

# Multiple-Source Context-Free Path Querying in Terms of Linear Algebra

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## ABSTRACT

Context-Free Path Querying (CFPQ) allows one to use context-free grammars to express paths constraints in navigational graph queries. Algorithms for CFPQ studied actively for a long time, but no one graph database provides full-stack support of CFPQ. In this work we provide a multiple-source version of Azimov's CFPQ algorithm, which, as shown by Arseniy Terekhov is applicable for real-world graph analysis. This step allows us to make the algorithm more practical and integrate it into RedisGraph graph database. In order to provide full-stack support, we also implement a Cypher graph query language extension that allows one to express context-free constraints. As a result, we provide the first, in our knowledge, full-stack support of CFPQ for graph databases. Our evaluation shows that the provided solution is applicable to real-world graph analysis.

## 1 INTRODUCTION

Language-constrained path querying [3] is a way to search for paths in edge-labeled graphs where constraints are formulated in terms of a formal language. The language restricts the set of accepted paths: the sentence formed by the labels of a path should be in the language. Regular languages are the most popular class of constraints used as navigational queries in graph databases. In some cases, regular languages are not expressive enough and context-free languages are used instead. Context-free path querying (CFPQ), can be used for RDF analysis [27], biological data analysis [22], static code analysis [20, 28], and in other areas.

CFPQ have been studied a lot since the problem was first stated by Mihalis Yannakakis in 1990 [26]. Jelle Hellings investigates various aspects of CFPQ in [8–10]. A number of CFPQ algorithms were proposed: (G)LL and (G)LR based algorithms by Ciro M. Medeiros et al. [15], Fred C. Santos et al. [21], Semyon Grigorev et al. [7], and Ekaterina Verbitskaia et al. [24]; CYK-based algorithm by Xiaowang Zhang et al. [27]; combinatorics-based approach to CFPQ by Ekaterina Verbitskaia et al. [25]. Nevertheless, the application of context-free constraints for real-world data analysis still faces many problems. The first problem is bad performance of the proposed algorithms on real-world data, as shown

by Jochem Kuijpers et al. [14]. The second problem is that no graph database provides full-stack support of CFPQ, since most effort was made in developing algorithms and researching their theoretical properties. This fact hinders research of problems which can be reduced to CFPQ, thus it hinders the development of new solutions for them. For example, graph segmentation in data provenance analysis was recently reduced to CFPQ [17], but evaluation of the proposed approach was complicated because no graph database supported CFPQ.

Rustam Azimov proposed a matrix-based algorithm for CFPQ in [2]. This algorithm provides a solution performant enough for real-world data analysis, as shown by Nikita Mishim et al. in [18] and Arseniy Terekhov et al. in [23]. This algorithm computes reachability or provides a single path which satisfies constraints for *every* vertex pair in the graph. Namely it solves *all-pairs* context-free path querying problem. In many real-world scenarios it is redundant to handle all possible pairs, instead one can provide one or a relatively small set of start vertices.

While all-pairs context-free path querying is a problem well studied, there is no, best to our knowledge, solutions for the single-source and multiple-source CFPQ. In this work we propose a matrix-based *multiple-source* (and *single-source* as a partial case) CFPQ algorithm.

We also provide full-stack support of CFPQ for the RedisGraph<sup>1</sup> [4] graph database. We implement a Cypher query language extension<sup>2</sup> that makes it possible to use context-free constraints, and extend the RedisGraph to support this extension. As far as we know, it is the first full-stack implementation of CFPQ.

To sum up, we make the following contributions in this paper.

- (1) We modify Azimov's matrix-based CFPQ algorithm and provide a multiple-source matrix-based CFPQ algorithm. As a partial case, it is possible to use our algorithm in a single-source scenario. Our modification is still based on linear algebra, hence it is simple to implementate and allows one to use high-performance libraries and utilize modern parallel hardware for queries evaluation.
- (2) We evaluate two versions of the proposed algorithm: with caching of results and without caching (naive). Caching is

<sup>1</sup>RedisGraph graph database Web-page: <https://redislabs.com/redis-enterprise/redis-graph/>. Access date: 19.07.2020.

<sup>2</sup>Proposal which describes path patterns specification syntax for Cypher query language: <https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc>. The proposed syntax allows one to specify context-free constraints. Access date: 19.07.2020.

aimed to reduce repeated calculation of the same data. Our evaluation shows that the naive version is more performant and memory-efficient than the version with results caching in almost all cases. We believe, it is a good choice for implementation in real-world graph database.

- (3) We provide full-stack support of CFPQ by extending the RedisGraph graph database. To do it, we extended Cypher with syntax for context-free constraints, implemented the proposed algorithm in a RedisGraph backend, and supported the new syntax in the RedisGraph query execution engine. Finally, we evaluate the proposed solution and show that it is performant and memory-efficient enough to be applicable for real-world graph querying.

## 2 PRELIMINARIES

In this section we introduce common definitions in graph theory and formal language theory which are used in this paper. Also, we provide a brief description of Azimov's algorithm which is used as a base of our solution.

### 2.1 Basic Definitions of Graph Theory

In this paper we use a labeled directed graph as a data model and define it as follows.

**Definition 2.1.** *Labeled directed graph* is a tuple of six elements  $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ , where

- $V$  is a finite set of vertices. For simplicity, we assume that the vertices are natural numbers from 0 to  $|V| - 1$ .
- $E \subseteq V \times V$  is a set of edges.
- $\Sigma_V$  and  $\Sigma_E$  are sets of labels of vertices and edges respectively, such that  $\Sigma_V \cap \Sigma_E = \emptyset$ .
- $\lambda_V : V \rightarrow 2^{\Sigma_V}$  is a function that maps a vertex to a set of its labels, which can be empty.
- $\lambda_E : E \rightarrow 2^{\Sigma_E} \setminus \{\emptyset\}$  is a function that maps an edge to a non-empty set of its labels, so each edge must have at least one label.

□

Labeled graph is a base of the widely-used *property graph* data model [1] and allows one to use not only edge labels but also vertex labels in navigation queries.

An example of the labeled directed graph  $D_1$  is presented in figure 1. Here the sets of labels  $\Sigma_V = \{x, y\}$  and  $\Sigma_E = \{a, b, c, d\}$ . We omit vertex labels set if it is empty.

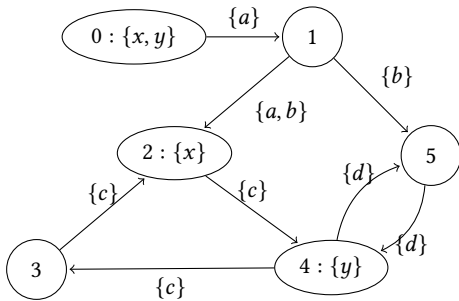


Figure 1: The input graph  $D_1$

**Definition 2.2.** Path  $\pi$  in the graph  $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$  is a finite sequence of vertices and edges  $(v_0, e_0, v_1, e_1, \dots, e_{n-1}, v_n)$ , where  $\forall i, 0 \leq i \leq n : v_i \in V; \forall j, 1 \leq j \leq n : e_j = (v_{j-1}, v_j) \in E$ .

