Формальные языки
Контрольная работа 1
16.10.2020

1. Привести три самых коротких различных строки, принадлежащих языку, описанному регулярным выражением; принадлежат ли строки $abbab$ и $bababa$ данному языку?

   1) $a((a \mid b)^*b)^*$
      - $a, ab, abb/aab,\ abbab$ — yes, $bababa$ — no.

   2) $(a(a \mid b)^*)^*b$
      - $b, ab, aab/abb,\ abbab$ — yes, $bababa$ — no.

   3) $(a \mid b)(a(a \mid b))^*(a \mid b)$
      - $aa, ab, ba, bb, aaa, aab, baa, bab,\ abbab$ — no, $bababa$ — yes.

   4) $(a \mid b)((a \mid b)b)^*(a \mid b)$
      - $aa, ab, ba, bb, aba, abb, bba, bbb,\ abbab$ — no, $bababa$ — yes.

   5) $(ba \mid b)^* \mid (bb \mid a)^*$
      - $\varepsilon, b, a,\ abbab$ — no, $bababa$ — yes.

   6) $(ab \mid b)^* \mid (bb \mid a)^*$
      - $\varepsilon, a, b,\ abbab$ — yes, $bababa$ — no.

   7) $(ba \mid a)^* \mid (bb \mid a)^*$
      - $\varepsilon, a, ba, bb, aa,\ abbab$ — no, $bababa$ — yes.

   8) $(ba \mid a)^* \mid (bb \mid b)^*$
      - $\varepsilon, a, b,\ abbab$ — no, $bababa$ — yes.

   9) $(a \mid b)^*b(a \mid \varepsilon)b(a \mid b)^*$
      - $bb, abb, bab, bba, bbb,\ abbab$ — yes, $bababa$ — yes.

   10) $(a \mid b)^*a(a \mid \varepsilon)b(a \mid b)^*$
      - $ab, aaa, aab, aba, abb, bab,,\ abbab$ — yes, $bababa$ — yes.

   11) $(a \mid b)^*b(a \mid \varepsilon)a(a \mid b)^*$
      - $ba, aba, baa, bab, bba,\ abbab$ — yes, $bababa$ — yes.

   12) $(a \mid b)^*a(a \mid \varepsilon)a(a \mid b)^*$
      - $aa, aaa, aab, baa,\ abbab$ — no, $bababa$ — no.

   13) $(a \mid b)^*b(b \mid \varepsilon)b(a \mid b)^*$
      - $bb, abb, bba, bbb,\ abbab$ — yes, $bababa$ — no.

   14) $(a \mid b)^*a(b \mid \varepsilon)b(a \mid b)^*$

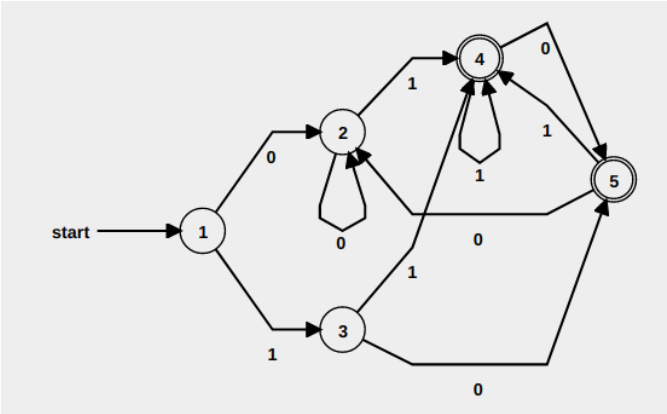
- $ab, aab, aba, abb, bab,\ abbab$ — yes, $bababa$ — yes.

15) $(a \mid b)^*b(b \mid \varepsilon)a(a \mid b)^*$
- $ba, aba, baa, bab, bba,\ abbab$ — yes, $bababa$ — yes.

