

Context-Free Path Querying Can be Fast if Cooked Properly

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ABSTRACT

Recently proposed matrix multiplication based algorithm for context-free path querying (CFPQ) offloads the most performance-critical parts onto boolean matrices multiplication. Thus, it is possible to achieve high performance of CFPQ by means of modern parallel hardware and software. In this paper, we provide results of empirical performance comparison of different implementations of this algorithm on both real-world data and synthetic data for the worst cases.

KEYWORDS

Context-free path querying, transitive closure, graph databases, linear algebra, context-free grammar, GPGPU, CUDA, boolean matrix, matrix multiplication

1 INTRODUCTION

Formal language constrained path querying, or formal language constrained path problem [3], is a graph analysis problem in which formal languages are used as a constraints for navigational path queries. In this approach a path is viewed as a word constructed by concatenation of edge labels. Paths of interest are constrained with some formal language: a query should find only paths labeled by words from the language. The class of language constraints which is most widely spread is regular: it is used in various graph query languages and engines. While being more expressive context-free path querying (CFPQ) [21] is still at the early stage of development. Context-free constraints allow one to express such important class of queries as *same generation queries* [1] which can not be expressed in terms of regular constraints.

Several algorithms for CFPQ based on such parsing techniques as (G)LL, (G)LR, and CYK are proposed recently [4, 5, 9, 11, 14, 16, 17, 19, 22]. But recent research by Jochem Kuijpers et.al. [13] shows that existing solutions are not applicable for real-world graph analysis because of significant running time and memory consumption. At the same time, Nikita Mishin et.al in [15] show that the matrix-based CFPQ algorithm demonstrates good performance on real-world data. Matrix-based algorithm is proposed by Rustam Azimov [2] and offloads the most critical computations onto boolean matrices multiplication. This algorithm is easy to implement, and employ modern massive-parallel hardware for CFPQ. The paper measures the performance of the algorithm, not integrating it with a graph storage, while J. Kuijpers provides an evaluation of algorithms which are integrated with Neo4j¹

graph database. Also, in [13] matrix-based algorithm is evaluated in simple single-thread Java implementation, while N. Mishin shows that the most efficient implementation should utilize high-performance matrix multiplication libraries which are highly parallel or better utilize GPGPU. Thus, evaluation of the matrix-based algorithm which is integrated with a graph storage and implemented in the appropriate way is required.

In this work we show that CFPQ in relational semantics (according to Hellings [10]) can be fast enough to be applicable to real-world graph analysis. We use RedisGraph² [6] graph database as a storage. This database uses adjacency matrices as a representation of a graph and GraphBLAS [12] for matrices manipulation. These facts allow us to integrate matrix-based CFPQ algorithm to RedisGraph with minimal effort. We make the following contributions in this paper.

- (1) We provide a number of implementations of the matrix multiplication based CFPQ algorithm which uses RedisGraph as graph storage. The first implementation is CPU-based and utilizes SuteSparse³ [7] implementation of GraphBLAS API for matrices manipulation. The second implementation is GPGPU-based and includes both the existing implementation from [15] and our own CUSP⁴-based implementation. The source code is available on GitHub⁵.
- (2) We extend the dataset presented in [15] with new real-world and synthetic cases of CFPQ⁶.
- (3) We provide evaluation which shows that matrix-based CFPQ implementation for RedisGraph database is fast enough for real-world data analysis.

2 MATRIX-BASED ALGORITHM FOR CFPQ

Matrix-based algorithm for CFPQ was proposed by Rustam Azimov [2]. This algorithm can be expressed in terms of operations over matrices boolean (see listing 1), and it is a sufficient advantage for implementation.

Here $D = (V, E)$ is the input graph and $G = (N, \Sigma, P)$ is the input grammar. For each matrix T^{A_i} , $T^{A_i}[i, j] = \text{true} \iff \exists \pi = v_i \dots v_j$ —path in D , such that $A_i \xrightarrow{*}_G \omega(\pi)$, where $\omega(\pi)$ is a word formed by the labels along the path π . Thus, this algorithm solves reachability problem, or, according to Hellings [10], processes CFPQs by using relational query semantics.