We denote the set of all paths in the graph  $D$  as  $\pi(D)$ . □

**Definition 2.3.** An *adjacency matrix*  $M$  of the graph  $D$  is a matrix of size  $|V| \times |V|$ , such that

$$M[i, j] = \begin{cases} \lambda_E((i, j)) & , (i, j) \in E \\ \emptyset & , \text{otherwise} \end{cases}$$

□

Adjacency matrix  $M$  of the graph  $D_1$  (fig. 1) is the following:

$$M = \begin{pmatrix} \emptyset & \{a\} & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \{a, b\} & \emptyset & \emptyset & \{b\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \{d\} & \emptyset \\ \emptyset & \emptyset & \{c\} & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \{c\} & \emptyset & \{d\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \{d\} & \emptyset \end{pmatrix}.$$

**Definition 2.4.** Let  $M$  be an adjacency matrix of the graph  $D$ . Then the *adjacency matrix of label  $l \in \Sigma_E$*  of graph  $D$  is a matrix  $\mathcal{E}^l$  of size  $|V| \times |V|$ , such that

$$\mathcal{E}^l[i, j] = \begin{cases} 1 & , l \in M[i, j] \\ 0 & , \text{otherwise} \end{cases}$$

□

**Definition 2.5.** A *boolean decomposition of adjacency matrix  $M$*  of the graph  $D$  is a set of Boolean matrices  $\mathcal{E} = \{\mathcal{E}^l \mid l \in \Sigma_E\}$ , where  $\mathcal{E}^l$  is the adjacency matrix of label  $l$ . □

For example, the boolean decomposition of the adjacency matrix  $M$  of the graph  $D_1$  is the set of Boolean matrices  $\mathcal{E}^a, \mathcal{E}^b, \mathcal{E}^c$  and  $\mathcal{E}^d$ :

$$\mathcal{E}^a = \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \mathcal{E}^b = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

$$\mathcal{E}^c = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \mathcal{E}^d = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix}.$$

**Definition 2.6.** A *vertices label matrix*  $H$  of the graph  $D$  is a matrix of size  $|V| \times |V|$ , such that

$$H[i, j] = \begin{cases} \lambda_V(i) & , i = j \\ \emptyset & , \text{otherwise} \end{cases}$$

□

The vertices label matrix  $H$  of the example graph  $D_1$  is

$$H = \begin{pmatrix} \{x, y\} & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \{x\} & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \{y\} & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & \emptyset \end{pmatrix}.$$

**Definition 2.7.** Let  $H$  be a vertices label matrix of graph  $D$ . Then the *vertices matrix of label  $l$*  is a matrix  $\mathcal{V}^l$  of size  $|V| \times |V|$ , such that

$$\mathcal{V}^l[i, j] = \begin{cases} 1 & , l \in H[i, j] \\ 0 & , \text{otherwise} \end{cases}$$

**Definition 2.8.** A *boolean decomposition of vertices label matrix  $H$*  of the graph  $D$  is the set of Boolean matrices  $\mathcal{V} = \{\mathcal{V}^l \mid l \in \Sigma\}$ , where  $\mathcal{V}^l$  is a vertices matrix of label  $l$ .

Vertices label matrix  $H$  of the graph  $D_1$  can be decomposed into a set of the following Boolean matrices:

$$\mathcal{V}^x = \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \mathcal{V}^y = \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

## 2.2 Basic Definitions of Formal Languages

We use context-free grammars as paths constraints, thus in this subsection we define context-free languages and grammars.

*Definition 2.9.* A context-free grammar is a tuple  $G = (N, \Sigma, P, S)$ , where

- $N$  is a finite set of nonterminals
- $\Sigma$  is a finite set of terminals,  $N \cap \Sigma = \emptyset$
- $P$  is a finite set of productions of the form  $A \rightarrow \alpha$ , where  $A \in N$ ,  $\alpha \in (N \cup \Sigma)^*$
- $S$  is the start nonterminal

□

*Definition 2.10.* A context-free language is a language generated by a context-free grammar  $G$ :

$$L(G) = \{w \in \Sigma^* \mid S \xRightarrow[G]{*} w\}$$

Where  $S \xRightarrow[G]{*} w$  denotes that a string  $w$  can be generated from a starting non-terminal  $S$  using some sequence of production rules from  $P$ .

□

*Definition 2.11.* A context-free grammar  $G = (N, \Sigma, P, S)$  is in *Chomsky normal form* if every production in  $P$  has one of the following forms:

- $A \rightarrow BC$ , where  $A \in N$ ,  $B, C \in N \setminus \{S\}$
- $A \rightarrow a$ , where  $A \in N$ ,  $a \in \Sigma$
- $S \rightarrow \varepsilon$ , where  $\varepsilon$  is an empty string.

□

*Definition 2.12.* A context-free grammar  $G = (N, \Sigma, P, S)$  is in *weak Chomsky normal form* if every production in  $P$  has one of the following forms:

- $A \rightarrow BC$ , where  $A, B, C \in N$
- $A \rightarrow a$ , where  $A \in N$ ,  $a \in \Sigma$
- $A \rightarrow \varepsilon$ ,  $A \in N$

□

In other words, weak Chomsky normal form differs from Chomsky normal form in the following:

- $\varepsilon$  can be derived from any non-terminal;
- $S$  can occur in the right-hand side of productions.

The matrix-based CFPQ algorithms process grammars only in weak Chomsky normal form, but every context-free grammar can be transformed into the equivalent grammar in this form.

Consider the example of the context-free grammar  $G_1 = (N, \Sigma, P, S)$ , where  $N = \{S\}$ ,  $\Sigma = \{c, d, y\}$ , and  $P$  has two rules:

$$\begin{aligned} S &\rightarrow c S d \\ S &\rightarrow c y d \end{aligned} \quad (1)$$

This grammar generates the context-free language:

$$L(G_1) = \{c^n y d^n, n \in \mathbb{N}\}.$$

The following grammar  $G_1^{\text{wcnf}}$  is a result of the transformation of  $G_1$  to weak Chomsky normal form:

$$\begin{aligned} S &\rightarrow C E & C &\rightarrow c \\ S &\rightarrow C S_1 & Y &\rightarrow y \\ E &\rightarrow Y D & D &\rightarrow d \\ S_1 &\rightarrow S D \end{aligned}$$

## 2.3 Context-Free Path Querying

*Definition 2.13.* Let  $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$  be a labeled graph,  $G = (N, \Sigma_V \cup \Sigma_E, P, S)$  be a context free grammar. Then a context free relation with grammar  $G$  on the labeled graph  $D$  is the relation  $R_{G,D} \subseteq V \times V$ :

$$R_{G,D} = \{(v_1, v_n) \in V \times V \mid \exists \pi = (v_1, e_1, v_2, \dots, e_n, v_n) \in \pi(D) : l(\pi) \cap L(G) \neq \emptyset\},$$

where  $l(\pi) \subset (\Sigma_V \cup \Sigma_E)^*$  is the set of labels along the path  $\pi$ :

$$l(\pi) = \lambda_V(v_1)^* \cdot \lambda_E(e_1) \cdot \lambda_V(v_2)^* \cdot \lambda_E(e_2) \cdot \dots \cdot \lambda_E(e_n) \cdot \lambda_V(v_n)^*$$

□

For example,  $\pi$  is a path from vertex 3 to vertex 6 in the labeled graph presented in figure 1:

$$\pi = 3 \xrightarrow{\{c\}} 5 : \{y\} \xrightarrow{\{d\}} 6.$$

Labels along  $\pi$  form the set of sequences  $l(\pi) = \{c y^n d \mid n \geq 0\}$ . Only one of these sequences satisfies context-free constraints of the grammar  $G_1$ :  $c y d$ . The derivation of the sequence is the following:

$$S \Rightarrow CE \Rightarrow cE \Rightarrow cYD \Rightarrow cyD \Rightarrow cyd$$

Hence  $l(\pi) \cap L(G_1) \neq \emptyset$  and the pair  $(3, 6) \in R_{G_1,D}$ .

Take a closer look at the definition of path labels, namely that it allows for zero or more repetitions of a label of each vertex. This makes it possible to omit vertex labels or, if there are many vertex labels, to use them in an arbitrary order. It also permits to write a query which uses one vertex label multiple times. This definition may appear strange in some cases, but it depends on the semantics of the graph query language. To formalize the semantics is future work, so we will stick to this definition in this paper.

Finally, we can define context-free path querying problem.

*Definition 2.14.* Context-free path querying problem is the problem of finding context-free relation  $R_{G,D}$  for a given directed labeled graph  $D$  and a context-free grammar  $G$ .

