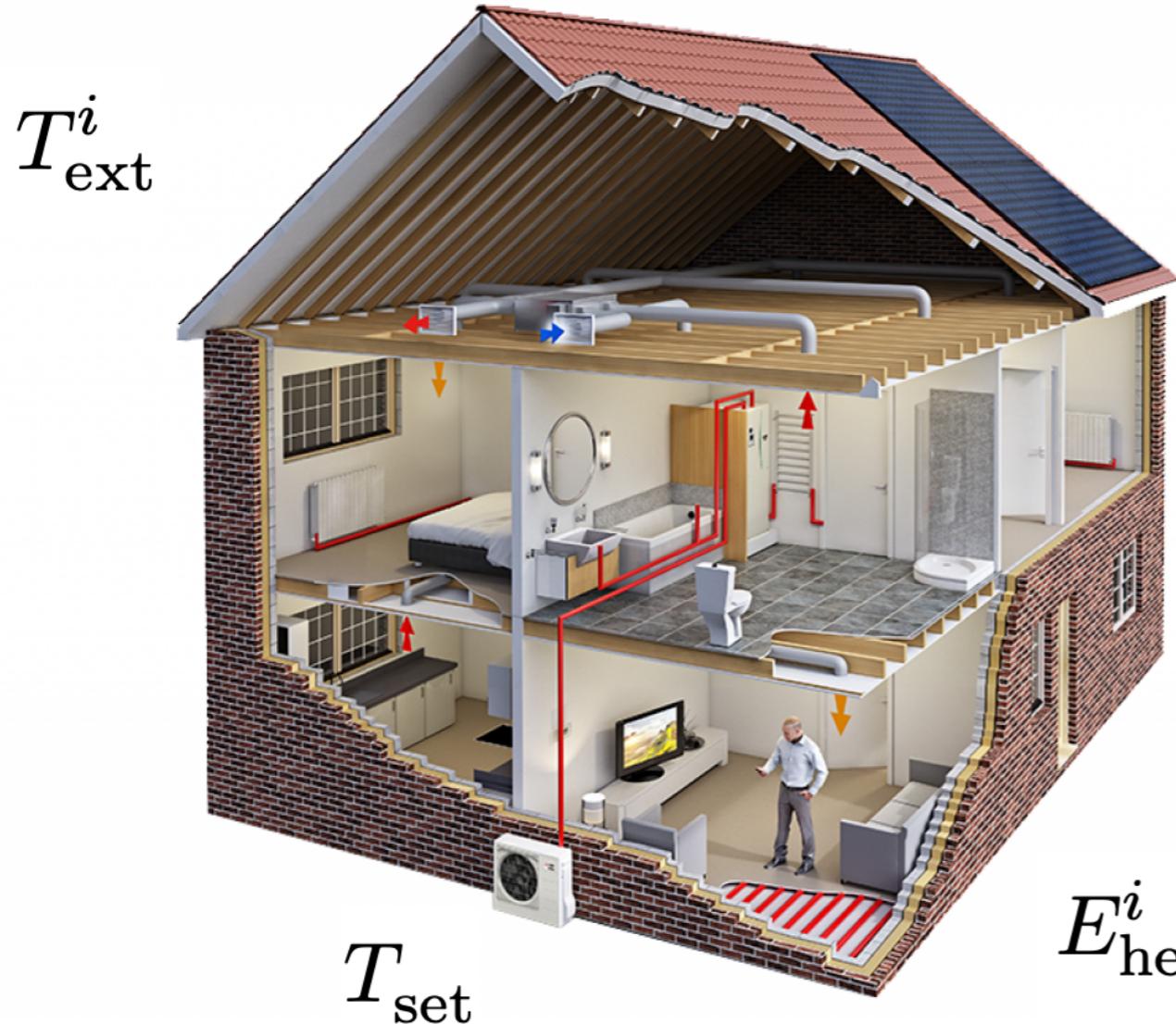


Prof. Alex Rogers
Department of Computer Science
University of Oxford
11th February 2023

