Rytter for CFPQ

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1 Linear input

Let the input grammar is

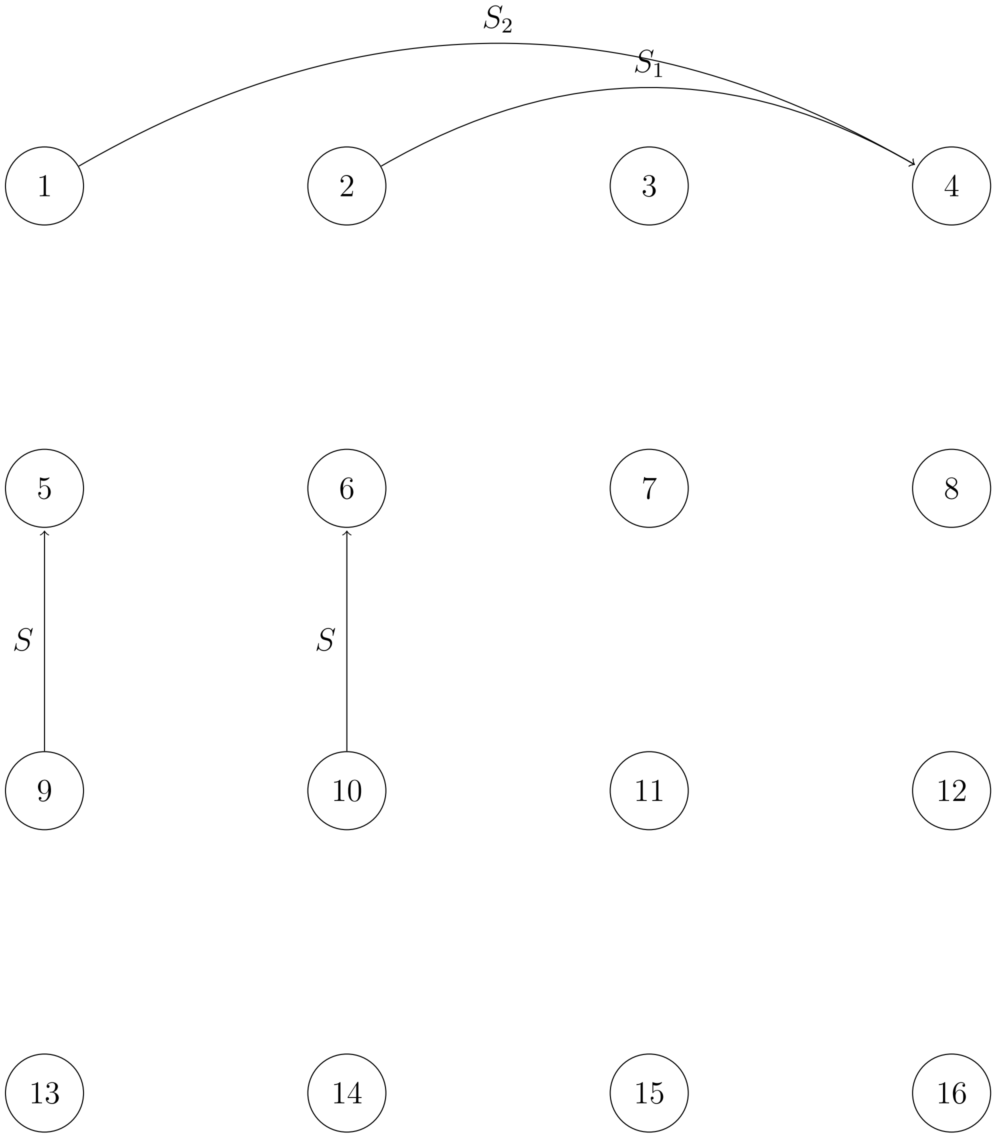
$$S \to a\ S\ b$$
$$S \to S\ S$$
$$S \to a\ b$$

The input grammar in CNF is

$$
\begin{aligned}
S &\to A\ S_1 \\
S_1 &\to S\ B \\
S &\to S\ S \\
S &\to A\ B \\
A &\to a \\
B &\to b
\end{aligned}
$$

Input: *abab*

Grid:



2 Graph input

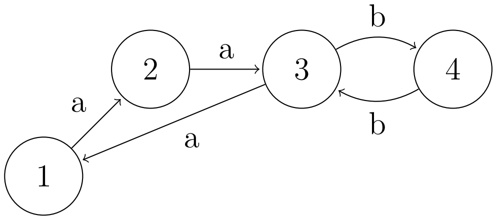
Let the input grammar is

$$
\begin{aligned}
S &\to a\ S\ b \\
S &\to a\ b
\end{aligned}
$$

The input grammar in CNF is

$$
\begin{aligned}
S &\to A\ S_1 \\
S_1 &\to S\ B \\
S &\to A\ B \\
A &\to a \\
B &\to b
\end{aligned}
$$

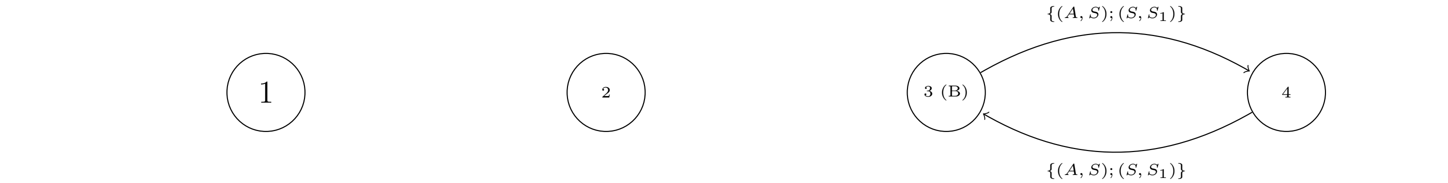
Let the input graph is

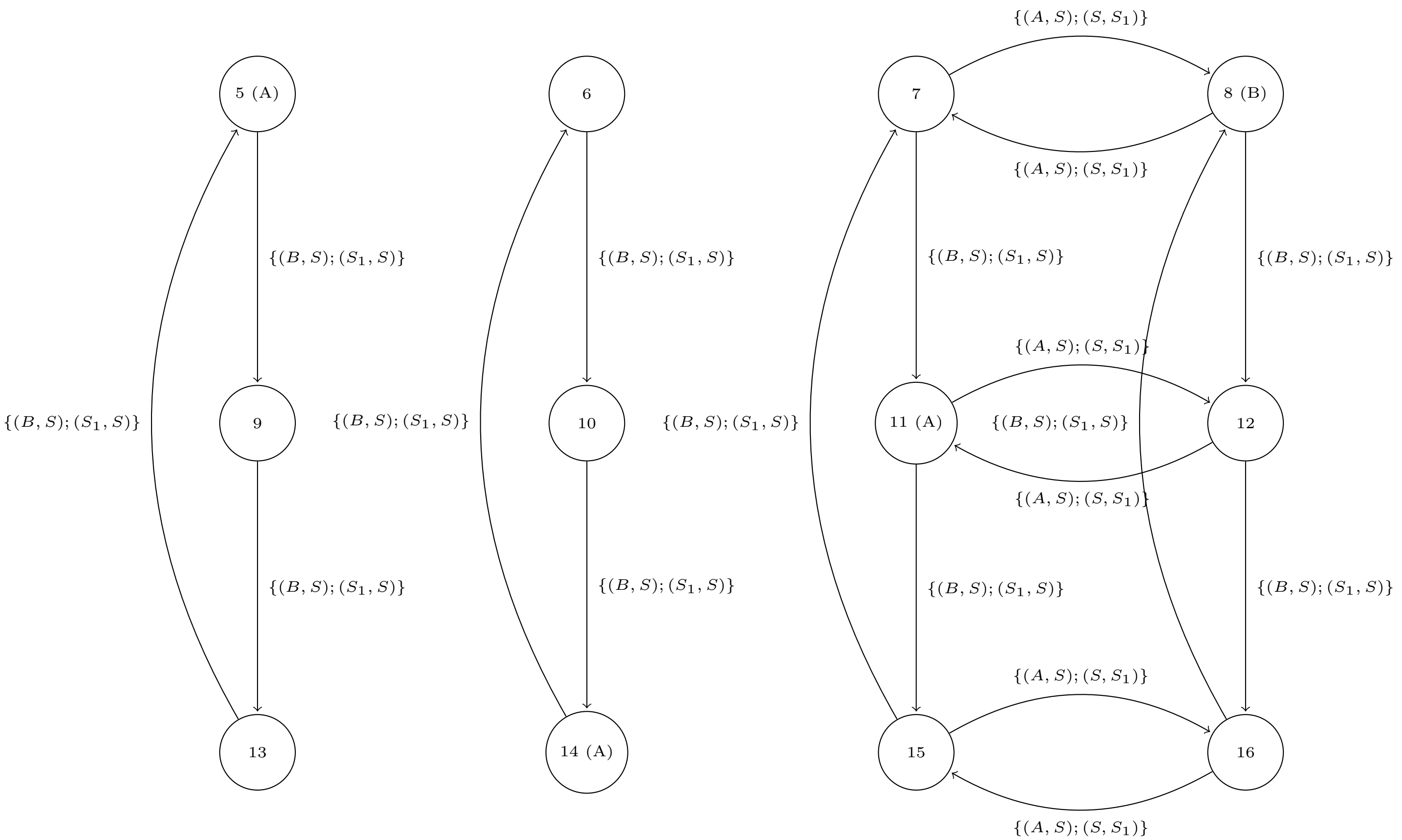


The *IMPLIED* relation:

$$
\begin{array}{llll}
(B,2,3) \Rightarrow (S,1,3) & (B,2,4) \Rightarrow (S,1,4) & (B,2,2) \Rightarrow (S,1,2) & (B,2,1) \Rightarrow (S,1,1) \\
(B,3,4) \Rightarrow (S,2,4) & (B,3,3) \Rightarrow (S,2,3) & (B,3,2) \Rightarrow (S,2,2) & (B,3,1) \Rightarrow (S,2,1) \\
(B,1,2) \Rightarrow (S,3,2) & (B,1,3) \Rightarrow (S,3,3) & (B,1,4) \Rightarrow (S,3,4) & (B,1,1) \Rightarrow (S,3,1) \\