□

In other words, the result of context-free path query evaluation is a set of vertex pairs such that there is a path between them and this path forms a word from the given language.

The context-free relation  $R_{G_1,D_1}$  for the graph  $D_1$  and the context-free free grammar  $G_1$  is the following:

$$R_{G_1,D_1} = \{(2, 4), (2, 5), (3, 4), (3, 5), (4, 4), (4, 5)\}.$$

Note that any relation  $R_{G,D}$  can be represented as a Boolean matrix:

$$T[i, j] = 1 \iff (i, j) \in R_{G,D}.$$

In our example,  $R_{G_1,D_1}$  can be represented as follows:

$$T = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

*Definition 2.15.* Suppose  $Src$  is a given set of start vertices, then *multiple-source context-free path querying problem* for the given  $Src$ , directed labeled graph  $D$  and context-free grammar  $G$  is to find a context-free relation

$$R_{G,D}^{Src} \subseteq Src \times V \subseteq R_{G,D}.$$

Thus we restrict start vertices of the paths of interest to be a vertices from the given set  $\square$

As a special case, a *single-source CFPQ* is when  $Src$  is a singleton set. If we set  $Src = \{2\}$  in the previous example, then the result is:

$$R_{G_1,D_1}^{\{2\}} = \{(2, 4), (2, 5)\}.$$

We can represent the  $R_{G_1,D_1}^{\{2\}}$  as a Boolean matrix:

$$T = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}.$$

## 2.4 Matrix-Based Algorithm

Our algorithm is based on the Azimov's CFPQ algorithm [2] which is based on matrix operations. This algorithm allows one to use high-performance linear algebra libraries and utilize modern parallel hardware for CFPQ.

Let  $G = (N, \Sigma, P, S)$  be the input grammar, the input edge-labeled graph  $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$  and language  $L$  over alphabet  $\Sigma$ . The matrix-based algorithm for CFPQ can be expressed in terms of operations over Boolean matrices as presented in Algorithm 1. Using Boolean matrices simplifies the implementation of the algorithm.

### Algorithm 1 Context-free path querying algorithm

```

1: function EVALCFPQ( $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ ,
    $G = (N, \Sigma, P, S)$ )
2:    $n \leftarrow |V|$ 
3:    $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i} \text{ is a matrix } n \times n, T_{k,l}^{A_i} \leftarrow \text{false}\}$ 
4:   for all  $(i, j) \in E, A_k \mid \lambda_E(i, j) = x, A_k \rightarrow x \in P$  do
      $T_{i,j}^{A_k} \leftarrow \text{true}$ 
5:   for all  $A_k \mid A_k \rightarrow \varepsilon \in P$  do
6:     for all  $i \in \{0, \dots, n-1\}$  do  $T_{i,i}^{A_k} \leftarrow \text{true}$ 
7:   while any matrix in  $T$  is changing do
8:     for  $A_i \rightarrow A_j A_k \in P$  do  $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$ 
9:   return  $T$ 

```

Note, that the provided algorithm computes not only the context-free relation  $R_{G,D}$  but a set of context-free relations  $R_{A,D} \subseteq V \times V$  for every  $A \in N$ . Thus it provides information about paths which form words derivable from any nonterminal in the given grammar. Also, this algorithm handles only the edge labels.

As was shown by Nikita Mishin et al. [18] and Arseniy Terekhov et al. [23], this algorithm can be implemented using various high-performance programming techniques (including GPGPU utilization), and it is applicable for real-world graph analysis. But this algorithm solves *all-pairs* version of CFPQ: it finds all pairs of vertices in the given graph, such that there exist a path between them which forms a word in the given language. Thus it is impractical in cases when we are only interested in paths

which start from the specific set of vertices, especially if this set is relatively small. Moreover, Azimov's algorithm operates over an adjacency matrix of the whole input graph, and as a result it requires a huge amount of memory, which may be a problem for a real-world graph database.

## 3 MATRIX-BASED MULTIPLE-SOURCE CFPQ ALGORITHM

In this section we introduce two versions of the multiple-source matrix-based CFPQ algorithm. This algorithm is a modification of Azimov's matrix-based algorithm for CFPQ and its core idea is to cut off those vertices which are not in the selected set of start vertices.

In order to simplify Azimov's algorithm modification and the final algorithm description, we assume the input graph to have only edge labels. Note, that we always can convert the original graph into such form. To do it, we should add loops into vertices in the following way: for the vertex  $i$  we add an edge  $i \xrightarrow{x} i$  iff  $\lambda_V(i) = x$  and  $x \neq \emptyset$ . This way we can switch to an edge-labeled graph with the same number of vertices while preserving the defined semantics of CFPQ.

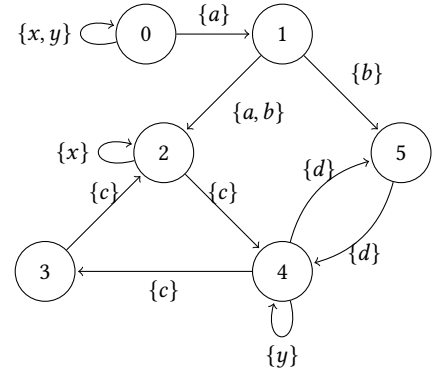


Figure 2: The example of  $D'_1$ : the modified input graph  $D_1$

The adjacency matrix  $M$  of the graph  $D'_1$  is

$$M = \begin{pmatrix} \{x, y\} & \{a\} & \emptyset & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \{a, b\} & \emptyset & \emptyset & \{b\} \\ \emptyset & \emptyset & \{x\} & \emptyset & \{c\} & \emptyset \\ \emptyset & \emptyset & \{c\} & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & \emptyset & \{c\} & \{y\} & \{d\} \\ \emptyset & \emptyset & \emptyset & \emptyset & \{d\} & \emptyset \end{pmatrix}.$$

Note that this transformation is impractical for real-world graphs, thus we use it only for algorithm description.

The first version of multiple-source algorithm is the Azimov's algorithm equipped with vertices filtering. Let  $G = (N, \Sigma, P, S)$  be the input context-free grammar,  $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$  be the input graph and  $Src$  be the input set of start vertices. The result of the algorithm is a Boolean matrix which represents relation  $R_{S,D}^{Src}$ .

In order to solve the single-source and multiple-source CFPQ problem, we modified the Azimov's algorithm. Each time, when a grammar rule is applied (see line 8 of Algorithm 1: boolean matrix multiplication  $T_A = T_A + T_B \cdot T_C$  for each  $A \rightarrow BC \in P$ ), only vertices of interest should be stored. To do it, we added one more matrix multiplication:  $T_A = T_A + (T_{Src^A} \cdot T_B) \cdot T_C$ , where  $T_{Src^A}$  — matrix of start vertices for the current iteration (lines 11-13 of the Algorithm 2). After every iteration of the while loop,

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**Algorithm 2** Multiple-source context-free path querying algorithm

