

①

example: assume  $n=6$ 

$$S = 1 + 2 + 3 + 4 + 5 + 6$$

$$S = 6 + 5 + 4 + 3 + 2 + 1$$

Two ways  
to write sum  
of ~~1-6~~ 1-6

②

now

$$S = 1 + 2 + 3 + \dots + N$$

$$S = N + (N-1) + (N-2) + \dots + 1$$

$$S + S = (1+N) + (2+N-1) + (3+N-2) + \dots + (N+1)$$

$$2S = (1+N) + (N+1) + (N+1) + \dots + (N+1)$$

$$2S = N(1+N)$$

$$\frac{2S}{2} = \frac{N(1+N)}{2}$$

$$S = \frac{N(N+1)}{2}$$

1	2	3	...	N
---	---	---	-----	---

N	n-1	n-2	...	1
---	-----	-----	-----	---

n+1	n+1	n+1	...	n+1
-----	-----	-----	-----	-----

$$\frac{2S}{2} = N(N+1)$$

$$S = \frac{N(N+1)}{2}$$



② Let's assume that there are 3 letters available; each time we want to select a pair of two letters out of the 3. Here is one way to write this;

A, B, C

$(A, B) (A, C)$

A, B, C

$(B, A) (B, C)$

A, B, C

$(C, A) (C, B)$

There are 6 ways to write it and if order of letters is not important, then there are 3 ways to write it.

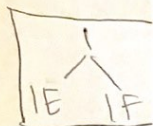
$$\# \text{ of ways} = \frac{3(3-1)}{2} = 3$$

$$\text{If } N \text{ letters are available} = \frac{N(N-1)}{2}$$

There are available.

2 letters numbers

E, F



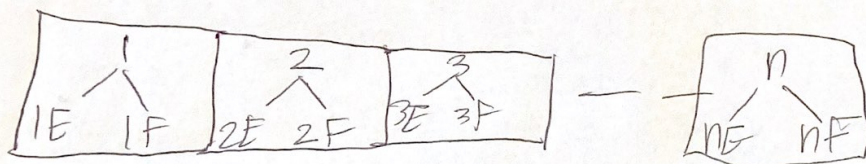
First

First

First



There are  $n$  numbers and  $A$  letters (values) available. Let's assume  $A=2$ . There are 2 letters of  $E$  &  $F$ . We assign each numbers from 1 to  $n$  to two letters of  $E$  &  $F$ .



First Box (2 ways)  $1E, 1F$   $2^1$

First + second (  $1E, 1F, 2E, 2F$  )  $2^1 \cdot 2^1 = 2^2$

First, second, third  $2^1 \cdot 2^1 \cdot 2^1 = 2^3$

There are  $2^n$  ways to write one numbers from 1 to  $n$  and 2 letters.

If there are  $n$  numbers and  $A$  letters than  $n$  for  $n$  boxes  $= A^n$