Developing and Describing Probabilistic Models

An AI to Help Reduce Heating Bills

An AI to Help Reduce Heating Bills



$$24 \times \text{HPLC} \times (T_{\text{set}} - T_{\text{ext}}^i)$$

$$E_{\text{heat}}^i$$

$$E_{\text{heat}}^i = 24 \times \text{HPLC} \times (T_{\text{set}} - T_{\text{ext}}^i)$$

Heating off

$$E_{\text{meter}}^i = E_{\text{app}}^i$$

$$T_{\text{ext}}^i > T_{\text{set}}$$

Heating on

$$E_{\text{meter}}^i = E_{\text{heat}}^i + E_{\text{app}}^i$$

$$T_{\text{ext}}^i \leq T_{\text{set}}$$

Methodology

Model Building Methodology

1. Understand the setting and the scientific question being asked of the data.
2. What are the sources of noise and imprecision in the process being considered.
 - What likelihood function is appropriate?
 - Are we making error-prone continuous measurements or are our observations discrete?
 - Do we know anything about the expected accuracy of the measurements?
3. Think about what prior information is available.
 - How should that be represented within the model?
 - What prior probability distributions should we use?
 - Are random variable discrete or continuous?

Last Exercise

Mixture Models and Hierarchical Models

Mixture models allow us to describe a huge range of real-world situations. We have seen several examples already (e.g. outliers in linear regression, different situations in the home heating model, and the two species model).

These can often be described as hierarchical models, where a decision parameters determines which other parameters are used to describe the data generation process.

Describing Models

Random Variables

When we describe probabilistic models we are interested in the probability distributions that describe likelihood function and prior distributions.

We typically describe the model mathematically in these terms, rather than discuss in detail how the model is then solved.

$$x_i \sim \text{Beta}(3, 5)$$

$$x_i \sim \text{Normal}(\mu, \sigma^2)$$

Latex and Overleaf



CTAN: Package Metafont

ctan.org/pkg/metafont?lang=en

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METAFONT – A system for specifying fonts

The program takes a programmatic specification of a font, and produces a bitmap font (whose properties are defined by a set of parameters of the target device), and metrics for use by \TeX .

The bitmap output may be converted into a format directly usable by a device driver, etc., by the tools provided in the parallel [mfware](#) distribution. Third parties have developed tools to convert the bitmap output to outline fonts.

The distribution includes the source of Knuth's *METAFONT book*; this source is there to read, as an example of writing \TeX — it should not be processed without Knuth's direct permission.

The mailing list tex-fonts@math.utah.edu is the best for general discussion of METAFONT usage; the tex@tug.org list is best for bug reports about building the software, etc.

Sources [/systems/knuth/dist/mf](#)
Support <https://lists.tug.org/tex-k>
Bug tracker <https://lists.tug.org/tex-k>
Repository <https://tug.org/svn/texlive/trunk/Build/source/texk/web2c/>
Version 2.71828182 2021-02-05
Licenses [Knuth License](#)
Copyright D. E. Knuth
Maintainer [Donald E. Knuth](#)
Contained in [TeX Live](#) as metafont
[MiKTeX](#) as miktex-metafont-bin-2.9



Suggestions

Maybe you are interested in the following packages as well.

- [web: The original literate programming system](#)
- [dvitype: Type out the content of a DVI file](#)
- [knuth-errata: Knuth's published errata](#)
- [texware: Utility programs for use with \TeX](#)

[more ➔](#)