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```

1: function MULTISRCFPQ(
     $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ ,
     $G = (N, \Sigma, P, S)$ ,
     $Src$ )
2:    $T \leftarrow \{T^A \mid A \in N, T_{i,j}^A \leftarrow \text{false, for all } i, j\}$ 
3:    $TSrc \leftarrow \{TSrc^A \mid A \in N, TSrc_{i,j}^A \leftarrow \text{false, for all } i, j\}$ 
4:   for all  $v \in Src$  do ▷ Input matrix initialization
5:      $TSrc_{v,v}^S \leftarrow \text{true}$ 
6:   for all  $A \rightarrow x \in P$  do ▷ Simple rules initialization
7:     for all  $(v, to) \in E, \lambda_E(v, to) = x$  do
8:        $T_{v,to}^A \leftarrow \text{true}$ 
9:   while  $T$  or  $TSrc$  is changing do ▷ Algorithm's body
10:    for all  $A \rightarrow BC \in P$  do
11:       $M \leftarrow TSrc^A * T^B$ 
12:       $T^A \leftarrow T^A + M * T^C$ 
13:       $TSrc^B \leftarrow TSrc^B + TSrc^A$ 
14:       $TSrc^C \leftarrow TSrc^C + \text{GETDST}(M)$ 
15:   return  $T^S$ 
16: function GETDST( $M$ )
17:    $A_{i,j} \leftarrow \text{false}$ 
18:   for all  $(v, to) \in V^2 \mid M_{v,to} = \text{true}$  do
19:      $A_{to,to} \leftarrow \text{true}$ 
20:   return  $A$ 

```

---

it is necessary to update the set of vertices paths from which we need to calculate. To do it, we call the function GETDST (see lines 17-21), in line 14. Thus, the modified algorithm supports the frontier of the vertices of interest and updates it on each iteration. As a result, it only computes the paths which starts from the small set of selected vertices.

In case when one has a sequence of similar queries to the single graph, it may be useful to cache results of the query evaluation and share them between queries. This may help to avoid recalculation of the already computed results. To introduce sharing of results between queries, we modify the previous version of the algorithm. The modified version stores all the vertices the paths from which have already been calculated in cache *index*, which is used to filter such vertices in line 11 of Algorithm 3. Thus, the modified algorithm calculates paths from the particular vertex only once. Note, that CREATEINDEX function should be called first, and the created index can be shared between multiple calls of MULTISRCFPQSMART after that.

### 3.1 Example

Consider the first few steps of the proposed algorithm (without caching) on the graph  $D'_1$ , grammar  $G_1^{\text{wcnf}}$ , and the set of start vertices  $Src = \{2\}$ . At the first step (lines 4-5)  $TSrc_{2,2}^S$  sets to *true*, all other cells have value *false*. Then the set of matrices  $T$  is

---

**Algorithm 3** Optimized multiple-source context-free path querying algorithm

---

