Context-Free Path Querying Can be Fast if Cooked Properly

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ABSTRACT

A recent study showed that the applicability of context-free path querying (CFPQ) algorithms integrated with Neo4j database is limited because of low performance and high memory consumption. In this work we implement a matrix-based CFPQ algorithm by using appropriate high-performance libraries for linear algebra and integrate it to RedisGraph graph database. Our evaluation shows that provided implementation is up to 1000 times faster than the best Neo4j-based one in some cases.

KEYWORDS

Context-free path querying, transitive closure, graph databases, linear algebra, context-free grammar, GPGPU, CUDA, boolean matrix, matrix multiplication

1 INTRODUCTION

Formal language constrained path querying, or formal language constrained path problem [4], is a graph analysis problem in which formal languages are used as a constraints for navigational path queries. In this approsch a path is viewed as a word constructed by concatenation of edge labels. Paths of interest are constrained with some formal language: a query should find only paths labeled by words from the language. The class of language constraints which is most widely spread is regular: it is used in various graph query languages and engines. While being more expressive context-free path queruing (CFPQ) [23] is still at the early stage of development. Context-free constraints allow one to express such important class of queries as *same generation queies* [1] which can not be expressed in terms of regular constraints.

Several algorithms for CFPQ based on such parsing techniques as (G)LL, (G)LR, and CYK are proposed recently [5, 6, 9, 11, 15, 17, 19, 21, 24]. But recent research by Jochem Kuijpers et.al. [14] shows that existing solutions are not applicable for real-world graph analysis because of significant running time and memory consumption. At the same time, Nikita Mishin et.al in [16] show that the matrix-based CFPQ algorithm demonstrates good performance on real-world data. Matrix-based algorithm is proposed by Rustam Azimov [3] and offloads the most critical computations onto boolean matrices multiplication. This algorithm is easy to implement, and employ modern massive-parallel hardware for CFPQ. The paper measures the performance of the algorithm, not integrating it with a graph storage, while J. Kuijpers provides an evaluation of algorithms which are integrated with Neo4j[1] graph database. Also, in [14] matrix-based algorithm is evaluated in simple single-thread Java implementation, while N. Mishin shows that the most efficient implementation should utilize high-performance matrix multiplication libraries which are highly parallel or better utilize GPGPU. Thus, evaluation of the matrix-based algorithm which is integrated with a graph storage and implemented in the appropriate way is required.

In this work we show that CFPQ in relational semantics (according to Hellings [10]) can be fast enough to be applicable to real-world graph analysis. We use RedisGraph[2] [7] graph database as a storage. This database uses adjacency matrices as a representation of a graph and GraphBLAS [13] for matrices manipulation. These facts allow us to integrate matrix-based CFPQ algorithm to RedisGraph with minimal effort. We make the following contributions in this paper.

(1) We provide a number of implementations of the matrix multiplication based CFPQ algorithm which uses RedisGraph as graph storage. The first implementation is CPU-based and utilizes SuteSparse[3] [8] implementation of GraphBLAS API for matrices manipulation. The second implementation is GPGPU-based and includes both the existing implementation from [16] and our own CUSP[4]-based implementation. The source code is available on GitHub[5].
(2) We extend the dataset presented in [16] with new real-world and synthetic cases of CFPQ[6].
(3) We provide evaluation which shows that matrix-based CFPQ implementation for RedisGraph database is fast enough for real-world data analysis.

2 MATRIX-BASED ALGORITHM FOR CFPQ

The matrix-based algorithm for CFPQ was proposed by Rustam Azimov [3]. This algorithm can be expressed in terms of operations over boolean matrices (see listing 1) which is an advantage for implementation.

Here $D = (V, E)$ is the input graph and $G = (N, \Sigma, P)$ is the input grammar. For each matrix T^{A_k}, $T^{A_k}_{i,j} = \texttt{true} \iff \exists \pi = v_i \dots v_j$ — path in D, such that $A_k \overset{*}{\underset{G}{\Rightarrow}} \omega(\pi)$, where $\omega(\pi)$ is a word formed by the labels along the path π. Thus, this algorithm solves the reachability problem, or, according to Hellings [10], implements relational query semantics.

[1]Neo4j graph database web page: https://neo4j.com/. Access date: 12.11.2019.

[2]RedisGraph is a graph database which is based on Property Graph Model. Project web page: https://oss.redislabs.com/redisgraph/. Access date: 12.11.2019.
[3]SuteSparse is a sparse matrix software which incudes GraphBLAS API implementation. Project web page: http://faculty.cse.tamu.edu/davis/suitesparse.html. Access date: 12.11.2019.
[4]CUSP is an open source library for sparse matrix multiplication on GPGPU. Project site: https://cusplibrary.github.io/. Access date: 12.11.2019.
[5]Sources of matrix-based CFPQ algorithm for RedisGraph database: https://github.com/YaccConstructor/RedisGraph. Access date: 12.11.2019.
[6]The CFPQ_Data data set fro CFPQ algorithms evaluation and comparison. GitHub page: https://github.com/JetBrains-Research/CFPQ_Data. Access date: 12.11.2019.