Rating Summary

★★★★★

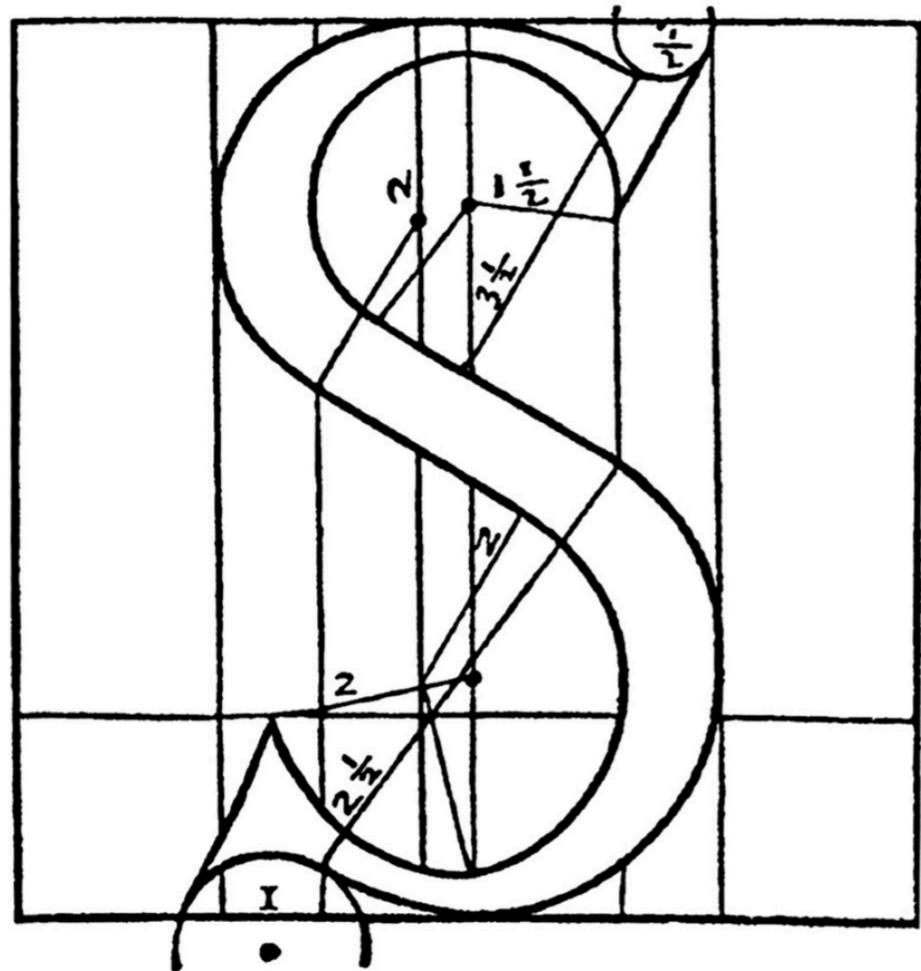


FIGURE 1. Francesco Torniello's method of "squaring the S" in 1517.