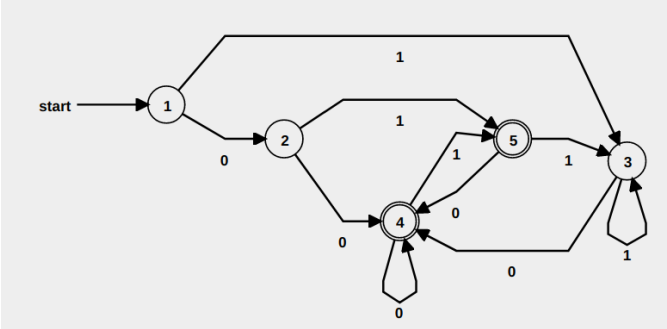
16) $(a \mid b)^*a(b \mid \varepsilon)a(a \mid b)^*$
- $aa, aaa, aab, aba, baa,\ abbab$ — no, $bababa$ — yes.

2. Построить минимальный детерминированный конечный автомат, распознающий язык:

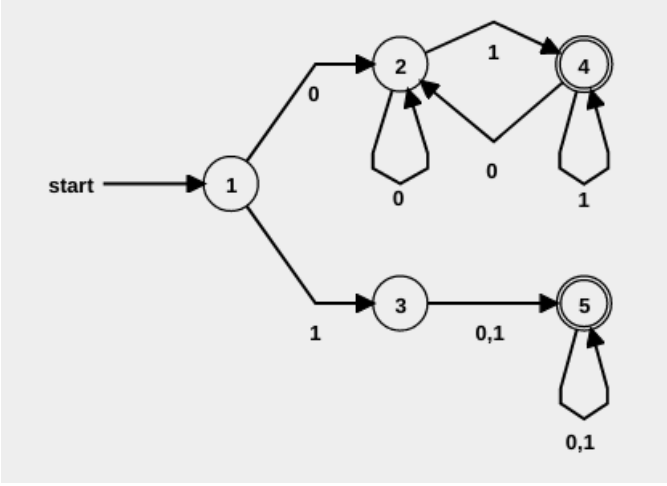
1) $\{\omega \cdot a \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a\ or\ b = 1\}$
- $(0 \mid 1)^*01 \mid (0 \mid 1)^*10 \mid (0 \mid 1)^*11$



2) $\{\omega \cdot a \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a\ and\ b = 0\}$
- $(0 \mid 1)^*01 \mid (0 \mid 1)^*10 \mid (0 \mid 1)^*00$

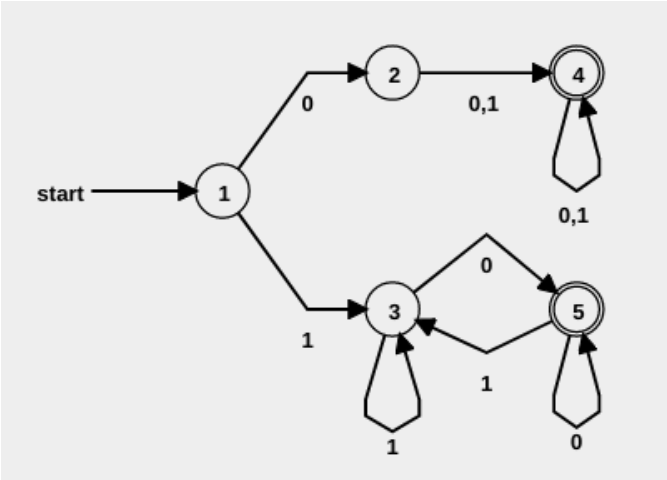


3) $\{a \cdot \omega \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a\ or\ b = 1\}$
- $0(0 \mid 1)^*1 \mid 1(0 \mid 1)^*0 \mid 1(0 \mid 1)^*1$



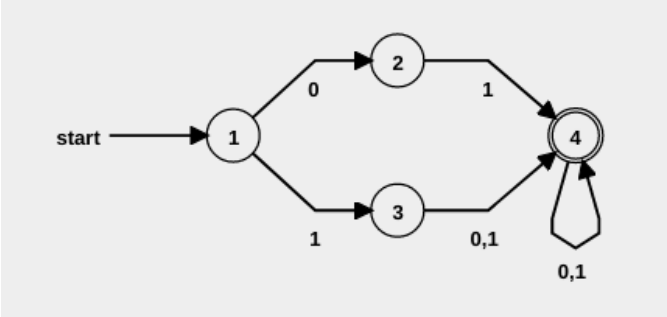
4) $\{a \cdot \omega \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a \; and \; b = 0\}$

