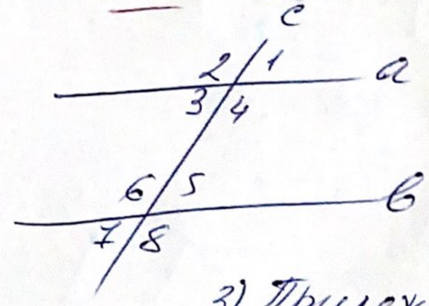
Геометрия

I Тема: Успоредни прави. Перпендикулярност. Еднакви триъгълници. Видове успоредници



Видове ъгли:

1) Кръстни: $(\sphericalangle 3, \sphericalangle 5)$; $(\sphericalangle 4, \sphericalangle 6)$; $(\sphericalangle 2; \sphericalangle 8)$ $(\sphericalangle 1; \sphericalangle 7)$
2) Съответни: $(\sphericalangle 1, \sphericalangle 5)$; $(\sphericalangle 4, \sphericalangle 8)$; $(\sphericalangle 2; \sphericalangle 6)$ $(\sphericalangle 3; \sphericalangle 7)$
3) Прилежащи ъгли: $(\sphericalangle 1; \sphericalangle 8)$; $(\sphericalangle 4, \sphericalangle 5)$; $(\sphericalangle 2; \sphericalangle 7)$; $(\sphericalangle 3, \sphericalangle 6)$

(T1) $a \parallel в \Rightarrow$ всички двойки кръстни ъгли са равни; всички двойки съответни ъгли са равни и всички двойки прилежащи имат сбор $180°$.

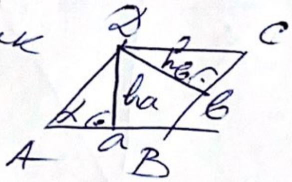
(T2) 1) Ако $\sphericalangle 1 = \sphericalangle 7 \Rightarrow a \parallel в$ (кр); 2) Ако $\sphericalangle 1 = \sphericalangle 5$ (съотв.) $\Rightarrow a \parallel в$;
3) ако $\sphericalangle 1 + \sphericalangle 8 = 180° \Rightarrow a \parallel в$.

Еднакви Δ. Признаци:

I пр. $\Delta ABC \cong \Delta A_1B_1C_1$, ако: 1) $AB = A_1B_1$; 2) $BC = B_1C_1$; 3) $\sphericalangle B = \sphericalangle B_1$
II пр. $\Delta ABC \cong \Delta A_1B_1C_1$, ако: 1) $AB = A_1B_1$; 2) $\sphericalangle A = \sphericalangle A_1$; 3) $\sphericalangle B = \sphericalangle B_1$
III пр. $\Delta ABC \cong \Delta A_1B_1C_1$, ако: 1) $AB = A_1B_1$; 2) $BC = B_1C_1$; 3) $CA = C_1A_1$
IV пр. $\Delta ABC \cong \Delta A_1B_1C_1$, ако: 1) $\sphericalangle C = \sphericalangle C_1 = 90°$; 2) $AB = A_1B_1$; 3) $AC = A_1C_1$

Успоредник:

def. $AB \parallel CD$ и $AD \parallel BC$, то $ABCD$ - успоредник



(T1) Ако $AB = CD$ и $AD = BC \Leftrightarrow ABCD$ - усп.
(T2) Ако $AB = CD$ и $AB \parallel CD \Leftrightarrow ABCD$ - усп.
(T3) $AC \cap BD = O$. $AO = OC$; $BO = OD \Leftrightarrow ABCD$ - усп.

$S = a h_a = в h_в = a в \sin\alpha = \frac{d_1 d_2 \sin\varphi}{2}$; $P = 2(a+в)$

(T4) $\boxed{\sphericalangle A = \sphericalangle C \quad \sphericalangle B = \sphericalangle D}$

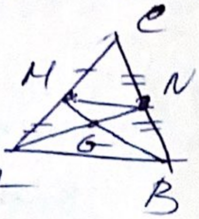
Ромб: успоредник с равни съседни страни.
(T1) $ABCD$ - ромб $\Leftrightarrow ABCD$ - усп. $AC \perp BD$
(T2) $ABCD$ - ромб $\Leftrightarrow ABCD$ - усп. AC - ъглопол. на $\sphericalangle BAD$

Правоъгълник: успоредник с прав ъгъл
(T1) $ABCD$ - правоъг. $\Leftrightarrow ABCD$ - усп. и $AC = BD$

Квадрат: Ромб + правоъгълник

$S_{ABC} = \dfrac{a h_a}{2} = \dfrac{ab\sin\gamma}{2} = p \cdot r = \sqrt{p(p-a)(p-b)(p-c)} = \dfrac{abc}{4R}$

Средна отсечка в $\triangle$:



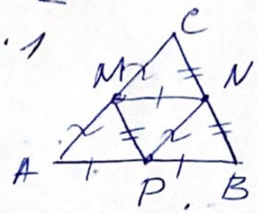
M - среда на AC

N - среда на BC

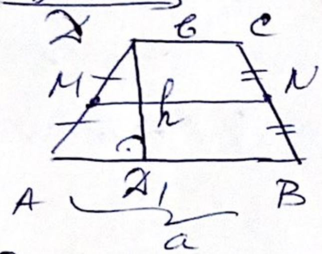
$\Rightarrow MN$ - ср. отс. $\Rightarrow MN \parallel AB;\ NM = \dfrac{AB}{2}$

$AN \cap BM = G$ - медицентър $\Rightarrow AG:GN = 2:1$

$S_{ABG} = S_{ACG} = S_{BCG} = \dfrac{S_{ABC}}{3};\ S_{MNP} = \dfrac{S_{ABC}}{4}$



Трапец: $AB \parallel CD$, $AB > CD$: $S = \dfrac{a+b}{2} \cdot h$

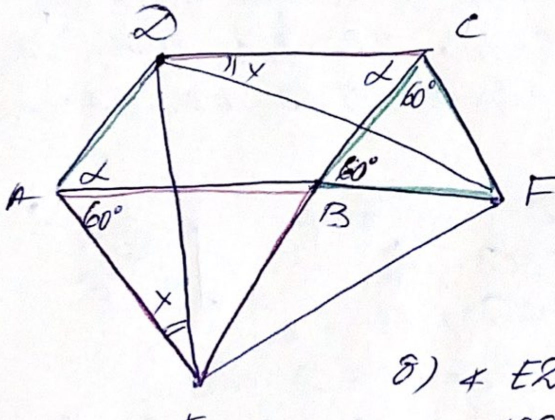


MN - ср. основа (отсечка)

$\Rightarrow MN = \dfrac{a+b}{2};\ MN \parallel AB \parallel CD$

Задачи:

① ABCD - успоредник. $\triangle ABE$ и $\triangle BCF$ равностранни.
Да се докаже, че: а) $\triangle ADE \cong \triangle CFD$; б) $\triangle FED$ - равностр.