(S_1,2,3) \Rightarrow (S,1,3) & (S_1,2,4) \Rightarrow (S,1,4) & (S_1,2,2) \Rightarrow (S,1,2) & (S_1,2,1) \Rightarrow (S,1,1) \\
(S_1,3,4) \Rightarrow (S,2,4) & (S_1,3,3) \Rightarrow (S,2,3) & (S_1,3,2) \Rightarrow (S,2,2) & (S_1,3,1) \Rightarrow (S,2,1) \\
(S_1,1,2) \Rightarrow (S,3,2) & (S_1,1,3) \Rightarrow (S,3,3) & (S_1,1,4) \Rightarrow (S,3,4) & (S_1,1,1) \Rightarrow (S,3,1) \\
(A,2,3) \Rightarrow (S,2,4) & (A,1,3) \Rightarrow (S,1,4) & (A,3,3) \Rightarrow (S,3,4) & (A,4,3) \Rightarrow (S,4,4) \\
(A,3,4) \Rightarrow (S,3,3) & (A,4,4) \Rightarrow (S,4,3) & (A,2,4) \Rightarrow (S,2,3) & (A,1,4) \Rightarrow (S,1,3) \\
(S,2,3) \Rightarrow (S_1,2,4) & (S,1,3) \Rightarrow (S_1,1,4) & (S,3,3) \Rightarrow (S_1,3,4) & (S,4,3) \Rightarrow (S_1,4,4) \\
(S,3,4) \Rightarrow (S_1,3,3) & (S,4,4) \Rightarrow (S_1,4,3) & (S,2,4) \Rightarrow (S_1,2,3) & (S,1,4) \Rightarrow (S_1,1,3)
\end{array}
$$

Grid:

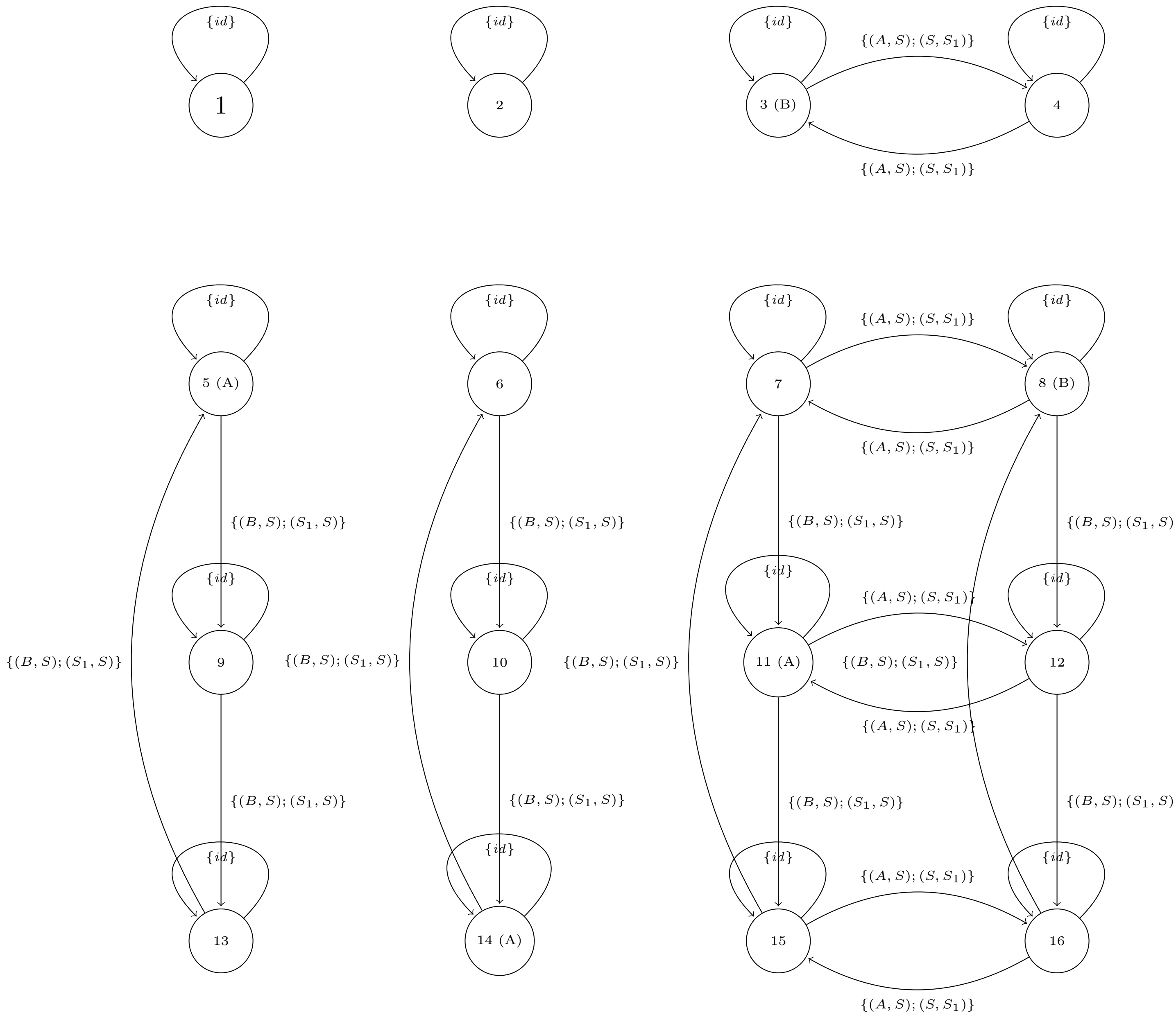




We should introduce the id implication such that for every $A \in \text{IMPLIED}$

- $id \times A = A \times id$

In order to compute transitive closure in logarithmic time we add self-loop with weight $\{id\}$ to each vertex.



Note that our graph is a Cartezian product of tho graph H and V with respective matrices.
$H =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(A,S);(S,S_1)\} \\ \varnothing & \varnothing & \{(A,S);(S,S_1)\} & \{id\} \end{pmatrix}$$

$V =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \{(B,S);(S_1,S)\} & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(B,S);(S_1,S)\} \\ \varnothing & \{(B,S);(S_1,S)\} & \varnothing & \{id\} \end{pmatrix}$$

Matrix of $G = V \otimes I + I \otimes H$ where I is identity matrix of size $n \times n$ and $\otimes$ is a Kronecker product.

