

# Parser Combinators for Context-Free Path Querying

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## ABSTRACT

A transparent integration of a domain-specific language for specification of context-free path queries (CFPQs) into a general-purpose programming language as well as static checking of errors in queries may greatly simplify the development of applications using CFPQs. LINQ and ORM can be used for the integration, but they have issues with flexibility: query decomposition and reusing of subqueries are a challenge. Adaptation of parser combinators technique for paths querying may solve these problems. Conventional parser combinators process linear input and only the Trails library is known to apply this technique for path querying. Trails suffers the common parser combinators issue: it does not support left-recursive grammars and also experiences problems in cycles handling. We demonstrate that it is possible to create general parser combinators for CFPQ which support arbitrary context-free grammars and arbitrary input graphs. We implement a library of such parser combinators and show that it is applicable for realistic tasks.

## CCS CONCEPTS

• **Information systems** → **Graph-based database models**; **Query languages for non-relational engines**; • **Software and its engineering** → *Functional languages*; • **Theory of computation** → *Grammars and context-free languages*;

## KEYWORDS

Graph Databases, Language-Constrained Path Problem, Context-Free Path Querying, Context-Free Language Reachability, Parser Combinators, Generalized LL, GLL, Neo4j, Scala

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## 1 INTRODUCTION

Graph querying is finding all paths in the graph which satisfy some constraints. If the constraints are specified with some language formalism, i.e. a grammar, it is called a language-constrained path query. The simplest query described with a grammar  $S \rightarrow a b$  being run against the graph in the Fig. 1 returns the only path 2, 3, 4 (shown in red).

A grammar  $S \rightarrow a S b \mid a b$  is a query for the paths of the form  $a^n b^n$ , where  $n \geq 1$ . Querying the graph in the Fig. 1 returns the infinite set of paths one of which starts and ends in the vertex 3 and goes around the cycles in the graph the appropriate number of times: 3, 1, 2, 3, 1, 2, 3, 4, 3, 4, 3, 4, 3, 4, 3.

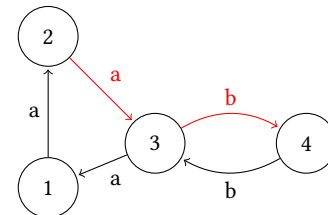


Figure 1: An example of input graph

Most existing graph traversing/querying languages, including SPARQL [19], Cypher<sup>1</sup>, and Gremlin [24] support only regular languages as constraints. For some applications regular languages are not expressive enough. Context-free path queries (CFPQ), on which we focus this paper, employ context-free languages for constraints specification. CFPQs are used in bioinformatics [28], static code analysis [2, 18, 21, 30, 34], and RDF processing [33]. Although there is a lot of problem-specific solutions and theoretical research on CFPQs [1, 6–8, 16, 23, 28, 31], cfSPARQL [33] is the single known graph query language to support CF constraints. Generic solution for the integration of CFPQs into general-purpose languages is not discussed enough.

When developing a data-centric application, one wants to use a general-purpose programming language and also to have a transparent and native access to data sources. One way to achieve this is to use string-embedded DSLs. In this approach, a query is written as a string, then passed on to a dedicated driver which executes it and returns a possibly untyped result. Despite the simplicity,

<sup>1</sup>Cypher language web page: <https://neo4j.com/developer/cypher-query-language/>. Access date: 16.01.2018

string-embedded DSLs have serious drawbacks. First of all, they require the developer to learn the language itself, its features, runtime and how the integration between the languages is implemented. DSLs are also a source of possible errors and vulnerabilities, static detection of which is a serious challenge [3]. Such techniques as the Object Relationship Mapping (ORM) or Language Integrated Query (LINQ) [4, 13, 15] partially solve these problems, but they still have issues with flexibility: both query decomposition and reusing of subqueries are a struggle. In this paper, we propose a transparent and natural integration of CFPQs into a general-purpose programming language.

Context-free path queries are known in various domains under different names. The *context-free language reachability framework* or *IFDS framework* is how they are called in the area of static code analysis. In [21, 22] Thomas Reps shows that the wide range of static code analysis problems can be formulated in terms of CFL-reachability in the graph. This framework is used for such problems as the taint analysis [9], the alias analysis [29, 30, 34], the label flow analysis [18], and the fix locations problem [5]. What we propose in the paper can be viewed as a core of such framework since it provides both problem and domain independent mechanism for CFPQ evaluation.

We view parser combinators as the best way to integrate context-free language specifications into a general-purpose programming language. Parser combinators not only provide a transparent integration but also compile-time checks of correctness and high-level techniques for generalization. An idea to use combinators for graph traversing has already been proposed in [12]. Unfortunately, the solution presented processes cycles in the input graph only approximately and is unable to handle left-recursive combinators, which is the most common issue of the approach. Authors pointed out that the idea described is similar to the classical parser combinators, but the language class supported or restrictions are not discussed.

Parser combinators are known to handle only a subset of context-free grammars: left recursion and ambiguity of the grammars are problematic. In [10], authors demonstrate a set of parser combinators which handles arbitrary context-free grammars by using ideas of the Generalized LL [25] algorithm (GLL). Meerkat<sup>2</sup> parser combinators library implements the ideas from the paper [10] and provides the parsing result in a compact form as a Shared Packed Parse Forest [20] (SPPF). SPPF is a suitable finite structural representation of a CFPQ result, even when the set of paths is infinite [6]. All the paths can be extracted from the SPPF—as the corresponding derivation trees—and further analysis can be done. It is also possible to run some further processing over the SPPF itself—without explicit paths extraction.