Док.:

а) Разгл. $\triangle ADE$ и $\triangle CFD$

1) $AD = CF$

2) $AE = DC$

3) $\sphericalangle DAE = \sphericalangle DCF = 60° + \alpha$

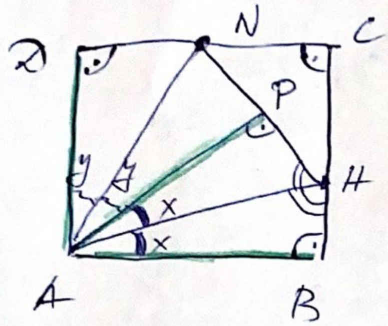
$\Rightarrow \triangle ADE \cong \triangle CFD$ (I пр.)

$\Rightarrow DE = CF$ (СЕЕТ) $\Rightarrow \triangle FED$ - равнобедр.

$\Rightarrow \sphericalangle AED = \sphericalangle CDF = x$

б) $\sphericalangle EDF = \sphericalangle ADC - \sphericalangle ADE - \sphericalangle CDF =$
$= 180° - \alpha - (180° - \alpha - x - 60°) - x =$
$= 180° - \alpha - 180° + \alpha + x + 60° - x = 60°$
$\Rightarrow \triangle DEF$ - равностранен

② ABCD-квадрат. $H \in BC$, $N \in DC$, $\sphericalangle AHB = \sphericalangle AHN$.
$\sphericalangle HAN = ?$



Решение:

Построяваме т. P: 1) $P \in HN$;
2) $AP \perp HN$

Разгл. $\triangle ABH$ и $\triangle APH$

1) AH-обща; 2) $\sphericalangle AHB = \sphericalangle AHP$ (по усл.)

3) $\sphericalangle BAH = \sphericalangle PAH = 90° - \sphericalangle AHB = 90° - \sphericalangle AHP =$
$= x$

$\Rightarrow \triangle AHB \cong \triangle AHP$ (II пр.)

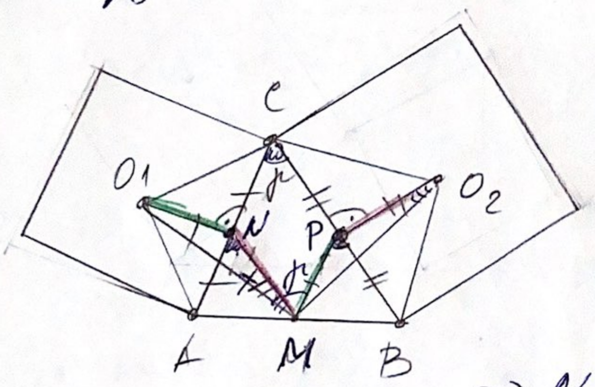
$\Rightarrow AB = AP = AD$

Разгл. $\triangle APN$ и $\triangle ADN$: 1) $AP = AD$; 2) AN-обща;
3) $\sphericalangle APN = \sphericalangle ADN = 90° \Rightarrow \triangle APN \cong \triangle ADN$ (IV пр)

$\Rightarrow \sphericalangle PAN = \sphericalangle DAN = y$

$\Rightarrow 2x + 2y = \sphericalangle BAD = 90° \;/:2 \Rightarrow x + y = \underline{\underline{\sphericalangle HAN = 45°}}$

③ Върху страните AC и BC на $\triangle ABC$ външно са построени квадрати с центрове O_1 и O_2. Точка M е среда на AB. Да се намерят $\sphericalangle O_2O_1M$ и $\sphericalangle O_1MO_2$.



Решение:

Нека N и P са среди на AC и BC. $\Rightarrow$ NP-ср. отс. $\Rightarrow NP =$
$= CP = PB = PO_2 = \frac{CB}{2}$

MP-ср. отс. $\Rightarrow MP = CN = NA =$
$= O_1N = \frac{AC}{2}$

$\Rightarrow$ NMPC-успоредник $\Rightarrow \sphericalangle ANM = \sphericalangle MPB =$
$= \sphericalangle ACB = \mu$

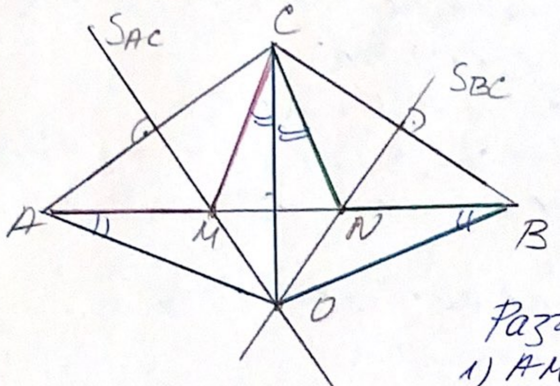
Разгл. $\triangle O_1NM$ и $\triangle O_2PM$

1) $O_1N = PM$
2) $NM = PO_2$
3) $\sphericalangle O_1NM = \sphericalangle O_2PM = 90° + \mu$

$\Big\} \Rightarrow \triangle O_1NM \cong \triangle O_2PM$ (I пр.)
$\Rightarrow O_1M = O_2M$
$\Rightarrow \triangle O_1O_2M$-равнобедрен

$\sphericalangle O_1MN = \sphericalangle MO_2P = x \Rightarrow \sphericalangle O_1MO_2 = \sphericalangle O_1MN + \sphericalangle NMP +$
$+ \sphericalangle PMO_2 = x + \mu + (180° - 90° - \mu - x) = 90°$
$\Rightarrow \sphericalangle O_2O_1M = 45°$

(4) $\triangle ABC: \measuredangle C > 90°$. $S_{AC} \cap AB = M$, $S_{BC} \cap AB = N$.
$S_{AC} \cap S_{BC} = O$. $\measuredangle MCN = 46°$. Да се намери $\measuredangle OBA$.



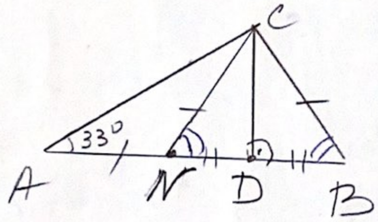
Решение:

$M \in S_{AC} \Rightarrow AM = MC$
$N \in S_{BC} \Rightarrow CN = NB$
$O \in S_{AC} \Rightarrow OA = OC$
$O \in S_{BC} \Rightarrow OC = OB = OA$
$\Rightarrow \measuredangle OAB = \measuredangle OBA$

Разгл. $\triangle AMO$ и $\triangle CMO$
1) $AM = MC$; 2) $AO = OC$; 3) MO – обща
$\Rightarrow \triangle AMO \cong \triangle CMO \Rightarrow \measuredangle MAO = \measuredangle MCO$
Разгл. $\triangle NCO$ и $\triangle NBO$: 1) $CN = NB$; 2) $CO = OB$; 3) NO – обща
$\Rightarrow \triangle CNO \cong \triangle BNO$ (III пр.) $\Rightarrow \measuredangle OBN = \measuredangle OCN = \measuredangle OAM = \measuredangle MCO =$
$= \dfrac{\measuredangle CMN}{2} = \underline{\underline{23°}}$

Следствие: Пресечната точка на симетралите на страните на $\triangle$ е равноотдалечена от върховете на $\triangle$, т.е. е център на описаната около $\triangle$ окръжност.

(5) $\triangle ABC: \measuredangle A = 33°$; $CD \perp AB$, $D \in AB$ и $AD - BD = BC$.
Да се намери $\measuredangle ABC$.