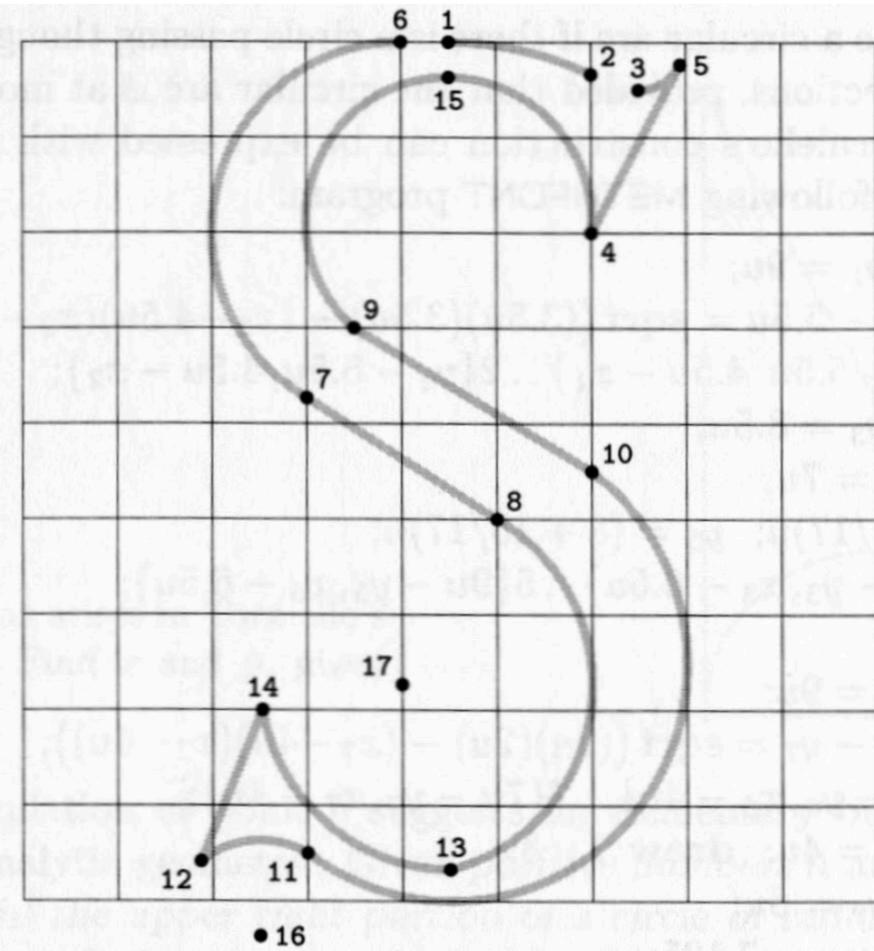


FIGURE 3. The METAFONT program in the text will produce this rendering of Torniello's S.

```
mode_setup;

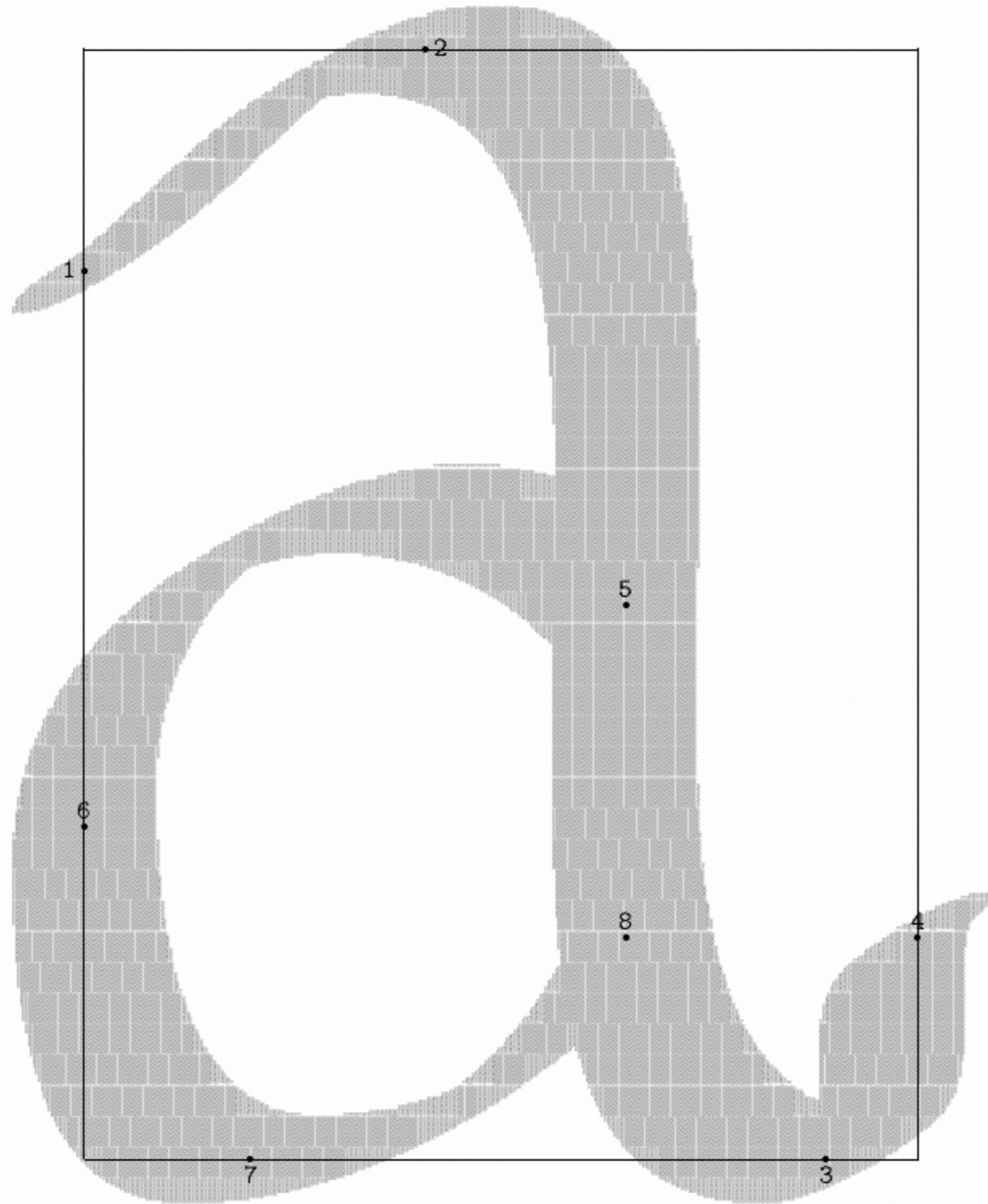
% parameters are name, width, height over base line, depth below base line
beginchar ("a", 9pt#, 12pt#, 0);
alpha:=0.65;
beta:=0.24;

% define points
x1 = 0; y1 = 0.8*h;
x4 = w; y4 = 0.2*h;
y2 = h; y3 = 0;
x2 = (alpha-beta)*w;
x5 = alpha*w;
x3 = (alpha+beta)*w;
y5 = 0.5*h;
x6 = 0; y6 = 0.3*h;
x7 = 0.2*w; y7 = 0;
x8 = x5; y8 = 0.2*h;

% define pen
pickup pencircle xscaled 0.2w yscaled 0.04w rotated 30;

% draw
draw z1{dir 30}..z2{right}..z5{down}..z3{right}..z4{dir 30};
draw z5..z6{down}..z7{right}..z8;
penlabels(1,2,3,4,5,6,7,8);
endchar;

end
```



