

Arseniy Terekhov simpletondl@yandex.ru Saint Petersburg State University St. Petersburg, Russia	Vlada Pogozhelskaya pogozhelskaya@gmail.com Saint Petersburg State University St. Petersburg, Russia	Vadim Abzalov !!!@!!! Saint Petersburg State University St. Petersburg, Russia
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Semyon Grigorev  
s.v.grigoriev@spbu.ru  
semyon.grigorev@jetbrains.com  
Saint Petersburg State University  
St. Petersburg, Russia  
JetBrains Research  
St. Petersburg, Russia

A long time ago in a galaxy far far away... Abstract is very abstract.  
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Language-constrained path querying [2] is a way to find paths in edge-labeled graphs when constraints are formulated in terms of language which restrict words formed by paths: the word formed by path’s labels concatenation should be in the specified language. This way is very natural for navigational queries in graph databases, and one of the most popular languages which are used for constraints is a regular language. But in some cases, regular languages are not expressive enough, as a result, context-free languages gain popularity. Constraints in the form of context-free languages, or context-free path querying (CFPQ), can be used for RDF analysis [11], biological data analysis [9], static code analysis [8, 12], and in other areas.

In [1] Rustam Azimov propose a matrix-based algorithm for CFPQ. This algorithm is one of promising way to solve the first problem and provide appropriate solution for real-world data analysis, as was shown by Nikita Mishim et al. in [7] and Arseniy Terekhov et al. in [10]. But this algorithm always computes information (reachability facts or single path which satisfies constraints) for all pairs of vertices in the graph, namely it solves *all-pairs* problem. It is unreasonable for some real-world scenarios when one can provide a relatively small set of start vertices or even single start vertex.

While all-pairs context-free path querying is a classical problem that investigates in a number of works, there is no, in our knowledge, solutions for single-source and multiple-source CFPQ. In this work we propose a matrix-based *multiple-source* (and *single-source* as a partial case) CFPQ algorithm.

Also, we provide full-stack support of CFPQ for the RedisGraph<sup>1</sup> [3] graph database. We implement a Cypher query language extension<sup>2</sup> that allows one to express context-free constraints, and extend the RedisGraph to support this extension. In our knowledge, it is the first full-stack implementation of CFPQ.

To summarize, we make the following contribution in this paper.

- (1) We modify Azimov’s matrix-based CFPQ algorithm and provide a multiple-source matrix-based CFPQ algorithm. As a partial case, it is possible to use our algorithm in a single-source scenario. Our modification still based on linear algebra, hence it is simple to implementation and allows one to use high-performance libraries for implementation.
- (2) We evaluate the proposed algorithm. Our evaluation shows that !!!
- (3) We provide full-stack support of CFPQ by extending the RedisGraph graph database. To do it, we extend Cypher with syntax allows one to express context-free constraints, implement the proposed algorithm in a RedisGraph backend, and support new syntax in the RedisGraph query execution engine. Finally, evaluate the poposed solution.

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<sup>1</sup>RedisGraph graph database Web-page: <https://redislabs.com/redis-enterprise/redis-graph/>. Access date: 19.07.2020.

<sup>2</sup>Proposal which describes path patterns specification syntax for Cypher query language: <https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc>. The proposed syntax allows one to specify context-free constraints. Access date: 19.07.2020.

## 2 PRELIMINARIES

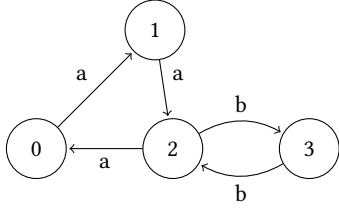
In this section we introduce common definitions in graph theory and formal language theory which will be used in this paper. Also, we provide brief description of Azimov's algorithm which is used as a base of our solution.

### 2.1 Graphs

In this work we use edge-labelled digraph as a data model and define it as follows.

*Definition 2.1.* Edge-labelled Digraph

An example of the graph is presented in figure 1.



**Figure 1: The example of input graph  $\mathcal{G}$**

We use adjacency matrix decomposed to a set of a boolean matrix as a representation of the graph.