```

1: function CREATEINDEX(
     $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ ,
     $G = (N, \Sigma, P, S)$ )
2:    $T \leftarrow \{T^A \mid A \in N, T_{i,j}^A \leftarrow \text{false, for all } i, j\}$ 
3:    $TSrc \leftarrow \{TSrc^A \mid A \in N, TSrc_{i,j}^A \leftarrow \text{false, for all } i, j\}$ 
4:   for all  $A \rightarrow x \in P$  do ▷ Simple rules initialization
5:     for all  $(v, to) \in E, \lambda_E(v, to) = x$  do
6:        $T_{v,to}^A \leftarrow \text{true}$ 
7:   return  $(T, TSrc)$ 
8:
9: function MULTISRCFPQSMART(
     $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ ,
     $G = (N, \Sigma, P, S)$ ,
     $Src$ ,
     $Index = (T, TSrc)$ )
10:   $TNewSrc \leftarrow \{TNewSrc^A \mid A \in N, TNewSrc^A \leftarrow \emptyset\}$ 
11:  for all  $v \in Src \mid TSrc_{v,v} = \text{false}$  do
12:     $TNewSrc_{v,v}^S \leftarrow \text{true}$ 
13:  while  $T$  or  $TNewSrc$  is changing do
14:    for all  $A \rightarrow BC \in P$  do
15:       $M \leftarrow TNewSrc^A * T^B$ 
16:       $T^A \leftarrow T^A + M * T^C$ 
17:       $TNewSrc^B \leftarrow TNewSrc^B + TNewSrc^A \setminus TSrc^B$ 
18:       $TNewSrc^C \leftarrow TNewSrc^C + \text{GETDST}(M) \setminus TSrc^C$ 
19:  return  $TSrc^S * T^S$  ▷ We want to return only relevant data, not all cached results

```

---

initialized using rules  $C \rightarrow c, D \rightarrow d, Y \rightarrow y$  as follows:

$$T^C = \mathcal{E}^c = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, T^D = \mathcal{E}^d = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot \end{pmatrix},$$

$$T^Y = \mathcal{V}^y = \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \end{pmatrix}$$

The following computations take place at the first iteration of the while loop in line 9 for the grammar rule  $S \rightarrow CE$ . First, the matrix  $M$  is computed as follows

$$M = TSrc^S * T^C = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Since matrix  $T^E$  is empty,  $T^S$  stays the same in line 12, as well as  $TSrc^C$  in line 13. But the matrix  $TSrc^E$  is updated (line 14):

$$TSrc^E = TSrc^E + \text{GETDST}(M) =$$

$$= \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

This means that we are interested in paths that start from the vertex 4 and satisfy the constraints specified by nonterminal  $E$ .

The second rule is  $S \rightarrow CS_1$  and its processing is similar to previous one. After processing,

$$T Src^{S_1} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

The third rule is  $E \rightarrow YD$ . Here  $M$  is computed as follows:

$$M = T Src^E * T^Y = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Then the algorithm updates  $T^E$  as follows:

$$T^E = T^E + M * T^D = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Thus we know that there exists a path from vertex 4 to vertex 5 such that it forms a word derivable from  $E$ .

The last rule is  $S_1 \rightarrow SD$ . During processing this rule only  $T Src^S$  is updated, since  $T^S$  is empty:

$$T Src^S = T Src^S + T Src^{S_1} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

This is the end of the first iteration.

At the second iteration, matrices  $M$  and then  $T^S$  are computed for rule  $S \rightarrow CE$  as follows:

$$M = T Src^S * T^C = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$T^S = M * T^E = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Thus, we found the first path that satisfies our query. After all iterations finished, we get the final result:

$$T^S = \begin{pmatrix} \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Thus only vertices 4 and 5 are reachable from the vertex 2 by a path which forms a word derivable from the start nonterminal  $S$ .

**Table 1: Graphs for CFPQ evaluation**

Graph	#V	#E	#subCalssOf	#type	#broaderTransitive
core	1323	3636	178	1412	0
pathways	6238	18 598	3117	3118	0
gohierarchy	45 007	980 218	490 109	0	0
enzyme	48 815	109 695	8163	14 989	0
eclass_514en	239 111	523 727	90 962	72 517	0
geospecies	450 609	2 311 461	0	89 062	20 867
go	272 770	534 311	90 512	58 483	0

### 3.2 Implementation Notes

All of the versions presented have been implemented<sup>3</sup> using GraphBLAS framework that allows one to represent graphs as matrices and to work with them in terms of linear algebra. For convenience, the code is written in Python using pygraphblas<sup>4</sup>, which is a Python wrapper for GraphBLAS API and is based on SuiteSparse:GraphBLAS<sup>5</sup> [5] — the complete implementation of the GraphBLAS standard. This library is specialized to work with sparse matrices which appear in real graphs most often.

### 3.3 Algorithm Evaluation

We evaluate both described versions of the multiple-source algorithm on real-world graphs. For evaluation, we use a PC with Ubuntu 20.04 installed. It has Intel core i7-4790 CPU, 3.60GHz, and DDR3 32Gb RAM. As far as we evaluate only the execution time of the algorithm, we store each graph fully in RAM as its adjacency matrix in sparse format. Note, that graph loading time is not included in the result time of evaluation.

For evaluation we use graphs and queries from CFPQ\_Data dataset<sup>6</sup> Detailed information, such as the number of vertices and edges, and the number of edges with the specific label, on graphs used in evaluation is provided in table 1. We use classical same-generation queries  $g_1$  (eq. 2) and  $g_2$  (eq. 3) which are used in other works for CFPQ evaluation. We also use *geo* (eq. 4) query provided by Jochem Kuijpers et. al [14] for *geospecies* RDF. Note that in queries we use  $\bar{x}$  notation to denote the inverse of the  $x$  relation and the respective edge.

$$S \rightarrow \overline{\text{subClassOf}} S \text{ subClassOf} | \overline{\text{type}} S \text{ type} \quad (2)$$

$$S \rightarrow \overline{\text{subClassOf}} S \text{ subClassOf} | \text{subClassOf} \quad (3)$$

$$S \rightarrow \overline{\text{broaderTransitive}} S \overline{\text{broaderTransitive}} \quad (4)$$

Our main goal is to compare the behavior of the two proposed versions of the algorithm. To do it, we measure query execution time of both versions for different sizes of star vertex set. Namely, for each graph we split all vertices into disjoint subsets of fixed size. After that, we evaluate queries using each subset as a set of start vertices. For algorithm with caching we initialize cache

