditional Probability and Independence

When knowledge that one event has happened does not change the likelihood that another event will happen, we say the two events are **independent**.

Definition:

Two events A and B are **independent** if the occurrence of one event has no effect on the chance that the other event will happen. In other words, events A and B are independent if $P(A | B) = P(A)$ and $P(B | A) = P(B)$.

Example:

Gender	Dominant Hand		Total
	Right	Left	
Male	20	3	23
Female	23	4	27
Total	43	7	50

Are the events “male” and “left-handed” independent? Justify your answer.

$$P(\text{left-handed} | \text{male}) = 3/23 = 0.13$$

$$P(\text{left-handed}) = 7/50 = 0.14$$

These probabilities are not equal, therefore the events “male” and “left-handed” are not independent.

■ Tree Diagrams

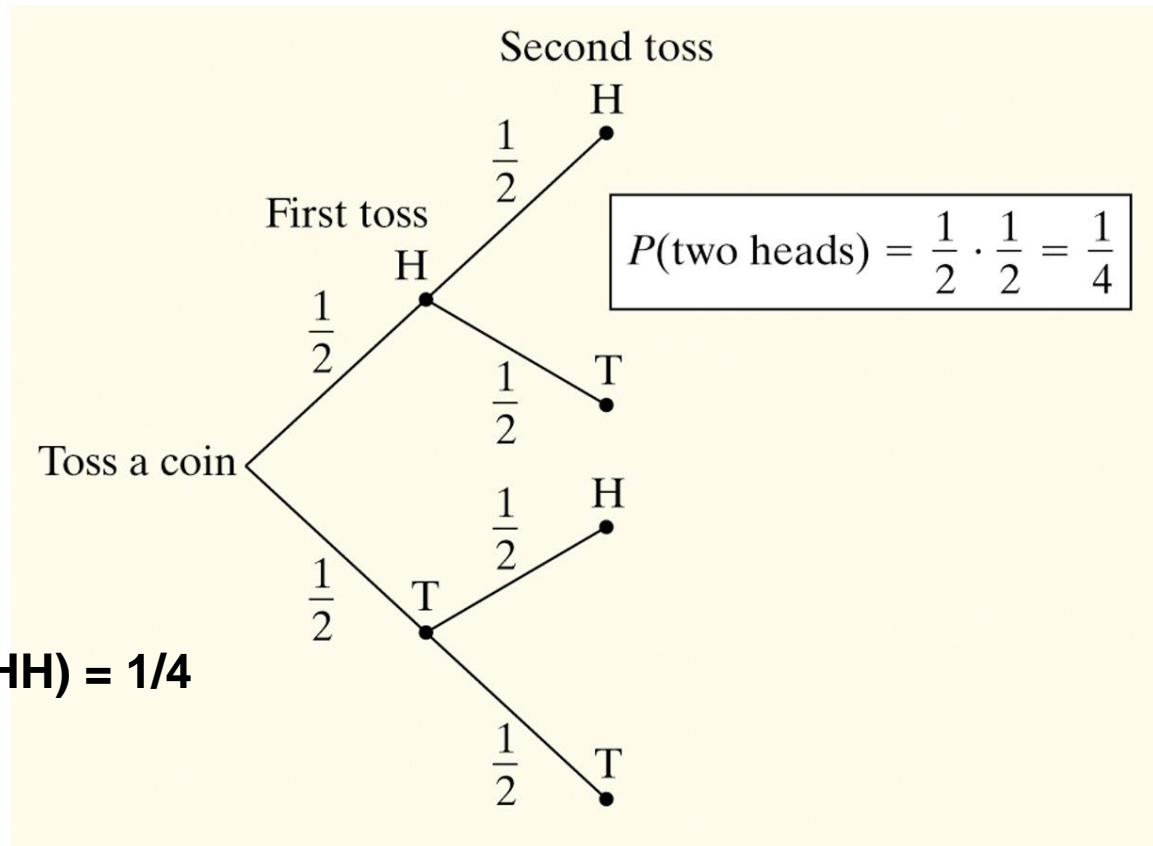
We learned how to describe the sample space S of a chance process in Section 5.2. Another way to model chance behavior that involves a sequence of outcomes is to construct a **tree diagram**.

Consider flipping a coin twice.

What is the probability of getting two heads?

Sample Space:
HH HT TH TT

So, $P(\text{two heads}) = P(HH) = 1/4$



■ General Multiplication Rule

The idea of multiplying along the branches in a tree diagram leads to a general method for finding the probability $P(A \cap B)$ that two events happen together.