One step is APSP (or transitive closure) of G. It can be computed as $(V \otimes I + I \otimes H)^{(n^2)}$. It can be "over approximated" as $M = (V^{(n^2)} \otimes I + V^{(n^2)} \otimes H^{(n^2)} + I \otimes H^{(n^2)})$. Now we should check validity of nonterminals. It can be don by multiplication of vector x and M. $x * (V^{(n^2)} \otimes I + V^{(n^2)} \otimes H^{(n^2)} + I \otimes H^{(n^2)}) = x * V^{(n^2)} \otimes I + x * V^{(n^2)} \otimes H^{(n^2)} + x * I \otimes H^{(n^2)}$. It is known that $(B \otimes C) * \mathrm{vec}(X) = \mathrm{Y} \equiv C * X * B^T = Y$. Hence $\mathrm{vec}(X) * (B \otimes C) = \mathrm{Y} \equiv C^T * X^T * B = Y$. As a result, we can compute distance matrix as $I^T * X * V^{(n^2)} + (H^{(n^2)})^T * X * V^{(n^2)} + (H^{(n^2)})^T * X * I$.

$H^2 =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \{id; (A, S_1)\} & \{(A, S); (S, S_1)\} \\ \varnothing & \varnothing & \{(A, S); (S, S_1)\} & \{id; (A, S_1)\} \end{pmatrix}$$

$H^4 = H^2$

$(H^2)^T =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \{id; (A, S_1)\} & \{(A, S); (S, S_1)\} \\ \varnothing & \varnothing & \{(A, S); (S, S_1)\} & \{id; (A, S_1)\} \end{pmatrix}$$

$V^2 =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \{(B, S); (S_1, S)\} & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(B, S); (S_1, S)\} \\ \varnothing & \{(B, S); (S_1, S)\} & \varnothing & \{id\} \end{pmatrix}$$

$V^4 = V^2$

$X =$

$$\begin{pmatrix} \varnothing & \varnothing & \{(\perp, B)\} & \varnothing \\ \{(\perp, A)\} & \varnothing & \varnothing & \{(\perp, B)\} \\ \varnothing & \varnothing & \{(\perp, A)\} & \varnothing \\ \varnothing & \{(\perp, A)\} & \varnothing & \varnothing \end{pmatrix}$$

$X^T =$

$$\begin{pmatrix} \varnothing & \{(\perp, A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp, A)\} \\ \{(\perp, B)\} & \varnothing & \{(\perp, A)\} & \varnothing \\ \varnothing & \{(\perp, B)\} & \varnothing & \varnothing \end{pmatrix}$$

$X^T * V^2 =$

$$\begin{pmatrix} \varnothing & \{(\perp, A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp, A)\} \\ \{(\perp, B)\} & \varnothing & \{(\perp, A)\} & \varnothing \\ \varnothing & \{(\perp, B)\} & \{(\perp, S)\} & \varnothing \end{pmatrix}$$