<sup>3</sup>GitHub repository with implemented algorithms: [https://github.com/JetBrains-Research/CFPQ\\_PyAlgo](https://github.com/JetBrains-Research/CFPQ_PyAlgo), last accessed 28.08.2020

<sup>4</sup>GitHub repository of PyGraphBLAS library: <https://github.com/michelp/pygraphblas>

<sup>5</sup>GitHub repository of SuiteSparse:GraphBLAS library: <https://github.com/DrTimothyAldenDavis/SuiteSparse>

<sup>6</sup>CFPQ\_Data is a dataset for CFPQ evaluation which contains both synthetic and real-world data and queries [https://github.com/JetBrains-Research/CFPQ\\_Data](https://github.com/JetBrains-Research/CFPQ_Data), last accessed 28.08.2020.

once for each chunk size and accumulate results for all chunks of the specific size.

We evaluate all three queries for each graph. The results of the evaluation are presented in figures 3–9. We use standard violin plot with median to show the distribution of results; time is measured in seconds. For several input graphs we provide additional figures for smaller chunks in order to analyze these cases carefully: figures 10–12.

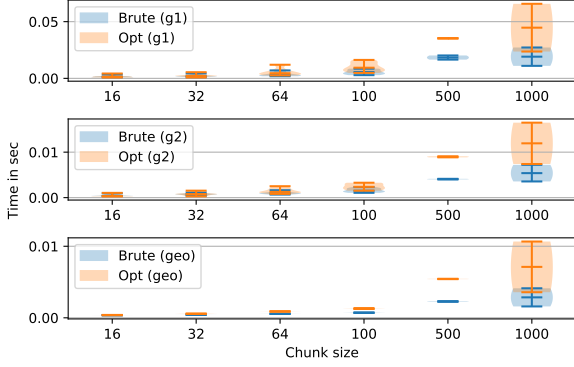


Figure 3: Performance of *core* graph querying

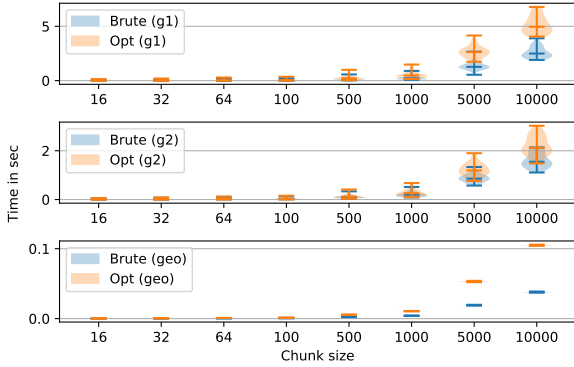


Figure 4: Performance of *go* graph querying

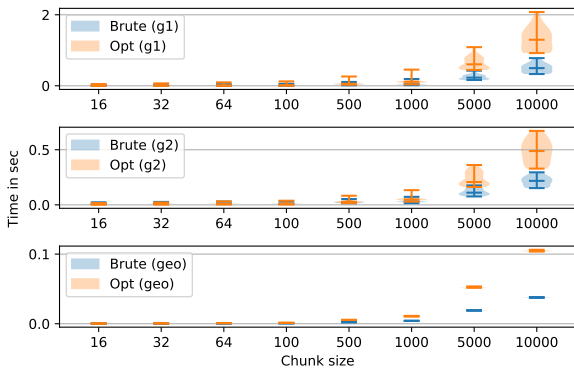


Figure 5: Performance of *eclclass\_514en* graph querying

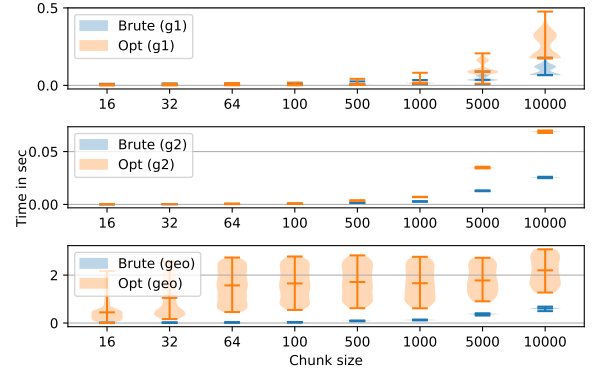


Figure 6: Performance of *geospecies* graph querying

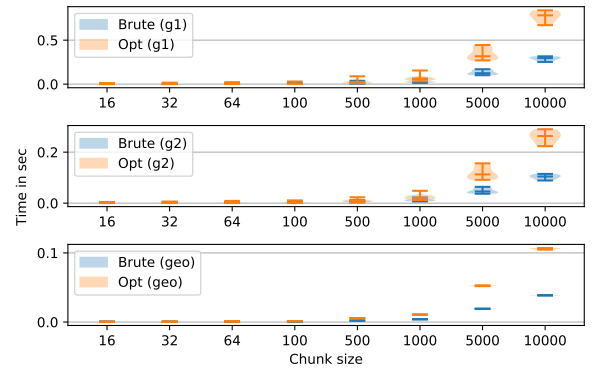


Figure 7: Performance of *enzyme* graph querying

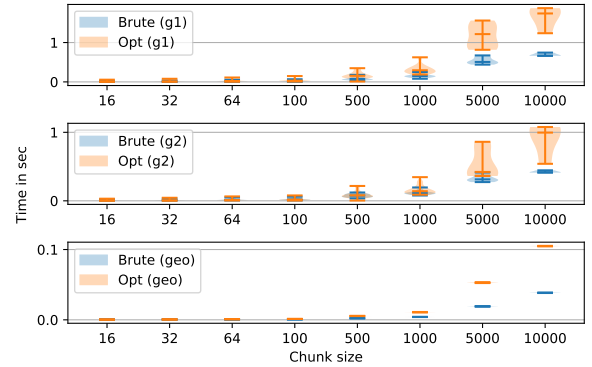


Figure 8: Performance of *gohierarchy* graph querying

First of all, we can see that even for the cases when the input graph does not contain edges used in a query, the chunk processing time grows with the size of the chunk. For example, consider the results for *geo* query for all graphs except *geospecies* and *enzyme*. Thus, the preliminary check of the existence of the edges of interest may improve performance.

We can also see, that the chunk processing time significantly depends on the graph structure. For example, *go* graph querying requires more than 5 seconds (fig. 4), while *geospecies* graph querying requires less than 0.5 seconds (fig. 6) for the chunks of size 10 000 and the query *g1*.

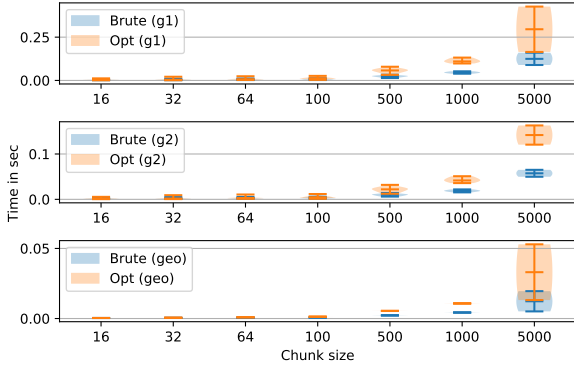


Figure 9: Performance of *pathways* graph querying

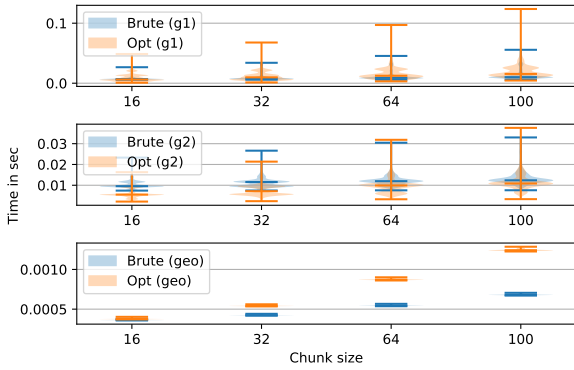


Figure 10: Performance of *eclass\_514en* graph querying with small chunks

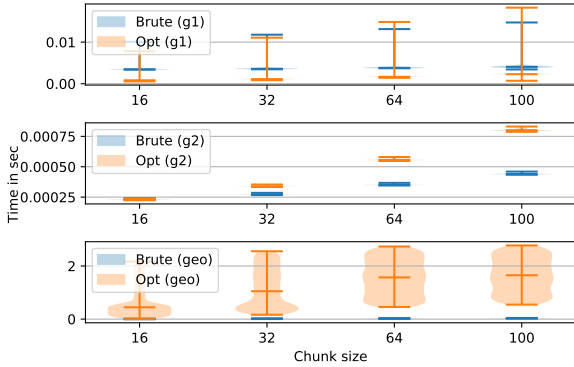


Figure 11: Performance of *geospecies* graph querying with small chunks

The comparison of two versions of the algorithm shows that the algorithm without caching is significantly faster in almost all cases, even when the graph does not contain edges of interest. Analysis of results for small chunks (fig. 10–12) shows that it is always true. For example, median time for algorithm with caching is slightly better than for the naive version for *eclass\_514en* graph and query  $g_2$  (fig. 11). On the other hand, algorithm with caching is drastically slower than the naive version for *geospecies* graph and *geo* query (fig. 11). At the same time, for *go* graph and  $g_2$  query, median time of both versions is comparable, while time for the

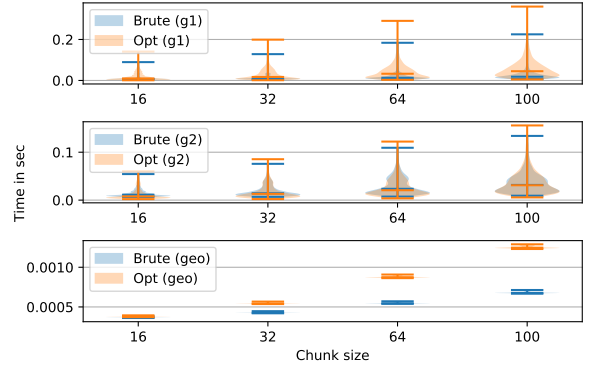


Figure 12: Performance of *go* graph querying with small chunks

worst queries is better for the naive version. Moreover, caching requires additional memory in comparison with the naive version of the algorithm. Thus we conclude, that the query results caching introduces significant overhead and does not lead to significant performance improvements. We can also conclude that small chunk processing using the naive version is fast enough: the worst time in our experiments is about 0.2 seconds (fig. 12, query  $G_1$ ).

As a result, we conclude that caching is not useful for multiple-source CFPQ for the evaluated cases even if one wants to process several chunks sequentially, or even process full graph chunk-by-chunk. Thus, we believe that the naive version of the algorithm is better for implementation in a real-world graph database.

## 4 CFPQ FULL-STACK SUPPORT

In order to provide full-stack support of CFPQ it is necessary to choose an appropriate graph database. It was shown by Arseniy Terekhov et al. in [23] that matrix-based algorithm can be naturally integrated into RedisGraph graph database because both, the algorithm and the database, operates over matrix representation of graphs. Moreover, RedisGraph supports Cypher as a query language and there is a proposal which describes Cypher extension which allows one to specify context-free constraints. Thus we choose RedisGraph as a base for our solution.

### 4.1 Cypher Extending

The first what we should do is to extend Cypher parser to be able to express context-free constraints. There is a description of the respective Cypher syntax extension<sup>7</sup>, proposed by Tobias Lindaaker, but this syntax does not implement yet in Cypher parsers.

This extension introduces path patterns, which are powerful alternative to the original Cypher relationship patterns. Path patterns allow one to express regular constraints over basic patterns such as relationship and node patterns. Like relationship patterns, they can be specified in the MATCH clause.

Main feature which allows one to specify context-free constraints is a *named path patterns*: one can specify a name for path pattern and after that use this name in other patterns, or in the same pattern. Named patterns can be defined in the PATH PATTERN clause. Using this feature, structure of query is pretty

<sup>7</sup>Formal syntax specification: <https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc#11-syntax>. Access date: 19.07.2020.



similar to context-free grammar in the Extended Backus-Naur Form (EBNF) [11].

**Listing 4** Query based on example grammar  $G_1$  (eq. 1) in Cypher with path patterns