*Definition 2.2.* An adjacency matrix  $M$  of the graph  $\mathcal{G} = (V, E)$  is a square  $|V| \times |V|$  matrix, such that  $M[i, j] = \{l \mid e = (i, l, j) \in E\}$ .

Adjacency matrix  $M$  of the graph  $\mathcal{G}$  is

$$M = \begin{pmatrix} \cdot & \{a\} & \cdot & \cdot \\ \cdot & \cdot & \{a\} & \cdot \\ \{a\} & \cdot & \cdot & \{b\} \\ \cdot & \cdot & \{b\} & \cdot \end{pmatrix}.$$

*Definition 2.3.* Boolean decomposition of adjacency matrix  $M$  of graph  $\mathcal{G}$  is set of Boolean matrix

$$\mathcal{M} = \{M^l \mid l \in L, M^l[i, j] = 1 \iff l \in M[i, j]\}.$$

Matrix  $M$  can be represented as a set of two Boolean matrices  $M^a$  and  $M^b$  where

$$M^a = \begin{pmatrix} \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}, M^b = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot \end{pmatrix} \quad (1)$$

### 2.2 Languages

Grammars, normal forms, ...

*Definition 2.4.* Grammar

*Definition 2.5.* Language

### 2.3 Matrix-Based Algorithm

Let  $D = (V, E)$  be the input graph and  $G = (N, \Sigma, P, S)$  be the input grammar. For a given graph and a context-free grammar, we define *context-free relations*  $R_A \subseteq V \times V$  for every  $A \in N$ , such that  $R_A = \{(n, m) \mid \exists n\pi m (l(\pi) \in L(G_A))\}$ . For the context-free path query evaluation, we must provide such a path for each node pair from  $R_A$ .

The matrix-based algorithm for CFPQ can be expressed in terms of operations over Boolean matrices (see listing 1) which is an advantage for implementation.

Comments and conclusion

#### Listing 1 Context-free path querying algorithm

```

1: function EVALCFPQ( $D = (V, E), G = (N, \Sigma, P)$ )
2:    $n \leftarrow |V|$ 
3:    $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i} \text{ is a matrix } n \times n, T_{k,l}^{A_i} \leftarrow \text{false}\}$ 
4:   for all  $(i, x, j) \in E, A_k \mid A_k \rightarrow x \in P$  do  $T_{i,j}^{A_k} \leftarrow \text{true}$ 
5:   for all  $A_k \mid A_k \rightarrow \varepsilon \in P$  do
6:     for all  $i \in \{0, \dots, n-1\}$  do  $T_{i,i}^{A_k} \leftarrow \text{true}$ 
7:   while any matrix in  $T$  is changing do
8:     for  $A_i \rightarrow A_j A_k \in P$  do  $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$ 
9:   return  $T$ 

```

## 3 MATRIX-BASED MULTIPLE-SOURCE CFPQ ALGORITHM

In this section we introduce two versions of multiple-source matrix-based CFPQ algorithm. This algorithm is a modification of Azimov's matrix-based algorithm for CFPQ and the idea is that we cut off those vertices from which we are not interested in paths.

Let  $D = (V, E)$  be the input graph,  $G = (N, \Sigma, P, S)$  be the input grammar and  $Src$  be the input set of vertices. For the multiple-source context-free path query evaluation, we must provide such a path for each selected node pair from  $R_A$ . In other words, for every  $A \in Src$  we want to find all node pairs  $(n, m)$  such that  $\exists n\pi m (l(\pi) \in L(G_A))$ .

