

# Measuring two vibrations using dual-external-cavity structure in a self-mixing system<sup>☆</sup>

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## ABSTRACT

This paper introduces a new dual-external-cavity structure in self-mixing interference system to be used with a semiconductor laser, and develops an improved algorithm for dominant and secondary harmonic order determination based on Fourier transform. The simulation and experimental results demonstrate that the proposed system can measure two vibrations simultaneously by using only a single detector and a single laser.

1个PD 1个LD实现2个振动测量

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## 1. Introduction

The output powers and frequencies of lasers can be altered by the feedback effect of the reflected light, which is portion of the output light that is reflected by the external cavity into the inner active cavity of the laser. The underlying phenomenon is self-mixing interference (SMI), which has been widely examined and applied in measuring several parameters, including distance [1], displacement [2,3], velocity [4,5], and vibration [6–8] because of its inherent simplicity [9], compactness [10,11], and self-alignment capability [12,13].

Several papers have examined the dual-channel SMI in fiber lasers. According to Wang et al. [14–16], applying SMI to parallel dual-channel fiber ring lasers can efficiently measure multiple displacements and velocities, although this approach does require use of two photodetectors (PDs) to measure two objects. Lu et al. [17] used the linear cavity of a fiber laser with a dual-channel SMI to measure vibrations, but this method required the use of an orthogonal Michelson interferometer in combination with a phase generated carrier demodulation method to demodulate the vibration signals. Moreover, both of these studies have employed complex devices and algorithms, thereby increasing the complexity and cost of the fiber laser structure with dual-channel SMI.

The SMI technique is intrinsically more efficient when using a class B laser (e.g., semiconductor laser) [17]. Nevertheless, semiconductor lasers with a dual-channel SMI are rarely used in measuring the multi-dimensional information of an object or the similar information of two objects.

This study proposed a new dual-external-cavity SMI structure for semiconductor lasers that can measure two vibrations with only a single detector and a single laser. To recover the vibrations from modulated SMI signals, this study also introduces an improved algorithm for dominant and secondary harmonic order determination based on Fourier transform (FT). Compared with the algorithm for dominant harmonic order determination proposed by Huang [18], the proposed improved algorithm can provide robust estimations of the vibration frequencies and amplitudes of two objects simultaneously.

**2. Theoretical analysis** 谐波阶数确定的改进算法，同时对两个物体的振动频率和振幅进行鲁棒性估计

### 2.1. A. SMI structures with a single cavity

The conventional single external cavity self-mixing model [19] is shown in Fig. 1, where  $r_1$  and  $r_2$  denote the reflection coefficients of laser facets  $R_1$  and  $R_2$ , respectively,  $L_D$  denotes the length of the laser active cavity,  $L$  denotes the length of the external cavity (calculated as the distance from  $R_2$  to the target  $M$ ), and  $\Delta L$  denotes the movement distance of the target  $M$ . A PD is typically used to detect any changes in power.

### 2.2. B. Amplitude and frequency solution for SMI with a single cavity

The output power detected by the PD can be expressed as follows [20]:

$$P = P_0[1 + m \cos(\omega \cdot \tau(t))] \quad (1)$$

where  $P$  and  $P_0$  are the light intensities with and without feedback, respectively, and  $m$  is the modulation index that reflects the visibility

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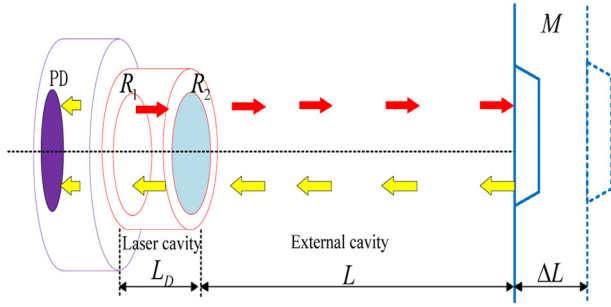


Fig. 1. Structure of the three-cavity F-P model for SMI.

of the interference fringes. A higher  $m$  corresponds to a higher signal-to-noise ratio for the interference signal. Here,  $\omega = 2\pi\nu$  is the angular frequency of the laser, where  $\nu$  denotes the optical frequency of the laser,  $\tau = 2L/c$  is the round trip time of the laser beam in the external cavity, and  $c$  denotes the speed of light.

The laser wavelength is no longer the initial wavelength  $\lambda_0$  but becomes equal to  $\lambda$  because of the optical feedback modulation. The change in wavelength is determined by solving the following phase equation:

$$\omega_0 \tau = \omega \tau + C \sin(\omega \tau + \arctan \alpha) \quad (2)$$

where  $\omega_0 = 2\pi\nu_0$  is the angular frequency without feedback,  $\nu_0$  denotes the optical frequency of the laser without feedback,  $C$  is an optical feedback parameter, and  $\alpha$  is the linewidth broadening factor.  $\omega_0 \tau$  and  $\omega \tau$  denote the phases without and with optical feedback, respectively.