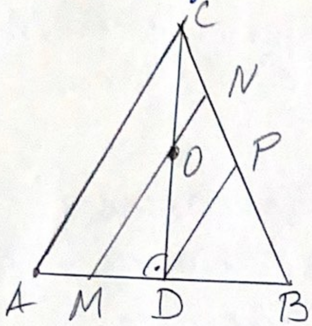
Решение:

$AD - BD = BC \Rightarrow AD = BC + BD$

$N: \begin{cases} N \in AB \\ ND = DB \end{cases} \Rightarrow CD$ – височина и медиана в $\triangle NBC$

$\Rightarrow CN = CB \Rightarrow AD = AN + ND =$
$= AN + DB \Rightarrow AN = CB = CN$
$\Rightarrow \triangle ANC$ – равнобедрен $\Rightarrow \measuredangle NAC = \measuredangle ACN = 33°$
$\Rightarrow \measuredangle CND = \measuredangle NBC = 2.33° = \underline{\underline{66°}}$

(6) Бедрото на равнобедрен $\triangle$ е 12 см. През средата на височината към основата е построена права, успоредна на бедрото. Да се намери частта от тази права, която е в триъгълника.



Решение:

Нека $CD \perp AB$ $(D \in AB)$

$AC = BC = 12$ см $\Rightarrow AD = DB = \frac{AB}{2}$.

O-среда на CD, $OM \parallel AC$

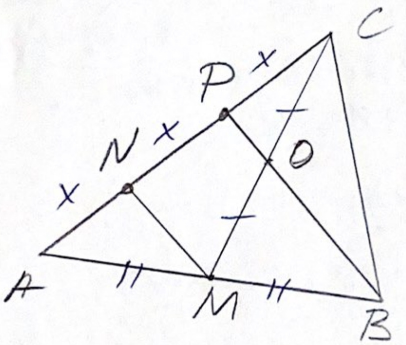
$\Rightarrow M$-среда на $AD \Rightarrow MO$-ср. отс. в $\triangle ADC \Rightarrow MO = \frac{AC}{2} = 6$ см.

Построяваме $DP \parallel MN \parallel AC$. O-среда на CD

$ON \parallel DP \Rightarrow ON$-ср. отс. в $\triangle CDP \Rightarrow ON = \frac{1}{2} DP$.

Но DP-ср. отс. в $\triangle ABC \Rightarrow DP = \frac{AC}{2} = 6$ см $\Rightarrow ON = \frac{DP}{2} =$
$= 3$ см $\Rightarrow MN = MO + ON = 6 + 3 = 9$ см

(7) Даден е $\triangle ABC$, M-среда на AB. т. O-среда на CM. $BO \cap AC = P$. $AC = 8{,}7$ см. Намерете AP и PC.



Решение:

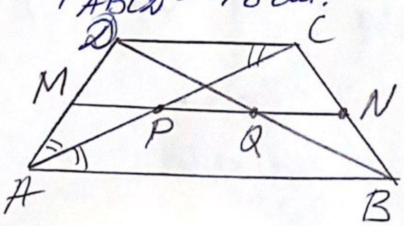
Построяваме т. N: $\begin{cases} N \in AC \\ MN \parallel BP \end{cases}$

Разгл. $\triangle ABP$: M-среда на AB, $MN \parallel BP \Rightarrow N$-среда на AP (MN-ср. отс.) $\Rightarrow AN = NP = x$

Разгл. $\triangle NMC$: O-среда на CM
$PO \parallel NM \Rightarrow PO$-ср. отс. $\Rightarrow NP = PC = x$

$\Rightarrow AC = 3x = 8{,}7$ см $\Rightarrow x = 2{,}9$ см $\Rightarrow AP = 2x = 5{,}8$ см
$PC = x = 2{,}9$ см

(8) $ABCD$-равнобедрен трапец. AC-ъглополовяща на $\measuredangle BAD$. P-среда на AC, Q-среда на BD. $PQ = 4$ см. $P_{ABCD} = 48$ см. Да се намерят страните на $ABCD$.



Решение:

$ABCD$-равнобедрен $\Rightarrow AD = BC$.

AC-ъглополовяща на $\measuredangle BAD$
$\Rightarrow \measuredangle BAC = \measuredangle CAD = x$. Но $\measuredangle DCA =$
$= \measuredangle CAB = x$ (кръстни) $\Rightarrow \measuredangle DAC = \measuredangle DCA$
$\Rightarrow AD = DC = CB = b$. Нека $AB = a$.

M-среда на AD, N-среда на BC.

$\Rightarrow MP$-ср.отс. в $\triangle DCA \Rightarrow MP = \frac{DC}{2} = \frac{b}{2}$

MQ-ср.отс. в $\triangle ABD \Rightarrow MQ = \frac{AB}{2} = \frac{a}{2} \Rightarrow PQ = MQ - MP =$

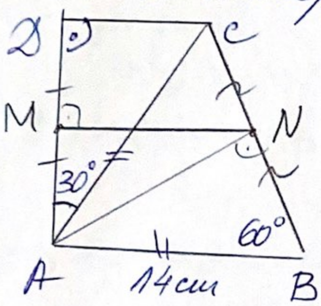
$= \frac{a-b}{2} = 4$ cm; $P_{ABCD} = 2AD + DC + AB = 3b + a = 48$ cm

$\Rightarrow \left| \begin{array}{l} a - b = 8 \mid \cdot(-1) \\ 3b + a = 48 \end{array} \right. (+) \Rightarrow 3b + b = 40 \Rightarrow b = 10$ cm

$\Rightarrow a = b + 8 = 18$ cm

$\Rightarrow AD = DC = CB = \underline{10\text{ cm}}$, $AB = 18$ cm

(9) Единият диагонал и едната основа на правоъгълен трапец са по 14 см, а единият от ъглите му е 120°. Да се намери средната основа на трапеца.



Решение:

Нека $\sphericalangle A = \sphericalangle D = 90° \Rightarrow \sphericalangle BCD = 120°$

$\Rightarrow \sphericalangle CBA = 60°$ (прилежащи)

DB-хипотенуза в $\triangle ADB$

$\Rightarrow DB > AB > CD$

$\Rightarrow AC = 14$ cm и $AB = 14$ cm

$\triangle ABC: AC = AB$ и $\sphericalangle B = 60° \Rightarrow \triangle ABC$-равностр.

$\Rightarrow \sphericalangle BAC = 60°$

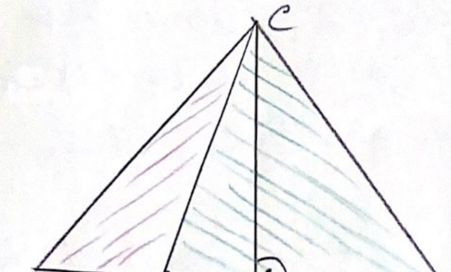
$\Rightarrow \sphericalangle BAD = \sphericalangle BAC + \sphericalangle CAD \Rightarrow 90° = 60° + \sphericalangle CAD \Rightarrow \sphericalangle CAD = 30°$

$\Rightarrow DC = \frac{AC}{2}$ (правоъг. △ с 30°) $\Rightarrow DC = 7$ cm

$\Rightarrow MN = \frac{AB+CD}{2} = \frac{14+7}{2} = \underline{10,5\text{ cm}}$

<u>Помощна задача:</u> $N \in AB$, $AN : NB = 2 : 7$.