In this paper, we compose these ideas and present a set of parser combinators for context-free path querying which handle arbitrary context-free grammars and provide a structural representation of the result. We make the following contributions in the paper.

- (1) We show that it is possible to create a set of parser combinators for context-free path querying which work on both arbitrary context-free grammars and arbitrary graphs and provide a finite structural representation of the query result.

- (2) We implement the parser combinators library in Scala. This library provides an integration to Neo4j<sup>3</sup> graph database. The source code is available on GitHub: <https://github.com/YaccConstructor/Meerkat>.
- (3) We perform an evaluation on realistic data and compare the performance of our library with another GLL-based CFPQ tool and with the Trails library. We conclude that our solution is expressive and performant enough to be applied to the real-world problems.

This paper is organized as follows. We introduce a formal definition of the CFPQ problem in section 2 and we provide a basic description of the Meerkat library and SPPF data structure in section 3. We describe our solution in section 4. In section 5 we present and discuss a set of classical queries (the same generation query, the queries to a movie dataset<sup>4</sup>) formulated in terms of our library. Evaluation of the library is described in section 6. Finally, In section 7 we conclude and discuss possible directions for further research.

## 2 CONTEX-FREE PATH QUERYING PROBLEM

In this section we formally describe the context-free path querying problem and the context-free reachability problem.

First, we introduce the necessary definitions.

- Context-free grammar is a quadruple  $G = (N, \Sigma, P, S)$ , where  $N$  is a set of nonterminal symbols,  $\Sigma$  is a set of terminal symbols,  $S \in N$  is a start nonterminal, and  $P$  is a set of productions.
- $\mathcal{L}(G)$  denotes a language specified by the grammar  $G$ , and is a set of terminal strings derived from the start nonterminal of  $G$ :  $\mathcal{L}(G) = \{\omega \mid S \Rightarrow_G^* \omega\}$ .
- Directed graph is a triple  $M = (V, E, L)$ , where  $V$  is a set of vertices,  $L \subseteq \Sigma$  is a set of labels, and a set of edges  $E \subseteq V \times L \times V$ . Note that there are not parallel edges with equal labels: if  $(v, l_1, u) \in E$  and  $(v, l_2, u) \in E$ , then  $l_1 \neq l_2$ .
- $tag : E \rightarrow L$  is a function which returns a tag of an edge.

$$tag((_, l, _)) = l$$

- $\oplus : L^+ \times L^+ \rightarrow L^+$  denotes a tag concatenation operation.
- $\Omega$  is a helper function which constructs a string produced by the given path. For every  $p$  path in  $M$

$$\Omega(p = e_0, e_1, \dots, e_{n-1}) = tag(e_0) \oplus \dots \oplus tag(e_{n-1}).$$

We define the context-free language constrained path querying as, given a query in the form of a grammar  $G$ , to construct the set of the paths from the input graph  $M$  which are also strings derivable in the grammar  $G$ :

$$\{p \mid p \text{ is path in } M, \Omega(p) \in \mathcal{L}(G)\}.$$

The CFL reachability problem is pretty similar, but here one is only concerned with the existence of the paths derived by the grammar. It is formulated as follows:

$$\{(v_0, v_n) \mid p \text{ is path in } M, p = v_0 \rightarrow \dots \rightarrow v_n, \Omega(p) \in \mathcal{L}(G)\}.$$

<sup>2</sup>Meerkat project repository: <https://github.com/meerkat-parser/Meerkat>. Access date: 16.01.2018

<sup>3</sup>Neo4j graph database site: <https://neo4j.com/>. Access date: 16.01.2018

<sup>4</sup>The movie database is a traditional dataset for graph databases. Detailed description is available here: <https://neo4j.com/developer/movie-database/>. Access date: 16.01.2018

Note that the query result can be an infinite set, hence it cannot be represented explicitly. We show how to construct a compact data structure which stores all the elements of the query result in a finite space; every path can be extracted from this representation.

### 3 GENERALIZED PARSER COMBINATORS

Combinators techniques are shown to be applicable for graph traversing [12], but it still suffers the common issue with left-recursive definitions. A general parser combinators library Meerkat [10], implemented in the Scala programming language, removes this restriction by using memoization, continuation-passing style, and the ideas of Johnson [11]. Basically, Meerkat converts parsers into a memoized versions of themselves. The memoization routine stores parsing results the first time they are computed and reuses them every time they are needed again. Every time a new parsing result for some combinator is calculated, all the combinators, which are dependent on it, are recomputed. This is the way Meerkat employs left-recursive parser combinators and it is also crucial for the handling of the cycles in the input graphs.

Meerkat supports the arbitrary (left-recursive and ambiguous) context-free specifications; it also supports the specification of action codes, and provides a `syn` macro for custom handling of the recursive nonterminal descriptions. Meerkat constructs the compact representation of the parse forest in the form of SPPF, which can be used for CFPQs results representation [6]. The worst case time and space complexity of the solution is cubic.

A Meerkat specification of the language  $\{a^n b^n \mid n \geq 1\}$  is `val S = syn("a" ~ S. ? ~ "b")`. Here `syn` is a macro which creates a parser: for example, it transforms string literals into parsers for those strings. The tilde `~` stands for a sequential parser combinator and the question mark `?` describes optional parsing. Other combinators available in the Meerkat library are shown in Table 1.

It is shown in [6] that Generalized LL parsing algorithm [25] can be generalized to effectively process CFPQs and the query result can be finitely represented. As the Meerkat library is closely related to the Generalized LL algorithm, it is also possible to adapt the Meerkat library for graph querying. It can be done by providing a function for retrieving the symbols which follow the specified position and utilizing it in the basic set of combinators. Details are described in the next section.