$(H^2)^T * X^T =$

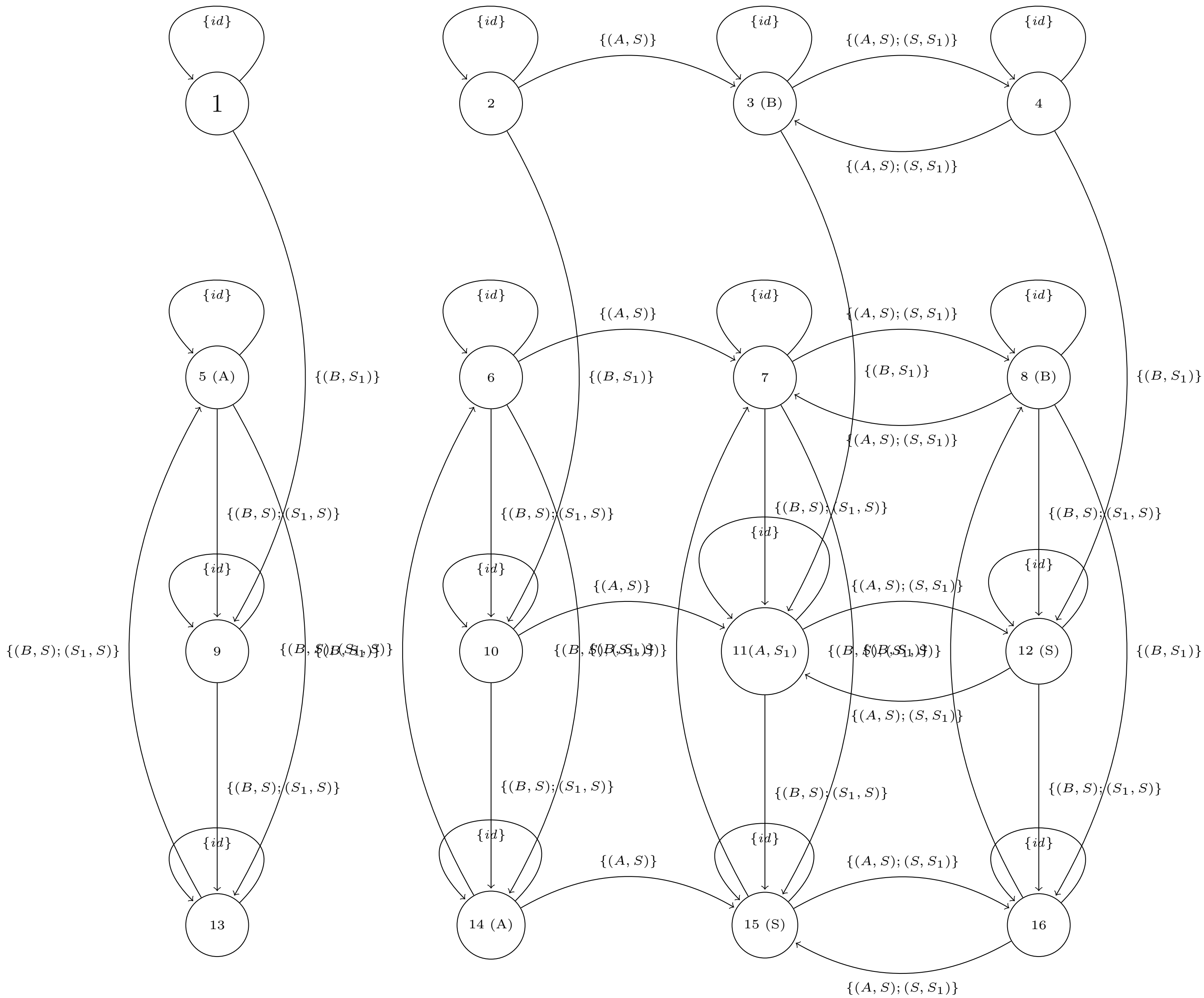
$$\begin{pmatrix} \varnothing & \{(\perp, A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp, A)\} \\ \{(\perp, B)\} & \varnothing & \{(\perp, A); (\perp, S_1)\} & \varnothing \\ \varnothing & \{(\perp, B)\} & \{(\perp, S)\} & \varnothing \end{pmatrix}$$

$(H^2)^T * X^T * V^2 =$

$$\begin{pmatrix} \varnothing & \{(\perp, A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp, A)\} \\ \{(\perp, B)\} & \varnothing & \{(\perp, A); (\perp, S_1)\} & \{(\perp, S)\} \\ \varnothing & \{(\perp, B)\} & \{(\perp, S)\} & \varnothing \end{pmatrix}$$

$(X^T * V^2 + (H^2)^T * X^T * V^2 + (H^2)^T * X^T)^T =$

$$\begin{pmatrix} \varnothing & \varnothing & \{(\perp, B)\} & \varnothing \\ \{(\perp, A)\} & \varnothing & \varnothing & \{(\perp, B)\} \\ \varnothing & \varnothing & \{(\perp, A); (\perp, S_1)\} & \{(\perp, S)\} \\ \varnothing & \{(\perp, A)\} & \{(\perp, S)\} & \varnothing \end{pmatrix}$$



$H =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \{(A,S)\} & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(A,S);(S,S_1)\} \\ \varnothing & \varnothing & \{(A,S);(S,S_1)\} & \{id\} \end{pmatrix}$$

$V =$

$$\begin{pmatrix} \{id\} & \varnothing & \{(B,S_1)\} & \varnothing \\ \varnothing & \{id\} & \{(B,S);(S_1,S)\} & \{(B,S_1)\} \\ \varnothing & \varnothing & \{id\} & \{(B,S);(S_1,S)\} \\ \varnothing & \{(B,S);(S_1,S)\} & \varnothing & \{id\} \end{pmatrix}$$

References

[1] Krishnendu Chatterjee, Bhavya Choudhary, and Andreas Pavlogiannis. 2017. *Optimal Dyck reachability for data-dependence and alias analysis*. Proc. ACM Program. Lang. 2, POPL, Article 30 (December 2017), 30 pages. DOI: https://doi.org/10.1145/3158118