$\frac{S_{ANC}}{S_{BNC}} = ?$



Решение: $CH \perp AB$, $H \in AB$

$S_{ANC} = \frac{AN.CH}{2}$; $S_{NBC} = \frac{NB.CH}{2}$

$\Rightarrow S_{ANC} : S_{BNC} = \frac{AN.CH}{2} : \frac{BN.CH}{2} =$

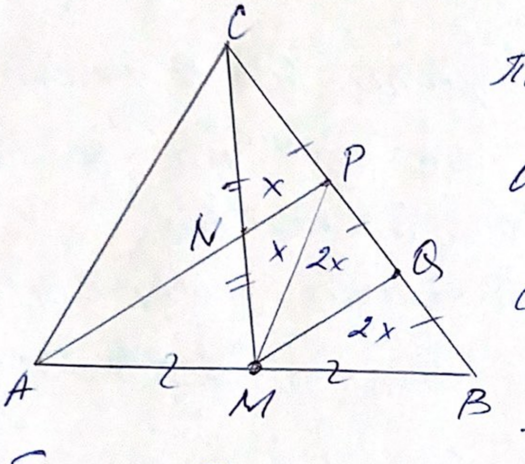
$= \frac{AN.\cancel{CH}}{\cancel{2}} \cdot \frac{\cancel{2}}{BN.\cancel{CH}} = AN : BN = 2 : 7$

$\Rightarrow \frac{S_{ANC}}{S_{BNC}} = \frac{AN}{NB}$

<u>Следствие:</u> Медианата разделя △ на две равнолицеви части.

(10) Точка M - среда на AB в $\triangle ABC$, а т. N е среда на CM. $CB \cap AN = P$. Да се намери $\frac{S_{BMNP}}{S_{ABC}} =$

Решение:



Построяване т. Q: $\begin{cases} Q \in BC \\ MQ \parallel AP \end{cases}$

От зад. (7) $\Rightarrow CP = PQ = QB = \frac{CB}{3}$

$CN = NM \Rightarrow S_{CNP} = S_{NPM} = x$

$CP = PQ \Rightarrow S_{CPM} = S_{MPQ} = 2x$

$PQ = QB \Rightarrow S_{PQM} = S_{MQB} = 2x$

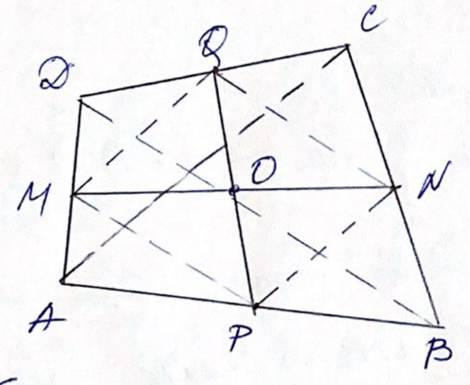
$\Rightarrow S_{BMNQ} = 5x$

$S_{CMB} = 6x = \frac{1}{2} S_{ABC}$ ($AM = MB$, CM-медиана)

$\Rightarrow S_{ABC} = 12x \Rightarrow \frac{S_{BMNP}}{S_{ABC}} = \frac{5\cancel{x}}{12\cancel{x}} = \frac{5}{12}$

(11) Да се намери лицето на четириъгълник, единият диагонал на който е 30см, а отсечките, съединяващи средите на двойките срещуположни страни, са 28 см и 26см.

Решение:



QN-ср.отс. в $\triangle DBC \Rightarrow QN = \frac{DB}{2}$ и $QN \parallel DB$

MP-ср.отс. в $\triangle ABD \Rightarrow MP = \frac{DB}{2}$ и $MP \parallel DB \Rightarrow$ PNQM-успоредник

Нека $AC = 30$см, $MN = 28$см, $PQ = 26$см.

Нека $S_{ABCD} = S$.

$S_{PAM} + S_{QCN} = \frac{1}{4} S_{ABD} + \frac{S_{BDC}}{4} = \frac{S_{ABD} + S_{BDC}}{4} = \frac{1}{4} S$

$S_{MDQ} + S_{BPN} = \frac{S_{DAC} + S_{BAC}}{4} = \frac{S}{4} \Rightarrow S_{PAM} + S_{BPN} + S_{QCN} + S_{MDQ} =$

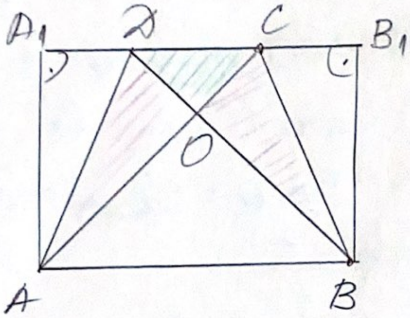
$= \frac{S}{4} + \frac{S}{4} = \frac{S}{2} \Rightarrow S_{PMQN} = \frac{S}{2}$. Но PMQN-усп.

$\Rightarrow S_{PON} = \frac{S_{MQNP}}{4}$. $PO = \frac{PQ}{2} = 13$см; $ON = \frac{MN}{2} = 14$см

$PN = \frac{AC}{2} = 15$см

$\Rightarrow S_{PON} = \sqrt{p(p-a)(p-b)(p-c)} = \sqrt{21(21-15)(21-14)(21-13)} =$
$= 84\,cm^2 \Rightarrow S_{PMQN} = 4.84 = 336\,cm^2 \Rightarrow S_{ABCD} = 2.S_{PMQN} =$
$= \underline{672\,cm^2}$ ($\Rightarrow$ стр. 13 - зад. 24)

(12) Нека т. $O = AC \cap BD$, $ABCD$-трапец. Да се докаже:
а) $S_{ADO} = S_{BCO}$; б) $S_{AOB} \cdot S_{DOC} = S_{ADO} \cdot S_{BCO} = S_{ADO}^2$.



Доказателство:

а) $ABCD$-трапец $\Rightarrow AA_1 = BB_1 = h$

$\Rightarrow S_{ACD} = S_{BCD} = \dfrac{DC.AA_1}{2} = \dfrac{DC.BB_1}{2}$

$S_{ACD} = S_{BCD} \;/ - S_{DOC}$ (обща част)

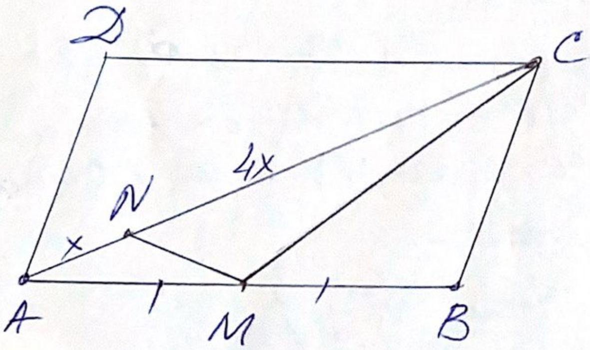
$\Rightarrow S_{ACD} - S_{DOC} = S_{BCD} - S_{DOC}$

$\Rightarrow S_{AOD} = S_{BOC}$.