#### 3.1 SPPF

Parsing of a string with respect to an ambiguous grammar can result in several derivation trees for a single string. The set of derivation trees is named a *derivation forest*. To store a derivation forest efficiently, the generalized parsing algorithms utilize a *Shared Packed Parse Forest* proposed by Joan Rekers [20]. The most efficient compact representation of derivation forests is a Binarized Shared Packed Parse Forest (we will abbreviate it to SPPF) [27]. The GLL algorithm, which employs this structure, achieves the worst-case cubic space complexity [26].

Binarized SPPF is a directed graph, each node of which has one of the four types described below. Almost every node of the SPPF is decorated with the *extension*: a pair  $(i, j)$  where  $i$  is a start position of a substring derivable from the node and  $j$ —its end position.

- **Terminal node** labelled  $(T, i, j)$ .

- **Nonterminal node** labelled  $(N, i, j)$ . This node denotes that there is at least one derivation  $N \Rightarrow_G^* \omega[i \dots j - 1]$ —a substring of the input from  $i$ -th to  $j$ -th position. Every derivation tree for the given substring and nonterminal can be extracted by left-to-right top-down traversal of SPPF started from the respective node.
- **Intermediate node**: a special kind of node used for the binarization of the SPPF. These nodes are labeled with  $(t, i, j)$ , where  $t$  is a grammar slot.
- **Packed node** labelled  $(N \rightarrow \alpha, k)$ , where  $k$  is a position in the input of the right end of leftmost subtree of this node. A subgraph with the root in such node is in turn a parse forest for which the first production is  $N \rightarrow \alpha$ .

An example of SPPF is presented in Fig. 3. We removed redundant intermediate and packed nodes for simplicity and to decrease the size of the figure.

SPPF finitely represents a possibly infinite set of paths in the context of the language-constrained graph querying [6]. Since the SPPF stores derivation trees for all paths, it is useful for postprocessing and further understanding of the query results. In static code analysis, for example, it is possible to map paths back onto the source code thus providing a human-readable result.

### 4 PARSER COMBINATORS FOR PATH QUERYING

Parser combinators is a way to specify both a language syntax and a parser for it in terms of higher-order functions. Parser in this framework is a function which consumes a prefix of an input and returns either a parsing result or an error if the input is erroneous. Parser combinators compose parsers to form more complex parsers. A parser combinators library usually provides a set of basic parser combinators, such as a combinator of the sequential application or of the choice, but there can also be user-defined combinators. Most parser combinators libraries, including the Meerkat library, can only process the linear input—strings or some kind of streams. We modify the Meerkat library to work on the graph input. The following ideas are at the core of the modification.

The intersection of a context-free and a regular language is context-free. There are several constructive proofs of this fact. The proposed solution is yet another constructive proof with the SPPF as a user-friendly representation of the context-free grammar for the intersection.

Linear input can be regarded as a linear directed graph with symbols of the input labeling the edges. A conventional parser moves a pointer in the input from the position  $i$  to the position  $i + 1$  and creates a new state when a token between the  $i$ -th and the  $i + 1$ -th positions matches what is required in the grammar. In case of graph processing, there are possibly multiple ways to move from the current vertex  $i$  and it is possible to produce multiple new states. Generalized parsing is designed to optimally handle the production of multiple new states thus it is suitable to handle graph processing.

Matching a token in the input can be viewed as a predicate, for example  $p_c(x) = x == c$ . We can generalize this observation allowing matching of an edge label of an arbitrary type with a predicate of some sort. If vertices of the graph contain any data of interest, we can treat them in the similar fashion as the edges.

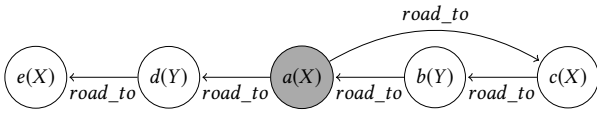
The formal definition of the context-free language constrained path querying changes only in a way a string is collected along a path in the input graph: the data stored in the vertices should also be taken into consideration.

Handling cycles in the input graphs imposes two challenges: not to get stuck in an infinite loop while processing the positions and if a new parsing state appears at some position, all paths which pass through this state, should be accounted for. Both of these challenges are solved by the Meerkat memoization routine. Parsing from a parser state at each position is only run if it has never been run before, so since there is only a finite number of parsing states and positions in the input, parsing terminates. Appearing of the new parsing state at a position which has been processed before triggers processing of every path possibly affected by it. Thanks to the memoization, each derivation of each subpath is analyzed only once, so there are no significant overhead at re-running.

Querying process in our library is inherited from generalized parsing and is done in two steps. The first step is “parsing”: the construction of the SPPF which contains all derivation trees for the paths satisfying syntactic constraints. The second step is semantic actions application which retrieves the necessary additional data about the paths from the SPPF.

#### 4.1 The Set of Combinators

We demonstrate the set of combinators by example: the input graph, which represents a map, is presented in Fig. 2. There are some cities connected by one-way roads represented by the edges labeled *road\_to*. Each city is labeled by its name and a country it belongs to.