By combining Eq. (1) and (2), the laser output power can be formulated as follows:

$$P = P_0 \{1 + m \cos[\varphi + a \sin(2\pi f_0 t + \phi)]\} \quad (3)$$

where  $f_0$  is the vibration frequency,  $\phi$  is the initial phase, and  $t$  is the time variable

$$\varphi = 4\pi L / \lambda \quad (4)$$

$$a = 4\pi A / \lambda \quad (5)$$

By expanding the normalized laser output power in a trigonometric series, we obtain the following:

$$\begin{aligned} \Delta P &= \cos \varphi J_0(a) \\ &+ \cos \varphi e^{j2n\phi} \sum_{n=1}^{\infty} J_{2n}(a) e^{j2n \cdot 2\pi f_0 t} \\ &+ \cos \varphi e^{-j2n\phi} \sum_{n=1}^{\infty} J_{2n}(a) e^{-j2n \cdot 2\pi f_0 t} \\ &- j \sin \varphi e^{j(2n+1)\phi} \sum_{n=0}^{\infty} J_{2n+1}(a) e^{j(2n+1) \cdot 2\pi f_0 t} \\ &- j \sin \varphi e^{-j(2n+1)\phi} \sum_{n=0}^{\infty} J_{2n+1}(a) e^{-j(2n+1) \cdot 2\pi f_0 t} \end{aligned} \quad (6)$$

where  $J_{2n}(a)$  and  $J_{2n+1}(a)$  are even- and odd-order Bessel functions of the first kind with argument  $a$ , respectively. The independent variable  $n$  is a positive integer. When only the direct-current (DC) and positive components of the frequency are considered, the FT model of Eq. (11) can be written as follows:

$$\begin{aligned} |F(f)| &= \cos \varphi |J_0(a)| \delta(f) \\ &+ \cos \varphi \sum_{n=1}^{\infty} |J_{2n}(a)| \delta(f - 2n f_0) \\ &+ \sin \varphi \sum_{n=0}^{\infty} |J_{2n+1}(a)| \delta[f - (2n+1) f_0] \end{aligned} \quad (7)$$

where  $\delta(f)$  is the impulse function, and  $f$  is the frequency. In Eq. (7),  $|F(f)|$  comprises  $f_0$  and its harmonic components. When  $J_n(a)$  reaches the maximum, the  $n$ th-order harmonic component is regarded as the maximum.  $n$  is then marked as  $n_d$ . The  $n_d$ th-order harmonic component

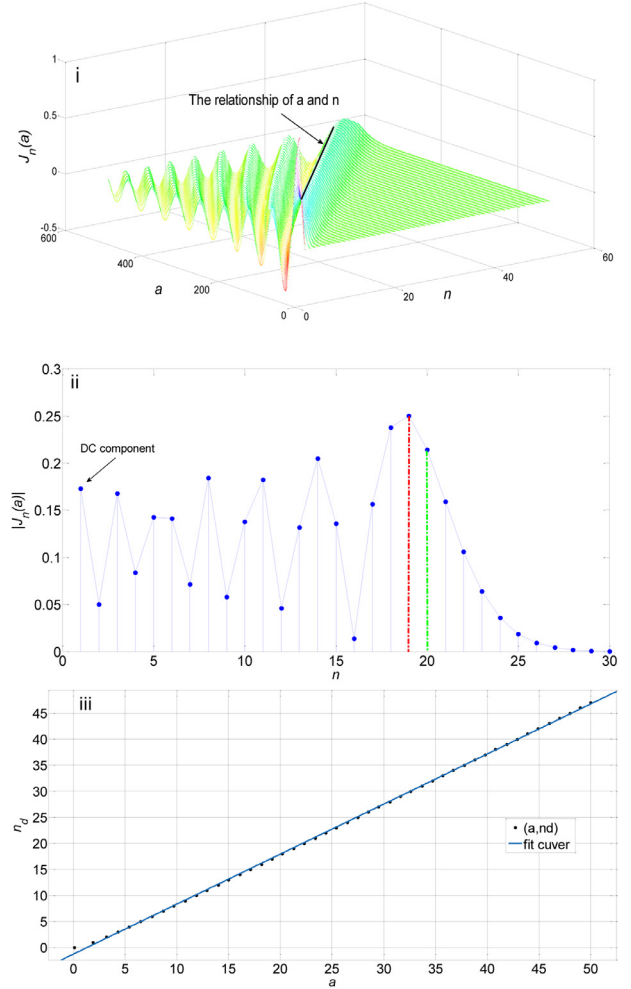


Fig. 2. Principle of the dominant harmonic order. (i) The Bessel function of the first kind that varies along with  $n$  and  $a$ ; (ii) the relationship between  $|J_n(a)|$  and  $n$ ,  $n_d$  (iii) the relationship between  $a$  and  $n_d$ .

is equivalent to the maximum frequency in the spectrum and is called the **dominant harmonic order**. By using the fast Fourier transform (FFT), the frequency of the vibration can be acquired easily but the amplitude can't be measured, so this paper further studied in order to measure vibration.

The value of the Bessel function with changing  $n$  and  $a$  is shown in Fig. 2(i). For a specific  $a$ ,  $J_n(a)$  changes along with  $n$ . The relationship between  $|J_n(a)|$  and  $n$  when  $a = 20$  is illustrated in Fig. 2(ii).

From Fig. 2(ii), it's clear that when  $a$  is a fixed value,  $J_n(a)$  has several maxima and one maxima in the range.

Record the corresponding  $n$  when  $J_n(a)$  is the maximum value as  $n_d$ . In the laser self-mixing interference spectrum generated by the vibrating object, the frequency corresponding to  $n_d$  is recorded as  $f_{n_d}$  which is considered the main frequency, and record the frequency corresponding to  $n_d + 1$  as  $f_{n_d+1}$ .