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The Comprehensive TeX Archive Network (CTAN) is the central place for all kinds of material around TeX. CTAN has currently 6332 packages. 2894 contributors have contributed to it. Most of the packages are free and can be downloaded and used immediately.

Announcements on CTAN-announce

You can see what's new and even get informed about new or updated packages on CTAN.

- 2022-11-10 New on CTAN: cvss
- 2022-11-10 CTAN update: fancyhdr
- 2022-11-10 CTAN update: glossaries-extra
- 2022-11-10 CTAN update: piton

[more ↗](#)

Activity on CTAN

An active TeX community takes care that CTAN is updated and extended regularly. CTAN receives usually more than 100 uploads per month.

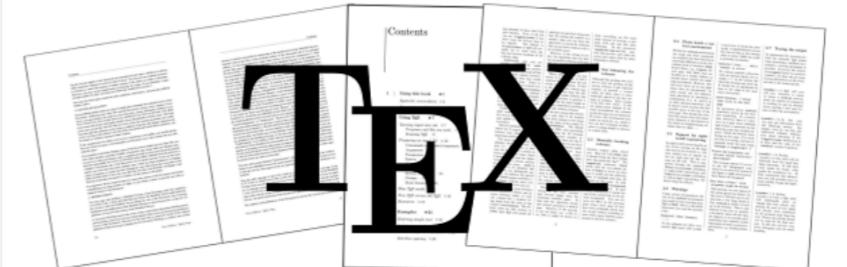


Did you know?

The topic [Font processor](#) in the TeX Catalogue has 49 packages for process fonts for use with TeX (includes installation).

[more ↗](#)