б) $\dfrac{S_{AOB}}{S_{AOD}} = \dfrac{OB}{OD} = \dfrac{S_{BOC}}{S_{DOC}} \Rightarrow \underline{S_{AOB} \cdot S_{DOC} = S_{AOD} \cdot S_{BCO} = (S_{ADO})^2}$

(13) $ABCD$-успоредник. M-среда на AB; $N \in AC$ и $AN = \frac{1}{5}AC$. Колко процента от лицето на $ABCD$ е лицето на $\triangle AMN$?

Решение:



$ABCD$-усп. $\Rightarrow \triangle ABC \cong \triangle ADC$ (III пр.)

$\Rightarrow S_{ABC} = S_{ACD} = \dfrac{S_{ABCD}}{2}$

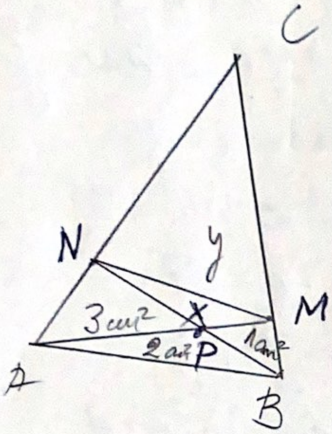
$AM = MB \Rightarrow S_{AMC} = S_{MBC} =$
$= \dfrac{S_{ABC}}{2} = \dfrac{S_{ABCD}}{4}$

$AN : AC = 1:5 \Rightarrow S_{ANM} = \dfrac{1}{5} \cdot S_{AMC}$

$\Rightarrow S_{ANM} = \dfrac{1}{4} \cdot \dfrac{1}{5} \cdot S_{ABCD} = \dfrac{1}{20} S_{ABCD} = \underline{5\% \cdot S_{ABCD}}$

(14) Върху BC и AC на $\triangle ABC$ са взети съответно т. M и N. $AM \cap BN = P$. $S_{BMP} = 1\,cm^2$; $S_{ABP} = 2\,cm^2$; $S_{ANP} = 3\,cm^2$
$S_{ABC} = ?$

Решение:



Нека $S_{CMN} = y$; $S_{PMN} = x$

$$\frac{S_{APN}}{S_{NPM}} = \frac{AP}{PM} \Rightarrow \frac{3}{x} = \frac{AP}{PM}$$

Но $\frac{S_{ABP}}{S_{BPM}} = \frac{AP}{PM} \Rightarrow \frac{2}{1} = \frac{AP}{PM}$

$$\Rightarrow \frac{3}{x} = \frac{2}{1} \Rightarrow x = 1{,}5\,cm^2$$

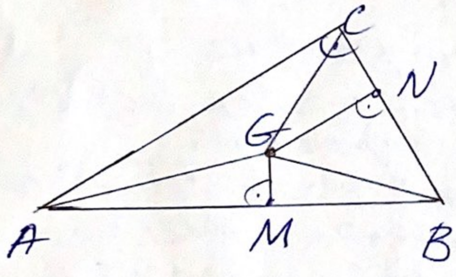
$\frac{S_{CMN}}{S_{ANM}} = \frac{NC}{AN} \Rightarrow \frac{y}{4{,}5} = \frac{NC}{AN}$. Но $\frac{S_{CNB}}{S_{ANB}} = \frac{CN}{AN}$

$$\Rightarrow \frac{y+2{,}5}{5} = \frac{CN}{AN} \Rightarrow \frac{y}{4{,}5} = \frac{y+2{,}5}{5} \Rightarrow y = 22{,}5\,cm^2$$

$$\Rightarrow S_{ABC} = x + y + 3 + 2 + 1 = \underline{30\,cm^2}$$

(15) $\triangle ABC$: $\sphericalangle C = 90^\circ$; G-медицентър $|G; AB| : |G; BC| = 1:2$
$\sphericalangle ABC = ?$

Решение:



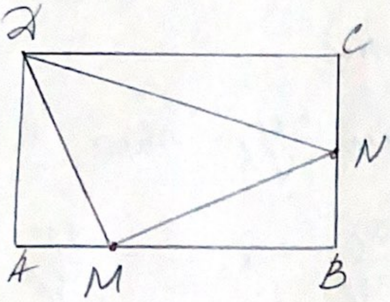
G-медицентър $\Rightarrow S_{AGB} = S_{GBC} = \frac{S_{ABC}}{3}$. $GM \perp AB$; $GN \perp BC$
$\Rightarrow GM : GN = 1:2$

$$S_{AGB} = \frac{AB.GM}{2}; \quad S_{GBC} = \frac{BC.GN}{2}$$

Нека $GM = x \Rightarrow GN = 2x$ $\Rightarrow S_{AGB} = \frac{AB.x}{2} = \frac{BC.2x}{2} = S_{GBC}$

$$\Rightarrow \frac{AB}{BC} = 2, \text{ т.е. } AB = 2BC \Rightarrow \sphericalangle CAB = 30^\circ \Rightarrow \underline{\sphericalangle ABC = 60^\circ}$$

(16) Даден е правоъгълник ABCD. $M \in AB$, $N \in BC$. $S_{AMD} = 10$, $S_{MBN} = 28$, $S_{NCD} = 12$. $S_{MND} = ?$ (M2-2020г)



Решение:

Нека $AB = a$; $AD = b \Rightarrow S_{ABCD} = ab$

$S_{AMD} = 10 = \dfrac{AM.b}{2} \Rightarrow AM = \dfrac{20}{b}$

$MB = AB - AM = a - \dfrac{20}{b}$

$S_{NCD} = \dfrac{NC.a}{2} \Rightarrow NC = \dfrac{24}{a}$

$NB = BC - NC = b - \dfrac{24}{a}$

$S_{MBN} = 28 = \dfrac{MB.BN}{2} \Rightarrow 56 = \left(a - \dfrac{20}{b}\right).\left(b - \dfrac{24}{a}\right)$

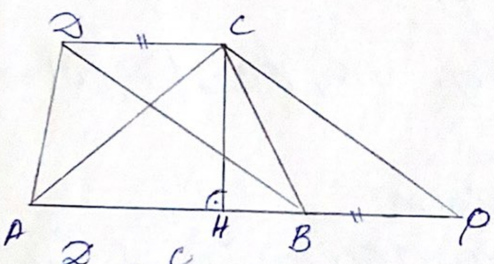
$\Rightarrow \dfrac{ab-20}{b} . \dfrac{ab-24}{a} = 56 \Rightarrow (ab-20)(ab-24) = 56ab$

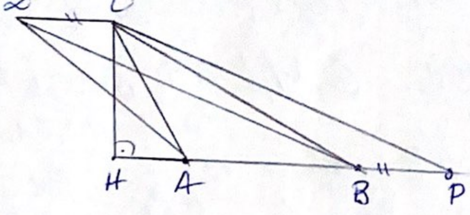
$\Rightarrow (ab)^2 - 100ab + 480 = 0 \Rightarrow ab = 50 \pm \sqrt{2020}$. Но $S_{ABCD} = ab >$

$> S_{AMD} + S_{BNM} + S_{NCD} = 50 \Rightarrow S_{ABCD} = 50 + \sqrt{2020}$

$\Rightarrow S_{MND} = ab - (S_{AMD} + S_{BNM} + S_{DNC}) = \sqrt{2020}$

(17) Да се пресметне лицето на трапеца ABCD ($AB \parallel CD$), ако $AC = 13$, $BD = 20$ и $h = 12$. (M2-2019г)





Решение:

$P \in AB$: $CP \parallel DB \Rightarrow BPCD$ - успоредник

$\Rightarrow \begin{cases} BP = DC \\ DB = CP \end{cases}$ Нека $CH \perp AB$, $H \in AB$ $\Rightarrow CH = 12$.