²RedisGraph is a graph database which is based on Property Graph Model. Project web page: <https://oss.redislabs.com/redisgraph/>. Access date: 12.11.2019.

³SuteSparse is a sparse matrix software which includes GraphBLAS API implementation. Project web page: <http://faculty.cse.tamu.edu/davis/suitesparse.html>. Access date: 12.11.2019.

⁴CUSP is an open source library for sparse matrix multiplication on GPGPU. Project site: <https://cusplibrary.github.io/>. Access date: 12.11.2019.

⁵Sources of matrix-based CFPQ algorithm for RedisGraph database: <https://github.com/YaccConstructor/RedisGraph>. Access date: 12.11.2019.

⁶The CFPQ_Data data set fro CFPQ algorithms evaluation and comparison. GitHub page: https://github.com/JetBrains-Research/CFPQ_Data. Access date: 12.11.2019.

¹Neo4j graph database web page: <https://neo4j.com/>. Access date: 12.11.2019.

Listing 1 Context-free path querying algorithm

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1: function CONTEXTFREEPATHQUERYING( $D, G$ )
2:    $n \leftarrow$  the number of nodes in  $D$ 
3:    $E \leftarrow$  the directed edge-relation from  $D$ 
4:    $P \leftarrow$  the set of production rules in  $G$ 
5:    $N \leftarrow$  the set of nonterminals in  $G$ 
6:    $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i} \text{ is a matrix } n \times n \text{ in which each}$ 
    $\text{element is false}\}$ 
7:   for all  $(i, x, j) \in E$  do ▷ Matrix initialization
8:     for  $A_k \mid A_k \rightarrow x \in P$  do
9:        $T_{i,j}^{A_k} \leftarrow \text{true}$ 
10:  for  $A_i \mid A_i \rightarrow \varepsilon \in P$  do
11:     $T_{i,i}^{A_i} \leftarrow \text{true}$ 
12:  while any matrix in  $T$  is changing do ▷ Transitive
  closure calculation
13:    for  $A_i \rightarrow A_j A_k \in P$  do
14:       $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$ 
15:  return  $T$ 

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The performance-critical part of the algorithm is matrix boolean multiplication. Note, that we can apply the next optimization: we can skip update if the matrices T_{N_j} and T_{N_k} have not been changed at the previous iteration. Also, it is important for applications that real-world data is almost sparse, so it should be better solution to use libraries which manipulates with sparse matrices.

As we can see, CFPQ can be naturally reduced to linear algebra. Linear algebra for graph problems is an actively developed area. One of the most important result is a GraphBLAS API which provides a way to operate over matrices and vectors over user-defined semirings. In this paper we use SuiteSparse implementation of GraphBLAS and boolean semiring. All our implementations are based on the optimized version of the algorithm.

3 IMPLEMENTATION

Previous works show [2, 15] that existing linear algebra libraries utilization is a right way to get high-performance CFPQ implementation in minimal effort. But none of these works do not provide evaluation with data storage, only pure time of algorithm execution was measured.

We provide a number of implementations of matrix-based CFPQ algorithm. All of them are based on RedisGraph — we use RedisGraph as a storage and implement CFPQ as an extension by using provided mechanism. Note that currently we do not provide full integration with querying mechanism: one can not use Cypher, which uses in RedisGraph as a query language. Instead, in current implementation query provided explicitly as an file with grammar in Chomsky normal form [?]. So, we can evaluate querying algorithms, but we should improve integration to make our solution applicable.

CPU-based implementation. Details on CPU implementation

GPGPU-based implementation. Details on GPGPU implementation

4 DATASET DESCRIPTION

In our evaluation we use combined dataset which contains the following parts.

- CFPQ_Data dataset which provided in⁷ [15] and contains both synthetic and real-world graphs and queries. Real-world data includes RDFs, syntactic cases include theoretical worst-case and random graphs.
- Dataset which provided in [13]. We integrate both Geospecies and Synthetic data sets into CFPQ_Data and use it in our evaluation.
- New bigger RDFs, such as *go-hierarchy* or *enzyme* — parts of UniProt database⁸. In [15] was shown that matrix-based algorithm is performed enough to handle bigger RDFs than used in initial data sets, such as [22]. So, we add a number of big RDFs to CFPQ_Data and use them in our evaluation.