As shown in Fig. 2(iii), the following relationship between  $n_d$  and  $a$  can be obtained after fitting:

$$n_d = 0.96a - 1.25 \quad (8)$$

By substituting Eq. (8) into Eq. (5) and by assuming that  $\lambda \approx \lambda_0$ , the following equation can be derived:

$$A = \frac{n_d + 1.25}{0.96} \frac{\lambda_0}{4\pi} \quad (9)$$

$f_0$  should be obtained afterward. In previous studies [18, 21],  $f_0$  was obtained on the basis that its value is equal to the fundamental frequency.

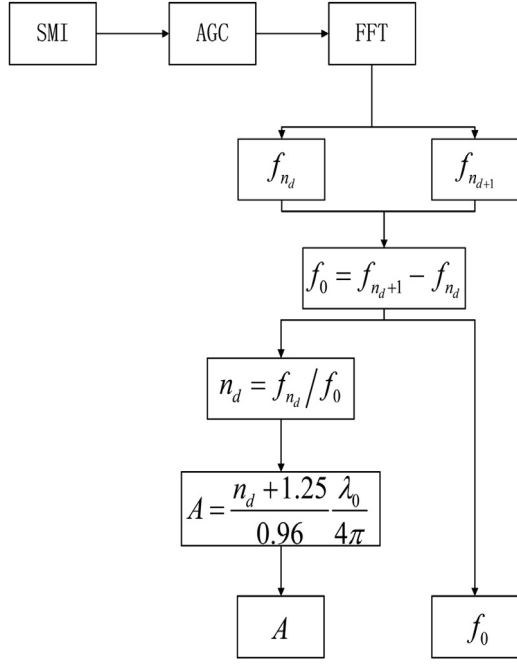


Fig. 3. Principle of the dominant and secondary harmonic order determination.

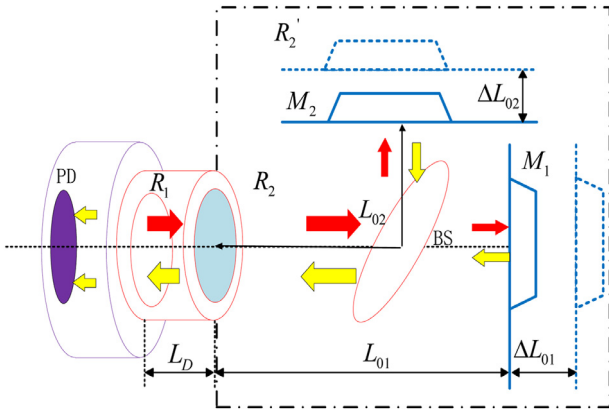


Fig. 4. Laser SMI structure with dual-external cavities.

However, in reality, the basic frequency may be submerged within the noise or spectrum of another frequency when the two signals are mixed. In such a case,  $f_0$  cannot be calculated accurately. To solve this problem,  $f_0$  is calculated by using Eq. (10), where the frequency of  $n_d$  is denoted by  $f_{n_d}$ , while the frequency of  $n_d + 1$  is denoted by  $f_{n_d+1}$ :

$$f_0 = (n_d + 1)f_0 - n_d f_0 = f_{n_d+1} - f_{n_d} \quad (10)$$

where  $f_{n_d}$  and  $f_{n_d+1}$  can be found within the SMI spectrum.

Thus, the improved vibration measurement algorithm called dominant and secondary harmonic order determination is illustrated in Fig. 3.

The condition  $\varphi = K\pi/2$ ,  $K = 1, 2, 3 \dots$  must be avoided during the operation because the amplitude of the basic frequency or the odd harmonic components in this case is 0. The initial phase  $\varphi$  can be varied by adjusting  $L$  (initial length).

### 2.3. C. SMI structures with a dual-external-cavity

In order to measure two vibrations simultaneously by using only a single detector and a single laser, SMI structures with a dual-external-cavity is proposed in this paper. The optical path obtained by using dual external cavities when SMI occurs is depicted in Fig. 4. A portion of the

light from the laser cavity passes through  $R_2$  and then travels toward the beam splitter (BS). Afterward, one part of the light travels toward the moving target  $M_1$ , while the another part travels toward the moving target  $M_2$ . The light reflected by  $M_1$  and  $M_2$  is subsequently transmitted through both BS and  $R_2$  before returning to the laser cavity. Here,  $t_2$  is the transmittance of  $R_2$ ,  $r_3$  and  $r_4$  are the reflectivity values of  $M_1$  and  $M_2$ , respectively,  $L_{01}$  is the length of external cavity 1 (from  $R_2$  to  $M_1$ ), and  $L_{02}$  is the length of external cavity 2 (from  $R_2$  to  $M_2$ ). In addition,  $\Delta L_{01}$  and  $\Delta L_{02}$  are the movement distances of targets  $M_1$  and  $M_2$ , respectively, while  $t_{BS}$  and  $r_{BS}$  are the transmittance and reflectance of the BS, respectively. We let  $R_2$  and the dual-external-cavity structure be equivalent to  $R'_2$ . The equivalent reflectance under the weak feedback condition  $\eta \leq 1$  can be written as follows:

$$\ln(|r'_2|) \approx \ln r_2 + \eta t_2^2 [t_{BS}^2 r_3 \cos \alpha_1 + r_{BS}^2 r_4 \cos \alpha_2] \quad (11)$$

$$\alpha = \arctan \frac{\eta t_2^2 [t_{BS}^2 r_3 \sin \alpha_1 + r_{BS}^2 r_4 \sin \alpha_2]}{r_2 + \eta t_2^2 [t_{BS}^2 r_3 \cos \alpha_1 + r_{BS}^2 r_4 \cos \alpha_2]} \approx \eta t_2^2 / r_2 [t_{BS}^2 r_3 \sin \alpha_1 + r_{BS}^2 r_4 \sin \alpha_2] \quad (12)$$

where  $\eta$  is the coupling coefficient of the feedback light,  $\alpha_1 = 2kL_{01}$  is the phase of external cavity 1, and  $\alpha_2 = 2kL_{02}$  is the phase of external cavity 2.

The threshold condition of the dual-external-cavity structure can be described as:

$$r_1 r_2' \exp(2gkL_D) \exp(i2n_c kL_D + i\alpha) = 1 \quad (13)$$

$$\begin{cases} gk = -\frac{1}{2L_D} \ln(r_1 |r'_2|) \\ 2nkL_D + \alpha = 2m\pi \end{cases} \quad (14)$$

From the F-P laser cavity, the power equation can be deduced as follows:

$$\omega_0 \tau_D = \omega \tau_D - [C_1 \sin(\omega \tau_1) + C_2 \sin(\omega \tau_2)] \quad (15)$$

$$P = P_0(1 - K\Delta G) = P_0[1 - \mu_1 \cos(\omega \tau_1) - \mu_2 \cos(\omega \tau_2)] \quad (16)$$

where  $\tau_D = 2n_0 L_D / c$  (in which  $\tau_D$  is the round trip time of the laser beam in the laser cavity, and  $c$  is the speed of light),  $\tau_1 = 2L_{01} / c$  (in which  $\tau_1$  is the round trip time of the laser beam in external cavity 1), and  $\tau_2 = 2L_{02} / c$  (in which  $\tau_2$  is the round trip time of the laser beam in external cavity 2). In addition,  $C_1 = \eta t_{BS}^2 r_3 t_2^2 / r_2$  and  $C_2 = \eta r_{BS}^2 r_4 t_2^2 / r_2$  are the external feedback strength parameters,  $K$  is the laser cavity constant,  $P_0$  is the initial light intensity without feedback, and  $\mu_1 = \eta t_{BS}^2 r_3 t_2^2$  and  $\mu_2 = \eta r_{BS}^2 r_4 t_2^2$  are the undulation coefficients of SMI.

Equations (15) and (16) indicate that using the SMI with a dual-external-cavity structure is equivalent to using two SMI systems and can therefore be applied in a multichannel measurement environment.

### 2.4. D. Solutions for the amplitudes and frequencies in SMI with a dual-external-cavity structure

Similar to Fig. 4,  $L_{01}$  represents the initial distance between  $R_2$  and  $M_1$ , while  $L_{02}$  represents the initial distance between  $R_2$  and  $M_2$ . Assuming that both  $M_1$  and  $M_2$  perform simple harmonic vibrations, the motions of the dual external cavities can be given as follows:

$$L_{01}(t) = L_{01} + \Delta L_{01} = L_{01} + A_1 \sin(2\pi f_1 t + \varphi_1) \quad (17)$$

$$L_{02}(t) = L_{02} + \Delta L_{02} = L_{02} + A_2 \sin(2\pi f_2 t + \varphi_2) \quad (18)$$

where  $A_1$  and  $A_2$  are the vibration amplitudes of  $M_1$  and  $M_2$ , respectively,  $f_1$  and  $f_2$  are the frequencies of the two vibrations, and  $\varphi_1$  and  $\varphi_2$  are the initial phases of  $M_1$  and  $M_2$ , respectively.

By combining Eqs. (16), (17), and (18), the laser output power can be written as follows:

$$P = P_0[1 - \mu_1 \cos(\psi_{01} + a_1 \sin(2\pi f_1 t + \varphi_1)) - \mu_2 \cos(\psi_{02} + a_2 \sin(2\pi f_2 t + \varphi_2))]$$

where

$$\psi_{01} = \frac{4\pi L_{01}}{\lambda} \quad (20)$$

$$\psi_{02} = \frac{4\pi L_{02}}{\lambda} \quad (21)$$

$$a_1 = \frac{4\pi A_1}{\lambda} \quad (22)$$

$$a_2 = \frac{4\pi A_2}{\lambda} \quad (23)$$

Similarly, the expansion of the normalized laser output power in a trigonometric series can be written as follows:

$$\Delta P(t) = \begin{bmatrix} \mu_1 \cos \psi_{01} J_0(a_1) \\ + \mu_1 \cos \psi_{01} \sum_{m=1}^{\infty} J_{2m}(a_1) e^{j2m\varphi_{01}} \\ + \mu_1 \cos \psi_{01} \sum_{m=1}^{\infty} J_{2m}(a_1) e^{-j2m\varphi_{01}} \\ + j\mu_1 \sin \psi_{01} \sum_{m=1}^{\infty} J_{2m-1}(a_1) e^{j2(m-1)\varphi_{01}} \\ - j\mu_1 \sin \psi_{01} \sum_{m=1}^{\infty} J_{2m-1}(a_1) e^{-j2(m-1)\varphi_{01}} \end{bmatrix} + \begin{bmatrix} \mu_2 \cos \psi_{02} J_0(a_2) \\ + \mu_2 \cos \psi_{02} \sum_{m=1}^{\infty} J_{2m}(a_2) e^{j2m\varphi_{02}} \\ + \mu_2 \cos \psi_{02} \sum_{m=1}^{\infty} J_{2m}(a_2) e^{-j2m\varphi_{02}} \\ + j\mu_2 \sin \psi_{02} \sum_{m=1}^{\infty} J_{2m-1}(a_2) e^{j2(m-1)\varphi_{02}} \\ - j\mu_2 \sin \psi_{02} \sum_{m=1}^{\infty} J_{2m-1}(a_2) e^{-j2(m-1)\varphi_{02}} \end{bmatrix} \quad (24)$$

where,  $\varphi_{01} = 2\pi f_1 t$ , and  $J_{2m}(a_1)$  and  $J_{2m-1}(a_1)$  are the even- and odd-order Bessel functions of the first kind with argument  $a_1$ . Similarly,  $J_{2m}(a_2)$  and  $J_{2m-1}(a_2)$  are the corresponding Bessel functions with the argument  $a_2$ . The independent variables  $a_1$ ,  $a_2$ , and  $m$  are all positive integers. By considering only the DC component and the positive component of the frequency, the residual  $|F_{12}(f)|$  can be written as follows:

$$|F_{12}(f)| = \begin{bmatrix} \mu_1 \cos \psi_{01} |J_0(a_1)| \delta(f) \\ + \mu_1 \cos \psi_{01} \sum_{m=1}^{\infty} |J_{2m}(a_1)| \delta(f - 2mf_1) \\ + j\mu_1 \sin \psi_{01} \sum_{m=1}^{\infty} |J_{2m-1}(a_1)| \delta(f - (2m-1)f_1) \end{bmatrix} + \begin{bmatrix} \mu_2 \cos \psi_{02} |J_0(a_2)| \delta(f) \\ + \mu_2 \cos \psi_{02} \sum_{m=1}^{\infty} |J_{2m}(a_2)| \delta(f - 2mf_2) \\ + j\mu_2 \sin \psi_{02} \sum_{m=1}^{\infty} |J_{2m-1}(a_2)| \delta(f - (2m-1)f_2) \end{bmatrix} \quad (25)$$

where the frequency spectrum is composed of  $f_1$ ,  $f_2$ , and their harmonic components. If the frequencies are not mixing of two vibrations, we can separate the two signals by filtering, and then measure the vibration of each signal, but when the frequencies are mixing, filtering is not effective, the signals cannot be separated clearly. Thus, this paper proposed an improved algorithm for dominant and secondary harmonic order determination based on FT to measure two vibrations using dual-external-cavity structure simultaneously.

For the target  $M_2$ , the frequency of  $n_{d1}$  is denoted by  $f_{n_{d1}}$ , while the frequency of  $n_{d1} + 1$  is denoted by  $f_{n_{d1}+1}$ . Meanwhile, for the target  $M_2$ , the frequency of  $n_{d2}$  is denoted by  $f_{n_{d2}}$ , while the frequency of  $n_{d2} + 1$  is denoted by  $f_{n_{d2}+1}$ . Equations (25) and (13) are then compared, and the first and second parts can be processed as follows by using the approach illustrated in Fig. 3:

$$f_1 = f_{n_{d1}+1} - f_{n_{d1}} \quad (26)$$

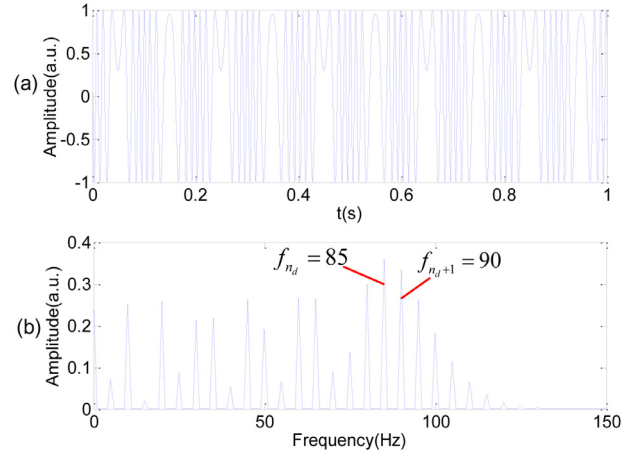


Fig. 5. Simulation results when  $A = 1\mu\text{m}$  and  $f_0 = 5\text{Hz}$ . (a) SMI signals; and (b) frequency spectrum.