```
1: PATH PATTERN S = ()-[:c ~S :d] | [:c (:y) :d] /->()
2: MATCH (v:x)-[:a | :c]->()-[:b ~S /->(to)
3: RETURN v, to
```

The example of query which uses named path patterns is presented in listing 4. This query is based on context-free grammar  $G_1$  (eq. 1). Namely, path pattern with name  $S$  specifies exactly the same constraint that specified by the grammar  $G_1$ . The MATCH clause uses pattern  $S$  in complex constraint which says that path of interest should starts in the vertex with label  $x$ , than in should goes throw edge with label  $a$  or  $c$ , and the end of path is a sequence of edges which starts from  $b$  and tail of this sequence matches with  $S$ .

For the example graph  $D_1$  this query returns the next pairs of vertices  $(v, to)$  (as specified in RETURN clause):

$\{(0, 4), (0, 5)\}$ .

Thus this Cypher extension allows one express more complex queries including context-free path queries. RedisGraph database supports subset of Cypher language and uses `libcypher-parser`<sup>8</sup> library to parse queries. We extend this library by introducing new syntax proposed. Note that we implement<sup>9</sup> full extension, not only part which is necessary for simple CFPQ.

## 4.2 RedisGraph Extending

This section describes the implementation of support for executing queries with the extended syntax in the RedisGraph. Throughout this section, we consider executing the example query from listing ?? for the graph  $D_1$  from Figure 1.  $\mathcal{E}$  and  $\mathcal{V}$  denotes boolean decompositions of adjacency and vertex label matrices of  $D_1$  respectively.

In the RedisGraph the main part of processing a query is building its execution plan. Execution plan consists of operations that perform basic processing such as filtering, pattern matching, aggregation and result construction. The diagram of its construction is shown in ??

After obtaining algebraic expressions they are used to construct execution plan operations. Each operation is derived from a single algebraic expression that is involved in the further execution of the corresponding operation. During the query execution this operation performs path pattern matching and solves context-free path reachability problem if necessary. This completes the part of the query execution plan building which concerns unnamed path patterns.

The remaining part of query processing is evaluation its execution plan.

Let's first consider the structure of the execution plan operations. Operations have parent-child relationships, so they are formed into a tree. For example, the part of execution plan that derived from example query is shown in ??. Each operation can consume a record from a child operation, process it and produce

<sup>8</sup>The `libcypher-parser` is an open-source parser library for Cypher query language. GitHub repository of the project: <https://github.com/cleishm/libcypher-parser>. Access date: 19.07.2020.

<sup>9</sup>The modified `libcypher-parser` library with support of syntax for path patterns: <https://github.com/YaccConstructor/libcypher-parser>. Access date: 19.07.2020.

another one for the parent. Records contain information necessary for the parent operation, as well as everything to restore the response, such as identifiers of accumulated vertices and edges.

## 4.3 Evaluation

In order to demonstrate applicability of the provided extension for RedisGraph we evaluate the proposed solution on the subset of cases provided in the section 3.3.

For RedisGraph evaluation, we used a PC with Ubuntu 18.04 installed. It has Intel Core i7-6700 CPU, 3.4GHz, and DDR4 64Gb RAM. RedisGraph with our extensions is installed from our GitHub repository<sup>10</sup>.

**4.3.1 Data preparing.** We use the same graphs which are presented in table 1 to evaluate RedisGraph-based solution.

Graphs are loaded into RedisGraph database such that each vertex has a field `id` which value is unique and is in  $[0 \dots |V| - 1]$ , where  $|V|$  is a number of vertices in the graph to load. This allows us to generate queries for specific chunk size using templates. The template for the  $g_1$  query is provided in listing 5. Here `{id_from}` and `{id_to}` are placeholders for lower and upper bounds for `id`. The example of the exact query for chunk of size 16 is presented in listing 6.

**Listing 5** Cypher query pattern for  $g_1$