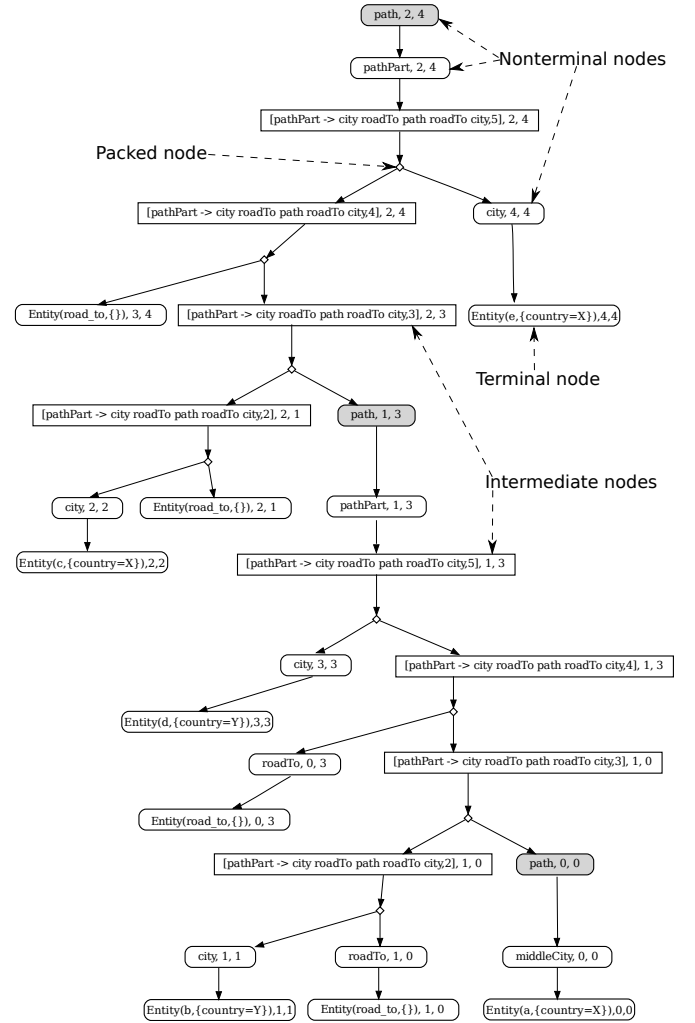


**Figure 2: Example graph. Vertex labels are in the form "city-name (country-name)"**

Two basic building blocks of queries are the combinators for dealing with edges and vertices.

- $V[N](\text{predicate}: N \Rightarrow \text{Boolean})$  the combinator for processing vertices, where  $N$  is a type of the node label. Parsing with this combinator succeeds iff the vertex satisfies the predicate.
- $E[L](\text{predicate}: L \Rightarrow \text{Boolean})$  the combinator for processing edges, where  $L$  is a type of the edge label. Parsing with this combinator succeeds iff the edge satisfies the predicate.

To select cities which belong to some country, we can use the function  $V: V[N]((e: \text{Entity}) \Rightarrow e.\text{country} = \text{"County\_Name"})$ . Here  $\text{Entity}$  is a property container for both edges and vertices. By using the Dynamic trait, all accesses to properties (like  $(e: \text{Entity}).\text{country}$ ) are converted to the accesses to the properties of either a vertex or an edge. For the sake of simplicity, we will omit  $\text{Entity}$  type specifications for predicates. To query the graph for the paths from a city in the country  $X$  to a city in the country  $Y$ , we need to sequentially compose the combinators for selecting the appropriate cities. A sequential combinator  $\sim$  does



**Figure 3: SPPF: result of applying cities query to the graph 2**

just that: it sequentially applies two queries one after the other. Let us denote a query for retrieving a city from the specific country  $\text{city}(\text{name}: \text{String})$  and a query for retrieving road edges  $\text{roadTo}$ . With this denotation, a query  $\text{city}("X") \sim \text{roadTo} \sim \text{city}("Y")$  returns the requested set of paths from the graph. The complete query with all necessary subqueries is shown in Fig. 4.

```
def city(country: String) = V(_.country == country)
val roadTo = E(_.value() == "road_to")
val ourPath = city("X") ~ roadTo ~ city("Y")
```

**Figure 4: Path query**

Having the query written, the next thing is to write a function to retrieve the actual data about the paths. If we care only about the names of the cities, we can return a pair of the cities for each path. First, we modify the query for vertices by adding the semantic action to it using the combinator  $\wedge$ :



`def city(name: String) = syn(V(e.value() == name) ^ (_.value()))5.`  
 Then we need to actually map a path to a pair of cities: this is done with the combinator `&`. The complete query is in Fig. 5. The result is the sequence of pairs of cities with the road between them.

```
def city(country: String) =
  syn(V(_.country() == country) ^ (_.name))
val roadTo = E(_.value() == "road_to")
val ourPath =
  syn(city("X") ~ roadTo ~ city("Y") &
    { case c0 ~ c1 => (c0, c1) })
```

Figure 5: Path query

The whole set of basic combinators, which our library provides, is presented in Table 1. There are two kinds of combinators: the first kind combines parsers to form new parsers, meanwhile the second one is dedicated to the processing of the query result. Whenever a string is used within a query, a parser which matches that string is implicitly generated.

Combinator	Description
<code>a ~ b</code>	sequential parsing: a then b
<code>a   b</code>	choice: a or b
<code>a ?</code>	optional parsing: a or nothing
<code>a *</code>	repetition of zero or more a
<code>a +</code>	repetition of at least one a
<code>a ^ f</code>	apply f function to a if a is a token
<code>a ^^</code>	capture output of a if a is a token
<code>a &amp; f</code>	apply f function to a if a is a parser
<code>a &amp;&amp;</code>	capture output of a if a is a parser

Table 1: Meerkat combinators

## 4.2 Generic Interface for Input

The combinators in our library are independent of the input representation. It is enough to specify two basic combinators which handle vertices and edges. Vertex handling is checking whether the vertex satisfies the given predicate. In case of the edges, one needs to check which of the edges outgoing from the given vertex satisfy the given predicate. These two functions form the trait for the input (Fig. 6). It has two type parameters: the type of edge labels `L` and the type of vertices labels `N`. Since the required functions are simple, we believe it is possible to support most storages of graph-structured data. We supported several different input sources:

- `Neo4jInput` — input source for the graph database Neo4j;
- `GraphxInput` — input source for the graph presented in memory using GraphX library;
- `LinearInput` — input source for the linear input data such as the ordinary strings.