$$f_2 = f_{n_{d2}+1} - f_{n_{d2}} \quad (27)$$

$$A_1 = \frac{n_{d1} + 1.25}{0.96} \frac{\lambda_0}{4\pi} \quad (28)$$

$$A_2 = \frac{n_{d2} + 1.25}{0.96} \frac{\lambda_0}{4\pi} \quad (29)$$

### 3. Numerical simulation

#### 仿真条件和结果

#### 3.1. A. Simulation analysis of SMI with a single cavity

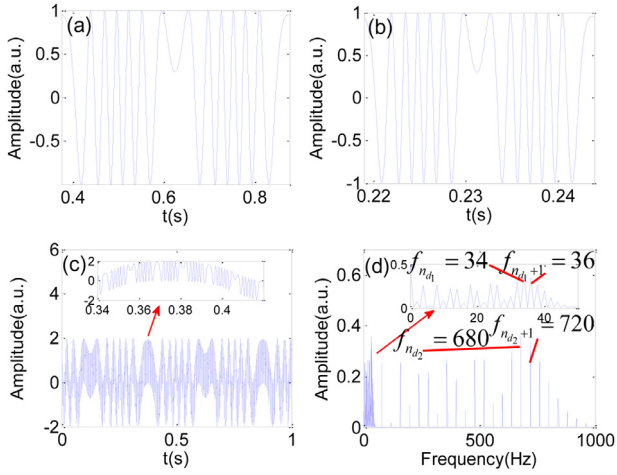
To analyze the practical aspects of the solutions for both amplitude and frequency in SMI when using a single cavity, as described in Section 2.2, we set the target to perform a sinusoidal motion with the amplitude of  $1\mu\text{m}$  and a frequency of  $5\text{Hz}$ . The sampling frequency is set to  $32\text{kHz}$ , while the number of sampling points is set to  $32\text{K}$ . Fig. 5(a) and 5(b) show the simulated displacement and frequency spectrum of the target, respectively. When using the improved algorithm, the demodulated frequency is  $5\text{Hz}$ , while the demodulated amplitude is  $0.984\mu\text{m}$ .

#### 3.2. B. Simulation analysis of SMI with a dual-external-cavity structure

To evaluate the feasibility of the proposed solution for the amplitudes and frequencies in SMI with a dual-external-cavity structure, as described in Section 2.3, we modulate  $L_1(t)$  and  $L_2(t)$  by using sinusoidal signals. These signals have frequencies of  $2\text{Hz}$  and  $20\text{Hz}$ , and have the same amplitude of  $1\mu\text{m}$  at  $\lambda_0 = 650\text{nm}$ . The simulated SMI signals are shown in Figs. 6(a) and 6(b), the simulated optical output power is illustrated in Fig. 6(c), and the frequency spectrum of the signal is shown in Fig. 6(d). The y axis of the four pictures is amplitude, the x axis of the first three pictures is time, the unit is second, and the abscissa of the last picture is frequency, the unit is hertz. In order to see the self-mixing interference fringes more intuitively, Fig. 5.2 (c) and Fig. 5.2 (d) respectively use the method shown in the figure to amplify the local signal.

According to the method of data processing in this article, we first find out  $f_{n_{d1}}$ ,  $f_{n_{d2}}$ ,  $f_{n_{d1}+1}$ ,  $f_{n_{d2}+1}$  in the FFT spectrum in Fig. 6(d). The  $f_{n_{d1}}$  of the spectrum is the marked point in the figure. The marked data's x axis is the coordinate of  $34\text{Hz}$ ,  $f_{n_{d1}+1}$  is the coordinate of the x axis of  $36\text{Hz}$ ;  $f_{n_{d2}}$  is the coordinate point of the x axis of  $680\text{Hz}$ , and  $f_{n_{d1}+1}$  is the coordinate point of the x axis of  $720\text{Hz}$ . The vibrations can be calculated by using the algorithm presented in Fig. 3. The demodulated





**Fig. 6.** Simulation results with  $A_1 = 1\mu\text{m}$ ,  $A_2 = 1\mu\text{m}$ ,  $f_1 = 2\text{Hz}$ , and  $f_2 = 40\text{Hz}$ . (a) SMI signal from external cavity 1, (b) SMI signal from external cavity 2, (c) SMI signal with a dual-external-cavity structure, and (d) frequency spectrum.

frequencies are 2 Hz and 40 Hz while the demodulated amplitude for both signals is  $0.984\mu\text{m}$ .

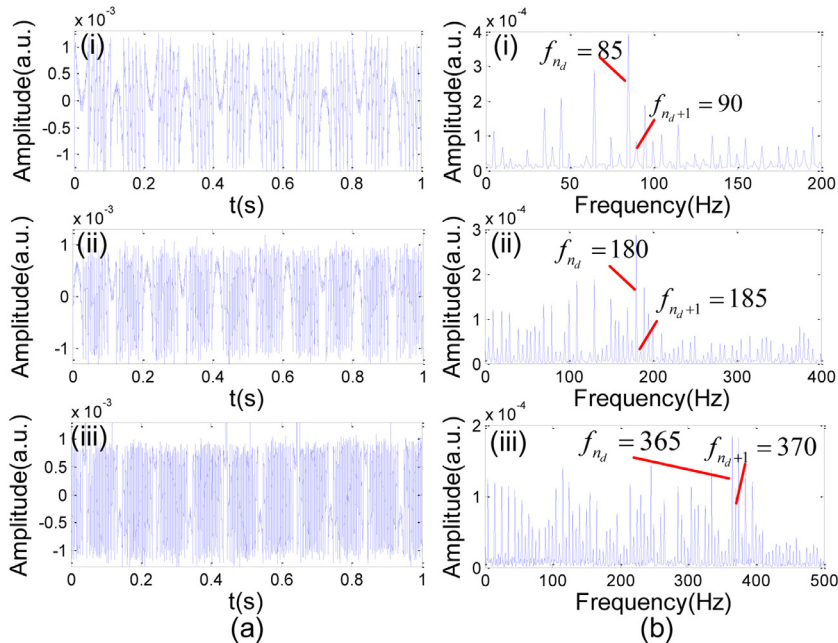
#### 4. Experimental procedures and results