#### Listing 2 Multiple-source context-free path querying algorithm

```

1: function MULTISRCFPQ( $D = (V, E), G = (N, \Sigma, P, S), Src$ )
2:    $T \leftarrow \{T^A \mid A \in N, T^A \leftarrow \emptyset\}$   $\triangleright$  Matrix in which every element is  $\emptyset$ 
3:    $TSrc \leftarrow \{TSrc^A \mid A \in N \setminus S, TSrc^A \leftarrow \emptyset\}$   $\triangleright$  Matrix for input vertices in which every element is  $\emptyset$ 
4:   for all  $v \in Src$  do  $\triangleright$  Input matrix initialization
5:      $TSrc_{v,v}^S \leftarrow \text{true}$ 
6:   for all  $A \rightarrow x \in P$  do  $\triangleright$  Simple rules initialization
7:     for all  $(v, x, to) \in E$  do
8:        $T_{v,to}^A \leftarrow \text{true}$ 
9:   while  $T$  or  $TSrc$  is changing do  $\triangleright$  Algorithm's body
10:    for all  $A \rightarrow BC \in P$  do
11:       $M \leftarrow TSrc^A * T^B$ 
12:       $T^A \leftarrow T^A + M * T^C$ 
13:       $TSrc^B \leftarrow TSrc^B + TSrc^A$ 
14:       $TSrc^C \leftarrow TSrc^C + \text{GETDST}(M)$ 
15:   return  $T$ 
16:
17: function GETDST( $M$ )
18:    $A \leftarrow \emptyset$ 
19:   for all  $(v, to) \in V^2 \mid M_{v,to} = \text{true}$  do
20:      $A_{to,to} \leftarrow \text{true}$ 
21:   return  $A$ 

```

The operation of transitive closure calculation from Azimov's algorithm is supplemented with one more matrix multiplication which saves only vertices we are interested in.

It is useful to add data caching to improve the performance of processing large graphs. We provide one more version of algorithm which has memory optimization. It is noticed that every time we want to find all paths from the certain set of vertices, the first version of algorithm calculates everything from

scratch. Since recalculating might take the significant amount of time, the second version is specified for such scenarios. This version stores all the vertices, the paths from which have already been calculated.