<sup>5</sup>Underscore in anonymous lambda functions serves as a placeholder for an argument: `(_.f)` is equivalent to `(x => x.f)`

```
trait Input[+L, +N] {
  def filterEdges(nodeId: Int,
    predicate: L => Boolean): Seq[(L, Int)]
  def checkNode(nodeId: Int,
    predicate: N => Boolean): Option[N]
}
```

Figure 6: Generalized input interface

## 4.3 Semantic Actions

Every path, which the query produces, has a derivation tree stored in the SPPF. The derivation tree is a very rich structure which can be hard to understand. To retrieve the data actually useful for the user, the library provides a mechanism of semantic actions. It is a way to apply some function to a parsed token or a subsequence.

Semantic action binder for the tokens—vertices and edges alike—is `^`. The most common use for it is to extract properties from the token and combine them in some fashion.

```
// Defined in Terminal[+L] (edge) parser
def ^[U](f: L => U) =
  new SymbolWithAction[L, Nothing, U] {...}
```

```
// Defined in Vertex[+N] parser
def ^[U](f: N => U) =
  new SymbolWithAction[Nothing, N, U] {...}
```

For the combination of parsers there is a `&` binder. Being applied to a sequence of tokens, it can collect and process the data returned by the terminal parsers.

```
// Defined in Symbol[+L, +N, +V] parser
def &[U](f: V => U) =
  new SymbolWithAction[L, N, U] {...}
```

They both produce a new parser that parses the input exactly like the given parser but also have a bound function. The function is referenced in each SPPF node produced by the corresponding parser.

The way semantic actions are executed has mostly remained the same as in the original Meerkat library: semantic actions are first executed for the children of the current node, then the results are collected and passed to a semantic action of the current node. If there are ambiguous nodes in the SPPF, the original Meerkat library just throws an exception. In our case, ambiguity can arise not only when there are multiple derivations of a string but ambiguous nodes can also represent several different subpaths which are derived from the respective nonterminal. We chose to provide a way to extract the derivation trees from the SPPF lazily, since the number of the paths can be infinite. Unambiguous trees are yielded with a breadth-first search.

The composition of the extraction of trees and the semantic action execution is called `executeQuery`. It parses the input graph from all positions, produces a list of SPPF roots, extracts all derivations from every root, executes semantic actions and returns a lazy stream of results.

## 5 EXAMPLES

In this section, we describe some examples of queries written with the library proposed. We show that the combinators are expressive enough for realistic queries and also ease their creation.

### 5.1 Complicated Query to Map

We would like to search for all the routes which pass through a fixed city  $a$  such that the countries of the cities on the route form a palindrome. This means that if a route starts at the city of the country  $X$ , then the last visited city should also belong to the country  $X$ . We also demand that the fixed city  $a$  is in the middle of the route. This query can be written as shown in Fig. 7. A combinator `reduceChoice` reduces a list of queries to a single query by means of the combinator `|`. The implementation of the `reduceChoice` is straightforward and can be found in Fig. 8. A subquery `middleCity` matches the fixed city  $a$  and a query `roadTo` matches a `roadTo` edge.

```
val countriesList = List("X", "Y")
val path =
  (reduceChoice(countriesList.map(pathPart)) |
   middleCity)
def pathPart(country: String) =
  syn(city(country) ~ roadTo ~ path ~
      roadTo ~ city(country))

val middleCity = V(_.value() == "a")
val roadTo = E(_.value() == "road_to")
def city(country: String) = V(_.country == country)
```

Figure 7: Path query

```
def reduceChoice(xs: List[Nonterminal]) =
  xs match {
    case x :: Nil => x
    case x :: y :: xs =>
      syn(xs.foldLeft(x | y)(_ | _))
  }
```

Figure 8: Reduce choice function implementation

```
val middleCity =
  syn(V(_.value() == "a") ^^) & (List(_))
def pathPart(country: String) = syn(
  (city(country) ~ roadTo ~
   path ~ roadTo ~ city(country) & {
     case a ~ (b: List[_]) ~ c => a +: b +: c})
```

Figure 9: Fixed queries

To filter out all the data but the list of the cities on the route, we can add the semantic actions as shown in Fig. 9

Executing the query for the graph in Fig. 2 returns the only three routes, which satisfy our restrictions.

- single-vertex path  $a$ ;
- $b \rightarrow a \rightarrow d$
- $c \rightarrow b \rightarrow a \rightarrow d \rightarrow e$

A simplified SPPF for this query is presented in Fig. 3. Rounded rectangles represent nonterminals and other rectangles represent productions. Every rectangle vertex contains a nonterminal name or a production rule, as well as the start and the end nodes in the input graph for the path derived from the corresponding SPPF vertex. The start nonterminals are drawn in grey.