General Multiplication Rule

The probability that events A and B both occur can be found using the **general multiplication rule**

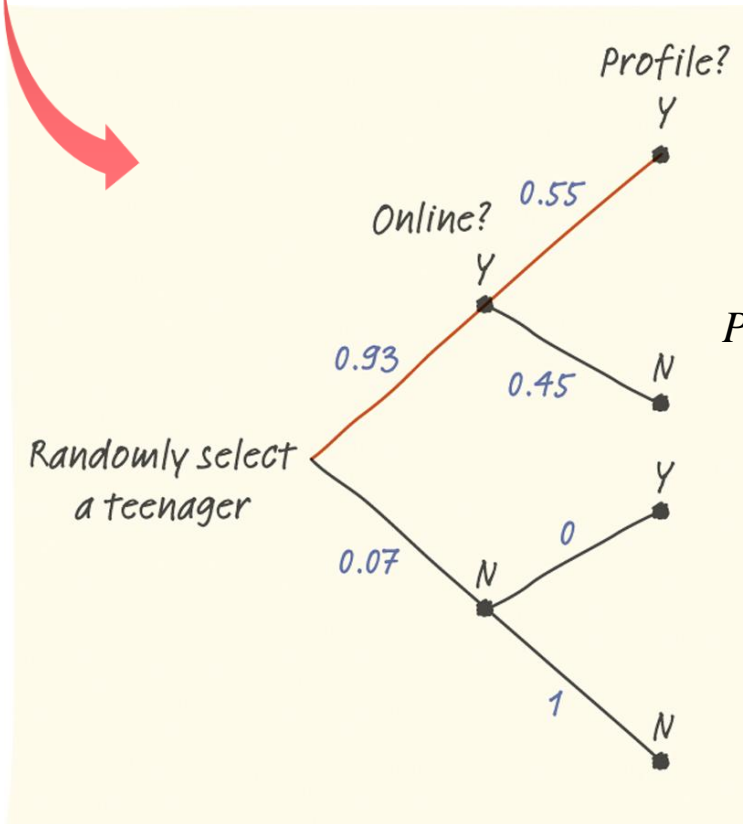
$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

where $P(B \mid A)$ is the conditional probability that event B occurs given that event A has already occurred.

■ Example: Teens with Online Profiles

The Pew Internet and American Life Project finds that 93% of teenagers (ages 12 to 17) use the Internet, and that 55% of online teens have posted a profile on a social-networking site.

What percent of teens are online *and* have posted a profile?



$$P(\text{online}) = 0.93$$

$$P(\text{profile} \mid \text{online}) = 0.55$$

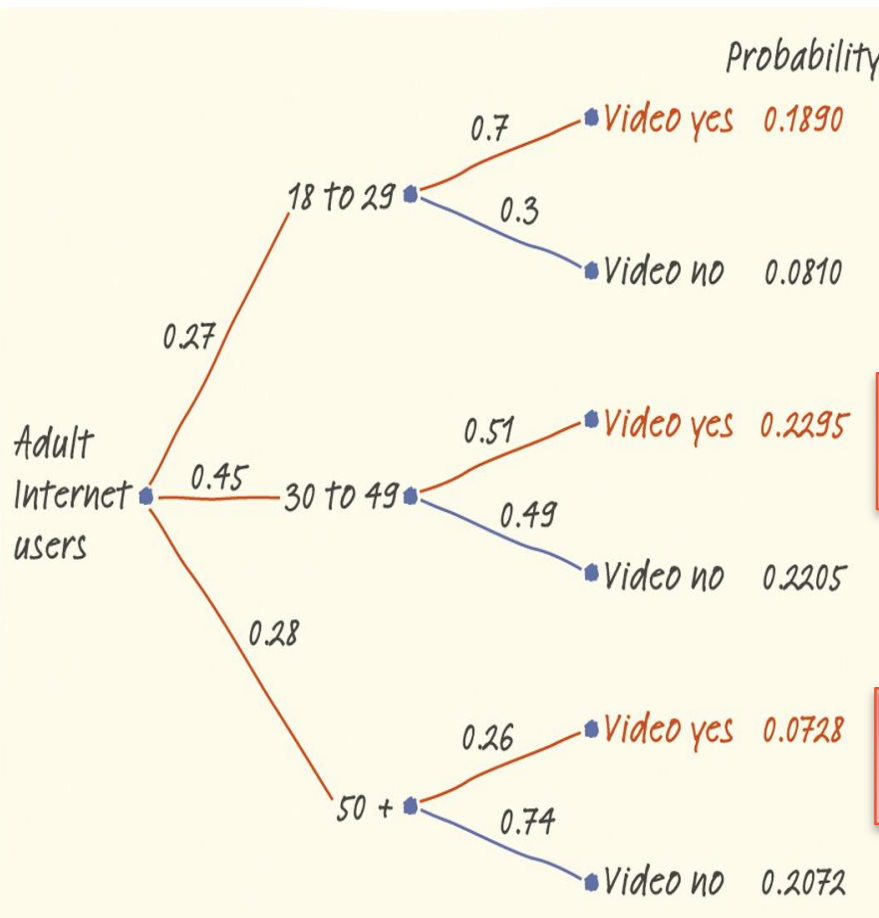
$$\begin{aligned} P(\text{online and have profile}) &= P(\text{online}) \cdot P(\text{profile} \mid \text{online}) \\ &= (0.93)(0.55) \\ &= 0.5115 \end{aligned}$$

51.15% of teens are online *and* have posted a profile.