- $0(0 \mid 1)^*1 \mid 1(0 \mid 1)^*0 \mid 0(0 \mid 1)^*0$



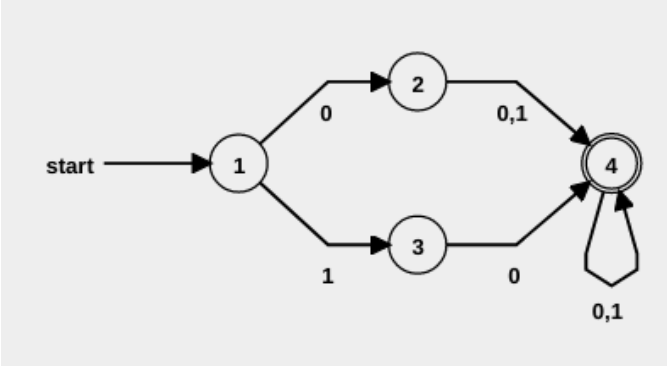
5) $\{a \cdot b \cdot \omega \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a \; or \; b = 1\}$

- $01(0 \mid 1)^* \mid 10(0 \mid 1)^* \mid 11(0 \mid 1)^*$



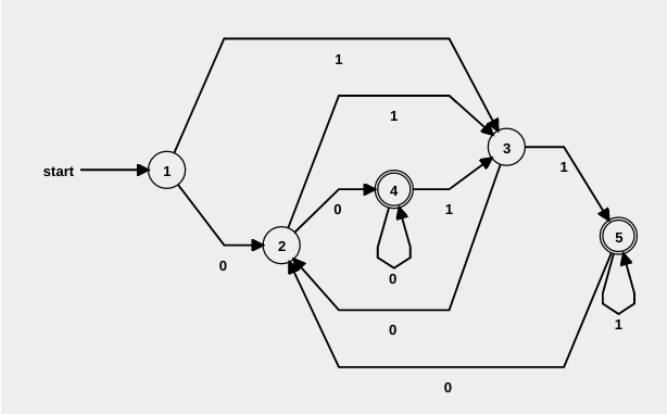
6) $\{a \cdot b \cdot \omega \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a \; and \; b = 0\}$

- $01(0 \mid 1)^* \mid 10(0 \mid 1)^* \mid 00(0 \mid 1)^*$

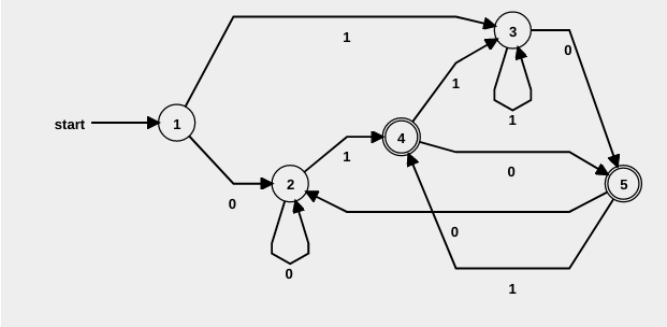


7) $\{\omega \cdot a \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a = b\}$