##### 4.1. A. Experimental results for SMI with a single cavity

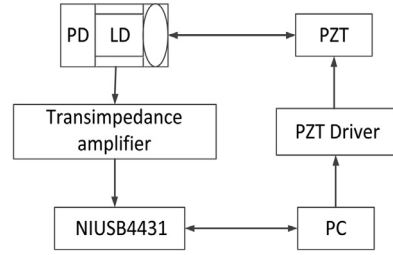
###### 单腔实验

To further confirm the validity of this measurement solution, we set up an experimental bench as shown in Fig. 7. In this experimental setup, a laser diode (LD; QL65D5SA, QSI Co. Ltd., Korea) is packaged with a PD to detect its output power and an aspherical collimating lens and is driven by a constant current source (LD1255).

To ensure the accuracy of the results, we used a high-precision piezoelectric ceramic transducer (PZT; P753.1 CD, Physik Instrumente, Karlsruhe, Germany) as an external target. The resolution of this PZT can reach  $0.05\text{ nm}$  when used in a closed-loop configuration. The current signal is converted into a voltage signal by using a transconductance amplifier, and the voltage signals are collected by using a data acquisition



**Fig. 8.** Experimental results for SMI with a single cavity when  $A = 1, 2, \text{ and } 4\mu\text{m}$  and  $f_0 = 5\text{Hz}$ . (a) SMI signals, and (b) frequency spectrum.



**Fig. 7.** Experimental setup for SMI with a single cavity.

card (USB-4431, National Instruments Corp.) before being sent to a computer.

A series of experiments was performed. First, the PZT was driven by using a sinusoidal waveform with a frequency of 5 Hz. Fig. 8(a) shows the SMI output signals with amplitudes of 1, 2, and  $4\mu\text{m}$ , while Fig. 8(b) shows the corresponding frequency spectra of these output signals. By using the method shown in Fig. 4, all demodulated frequencies were equal to 5 Hz, while their corresponding demodulated amplitudes were equal to  $0.984, 2.008, \text{ and } 4.002\mu\text{m}$ , respectively.

Fig. 9(a) and (b) show the output signals and their corresponding frequency spectra for different PZT drive frequencies at a vibration amplitude of  $1\mu\text{m}$ . The PZT drive frequencies are set to 5, 10, and 20 Hz, which have corresponding demodulated frequencies of 5, 10, and 20 Hz as well as demodulated amplitudes of  $0.984, 1.038, \text{ and } 1.038\mu\text{m}$ , respectively.

##### 4.2. B. Experimental results for SMI with a dual-external-cavity structure

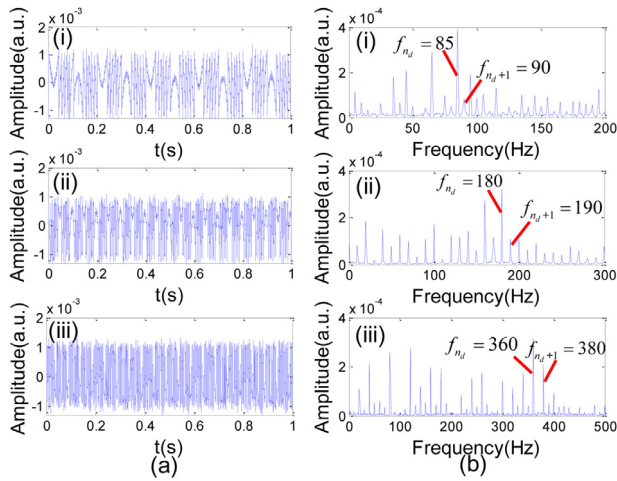
###### 双外腔实验

The experimental setup for SMI with a dual-external-cavity structure is shown in Fig. 10. In addition to the setup shown in Fig. 7, a BS that divides the laser beam into two beams and a loudspeaker driven by a signal generator are included in this setup.

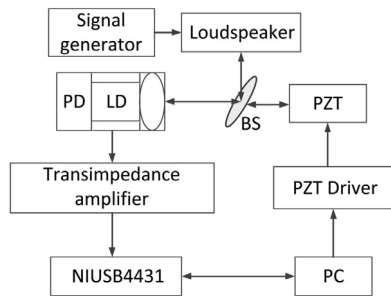
First, the PZT was driven by using a sinusoidal waveform with a frequency of 2 Hz and an amplitude of  $1\mu\text{m}$ , while the loudspeaker was driven by using a sinusoidal waveform with a frequency of 40 Hz and an amplitude of  $1\mu\text{m}$ . The SMI signal when using the dual-external-cavity structure is presented in Fig. 11(a), and the frequency spectrum that is determined by FT is shown in Fig. 11(b).