■ Example: Who Visits YouTube?

See the example on page 320 regarding adult Internet users.

What percent of all adult Internet users visit video-sharing sites?



$$P(\text{video yes} \cap 18 \text{ to } 29) = 0.27 \cdot 0.7 = 0.1890$$

$$P(\text{video yes} \cap 30 \text{ to } 49) = 0.45 \cdot 0.51 = 0.2295$$

$$P(\text{video yes} \cap 50 +) = 0.28 \cdot 0.26 = 0.0728$$

$$P(\text{video yes}) = 0.1890 + 0.2295 + 0.0728 = 0.4913$$

■ Independence: A Special Multiplication Rule

When events A and B are independent, we can simplify the general multiplication rule since $P(B|A) = P(B)$.

Definition:

Multiplication rule for independent events

If A and B are independent events, then the probability that A and B both occur is

$$P(A \cap B) = P(A) \cdot P(B)$$

Example:

Following the Space Shuttle *Challenger* disaster, it was determined that the failure of O-ring joints in the shuttle's booster rockets was to blame. Under cold conditions, it was estimated that the probability that an individual O-ring joint would function properly was 0.977. Assuming O-ring joints succeed or fail independently, what is the probability all six would function properly?

$$\begin{aligned} &P(\text{joint 1 OK and joint 2 OK and joint 3 OK and joint 4 OK and joint 5 OK and joint 6 OK}) \\ &= P(\text{joint 1 OK}) \cdot P(\text{joint 2 OK}) \cdot \dots \cdot P(\text{joint 6 OK}) \\ &= (0.977)(0.977)(0.977)(0.977)(0.977)(0.977) = 0.87 \end{aligned}$$

■ Calculating Conditional Probabilities

If we rearrange the terms in the general multiplication rule, we can get a formula for the conditional probability $P(B | A)$.

General Multiplication Rule

$$P(A \cap B) = P(A) \cdot P(B | A)$$

Conditional Probability Formula

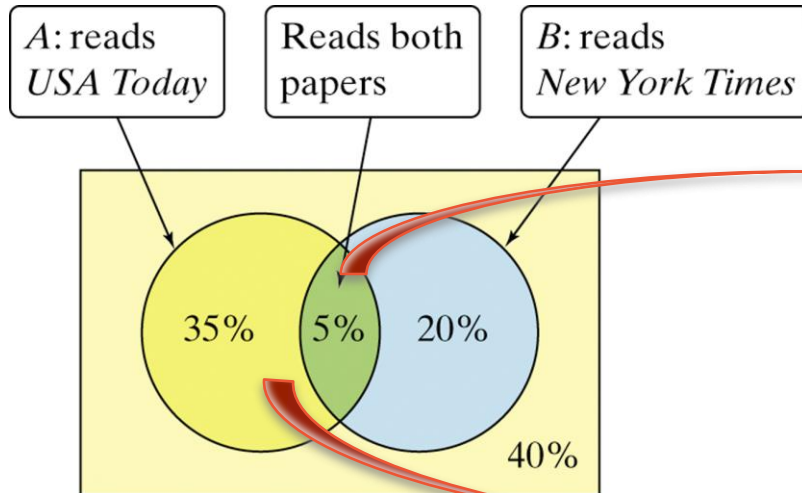
To find the conditional probability $P(B | A)$, use the formula

$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

■ Example: Who Reads the Newspaper?

In Section 5.2, we noted that residents of a large apartment complex can be classified based on the events A : reads *USA Today* and B : reads the *New York Times*. The Venn Diagram below describes the residents.

What is the probability that a randomly selected resident who reads *USA Today* also reads the *New York Times*?



$$P(B | A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = 0.05$$

$$P(A) = 0.40$$

$$P(B | A) = \frac{0.05}{0.40} = 0.125$$

There is a 12.5% chance that a randomly selected resident who reads *USA Today* also reads the *New York Times*.



Section 5.3

Conditional Probability and Independence

Summary

In this section, we learned that...

- ✓ If one event has happened, the chance that another event will happen is a **conditional probability**. $P(B|A)$ represents the probability that event B occurs given that event A has occurred.
- ✓ Events A and B are **independent** if the chance that event B occurs is not affected by whether event A occurs. If two events are mutually exclusive (disjoint), they cannot be independent.
- ✓ When chance behavior involves a sequence of outcomes, a **tree diagram** can be used to describe the sample space.
- ✓ The **general multiplication rule** states that the probability of events A and B occurring together is $P(A \cap B) = P(A) \cdot P(B|A)$
- ✓ In the special case of *independent* events, $P(A \cap B) = P(A) \cdot P(B)$
- ✓ The **conditional probability formula** states $P(B|A) = P(A \cap B) / P(A)$



Looking Ahead...

In the next Chapter...

We'll learn how to describe chance processes using the concept of a **random variable**.

We'll learn about

- ✓ **Discrete and Continuous Random Variables**
- ✓ **Transforming and Combining Random Variables**
- ✓ **Binomial and Geometric Random Variables**