TeX

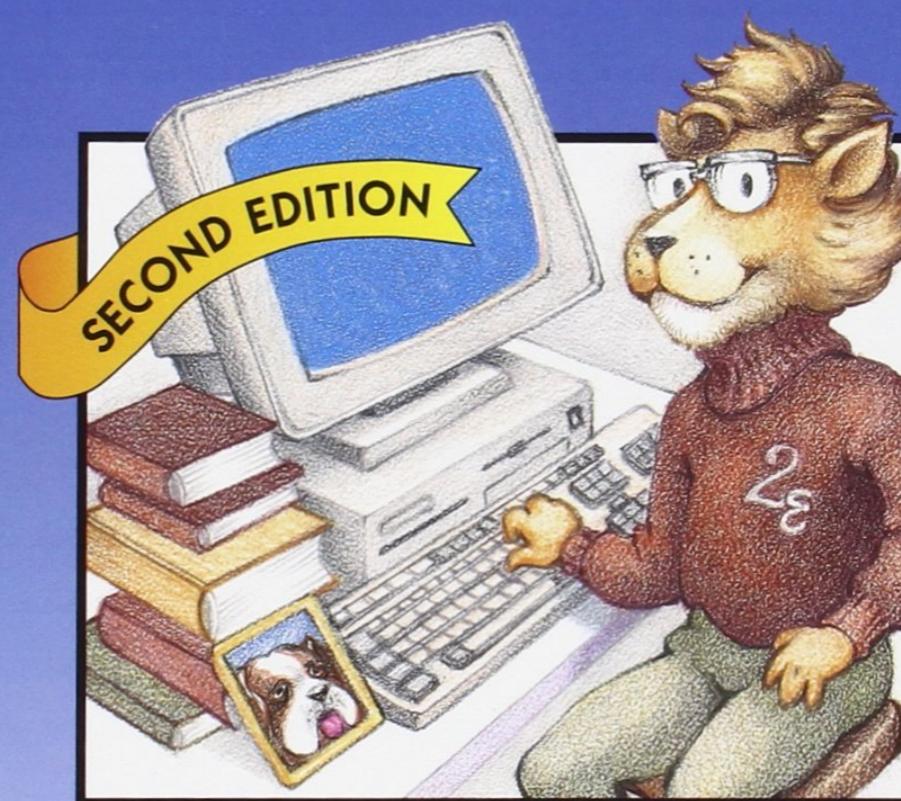


TeX is a typesetting program designed for high-quality composition of material that contains a lot of mathematical and technical expressions. It has been adopted by many authors and

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USER'S GUIDE AND
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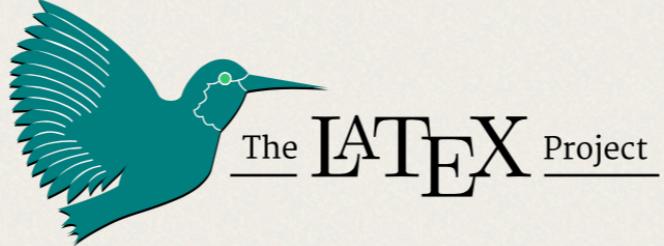


Leslie Lamport

Updated for
LATEX 2^E

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LaTeX – A document preparation system

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report

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format.tex

report.bib

report.tex

File outline

Overview

Millikan's Oil Drop Expe...

Model Description

Solving the Model

```

1 \documentclass[10pt, twocolumn, a4paper]{article}
2
3 \input{format.tex}
4
5 \usepackage{amsmath}
6 \usepackage{amsfonts}
7 \usepackage{amssymb}
8 \usepackage{listings}
9
10 %-----
11 %-----
12 \title{An Example Report}
13
14 \author{
15   \authorstyle{Alex Rogers, 30th July 2022}
16 }
17
18 %-----
19 %-----
20 \date{}
21
22 \begin{document}
23
24 \maketitle
25
26 \setcounter{page}{1}
27
28 \thispagestyle{firstpage}
29
30 %-----
31 %-----
32 \section{Overview}
33
34 This document provides a template for you to use to write your reports and also
an example of the description of a model for the Millikan oil drop experiment.
The modelling approaches are described further in the two course books
\citet[bayesian_hackers, bayesian_analysis, new_paper]{}.
35

```

Recompile

An Example Report

Alex Rogers, 30th July 2022

1 Overview

This document provides a template for you to use to write your reports and also an example of the description of a model for the Millikan oil drop experiment. The modelling approaches are described further in the two course books (Davidson-Pilon, 2015; Martin, 2018; Ramchurn et al., 2012).

2 Millikan's Oil Drop Experiment

The experimental set-up of Millikan's original oil drop experiment is shown in Figure 1.