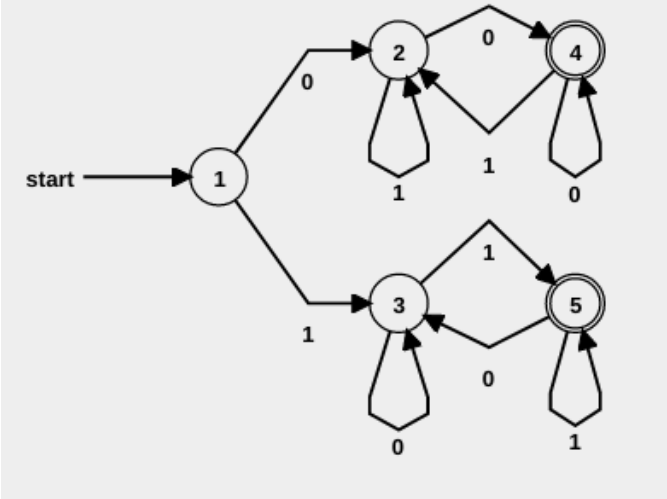
- $(0 \mid 1)^*00 \mid (0 \mid 1)^*11$



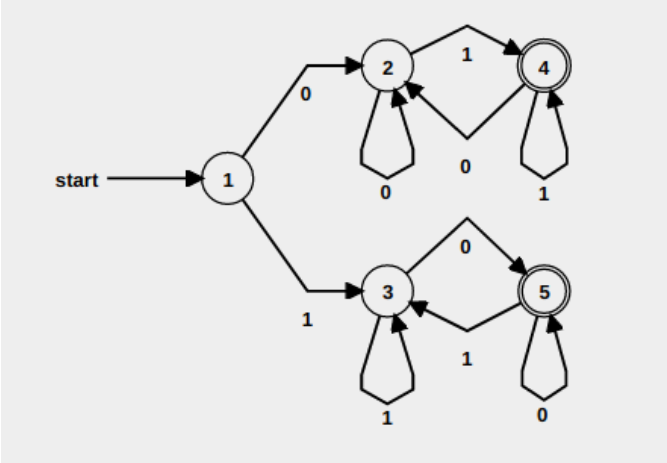
8) $\{\omega \cdot a \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a \neq b\}$
   - $(0 \mid 1)^*01 \mid (0 \mid 1)^*10$



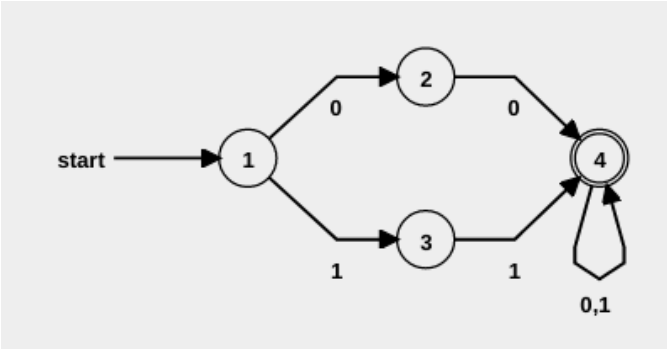
9) $\{a \cdot \omega \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a = b\}$
   - $0(0 \mid 1)^*0 \mid 1(0 \mid 1)^*1$



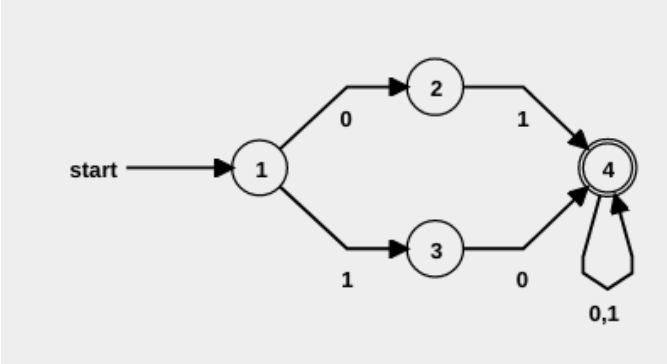
10) $\{a \cdot \omega \cdot b \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a \neq b\}$
   - $0(0 \mid 1)^*1 \mid 1(0 \mid 1)^*0$



11) $\{a \cdot b \cdot \omega \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a = b\}$
    - $00(0 \mid 1)^* \mid 11(0 \mid 1)^*$



12) $\{a \cdot b \cdot \omega \mid \omega \in \{0,1\}^*, a \in \{0,1\}, b \in \{0,1\}, a \neq b\}$
    - $01(0 \mid 1)^* \mid 10(0 \mid 1)^*$



3. Построить регулярную грамматику, задающую язык:

1) $\{\alpha \cdot 100 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 000 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 000 \xi 100 \xi \mid \xi 100 \xi 000 \xi \mid \xi 1000 \xi$

2) $\{\alpha \cdot 100 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 000 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 100 \xi \mid \xi 000 \xi$

3) $\{\alpha \cdot 001 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 000 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 001 \xi 000 \xi \mid \xi 000 \xi 001 \xi \mid \xi 0001 \xi$

4) $\{\alpha \cdot 001 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 000 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$

- $\xi 001 \xi \mid \xi 000 \xi$

5) $\{\alpha \cdot 010 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 000 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
   - $\xi 010 \xi 000 \xi \mid \xi 000 \xi 010 \xi \mid \xi 00010 \xi \mid \xi 01000$

6) $\{\alpha \cdot 010 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 000 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
   - $\xi 010 \xi \mid \xi 000 \xi$

7) $\{\alpha \cdot 001 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 100 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
   - $\xi 001 \xi 100 \xi \mid \xi 100 \xi 001 \xi \mid \xi 00100 \xi \mid \xi 1001 \xi \mid \xi 10001 \xi$

8) $\{\alpha \cdot 001 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 100 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
   - $\xi 001 \xi \mid \xi 100 \xi$

9) $\{\alpha \cdot 101 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 010 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
   - $\xi 101 \xi 010 \xi \mid \xi 010 \xi 101 \xi \mid \xi 0101 \xi \mid \xi 1010 \xi$

10) $\{\alpha \cdot 101 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 010 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 101 \xi \mid \xi 010 \xi$

11) $\{\alpha \cdot 011 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 111 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 011 \xi 111 \xi \mid \xi 111 \xi 011 \xi \mid \xi 0111 \xi$

12) $\{\alpha \cdot 011 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 111 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 011 \xi \mid \xi 111 \xi$

13) $\{\alpha \cdot 110 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 111 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 110 \xi 111 \xi \mid \xi 111 \xi 110 \xi \mid \xi 1110 \xi$

14) $\{\alpha \cdot 110 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 111 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 110 \xi \mid \xi 111 \xi$

15) $\{\alpha \cdot 101 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 111 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 101 \xi 111 \xi \mid \xi 111 \xi 101 \xi \mid \xi 11101 \xi \mid \xi 10111 \xi$

16) $\{\alpha \cdot 101 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 111 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 101 \xi \mid \xi 111 \xi$

17) $\{\alpha \cdot 110 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 011 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 110 \xi 011 \xi \mid \xi 011 \xi 110 \xi \mid \xi 01110 \xi \mid \xi 0110 \xi \mid \xi 11011 \xi$

18) $\{\alpha \cdot 110 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 011 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 110 \xi \mid \xi 011 \xi$

19) $\{\alpha \cdot 010 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cap \{\gamma \cdot 101 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 010 \xi 101 \xi \mid \xi 101 \xi 010 \xi \mid \xi 1010 \xi \mid \xi 0101 \xi$

20) $\{\alpha \cdot 010 \cdot \beta \mid \alpha, \beta \in \{0,1\}^*\} \cup \{\gamma \cdot 101 \cdot \delta \mid \gamma, \delta \in \{0,1\}^*\}$
    - $\xi 010 \xi \mid \xi 101 \xi$

4. Проверить регулярность языка (если регулярный, построить автомат, регулярное выражение или регулярную грамматику, иначе — доказать нерегулярность)

   1) $\{\omega \in \{a,b\}^* \mid |\omega|_a = |\omega|_b\}$

   2) $\{\omega \in \{a,b\}^* \mid |\omega|_a \geq |\omega|_b\}$

   3) $\{\omega \in \{a,b\}^* \mid |\omega|_a \leq |\omega|_b\}$

   4) $\{\omega \in \{a,b\}^* \mid |\omega|_a \neq |\omega|_b\}$

   5) $\{\alpha \cdot a \cdot \beta \mid \alpha, \beta \in \{a,b\}^*, |\alpha|_b \geq |\beta|_a\}$