$\triangle ACH$ - Питагорова т-ма

$AH^2 + HC^2 = AC^2 \Rightarrow AH = \sqrt{AC^2 - HC^2} = 5$

$\triangle HPC$ - Питагорова т-ма

$\Rightarrow HP^2 + HC^2 = CP^2 \Rightarrow HP = \sqrt{CP^2 - CH^2} = 16$

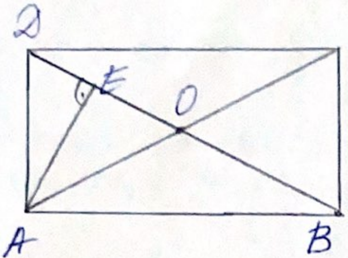
$\Rightarrow AP = AH + HP = 5 + 16 = 21$, ако

I) $\angle PAC < 90° \Rightarrow S_{ABCD} = \dfrac{AB+CD}{2}.h =$

$= \dfrac{AB+BP}{2}.h = \dfrac{AP}{2}.h = 6AP = 126$

II) Нека $\angle PAC > 90° \Rightarrow AP = HP - AH = 16 - 5 = 11 \Rightarrow S_{ABCD} = 6AP = 66$

(18) ABCD-правоъгълник. $AE \perp BD$, $E \in BD$, $BE = 3DE$, $AD = 6\,cm$
$BD = ?$, $S_{ABCD} = ?$



Решение:

$AE \perp DB$. Нека $DE = x$; $BE = 3x$

$O = AC \cap BD \Rightarrow AO = OB = OC = OD = 2x$

Разгл. $\triangle DAO$: AE - височина, но $DE = EO = x$

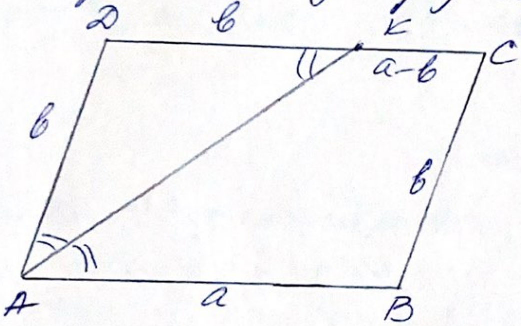
$\Rightarrow AE$ - медиана $\Rightarrow AD = AO$. Но $DO = AO$

$\Rightarrow \triangle AOD$ - равностранен $\Rightarrow AO \cdot 2 = AC = BD =$

$= 2AD = 12\,cm$. $\sphericalangle AOD = 60^\circ$

$\Rightarrow S_{ABCD} = \frac{AC.BD\sin 60^\circ}{2} = \frac{12.\cancel{12}^3\sqrt{3}}{\cancel{2.2}} = \underline{\underline{36\sqrt{3}\,cm^2}}$

(19) Периметърът на успоредник е 44 cm, а ъглополовящата на един от ъглите му го дели на две фигури, разликата на периметрите на които е 4 cm. Да се намерят страните на успоредника.



Решение:

Нека $AB = CD = a$; $BC = AD = b$, $a > b$

AK - ъглоп. на $\sphericalangle BAD$, $K \in DC$

$\Rightarrow \sphericalangle BAK = \sphericalangle KAD$, но $\sphericalangle BAK = \sphericalangle AKD$ (кр.)

$\Rightarrow \sphericalangle KAD = \sphericalangle AKD \Rightarrow DK = AD = b$

$\Rightarrow KC = a - x$

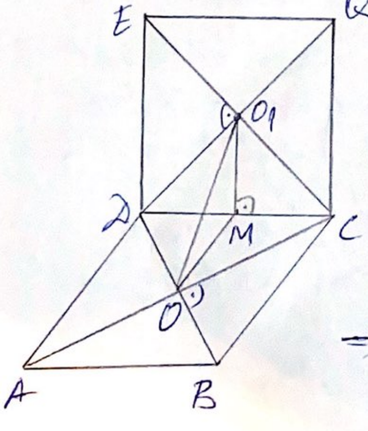
$P_{AKD} = 2b + AK$; $P_{ABCK} = 2a + AK$

$\Rightarrow P_{ABCK} - P_{AKD} = 2a + AK - 2b - AK = 2(a - b) = 4\,cm$

$\Rightarrow a - b = 2\,cm$; $P_{ABCD} = 2(a + b) = 44 \;|:2 \Rightarrow a + b = 22$

$\Rightarrow \left| \begin{array}{l} a - b = 2 \\ a + b = 22 \end{array} \right. (+) \Rightarrow 2a = 24 \Rightarrow \underline{\underline{a = 12\,cm}}$, $\underline{\underline{b = BC = 10\,cm}}$

(20) ABCD-ромб, $\sphericalangle BAD = 50^\circ$. Външно за ромба е построен квадрат DCQE. $AC \cap BD = O$, $DQ \cap CE = O_1$, M - среда на DC. Намерете ъглите на $\triangle OMO_1$.



Решение:

ABCD-ромб $\Rightarrow AC \perp BD$; AC и BD-ъглоп. на ъглите $\Rightarrow \sphericalangle ODC = \frac{180^\circ - 50^\circ}{2} = 65^\circ$

OM - медиана в $\triangle ODC \Rightarrow DM = OM = MC = \frac{DC}{2}$

$\Rightarrow \sphericalangle DMO = 180^\circ - 2.\sphericalangle ODM = 50^\circ$

DCQE-квадрат $\Rightarrow DQ \perp EC$, $DQ = EC$

$\Rightarrow \triangle DCO_1$ - правоъгълен и равнобедрен

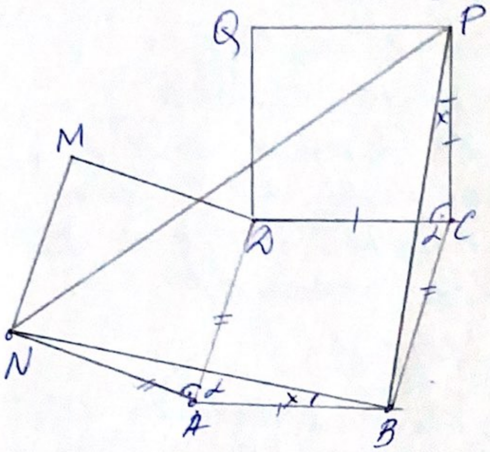
$\Rightarrow O_1M \perp DC$ и $DM = MO_1 = DM = \frac{DC}{2}$

$\Rightarrow OM = O_1M$ и $\sphericalangle OMO_1 = \sphericalangle OMD + \sphericalangle DMO_1 =$

$= 50^\circ + 90^\circ = 140^\circ \Rightarrow \sphericalangle MOO_1 = \sphericalangle MO_1O = \frac{180 - 140^\circ}{2} =$

$\underline{\underline{= 20^\circ}}$

(21) Даден е успоредник ABCD. Външно за него са построени квадратите ADMN и DCPQ. Да се намерят ъглите на $\triangle NBP$.