**Table 1**  
Results of the three groups of experiments.

	Set				Measured				Error	
	PZT		Loudspeaker		PZT		Loudspeaker		PZT	Loudspeaker
	$A_1$	$f_1$	$A_2$	$f_2$	$\bar{A}_1$	$\bar{f}_1$	$\bar{A}_2$	$\bar{f}_2$	Error ( $A_1$ )	Error ( $A_2$ )
I	1	2	1	40	0.98	2	0.98	40	0.02	0.02
	2	2	1	40	1.95	2	0.98	40	0.05	0.02
	4	2	1	40	4.11	2	0.98	40	0.11	0.02
III	1	2	1	40	0.98	2	0.98	40	0.02	0.02
	1	2	2	40	1.04	2	2.01	40	0.04	0.01
	1	2	4	40	1.04	2	4	40	0.04	0.00
III	1	2	1	40	0.98	2	0.98	40	0.02	0.02
	1	2	1	60	0.98	2	0.98	60	0.02	0.02
	1	2	1	80	0.98	2	0.98	80	0.02	0.02



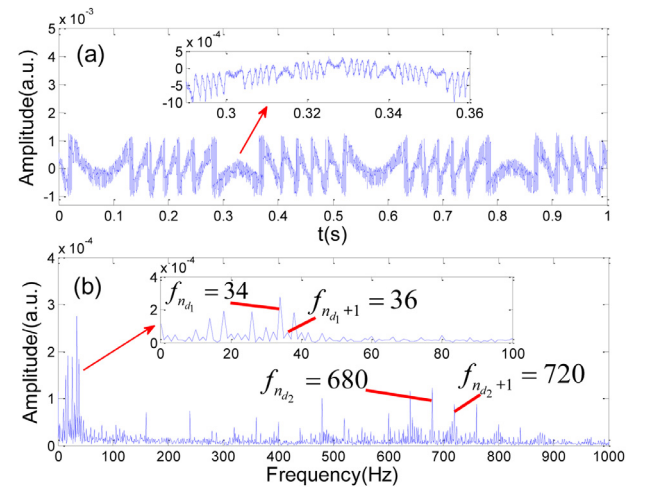
**Fig. 9.** Experimental results for SMI with a single cavity when  $A = 1\mu\text{m}$  and  $f_0 = 5, 10, \text{ and } 20\text{ Hz}$ . (a) SMI signals, and (b) frequency spectrum.



**Fig. 10.** Experimental setup for SMI with a dual-external-cavity structure.

As can be seen in Fig. 11(b), the demodulated frequencies are 2 Hz and 40 Hz, while the corresponding demodulated amplitudes are 0.984  $\mu\text{m}$  and 0.984  $\mu\text{m}$ , respectively.

Second, further experiments were conducted on a series of states. The vibration amplitude of the PZT was varied and its frequency was maintained at 2 Hz, while the loudspeaker was prevented from being driven by the signal with an amplitude of 1  $\mu\text{m}$  and frequency of 40 Hz. The measurement results are listed in Table 1(I). The amplitude of the loudspeaker was then varied and its frequency was maintained at 40 Hz while the PZT was driven by an amplitude of 1  $\mu\text{m}$  and frequency of 2 Hz. The measurement results are listed in Table 1(III). The PZT was then driven by an amplitude of 1  $\mu\text{m}$  at frequency of 2 Hz, and the vibration frequency of the loudspeaker with a signal amplitude of 1  $\mu\text{m}$  was driven invariably. The measurement results are listed in Table 1(III).



**Fig. 11.** Experimental results for SMI with a dual-external-cavity structure when,  $A_1 = 1\mu\text{m}$ ,  $A_2 = 1\mu\text{m}$ ,  $f_1 = 2\text{Hz}$ , and  $f_2 = 40\text{Hz}$ . (a) SMI signal, and (b) frequency spectrum.

The measured frequencies are consistent with the set frequencies, and the measured amplitudes show a favorable agreement with the set amplitudes. However, the measured amplitudes show slight errors that may be caused by numerical quantification and electronic noise [22].

## 5. Discussion

This paper uses a single laser to achieve dual-channel vibration measurement. In theory, when the frequency of the measured vibration is quite different, this method can be used to measure more than two targets, and is not limited to dual-channel measurement. In the measurement, the intervention of the speckle random signal will cause complicated interference in the time domain analysis. In this paper, by converting the time domain analysis into a simpler and more intuitive frequency domain analysis, it is more robust. This is because in the time domain analysis, due to the intervention of speckle random signals, there is too much interference in the time domain, which is very complicated. However, studying the frequency components of the signal in the frequency domain reduces the time complexity in the signal analysis and processing process, is more robust, and is less affected by the speckle signal.

In actual industrial production, this technology can be developed as a two-dimensional sensor, which has a wide range of application prospects, such as the detection of horizontal and vertical axis of pipeline leaks, the multi-directional simultaneous measurement of stones in the human body in medical field, and the detection of underground oil in oil and gas development sector.

## 6. Conclusions

In this study, a new SMI system with a dual-external-cavity structure is developed to be used with a semiconductor laser, and an improved algorithm for dominant and secondary harmonic order determination is proposed based on FT. A series of simulations and experiments were also performed to verify the feasibility of the proposed structure and the effectiveness of the algorithm. The results of these simulations and experiments demonstrated that the algorithm could measure two vibrations robustly by using only a single detector and a single laser.

## Declaration of Competing Interest

The authors declared that they have no conflicts of interest to this work. We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

## CRedit authorship contribution statement

**Xiangyu Cui:** Conceptualization, Methodology, Software, Data curation, Writing - original draft. **Yuwei Liu:** Software, Validation. **Peng Chen:** Supervision. **Chunsheng Li:** Visualization, Investigation, Writing - review & editing.

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