Session 2 - Bayesian Inference and Markov Chain Monte Carlo - Exercises

1. [**Advanced**] In the session notes we showed that the posterior distribution that results from applying Bayes rule with an uninformed prior and a Bernoulli likelihood function is a beta probability density function. Show that this is also the result when the prior is already a beta probability density function and it is updated with a further Bernoulli likelihood function.

2. A coin is flipped 8 times and comes up heads just two times. Use PyMC to model this setting and generate a trace representing the posterior probability of the coin coming up heads. Use that trace to calculate:

   (a) The probability of the next coin flip coming up heads.
   (b) The probability of the next two coin flips both coming up heads.
   (c) The probability of any three out of the next five coin flips coming up heads.

3. Consider the case where a coin has been flipped 6 times and has come up heads twice. This can be represented as a prior with a beta probability density function using the built-in beta function as below:

   ```
   p = pm.Beta('p', 3, 5)
   ```

   Build an appropriate model in PyMC using this prior and generate a trace representing the posterior probability of the coin coming up heads assuming that two more coin flips are made and both of them come up heads.

4. For what values of a and b does the beta probability density function have a value greater than 0 at $p = 0$ and $p = 1$. What does this mean in terms of the results of the coin flipping experiments discussed in this session.