Решение:

ABCD-усп. $\Rightarrow AB = DC;\ BC = AD;\ \angle BAD = \angle DCB = \alpha$

ADMN-квадрат $\Rightarrow NA = AD,\ \angle NAD = 90^\circ$

DCPQ-квадрат $\Rightarrow DC = CP,\ \angle DCP = 90^\circ$

Разг. $\triangle NAB$ и $\triangle BCP$

1) $NA = CB$; 2) $AB = PC$; 3) $\angle NAB = \angle BCP = 90^\circ + \alpha$

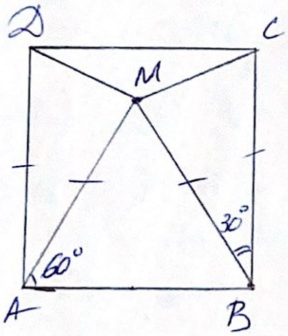
$\Rightarrow \triangle NAB \cong \triangle BCP$ (I пр)

$\Rightarrow NB = BP;\ \angle ABN = \angle CPB = x$

$\Rightarrow \triangle NBP$-равнобедрен; $\angle NBP = \angle ABC - (\angle ABN + \angle PBC) = 180^\circ - \alpha - (x + 180^\circ - \alpha - x - 90^\circ) = 90^\circ \Rightarrow \angle BNP = \angle BPN = 45^\circ$

(22) Във вътрешността на квадрат ABCD е взета т. M така, че $\triangle ABM$ е равностранен. $\angle DMC = ?$

Решение:



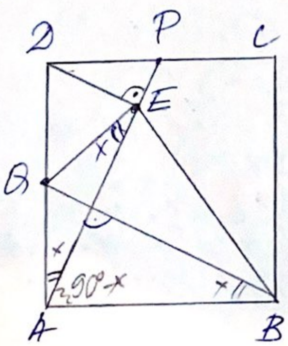
$\triangle ABM$ - равностранен $\Rightarrow AM = MB = AB = AD = BC$ и $\angle MAB = \angle MBA = 60^\circ$

$\Rightarrow \angle MAD = \angle MBC = 90^\circ - 60^\circ = 30^\circ$

Но $\triangle MBC$-равнобедрен $\Rightarrow \angle BMC = \angle BCM = \dfrac{180^\circ - 30^\circ}{2} = 75^\circ$. Аналогично $\angle DMA = \angle MDA = 75^\circ \Rightarrow \angle MDC = \angle MCD = 90^\circ - 75^\circ = 15^\circ \Rightarrow \angle DMC = 180^\circ - 2.15^\circ = 150^\circ$

(23) P-среда на CD-страна на квадрат ABCD. От върха D е построен перпендикуляр DE към AP; $E \in AP$. Да се докаже, че $BE = CD$.

Доказателство:



Нека т. Q-среда на AD. Разг. $\triangle APD$ и $\triangle AQB$:

1) $AD = AB$; 2) $DP = AQ$; 3) $\angle PDA = \angle QAB = 90^\circ$

$\Rightarrow \triangle APD \cong \triangle AQB$ (I пр) $\Rightarrow \angle DAP = \angle ABQ = x$

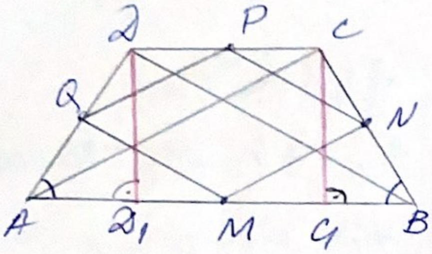
$\Rightarrow \angle BAP = 90^\circ - x \Rightarrow AP \perp BQ$

QE-медиана в $\triangle AED$-правоъгълен

$\Rightarrow QE = QA \Rightarrow \angle QEA = x \Rightarrow QB = S_{AE}$

$\Rightarrow AB = BE = CD.$

(24) Основите на равнобедрен трапец са $AB=40$ см и $CD=10$ см, а бедрото $AD=25$ см. Точките M, N, P и Q са средите на страните на $ABCD$. Да се намери лицето на $MNPQ$. (ДЗИ²-2021г)



Решение: $AD = BC \Rightarrow AC = BD$, $\angle A = \angle B$

Нека DD_1 и CC_1 - височини.

$\triangle DD_1A \cong \triangle CC_1B$ (IV пр)

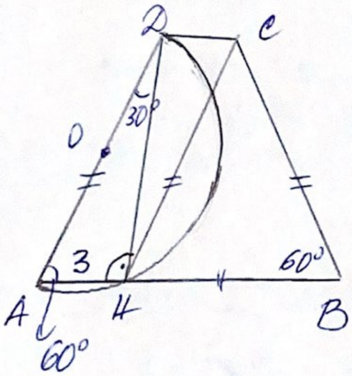
$\Rightarrow AD_1 = BC_1 = x \Rightarrow AB = 2AD_1 + D_1C_1$

$\Rightarrow 40 = 10 + 2x \Rightarrow x = 15$ см

$\triangle ADD_1$ - Питагорова т-ма

$\Rightarrow DD_1 = \sqrt{AD^2 - AD_1^2} = \sqrt{25^2 - 15^2} = 20$ см. В зад. (11) доказахме, че $S_{MNPQ} = \frac{1}{2} S_{ABCD} = \frac{1}{\cancel{2}} \cdot \frac{40+10}{\cancel{2}} \cdot \cancel{20}\,5 = 250$ см²

(25) В равнобедрен трапец $ABCD$ $(AB \parallel CD)$ е построена полуокръжност с диаметър AD, която пресича AB в т. H. $AH = 3$ см, а $\triangle BHC$ е равностранен. Намерете лицето на $ABCD$. (ДЗИ²-2021г.)



Решение:

AD-диаметър $\Rightarrow \angle AHD = 90°$

$CB = AD$ (равнобедрен), но $CB = CH$

и $\angle B = \angle A = 60° \Rightarrow \angle ADH = 30°$

$\Rightarrow AD = 2AH = 6$ см $\Rightarrow HB = CB = HC = 6$ см

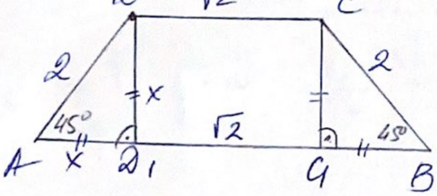
$\Rightarrow AB = AH + HB = 9$ см, $AH \parallel DC$, $AD \parallel HC$

$\Rightarrow AHCD$ - успоредник $\Rightarrow DC = AH = 3$ см

$\triangle AHD$: $DH = \sqrt{AD^2 - AH^2} = 3\sqrt{3}$ см

$\Rightarrow S_{ABCD} = \frac{AB+CD}{2} \cdot DH = 18\sqrt{3}$ см²

(26) Бедрото на равнобедрен трапец е 2 см, малката основа е $\sqrt{2}$ см, а ъгълът при нея е $135°$. Лицето на трапеца се търси. (ДЗИ¹-2021г.)



Решение:

Спускаме височините DD_1 и CC_1.