The variants of the *same generation query* [1] is used in almost all cases because it is an important example of real-world queries that are context-free but not regular. So, variations of the same generation query is used in our evaluation. All queries are added to the CFPQ_Data data set.

5 EVALUATION AND DISCUSSION

We evaluate all the described implementations on all the datasets and the queries presented. We compare our implementations with [15] and [13]. We measure full time of query execution, so all overhead on data preparing is included. Thus we can estimate applicability of matrix-based algorithm for real-world solutions.

For evaluation, we use a PC with Ubuntu 18.04 installed. It has Intel core i7 CPU, DDR4 32 Gb RAM, and Geforce GTX 1080 GPGPU with 8Gb RAM.

The results of the evaluation are summarized in the tables below. Time is measured in seconds unless specified otherwise. Note that for all implementations except our own we provide results from related paper. The cell is left blank if the time limit is exceeded, if there is not enough memory to allocate the data, or information is not available.

The results of the first dataset [RDF] are presented in table 1. We can see, that in this case the running time of all our implementations is smaller than of the reference implementation, and all implementations but [CuSprs] demonstrate similar performance. It is obvious that performance improvement in comparison with the first implementation is huge and it is necessary to extend the dataset with new RDFs of the significantly bigger size.

Table 2: Evaluation results for the worst case

| #V | Scipy | M4RI | GPU4R | GPU_N | GPU_Py | CuSprs |
|------|----------|---------|--------|---------|---------|----------|
| 16 | 0.032 | < 1 | 0.008 | 0.002 | 0.027 | 0.309 |
| 32 | 0.118 | 0.001 | 0.034 | 0.008 | 0.136 | 0.441 |
| 64 | 0.476 | 0.041 | 0.133 | 0.032 | 0.524 | 0.988 |
| 128 | 2.194 | 0.226 | 0.562 | 0.129 | 2.751 | 3.470 |
| 256 | 15.299 | 1.994 | 3.088 | 0.544 | 11.883 | 15.317 |
| 512 | 121.287 | 23.204 | 13.685 | 2.499 | 43.563 | 102.269 |
| 1024 | 1593.284 | 528.521 | 88.064 | 19.357 | 217.326 | 1122.055 |
| 2048 | - | - | - | 325.174 | - | - |

Results of the theoretical worst case ([Worst] dataset) are presented in table 2. This case is really hard to process: even for a graph of 1024 vertices, the query evaluation time is greater than

⁷CFPQ_Data data set GitHub repository: https://github.com/JetBrains-Research/CFPQ_Data. Access date: 12.11.2019.

⁸Protein sequences data base: <https://www.uniprot.org/>. RDFs with data are available here: ftp://ftp.uniprot.org/pub/databases/uniprot/current_release/rdf. Access date: 12.11.2019

Table 1: RDFs querying results (time in milliseconds)