**Listing 3** Optimized multiple-source context-free path querying algorithm

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```

1: function    MULTISRCFPQSMART( $index$       =
   ( $D, G, T, TSrc$ ),  $Src$ )
2:    $TNewSrc \leftarrow \{TNewSrc^A \mid A \in N \setminus S, TNewSrc^A \leftarrow \emptyset\}$ 
3:   for all  $v \in Src \mid index.TSrc_{v,v} = false$  do
4:      $TNewSrc_{v,v}^S \leftarrow true$ 
5:   while  $index.T$  or  $TNewSrc$  is changing do
6:     for all  $A \rightarrow BC \in P$  do
7:        $M \leftarrow TNewSrc^A * index.T^B$ 
8:        $index.T^A \leftarrow index.T^A + M * index.T^C$ 
9:        $TNewSrc^B \leftarrow TNewSrc^B + TNewSrc^A \setminus$ 
 $index.TSrc^B$ 
10:       $TNewSrc^C \leftarrow TNewSrc^C + GETDST(M) \setminus$ 
 $index.TSrc^C$ 

```

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### 3.1 Implementation Details

All of the above versions have been implemented<sup>3</sup> using GraphBLAS framework that allows you to represent graphs as matrices and work with them in terms of linear algebra. For convenience, all the code is written in Python using pygraphblas<sup>4</sup>, which is Python wrapper around GraphBLAS API and based on SuiteSparse:GraphBLAS<sup>5</sup> [4] — the full implementation of GraphBLAS standart. This library is specialized for working with sparse matrices, which most often appear in real graphs.

### 3.2 Algorithm Evaluation

And comparison. With combinators, GLL (.NET version).

Evaluation setup. Hardware basic description.

Graphs and queries from CFPQ\_Data<sup>6</sup> Graphs and queries description: #V, #E, types of queries.

Tables.

Graphics (boxes). 1,2,4,8,16,32,50,100,500,1000,5000

Results.

Conclusion.

## 4 CFPQ FULL-STACK SUPPORT

In order to provide full-stack support of CFPQ it is necessary to choose an appropriate graph database. It was shown by Arseniy Terekhov et al. in [10] that matrix-based algorithm can be naturally integrated into RedisGraph graph database because both, the algorithm and the database, operates over matrix representation of graphs. Moreover, RedisGraph supports Cypher as a query language and there is a proposal which describes Cypher extension which allows one to specify context-free constraints. Thus we choose RedisGraph as a base for our solution.

### 4.1 Cypher Extending

The first what we should do is to extend Cypher to be able to express context-free constraints. There is a description of the respective Cypher syntax extension<sup>7</sup>, proposed by Tobias Lindaa, but this syntax does not implement yet in Cypher parsers.

RedisGraph database supports subset of Cypher language and useses libcypher-parser<sup>8</sup> library to parse queries. We extend this library by introducing new syntax proposed !!! We implement<sup>9</sup> full extension, not only part which is necessary for simple CFPQ.

Main feature which allows one to specify context-free constraints is a *named path patterns*: one can specify a name for pattern and after that use it in other patterns, or in the same pattern. Using this feature, structure of query is pretty similar to context-free grammar. For example !!!

Examples of queries. Description of examples.

### 4.2 RedisGraph Extending

CFPQ to matrix expressions, etc. General schema of integration.

Limits, restrictions, examples, etc.

### 4.3 Evaluation

Small basic evaluation on real-world graph (geo?). In order to show, that performance is reasonable.

Regular queries. Comparison with other DB?

## 5 CONCLUSION

In this paper we propose a number of multiple-source modifications of Azimov's CFPQ algorithm. Evaluation of the proposed modifications on the real-world examples shows that !!!! Finally, we provide the full-stack support of CFPQ. For our solution we implement corresponding Cypher extension as a part of libcypher-parser, integrate the proposed algorithm into RedisGraph, and extend RedisGraph execution plan builder to support extended Cypher queries. We demonstrate, that our solution allows one evaluate not only context-free queries, but also regular one.

In the future, it is necessary to provide formal translation of Cypher to linear algebra, or find a maximal subset of Cypher which can be translated to linear algebra. There is a number of work on a subset of SPARQL to linear algebra translation, such as [?], but they are very limited. Deep investigation of this topic helps one to realize limits and restrictions of linear algebra utilization for graph databases. Moreover, it helps to improve existing solutions.

We show that evaluation of regular queries is possible in practice by using CFPQ algorithm, as far as regular queries is a partial case of the context-free one. But it seems, that the proposed solution is not optimal. For real-world solutions it is important to provide an optimal unified algorithm for both RPQ and CFPQ. One of possible way to solve this problem is to use tensor-based algorithm [?].

Another important task is to compare non-linear-algebra-based approaches to multiple-source CFPQ with the proposed solution. In [?] Jochem Kuyper et.al. shows that all-pairs CFPQ algorithms

<sup>3</sup>GitHub repository with implemented algorithms: [https://github.com/JetBrains-Research/CFPQ\\_PyAlgo](https://github.com/JetBrains-Research/CFPQ_PyAlgo), last accessed 28.08.2020

<sup>4</sup>GitHub repository of PyGraphBLAS library: <https://github.com/michelp/pygraphblas>

<sup>5</sup>GitHub repository of SuiteSparse:GraphBLAS library: <https://github.com/DrTimothyAldenDavis/SuiteSparse>

<sup>6</sup>!!!

<sup>7</sup>Formal syntax specification: <https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc#11-syntax>. Access date: 19.07.2020.

<sup>8</sup>The libcypher-parser is an open-source parser library for Cypher query language. GitHub repository of the project: <https://github.com/cleishm/libcypher-parser>. Access date: 19.07.2020.

<sup>9</sup>The modified libcypher-parser library with support of syntax for path patterns: <https://github.com/YaccConstructor/libcypher-parser>. Access date: 19.07.2020.

implemented in Neo4j demonstrate unreasonable performance on real-world data for Neo4j. At the same time, Arseniy Terekhov et.al. shows that matrix-based all-pairs CFPQ algorithm implemented in appropriate linear algebra based graph database (RedisGraph) demonstrates good performance. But in the case of multiple-source scenario, when a number of sources is relatively small, non-linear-algebra-based solutions can be better, because such solutions naturally handle small required subgraph.

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