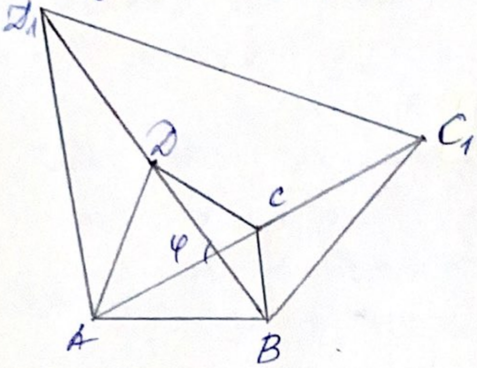
$\Rightarrow AD_1 = C_1B = x$. Но $\angle DAD_1 = 180° - 135° = 45°$

$\Rightarrow DD_1 = AD_1 = x = AD \sin 45° = \frac{2\sqrt{2}}{2} = \sqrt{2}$ см

$\Rightarrow AB = 3\sqrt{2}$ см $\Rightarrow S_{ABCD} = \frac{AB+CD}{2} \cdot h =$

$= \frac{3\sqrt{2}+\sqrt{2}}{2} \cdot \sqrt{2} = 4$ см²

(27) Върху продълженията на AC и BD на четириъгълника $ABCD$ са построени отсечки CC_1 и DD_1, съотв. равни на AC и BD (C е м/у A и C_1, т. D е между B и D_1). Ако $S_{ABCD}=3\,cm^2$, то да се намери $S_{ABC_1D_1}$. (ДЗИ¹-2020г).



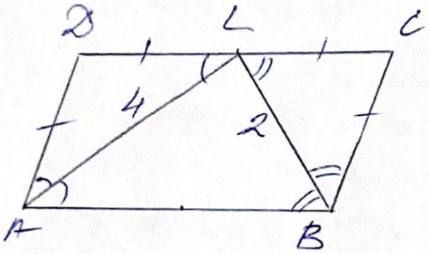
Решение:

$$S_{ABCD} = \frac{AC.BD.\sin\varphi}{2} = 3\,cm^2$$

$$S_{ABC_1D_1} = \frac{AC_1.BD_1.\sin\varphi}{2} = \frac{2AC.2BD.\sin\varphi}{2} =$$

$$= 4.\frac{AC.BD.\sin\varphi}{2} = 4.3 = 12\,cm^2$$

(28) В успоредника $ABCD$ ъглополовящите на ъглите при върховете A и B се пресичат в т. $L \in DC$. Ако $AL=4\,dm$, $BL=2\,dm$, то да се намери BC. (ДЗИ¹-2020г)



Решение:

$\sphericalangle A + \sphericalangle B = 180°$ ($ABCD$-усп.)

$\Rightarrow \frac{\sphericalangle A}{2} + \frac{\sphericalangle B}{2} = 90°$

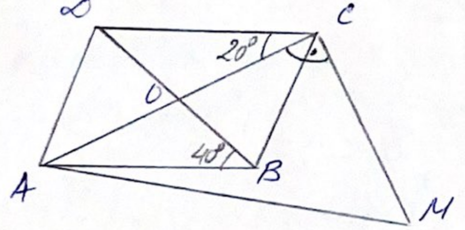
$\Rightarrow AB^2 = AL^2 + LB^2 = 16+4=20$

$\Rightarrow AB = 2\sqrt{5}\,dm$

$\sphericalangle BAL = \sphericalangle LAD = \sphericalangle ALD$ (кр., ъгли.) $\Rightarrow AD = DL = BC = LC$

$\Rightarrow BC = \frac{AB}{2} = \sqrt{5}\,dm$

(29) На чертежа $ABCD$-успоредник, $\sphericalangle ABD=40°$, $\sphericalangle ACD=20°$, $\sphericalangle ACM=90°$. $CM=BD$. $S_{AMC}=12\,cm^2$. $S_{ABCD}=?$ (ДЗИ¹-2019г)



Решение:

$$S_{ABCD} = \frac{AC.\ BD \sin\sphericalangle BOC}{2}, \text{ но } BD=CM$$

$$\Rightarrow S_{ABCD} = \frac{AC.\ CM \sin 60°}{2} =$$

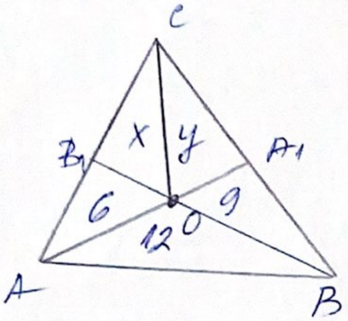
$$= \frac{AC.CM}{2}.\frac{\sqrt{3}}{2} = S_{ACM}.\frac{\sqrt{3}}{2} = 6\sqrt{3}\,cm^2$$

(30) $\triangle ABC$: $A_1 \in BC$, $B_1 \in AC$, $AA_1 \cap BB_1 = O$, $S_{AOB_1} = 6$

$S_{ABO} = 12$, $S_{BOA_1} = 9$. $S_{ABC} = ?$ (М2-2021)

Решение:

Нека $S_{CB_1O} = x$; $S_{COA_1} = y$



$$\frac{S_{AOB_1}}{S_{AOB}} = \frac{B_1O}{OB} = \frac{S_{COB_1}}{S_{COB}} \Rightarrow \frac{6}{12} = \frac{x}{y+9} \Rightarrow \frac{x}{y+9} = \frac{1}{2}$$

$$\Rightarrow y = 2x - 9$$

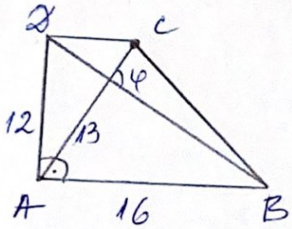
$$\frac{S_{ABA_1}}{S_{AA_1C}} = \frac{BA_1}{A_1C} = \frac{S_{OBA_1}}{S_{OCA_1}} \Rightarrow \frac{\cancel{21}^{\,7}}{x+y+6} = \frac{\cancel{9}^{\,3}}{y}$$

$$\Rightarrow 4y = 3x + 18 \Rightarrow x = \frac{54}{5};\ y = \frac{63}{5}$$

$$\Rightarrow S_{ABC} = x + y + 6 + 9 + 12 = 50{,}4$$

(31) Даден е трапец $ABCD$, $AB \parallel CD$, $AB = 16$, $AD = 12$, $\measuredangle A = 90^\circ$; $AC = 13$

Да се намери $\sin \measuredangle(AC, BD)$.



Решение:

$\triangle ADB$: $DB = \sqrt{AD^2 + AB^2} = \sqrt{12^2 + 16^2} = 20$

$\triangle ADC$: $DC = \sqrt{AC^2 - AD^2} = 5$

$$\Rightarrow S_{ABCD} = \frac{AB + CD}{2} \cdot AD = \frac{(16+5)}{\cancel{2}} \cdot \cancel{12}^{\,6} = 126$$

$$S_{ABCD} = \frac{AC \cdot DB \cdot \sin\varphi}{2} \Rightarrow 126 = \frac{\cancel{20}^{\,10} \cdot 13 \cdot \sin\varphi}{\cancel{2}}$$

$$\Rightarrow \sin \measuredangle(AC; BD) = \frac{63}{65}$$