2.1 Model Description

We assume that we have N measurements of the charge on N individual oil drops. We denote this measurement as c_i for $i \in \{0, \dots, N\}$. We assume that each measurement is sampled independent from a Gamma distribution such that:

$$c_i \sim \text{Gamma}(n_i \times e, \sigma^2) \quad (1)$$

where n_i is the number of additional electrons on each oil drop and e is the charge on an electron. We do not directly observe n_i and thus we assign it a Poisson prior given by:

$$n_i \sim 1 + \text{Poisson}(1) \quad (2)$$

Similarly, we model σ and e with appropriate priors such that:

$$\sigma \sim \text{Exponential}(10) \quad (3)$$

$$e \sim \text{Uniform}(1, 2) \quad (4)$$

2.2 Solving the Model

We solve the model using PyMC.¹ Figure 2 shows the posterior probability distribution for the charge on the electron and the accuracy of the experimental measurements.

References

Davidson-Pilon, Cameron (2015). *Bayesian Methods for Hackers: Probabilistic Programming and Bayesian Inference*. Addison-Wesley Professional.

Martin, Osvaldo (2018). *Bayesian Analysis with Python: Introduction to statistical modeling and probabilistic programming using PyMC3 and ArviZ*. Packt Publishing.

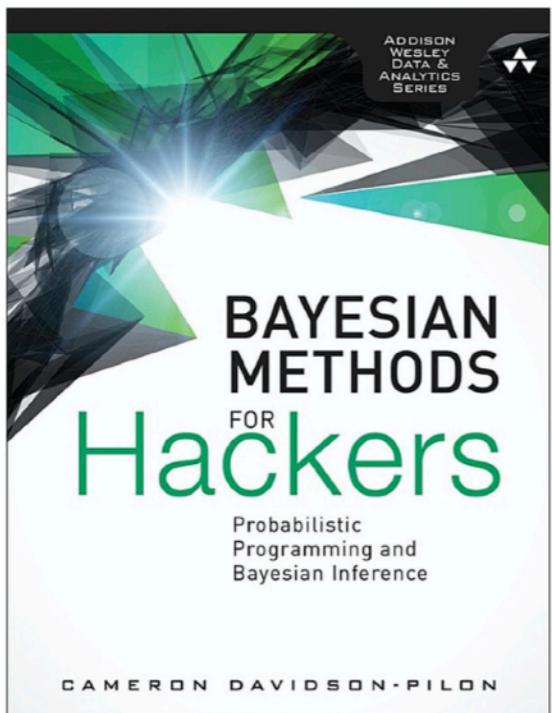
¹<https://docs.pymc.io>

Figure 1: Millikan's oil drop experiment.

Figure 2: Posterior probability density functions for the charge on the electron (e) and the measurement error (σ).

Take Home Quiz

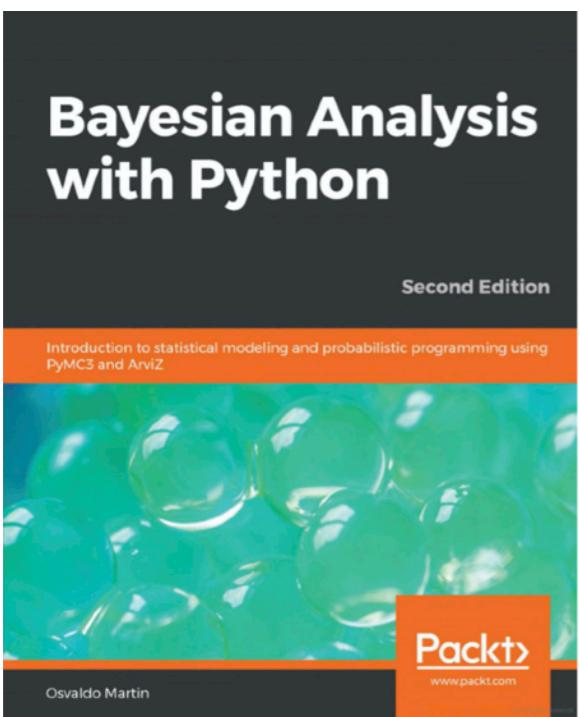
Follow On Work



Probabilistic Programming and Bayesian Methods for Hackers

Cameron Davidson-Pilon

[https://camdavidsonpilon.github.io/
Probabilistic-Programming-and-Bayesian-
Methods-for-Hackers/](https://camdavidsonpilon.github.io/Probabilistic-Programming-and-Bayesian-Methods-for-Hackers/)



Bayesian Analysis with Python

Osvaldo Martin

Questions