| RDF | | | Query G_4 | | | | | | Query G_5 | | | | | |
|-------------|-----|------|-------------|------|-------|-------|--------|--------|-------------|------|-------|-------|--------|--------|
| Name | #V | #E | Scipy | M4RI | GPU4R | GPU_N | GPU_Py | CuSprs | Scipy | M4RI | GPU4R | GPU_N | GPU_Py | CuSprs |
| atm-prim | 291 | 685 | 3 | 2 | 2 | 1 | 5 | 269 | 1 | < 1 | 1 | < 1 | 2 | 267 |
| biomed | 341 | 711 | 3 | 5 | 2 | 1 | 5 | 283 | 4 | < 1 | 1 | < 1 | 5 | 280 |
| foaf | 256 | 815 | 2 | 9 | 2 | < 1 | 5 | 270 | 1 | < 1 | 1 | < 1 | 2 | 263 |
| funding | 778 | 1480 | 4 | 7 | 4 | 1 | 5 | 279 | 2 | < 1 | 3 | < 1 | 4 | 274 |
| generations | 129 | 351 | 3 | 3 | 2 | < 1 | 5 | 273 | 1 | < 1 | 1 | < 1 | 2 | 263 |
| people_pets | 337 | 834 | 3 | 3 | 3 | 1 | 7 | 284 | 1 | < 1 | 1 | < 1 | 3 | 277 |
| pizza | 671 | 2604 | 6 | 8 | 3 | 1 | 6 | 292 | 2 | < 1 | 2 | < 1 | 5 | 278 |
| skos | 144 | 323 | 2 | 4 | 2 | < 1 | 5 | 273 | < 1 | < 1 | 1 | < 1 | 2 | 265 |
| travel | 131 | 397 | 3 | 5 | 2 | < 1 | 6 | 268 | 1 | < 1 | 1 | < 1 | 3 | 271 |
| unv-bnch | 179 | 413 | 2 | 4 | 2 | < 1 | 5 | 266 | 1 | < 1 | 1 | < 1 | 3 | 266 |
| wine | 733 | 2450 | 7 | 6 | 4 | 1 | 7 | 294 | 1 | < 1 | 3 | < 1 | 3 | 281 |

10 seconds even for the most performant implementation. We can see, that the running time grows too fast with the number of vertices.

Table 3: Sparse graphs querying results

| Graph | Scipy | M4RI | GPU4R | GPU_N | GPU_Py | CuSprs |
|------------|---------|----------|--------|--------|--------|---------|
| G5k-0.001 | 10.352 | 0.647 | 0.113 | 0.041 | 0.216 | 5.729 |
| G10k-0.001 | 37.286 | 2.395 | 0.435 | 0.215 | 1.331 | 35.937 |
| G10k-0.01 | 97.607 | 1.455 | 0.273 | 0.138 | 0.763 | 47.525 |
| G10k-0.1 | 601.182 | 1.050 | 0.223 | 0.114 | 0.859 | 395.393 |
| G20k-0.001 | 150.774 | 11.025 | 1.842 | 1.274 | 6.180 | - |
| G40k-0.001 | - | 97.841 | 11.663 | 8.393 | 37.821 | - |
| G80k-0.001 | - | 1142.959 | 88.366 | 65.886 | - | - |

The next is the **[Sparse]** dataset presented in table 3. The evaluation shows that sparsity of graphs (value of parameter p) is important both for implementations which use sparse matrices and for implementations which use dense matrices. Note that the behavior of the sparse matrices based implementation is as expected, but for dense matrices we can see, that more sparse graphs are processed faster. Reasons for such behavior demand further investigation. Note that we estimate only the query execution time, so it is hard to compare our results with the results presented in [8]. Nevertheless, the running time of our **[GPU_N]** implementation is significantly smaller than the one provided in [8].

The last dataset is **[Full]**, and results are shown in table ?? . As we expect, this case is very hard for sparse matrices based implementations: the running time grows too fast. This dataset also demonstrates the impact of the grammar size. Both queries specify the same constraints, but the grammar G_3 in CNF contains 2 times more rules than the grammar G_2 , so, the running time for big graphs differs by more than twice.

Finally, we can conclude that GPGPU utilization for CFPQ can significantly improve performance, but more research on advanced optimization techniques should be done. On the other hand, the high-level implementation (**[GPU_Py]**) is comparable with other GPGPU-based implementations. So, it may be a balance between implementation complexity and performance. Highly optimized existing libraries can be of some use: the implementation based on m4ri is faster than the reference implementation and the other CPU-based implementation. Moreover, it is comparable with some GPGPU-based implementations in

some cases. Sparse matrices utilization demands more thorough investigation. The main question is if we can create an efficient implementation for sparse boolean matrices multiplication.