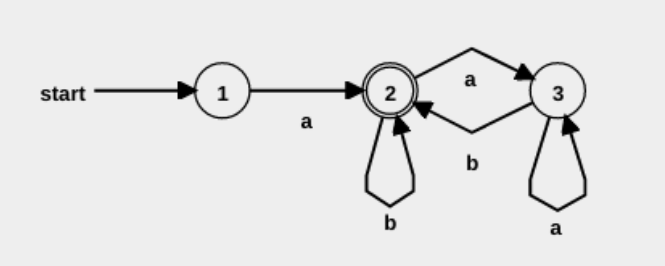
   6) $\{\alpha \cdot a \cdot \beta \mid \alpha, \beta \in \{a,b\}^*, |\alpha|_b > |\beta|_a\}$

   7) $\{a^m \cdot \omega \mid 1 \leq |\omega|_b \leq m\}$

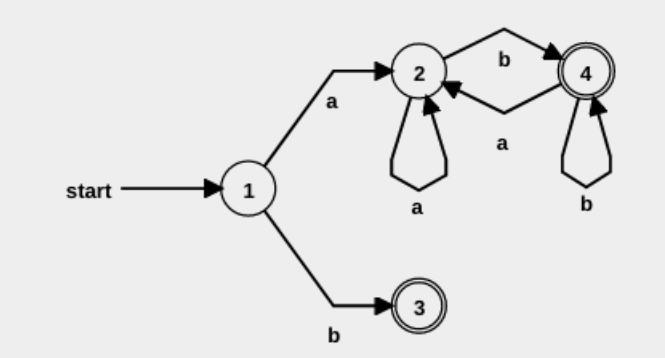
   8) $\{\omega \cdot a^m \mid 1 \leq |\omega|_b \leq m\}$

5. По регулярному выражению построить недетерминированный конечный автомат без эпсилон-переходов

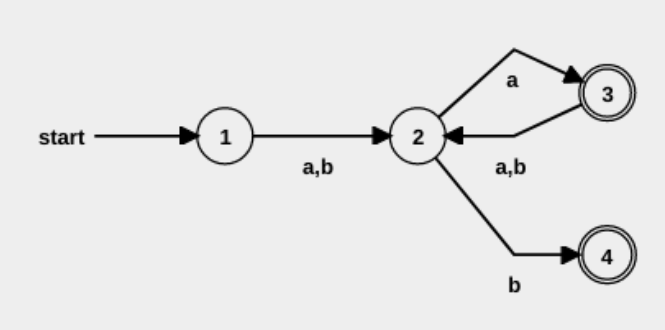
   1) $a((a \mid b)^* b)^*$



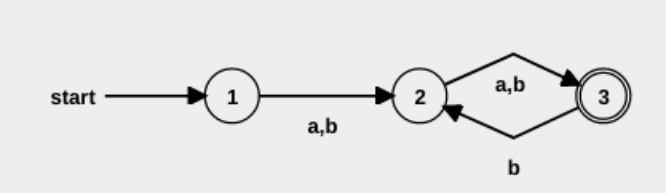
   2) $(a(a \mid b)^*)^* b$



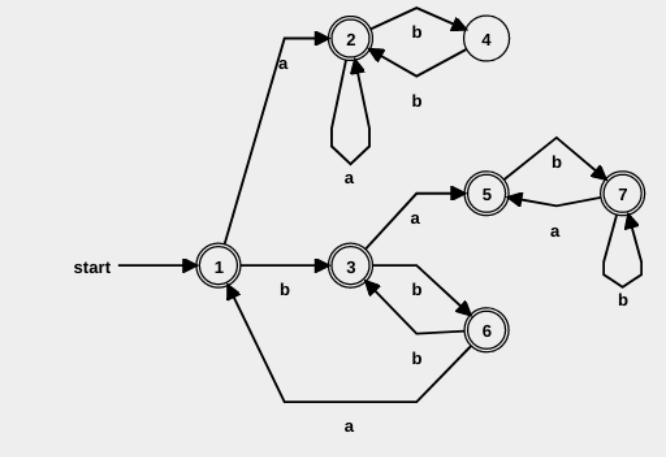
   3) $(a \mid b)(a(a \mid b))^*(a \mid b)$