```
1: PATH PATTERN S =
   ()-[:<SubClassOf [~S | ()] :SubClassOf]
   | [:<Type [~S | ()] :Type] /->()
2: MATCH (src)-/ ~S /->()
3: WHERE {id_from} <= src.id and src.id <= {id_to}
4: RETURN count(*)
```

**Listing 6** Query  $g_1$  in Cypher using the template from listing 5

```
1: PATH PATTERN S =
   ()-[:<SubClassOf [~S | ()] :SubClassOf]
   | [:<Type [~S | ()] :Type] /->()
2: MATCH (src)-/ ~S /->()
3: WHERE 15 <= src.id and src.id <= 31
4: RETURN count(*)
```

Queries generator for all three queries ( $g_1$ ,  $g_2$ , and  $geo$ ) was implemented and used to create queries for all chunks which are used in the previous experiment.

**4.3.2 Evaluation results.** For evaluation we select  $geo$  query for *geospecies* graph as one of the hardest queries, and  $g_1$  query for other graphs. Time and memory consumption are measured for each chunk processing. Results of time measurement are presented in figures 13–19.

We can see, that results is comparable with one given in section 3.3. Processing time for all chunks, except chunk of size 10 000 for *geospecies* graph (fig. 17) is less then 1 second. Moreover, for chunks of size 16 processing median time is less then 0.1 second, except *geospecies* graph.

Memory consumption for two big graphs *eclass\_514en* and *geospecies* is presented in figures 20 and 21 respectively. We can see, that amount of used memory depends on graph and query,

<sup>10</sup>Sources of RedisGraph database with full-stack CFPQ support: [https://github.com/YaccConstructor/RedisGraph/tree/path\\_patterns\\_dev](https://github.com/YaccConstructor/RedisGraph/tree/path_patterns_dev). Access date: 19.07.2020.

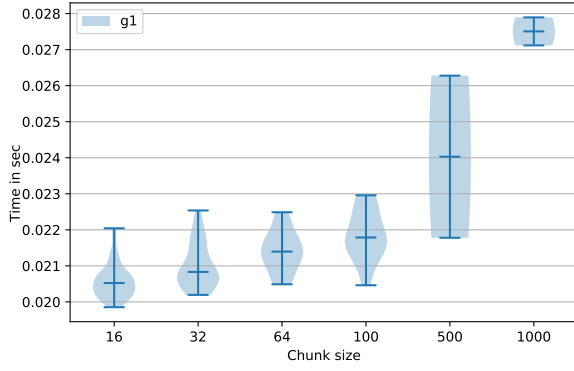


Figure 13: RedisGraph performance on *core* graph

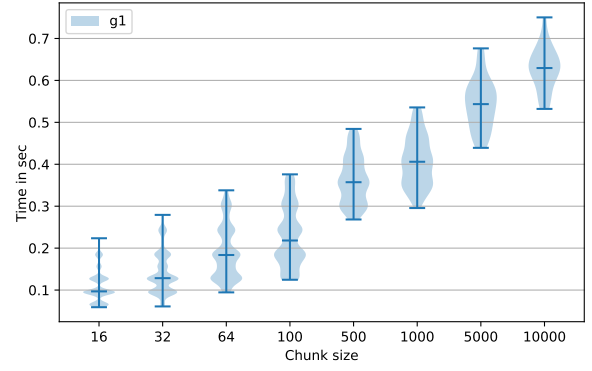


Figure 16: RedisGraph performance on *go* graph

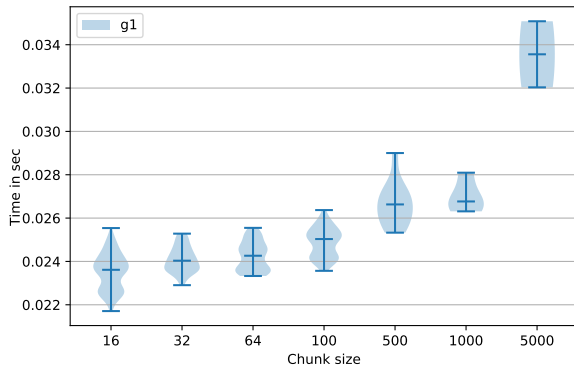


Figure 14: RedisGraph performance on *pathways* graph

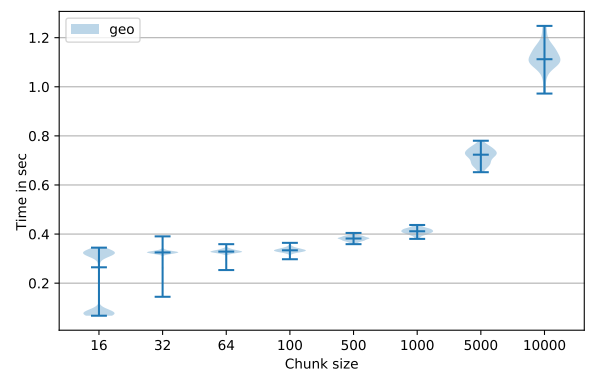


Figure 17: RedisGraph performance on *geospecies* graph

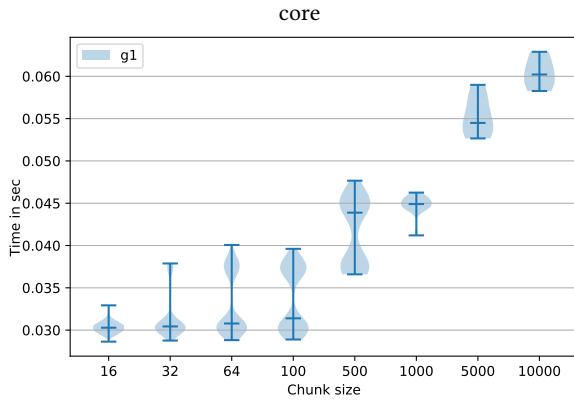


Figure 15: RedisGraph performance on *enzyme* graph

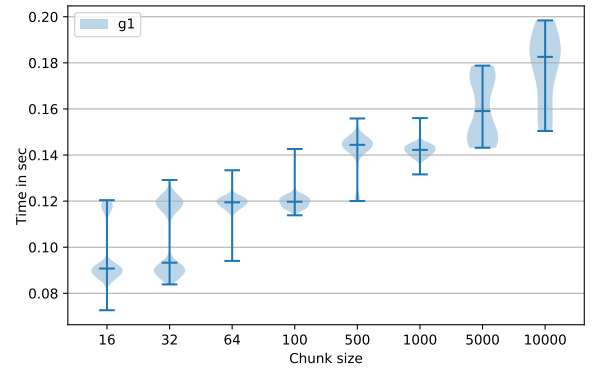


Figure 18: RedisGraph performance on *eclass\_514en* graph

but for relatively small chunks ( $\leq 1000$ ) RedisGraph uses less than 50Mb of RAM to process one chunk. Note that RedisGraph includes memory management system, thus in our experiments all allocated memory is measured, not only really used for query evaluation. As a result, we can conclude that multiple-source CFPQ is significantly more memory efficient than creation of full reachability index and its filtering: processing the chunk of size 10 000 on *geospecies* graph requires less than 200Mb, while full index creation requires 16Gb [23].

Additionally, we measure the time required to process full graph (to solve all-pairs reachability problem) by chunks of size

**Listing 7** Query  $g_1$  in Cypher for all-pairs scenario evaluation

- 1: PATH PATTERN S =  
 $() \text{--} [ < \text{SubClassOf} [\sim S \mid ()] : \text{SubClassOf} ]$   
 $| [ < : \text{Type} [\sim S \mid ()] : \text{Type} ] \text{--} ()$
- 2: MATCH  $() \text{--} \sim S \text{--} ()$
- 3: RETURN count(\*)

1000. Namely, we evaluate the query, presented in listing 7. It is

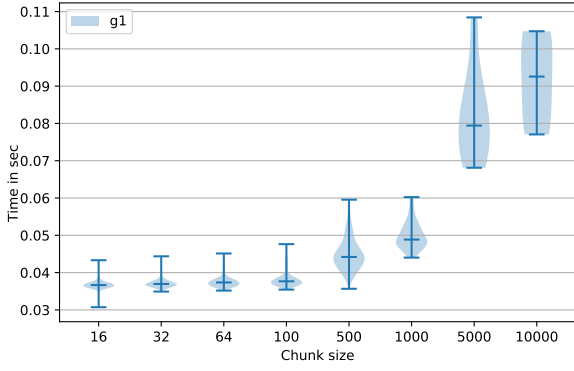


Figure 19: RedisGraph performance on *gohierarchy* graph

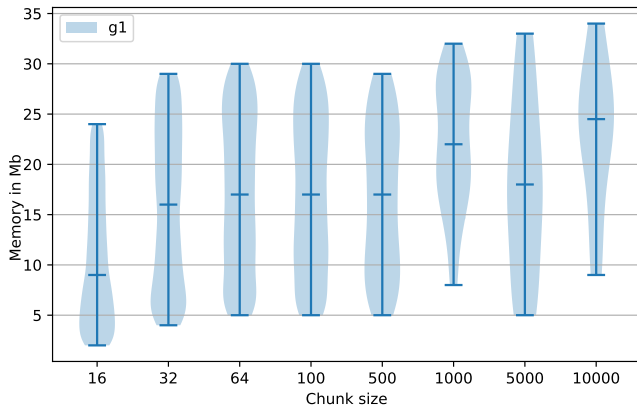


Figure 20: RedisGraph memory consumption on *eclass\_514en* graph

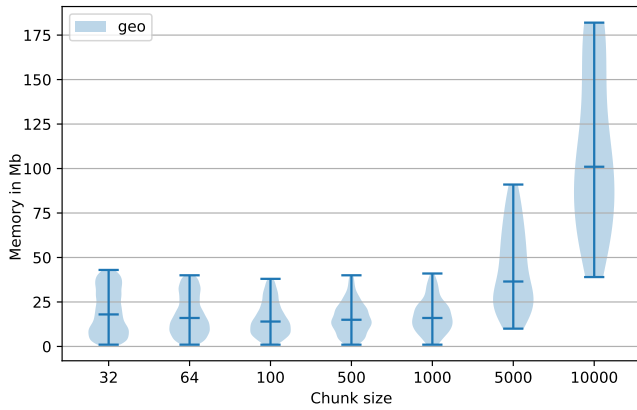


Figure 21: RedisGraph memory consumption on *geospecies* graph

similar to query form the previous scenario, but without constraints on vertices ids (without WHERE clause). Total processing time (in seconds) and total required memory (in Mb) are measured. Also, we compare our solution with results of Arseniy Terekhov et al. from [23] which were measured for RedisGraph deployed on the similar hardware and for the same graphs and queries. In [23] Azimov’s algorithm was naively integrated with

Table 2: Full graph processing time by RedisGraph with chunks of size 1000, time is measured in seconds, memory in Mb (Chunks — the proposed solution, Mono — results from [23])

Graph	#V	#E	Query	Chunks Time	Chunks Mem	Mono
core	1323	4342	$g_1$	0.003	2	0.004
pathways	6238	18 598	$g_1$	0.031	6	0.011
gohierarchy	45 007	980 218	$g_1$	0.847	62	0.091
enzyme	48 815	109 695	$g_1$	0.698	13	0.018
eclass_514en	239 111	523 727	$g_1$	18.825	35	0.067
geospecies	450 609	2 311 461	$geo$	80.979	196	7.146
go	272 770	534 311	$g_1$	72.034	40	0.604

RedisGraph storage without support of query language and other mechanisms such as lazy query evaluation. Results are provide in the table 2.

We can see, that chunk-by-chunk processing is slower, but it is still require reasonable time. First of all, if chunk size is comparable with graph size (*core* and *pathways* graphs) then time to processing is comparable with monolithic processing. Thus one can decrease time to process by increasing of chunk size and gets near optimal time. On the other hand, even with relatively small chunks (*eclass\_514*, *go* and *geospecies* graphs), when for chunk-by-chunk processing requires more than 100 times more time, our results still reasonable for some cases. For example, it requires more than 70 times less time for *geospecies* grap processing than solution of Jochem Kuijpers et al. [14] which is based on Neo4j and requires more than 6000 seconds. Moreover, while solution from [23] requires huge amount of memory (more than 16Gb for *geospecies* graph and *geo* query), our solution requires only 196Mb in the same scenario. Thus it is more suitable for general-purpose graph databases: main scenario—relatively small start vertices set—can be handled efficiently, and all-pairs reachability, which is not a massive case, can be solved in reasonable time with low memory consumption. In specific cases one can easy tune our solution to get optimal time and memory consumption.

Finally we can conclude that provided solution is a promising way to implement CPFQ for real-world graph databases.

## 5 CONCLUSION AND FUTURE WORK

In this paper we propose two version of multiple-source modifications of Azimov’s CFPQ algorithm. The evaluation of the proposed modifications on the real-world examples shows that caching of the queries results is not useful in the evaluated scenarios and the naive implementation is the best choice for the integration with a rel-world graph database. We also provide the full-stack support of CFPQ. For our solution, we implement a Cypher extension as a part of libcypher-parser, integrate the proposed algorithm into RedisGraph, and extend RedisGraph execution plan builder to support the extended Cypher queries. We demonstrate that our solution is applicable for real-world graph analysis.

In the future, it is necessary to provide formal translation of Cypher to linear algebra, or to determine a maximal subset of Cypher which can be translated to linear algebra. There is a number of works on a subset of SPARQL to linear algebra translation, such as [6, 12, 13, 16]. Most of them are practical-oriented and do not provide full theoretical basis to translate querying language to linear algebra. Others discuss only partial cases and should

be extended to cover real-world query languages. Deep investigation of this topic can help to determine the restrictions of linear algebra utilization for graph databases. Moreover, it can also improve the existing solutions.

We show that the evaluation of the regular queries is possible in practice by using the CFPQ algorithm, since the regular queries are a partial case of the context-free queries. But it seems that the proposed solution is not optimal. It is important to provide an optimal unified algorithm for both RPQ and CFPQ to create a tool applicable to a real-world tasks. One of possible way to solve this problem is to use the tensor-based algorithm [19].

Another important task is to compare non-linear-algebra-based approaches to the multiple-source CFPQ with the proposed solution. In [14] Jochem Kuijpers et al. show that all-pairs CFPQ algorithms implemented in Neo4j demonstrate unreasonable performance on real-world data. At the same time, Arseniy Terekhov et.al. shows that matrix-based all-pairs CFPQ algorithm implemented in the appropriate linear algebra based graph database (RedisGraph) demonstrates good performance. But in the case of multiple-source scenario, when a number of start vertices is relatively small, non-linear-algebra-based solutions can be better, because such solutions naturally handle only subgraph required to answer the query. Thus detailed investigation and comparison of other approaches to evaluate multiple-source CFPQ is required in the future.

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