### 5.2 Same Generation Query

The generalization of the classical same generation query benefits from utilizing the first-order functions for querying. Such a query can be used for the hierarchy analysis in RDF storages [33]. Let's consider RDF graphs which have two pairs of relations between objects:  $(subClassOf; subClassOf^{-1})$  and  $(type; type^{-1})$ . Each relation has its reverse denoted by the  $-1$  superscript. To search for the vertices which are on the same level of hierarchy one can use the grammars  $G_1$  (Fig. 10) and  $G_2$  (Fig. 11).

$$\begin{aligned} S &\rightarrow subClassOf^{-1} S? subClassOf \\ &| type^{-1} S? type \end{aligned}$$

Figure 10: Context-free grammar  $G_1$  for query 1

$$\begin{aligned} S &\rightarrow B? subClassOf \\ B &\rightarrow subClassOf^{-1} B? subClassOf \end{aligned}$$

Figure 11: Context-free grammar  $G_2$  for query 2

```
val query1: Nonterminal = syn(
  "subclassof-1" ~ query1.? ~ "subclassof" |
  "type-1" ~ query1.? ~ "type")
```

Figure 12: The same generation query (Query 1) in Meerkat

```
val S = syn("subclassof-1" ~ S ~ "subclassof")
val query2 = syn(S ~ "subclassof")
```

Figure 13: The same generation query (Query 2) in Meerkat

These two queries are context-free, so they can be easily written in Meerkat: the code is presented in Fig. 12 and Fig. 13.

The implementations of the queries are similar and we can get rid of the code duplication by generalizing them. The function `sameGen` presented in Fig. 14 generalize the same generation query and is independent of the environment such as the input graph structure or other parsers. Other queries can be written with this function, including the one presented in Fig. 12: it is the result of the application of `sameGen` to the appropriate relations (which can be

```
def sameGen(brs) =
  reduceChoice(
    bs.map {case (lbr, rbr) =>
      lbr ~ syn(sameGen(bs).?) ~ rbr}}
```

Figure 14: Generic function for the same generations query

```
val query1 = syn(sameGen(List(
  ("subclassof-1", "subclassof"),
  ("type-1", "type"))))
```

Figure 15: Query 1 as an application of sameGen

```
val query2 = syn(
  sameGen(List(("subclassof-1", "subclassof"))) ~
  "subclassof")
```

Figure 16: Query 2 as an application of sameGen

treated as the opening and closing brackets). Another application of the sameGen is the Query 2, presented in Fig. 16.

We illustrated that the generic queries are easily written by means of the parser combinators. It is possible to create a library of standard templates for most popular generic queries, such as the same generation query or other domain-specific queries (for example, for specific static code analysis problem).

### 5.3 Classical Movies Queries

We also implemented some queries to the movie database used in the Neo4j tutorial<sup>6</sup>. The database contains the data about movies, actors, directors and the relations between them. In this set of queries we demonstrate more semantic actions for the results processing.

Several helper functions were implemented to simplify the processing of the nodes and the edges. They are built upon the basic combinators and are presented in Fig. 25.

```
def LV(labels: String*) =
  V(e => labels.forall(e.hasLabel))
def outLE(label: String) =
  outE(_.label() == label)
def inLE(label: String) =
  inE(_.label() == label)
```

Figure 17: Helpers for edges and nodes processing

Having the helper functions implemented, we can start building queries. The first query is selects *actors who played in some film*. Let us compare the Cypher (Fig. 18) and the Meerkat versions (Fig. 19) of this query. The structure of them both is similar: the first part is a path specification and the second part is a specification of the value to return; in the Meerkat version we use semantic actions to calculate it.

<sup>6</sup>The set of classical queries to movie dataset in Cypher language: <https://neo4j.com/developer/movie-database/>.

```
MATCH (m:Movie {title: 'Forrest_Gump'})
  <-[:ACTS_IN]-(a:Actor)
RETURN a.name, a.birthplace;
```

Figure 18: The query *actors who played in some film* in Cypher

```
val query =
  syn((
    (LV("Movie")::V(_.title == "Forrest_Gump")) ~
    inLE("ACTS_IN") ~
    syn(
      LV("Actor") ^ (e =>
        (e.name,
          if (e.hasProperty("birthplace"))
            e.birthplace
          else "")))) &&))
```

```
executeQuery(query, input).foreach(println)
```

Figure 19: The query *actors who played in some film* in Meerkat

In the *most prolific actors* query (Fig. 21) we need to use the postprocessing of the paths set to express ordering: executeQuery returns a lazy set of paths which can be processed by using standard Scala functions as shown in Fig. 21.

```
MATCH (a:Actor)-[:ACTS_IN]->(m:Movie)
RETURN a, count(*)
ORDER BY count(*) DESC LIMIT 10
```

Figure 20: The query *most prolific actors* in Cypher

```
val query =
  syn((
    syn(LV("Actor") ^^) ~
    outLE("ACTS_IN") ~
    LV("Movie")) & (a => (a.name, a.toInt)))

executeQuery(query, input)
  .groupBy {case (a, i) => i}
  .toIndexedSeq
  .map {case (i, ms) => (ms.head._1, ms.length)}
  .sortBy {case (a, mc) => -mc}
  .take(10)
  .foreach(println)
```

Figure 21: The query *most prolific actors* in Meerkat

For query presented in Fig. 22 it is necessary to use subqueries. First of all, we specify the user subquery to find the user with the

specific login. Then we specify the friendsWith subquery. Finally, we combine these subqueries to form the resulting query.

```
MATCH (u:User {login: 'adilfulara'})-
      [:FRIEND]-(f:Person)-[r:RATED]->(m:Movie)
WHERE r.stars > 3
RETURN f.name, m.title, r.stars, r.comment;
```