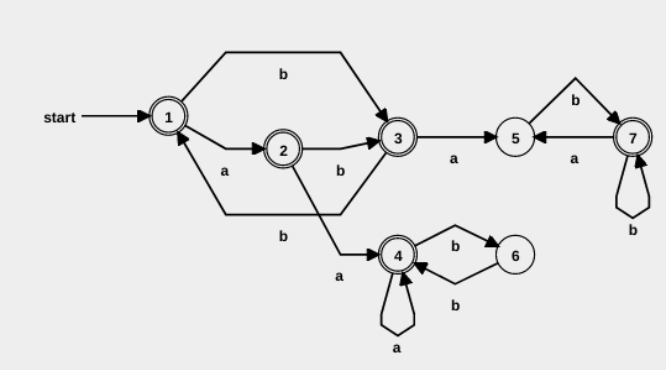
4) $(a \mid b)((a \mid b)b)^*(a \mid b)$



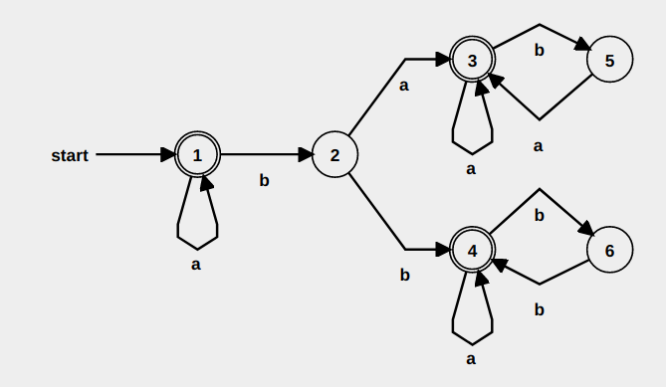
5) $(ba \mid b)^* \mid (bb \mid a)^*$



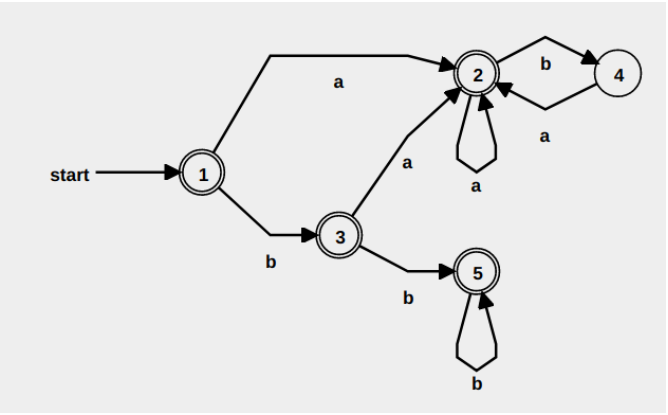
6) $(ab \mid b)^* \mid (bb \mid a)^*$



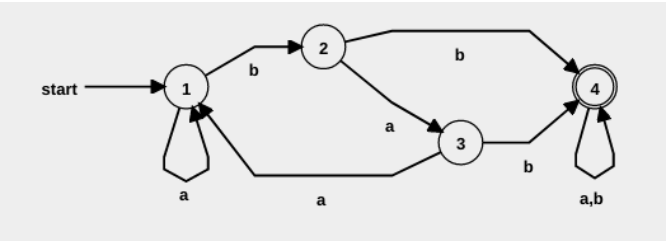
7) $(ba \mid a)^* \mid (bb \mid a)^*$



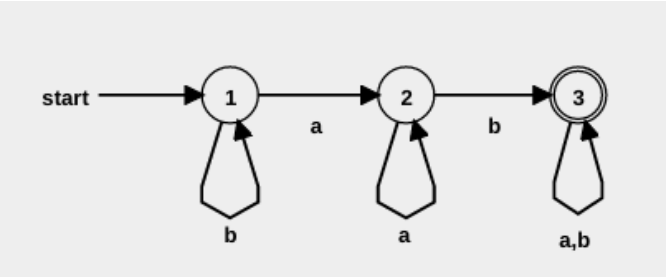
8) $(ba \mid a)^* \mid (bb \mid b)^*$



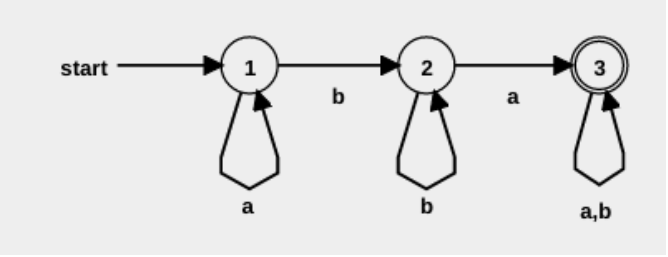
9) $(a \mid b)^* b (a \mid \varepsilon) b (a \mid b)^*$



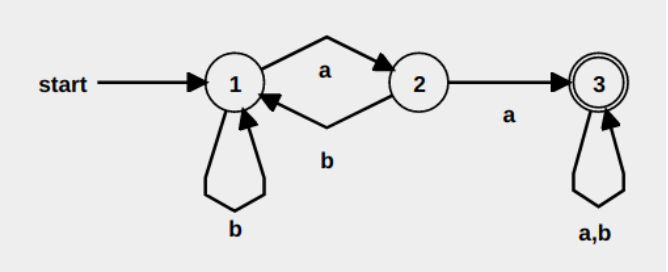