6 CONCLUSION AND FUTURE WORK

We provide an CPU and GPGPU based context-free path querying implementations for RedisGraph and show that CFPQ can be fast enough to analyze real-world data. But our implementations are on prototype stage and we should provide full integration of CFPQ to RedisGraph. First of all it is necessary to extend Cupher graph query language, which uses in RedisGraph, to support respective syntax for context-free constraints specification. There is a proposal which describes such syntax extension⁹ and we plan to support proposed syntax in libcypher-parser¹⁰ which uses in RedisGraph.

In current version we use CUSP matrix multiplication library for GPGPU utilization, but it may be better to use GraphBLAST¹¹ [20] – Gunrock¹² [18] based implementation of GraphBLAS API for GPGPU. First of all, we should evaluate GraphBlast based implementation of CFPQ. Also, we should investigate to implement multi-GPU support for GraphBlast, because it should improve performance of huge real-world data processing.

Our implementations calculate queries in respect to relational semantics, but in some cases it is necessary to provide a path which satisfied constraints. As we know, matrix based algorithm for single path or all paths semantics is not provided yet, and it is a direction for future research.

Another important question for future research is how to update query result dynamically when data changes. Mechanism for result updating allows one to recalculate query faster and use result as an index for other queries.

Also, further improvements of the dataset are required. For example, it is necessary to include real-world cases from static code analysis [?].

⁹Proposal with path pattern syntax for openCypher: <https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc>. It is shown that context-free constraints are expressible in proposed syntax. Access date: 12.11.2019

¹⁰Web page of libcypher-parser project: <http://cleishm.github.io/libcypher-parser/>. Access date: 12.11.2019

¹¹GraphBLAST project: <https://github.com/gunrock/graphblast>. Access date: 12.11.2019.

¹²Gunrock project web page: <https://gunrock.github.io/docs/>. Access date: 12.11.2019.

Table 4: Free scale graphs querying results

| Graph | CPU | | m4ri | | CUSP | | neo4j | |
|-------------|---------|---------|---------|---------|---------|----------|----------|---------|
| | Time | Mem | Time | Mem | Time | Mem | Time | Mem |
| G(100,1) | < 1 | < 1 | 0.002 | < 1 | 0.003 | 0.278 | 0.023 | 0.076 |
| G(100,3) | < 1 | < 1 | 0.002 | 0.001 | 0.004 | 0.279 | 0.105 | 0.098 |
| G(100,5) | < 1 | < 1 | 0.003 | 0.001 | 0.004 | 0.329 | 1.636 | 0.094 |
| G(100,10) | < 1 | < 1 | 0.005 | 0.001 | 0.006 | 0.571 | 13.071 | 0.106 |
| G(500,1) | < 1 | < 1 | 0.019 | 0.003 | 0.017 | 1.949 | 93.676 | 0.108 |
| G(500,3) | 0.003 | < 1 | 0.125 | 0.038 | 0.150 | 99.651 | 1205.421 | 0.851 |
| G(500,5) | 0.005 | < 1 | 0.552 | 0.315 | 0.840 | 1029.042 | - | 4.690 |
| G(500,10) | 1.239 | 7202 | 7.252 | 5.314 | 15.521 | - | - | 70.823 |
| G(2500,1) | 40.309 | 0.063 | 0.019 | 0.003 | 0.017 | 1.949 | 93.676 | 0.108 |
| G(2500,3) | 651.343 | 0.366 | 0.125 | 0.038 | 0.150 | 99.651 | 1205.421 | 0.851 |
| G(2500,5) | - | 1.932 | 0.552 | 0.315 | 0.840 | 1029.042 | - | 4.690 |
| G(2500,10) | - | 360.035 | 58.751 | 44.611 | 129.641 | - | - | 775.765 |
| G(10000,1) | 0.009 | 1024 | 0.019 | 0.003 | 0.017 | 1.949 | 93.676 | 0.108 |
| G(10000,3) | 5.439 | 4353 | 0.125 | 0.038 | 0.150 | 99.651 | 1205.421 | 0.851 |
| G(10000,5) | 7.978 | 8193 | 0.552 | 0.315 | 0.840 | 1029.042 | - | 4.690 |
| G(10000,10) | 13.180 | 47362 | 256.579 | 190.343 | 641.260 | - | - | - |

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