**Figure 22: The query *mutual friend recommendations in Cypher***

```
val user = syn(
  LV("User")::V(_.login == "adilfulara"))

val friendsWith =
  syn(inLE("FRIEND") | outLE("FRIEND"))

val query = syn((
  user ~ friendsWith ~ syn(LV("Person") ^^) ~
  syn(outLE("RATED") ^^) ~ syn(LV("Movie") ^^)) &
  {case p ~ r ~ m =>
    (p.name,
     m.title,
     r.stars.toInt,
     if (r.hasProperty("comment")) r.comment
     else "")})

executeQuery(query, input)
  .filter {case (_, _, s, _) => s > 3}
  .foreach(println)
```

**Figure 23: The query *mutual friend recommendations in Meerkat***

We also can compose all of the features used in the last two queries. To express query presented in Fig. 24, we need to use not only postprocessing but also subquerying with post-processing. First, we evaluate the directors subquery and build directorsMap which is further used in the semantic actions for the actor\_prof\_director subquery. In the posprocessing step of the directors subquery, we implement the filtering which relates to WHERE length(directed) >=2 condition of the Cypher query. In order to get the final result, we also need to use postprocessing as far as it is necessary to implement the filtering and sorting.

This shows that our library is expressive enough to formulate realistic queries. The main drawback of our library as compared to the Cypher language is that all additional logic such as filtering, sorting or grouping has to be implemented manually as a separate step.

## 6 EVALUATION

In this section, we present an evaluation of our graph querying library. We measure its performance on a classical set of ontology graphs [33]: both when the graph is loaded into the RAM and for

```
MATCH (a:Actor:Director)-[:ACTS_IN]->(m:Movie)
WITH a, count(1) AS acted WHERE acted >= 10
WITH a, acted
      MATCH (a:Actor:Director)-[:DIRECTED]->(m:Movie)
WITH a, acted, collect(m.title)
      AS directed WHERE length(directed) >= 2
RETURN a.name, acted, directed
ORDER BY length(directed) DESC, acted DESC;
```

**Figure 24: The query *Directed more than 2 films, acted in more than 10 in Cypher***

```
val directors = syn((
  syn(LV("Actor", "Director") ^^) ~
  outLE("DIRECTED") ~ LV("Movie")) & (_.id.toInt))

val directorsMap = executeQuery(directors, input)
  .groupBy(i => i)
  .map {case (i, ms) => (i, ms.length)}
  .filter {case (_, ms) => ms >= 2}

val actor_prof_director = syn(
  LV("Actor", "Director") ::
  V(e => directorsMap.contains(e.id.toInt)) ^^)

val acts = syn(
  (actor_prof_director ~ outLE("ACTS_IN") ~
  LV("Movie")) & (a => (a.name, a.id.toInt)))

executeQuery(acts, input)
  .groupBy {case (a, i) => i}
  .toStream
  .map {case (i, ms) => (i, ms.head._1, ms.length)}
  .filter {case (i, a, mc) => mc >= 10}
  .map {case (i, a, mc) => (a, mc, directorsMap(i))}
  .sortBy {case (a, mc, dc) => (-dc, -mc)}
  .foreach(println)
```

**Figure 25: The query *Directed more than 2 films, acted in more than 10 in Meerkat***

the integration with the Neo4j database and compare it with the solution based on the GLL parsing algorithm [6] and the Trails [12] library for graph traversals. We also show how may-alias static code analysis can be done by the means of the library.

All tests have been performed on a computer running Fedora 27 with quad-core Intel Core i7 2.5 GHz CPU and 8 GB of RAM.

### 6.1 Ontology Querying

Querying for ontologies is a well-known graph querying problem. We evaluate our library on some popular ontologies in form of RDF files from the paper [33]. First, we convert RDF files to a labelled directed graph in the following manner: we create two edges (*subject*, *predicate*, *object*) and (*object*, *predicate*<sup>-1</sup>, *subject*) for every RDF triple (*subject*, *predicate*, *object*). Then the graph is



Ontology	#nodes	#edges	Query 1					Query 2				
			#results	In memory graph (ms)	DB query (ms)	Trails (ms)	GLL (ms)	#results	In memory graph (ms)	DB query (ms)	Trails (ms)	GLL (ms)
atom-primitive	291	685	15454	64	67	2849	232	122	29	79	453	19
biomedical-measure-primitive	341	711	15156	112	108	3715	482	2871	18	18	60	26
foaf	256	815	4118	11	11	432	29	10	1	1	1	1
funding	778	1480	17634	69	68	367	179	1158	9	9	76	13
generations	129	351	2164	4	4	9	12	0	1	0	0	0
people_pets	337	834	9472	37	37	75	80	37	1	1	2	1
pizza	671	2604	56195	333	325	7764	793	1262	17	18	905	50
skos	144	323	810	1	2	6	6	1	1	0	0	0
travel	131	397	2499	11	13	34	21	63	1	1	1	2
univ-bench	179	413	2540	10	10	31	24	81	1	1	2	1
wine	733	2450	66572	405	401	3156	606	133	2	3	4	5

Table 2: Comparison of Meerkat, Trails and GLL performance on ontologies

either loaded into the Neo4j database or is loaded directly into the memory. Then we run two queries from the paper [6] for these graphs. The grammars for the queries are presented in Fig. 15 and Fig. 16.

The performance results are shown in Table 2 where *#results* is the number of pairs of the nodes between which exists at least one S-path.

The Meerkat-based and the GLL-based [6] solutions show the same results (column *#results*) and the Queries 1 and 2 run at least 1.5 times faster on the Meerkat-based solution than on the GLL-based. Querying the Neo4j database is as performant as the querying the graphs located in the RAM.