10) $(a \mid b)^* a (a \mid \varepsilon) b (a \mid b)^*$



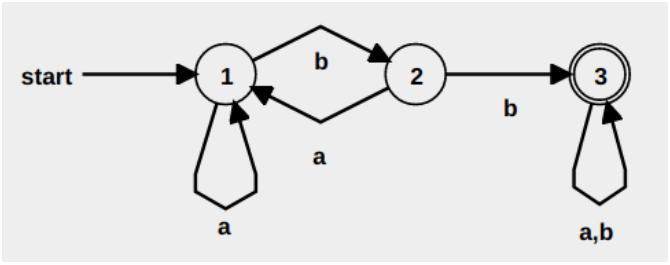
11) $(a \mid b)^* b (a \mid \varepsilon) a (a \mid b)^*$



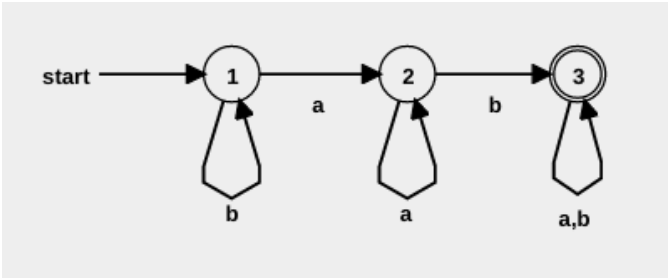
12) $(a \mid b)^* a (a \mid \varepsilon) a (a \mid b)^*$



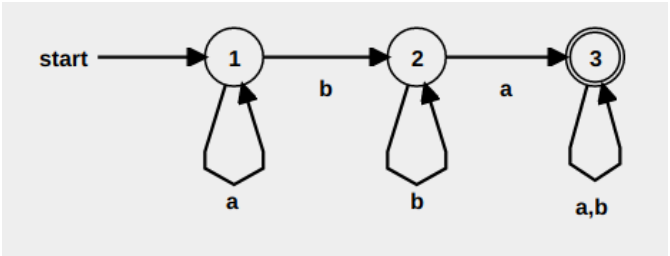
13) $(a \mid b)^* b (b \mid \varepsilon) b (a \mid b)^*$



14) $(a \mid b)^* a (b \mid \varepsilon) b (a \mid b)^*$



15) $(a \mid b)^* b (b \mid \varepsilon) a (a \mid b)^*$



16) $(a \mid b)^* a (b \mid \varepsilon) a (a \mid b)^*$