## 6.2 Static Code Analysis

Alias analysis is a fundamental static analysis problem [14]: it checks may-alias relations between code expressions and can be formulated as a context-free language (CFL) reachability problem [21], which is closely related to the context-free path querying problem. In this analysis, a program is represented as a Program Expression Graph (PEG) [34]. Vertices in a PEG correspond to program expressions and edges are relations between them. There are two types of edges possible while analyzing source code written in the C programming language: **D**-edge and **A**-edge.

- Pointer dereference edge (**D**-edge). For each pointer dereference  $*e$  there is a directed **D**-edge from  $e$  to  $*e$ .
- Pointer assignment edge (**A**-edge). For each assignment  $*e_1 = e_2$  there is a directed **A**-edge from  $e_2$  to  $*e_1$ .

For the sake of simplicity, we add edges labelled by  $\bar{D}$  and  $\bar{A}$  which correspond to the reversed **D**-edge and **A**-edge, respectively.

The grammar for may-alias problem from [34] is shown in Fig. 26. It contains two nonterminals **M** and **V**, which we can query for.

- **M**-production means that two l-value expressions are memory aliases i.e. may refer to the same memory location.
- **V**-production means that two expressions are value aliases i.e. may evaluate to the same pointer value.

$$M \rightarrow \bar{D} V D$$

$$V \rightarrow (M? \bar{A})^* M? (A M?)^*$$

Figure 26: Context-Free grammar for the may-alias problem

We run the **M** and **V** queries on some open-source C projects. We used Crystal<sup>7</sup> to construct Program Expression Graph and ran the queries over it. The results are shown in Table 3. We conclude that our solution is expressive and performant enough to be used for static analysis problems, which can be expressed as CFPQs.

```

val M = syn("nd" ~ V ~ "d")
val V = syn((M.? ~ "na").* ~ M.? ~ ("a" ~ M.?).*)

```

Figure 27: Meerkat representation of may-alias problem grammar

## 6.3 Classical Movies Queries

In order to examine the applicability of our library for the real data processing we evaluate queries presented in the section 5 on the classical movie database which contains more than 60000 nodes and more than 100000 edges.

All queries are performed using the Neo4j database connected to Meerkat. During evaluation of these queries, caching in Neo4j was disabled to simplify time measurement.

Our library implements a general parsing algorithm which supports arbitrary context-free queries. Unfortunately, when the queries are regular, there is a big overhead for processing them. The queries mentioned in this section are all regular, thus it is not a surprise that our implementation is dramatically slower than the

<sup>7</sup>Crystal: a program analysis system for C <https://www.cs.cornell.edu/projects/crystal/>. Access date: 30.07.2018.

Program	#edges	#nodes	Code Size (KLOC)	In memory graph (ms)	Neo4j graph (ms)	Trails graph (ms)	M aliases	V aliases
wc-5.0	332	770	0.5	0	2	3	174	107
pr-5.0	815	2062	1.7	11	12	14	1131	63
ls-5.0	1687	4734	2.8	43	51	170	5682	253
bzip2-1.0.6	632	1508	5.8	8	13	21	813	71
gzip-1.8	2687	7510	31	111	120	537	4567	227

**Table 3: Running may-alias queries on Meerkat on some C open-source projects**

Query	Neo4j + Meerkat (ms)	Neo4j + Cypher (ms)
Mutual friend recommendations	106.86	17.26
Directed more than 2 films, Acted in more than 10	348.11	44.82
Find the most prolific actors	2333.37	189.32
Actors who played in some film	89.78	11.31

**Table 4: Running regular queries using Meerkat and Cypher**

native Neo4j solution. Besides the overhead itself, our library does not take any advantages from the internal Neo4j optimizations.

Despite the seeming failure of this test, we still can conclude that the combinators can process quite big datasets in a reasonable time: 6 times bigger in terms of the number of vertices than the other tests. It also shows that further optimization of the implementation is justified.

## 6.4 Comparison with Trails

Trails [12] is a Scala graph combinator library. It provides traversers for describing paths in graphs which resemble parser combinators and calculates possibly infinite stream of all possible paths described by the composition of the basic traversers. Both Trails and Meerkat-based solution support parsing of the graphs located in RAM, so we compare the performance of the Trails and the Meerkat-based solution on the ontology queries described above: the results are presented in Table 2. Trails and Meerkat-based solution compute the same results, but Trails is up to 10 times slower than the Meerkat-based solution.

To summarize, we demonstrated that parser combinators suit for implementing real queries. Our implementation is as performant as the other existing combinators library and is comparable to the GLL-based solution.

## 7 CONCLUSION

We propose a native way to integrate a language for context-free path querying into a general-purpose programming language. Our solution handles arbitrary context-free grammars and arbitrary input graphs. The proposed approach is language-independent and may be implemented in nearly every general-purpose programming language. We implement it in Scala as a modification to the parser combinator library Meerkat and show that our approach can be applied to the real world problems.

We can propose some possible directions for the future work. In order to improve the performance and investigate the scalability of the solution, we plan to implement a parallel single machine and distributed GLL. It is a challenge from both the theoretical and the practical standpoint.

Some important problems in the realm of the static code analysis cannot be expressed in terms of context-free path querying. For example, the context-sensitive data-dependence analysis may be precisely expressed in terms of the linear-conjunctive language [17] reachability but not context-free [32]. How to support the arbitrary conjunctive grammars is also worth research. This technique can be employed as a static analysis framework.

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