Listing 1 Context-free path quering algorithm

```
function evalCFPQ($D = (V, E)$, $G = (N, \Sigma, P)$)
    $n \leftarrow |V|$
    $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i}$ is a matrix $n \times n$, $T^{A_i}_{k,l} \leftarrow$ false$\}$
    for all $(i, x, j) \in E, A_k \mid A_k \to x \in P$ do $T^{A_k}_{i,j} \leftarrow$ true
    for $A_k \mid A_k \to \varepsilon \in P$ do $T^{A_k}_{i,i} \leftarrow$ true
    while any matrix in $T$ is changing do
        for $A_i \to A_j A_k \in P$ do $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$
    return $T$
```

The performance-critical part of the algorithm is boolean matrix multiplication. Note, that if the matrices T_{N_j} and T_{N_k} have not been changed at the previous iteration, then we can skip update operation. Such optimization can improve performance. Also, it is important for applications that real-world data is often sparse, so it should be a better solution to use libraries which manipulate with sparse matrices.

3 IMPLEMENTATION

We showed that CFPQ can be naturally reduced to linear algebra. Linear algebra for graph problems is an actively developed area. One of the most important results is a GraphBLAS API which provides a way to operate over matrices and vectors over user-defined semirings.

Previous works show [3, 16] that existing linear algebra libraries utilization is the right way to get high-performance CFPQ implementation with minimal effort. But neither of these works do provide an evaluation with data storage, only pure time of algorithm execution was measured.

We provide a number of implementations of the matrix-based CFPQ algorithm. All of them a based on RedisGraph — we use RedisGraph as storage and implement CFPQ as an extension by using a provided mechanism. We choose this database because it uses sparse adjacency matrices for graphs representation which is an appropriate representation for the matrix-based algorithm. Note that currently, we do not provide full integration with querying mechanism: one cannot use Cypher, which uses in RedisGraph as a query language. Instead, query should be provided explicitly as a file with grammar in Chomsky normal form. This is enough to evaluate querying algorithms, but in the future we should improve integration to make our solution applicable.

CPU-based implementation (RG_CPU) uses SuteSparse implementation of GraphBLAS, which is used in RedisGraph, and a predefined boolean semiring. Thus we avoid data format problems: we use native RedisGraph representation of the adjacency matrix in our algorithm.

GPGPU-based implementation has two versions. The first one (**RG_M4RI**) uses the Method of Four Russians implemented in [16], and the second one (**RG_CUSP**) utilizes a modified CUSP library for matrix operations. Both implementations require matrix format conversion.

4 DATASET DESCRIPTION

In our evaluation we use combined dataset which contains the following parts.

- CFPQ_Data dataset which is provided in[7] [16] and contains both synthetic and real-world graphs and queries. Real-world data includes RDFs, synthetic cases include theoretical worst-case and random graphs.
- Dataset which is provided in [14]. Both Geospecies — RDF which contains information about biological hierrarchy[8] and same generation query over *broaderTransitive* relation, and Synthetic — the set of graphs generated by using the Barabási-Albert model [2] of scale-free networks and same generation quey, are integrated to CFPQ_Data and used it in our evaluation.
- In [16] was shown that matrix-based algorithm is performant enough to handle bigger RDFs than those used in initial data sets, such as [24]. So, we add a number of big RDFs to CFPQ_Data and use them in our evaluation. New RDFs: *go-hierarchy, go, enzime, core, pathways* are from UniProt database[9], and *eclass-514en* is from eClassOWL project[10].

The variants of the *same generation query* [1] are used in almost all cases because it is an important example of real-world queries that are context-free but not regular. So, variations of the same generation query are used in our evaluation. All queries are added to the CFPQ_Data data set.

For RDFs (**[RDF]** dataset) we use two queries over *subClassOf* and *type* relations. The first query is the grammar G_1:

$$s \to subClassOf^{-1}\ s\ subClassOf \qquad s \to type^{-1}\ s\ type$$
$$s \to subClassOf^{-1}\ subClassOf \qquad s \to type^{-1}\ type$$

The second one is the grammar G_2:

$$s \to subClassOf^{-1}\ s\ subClassOf \mid subClassOf.$$

For geospecies and free scale graphs querying we use same-generation queries form the original paper.

5 EVALUATION AND DISCUSSION

We evaluate all the described implementations on all the datasets and the queries presented. We compare our implementations with [16] and [14]. We measure the full time of query execution including all overhead on data preparation. Thus we can estimate the applicability of the matrix-based algorithm for real-world solutions.

For evaluation, we use a PC with Ubuntu 18.04 installed. It has Intel core i7-6700 CPU, 3.4GHz, DDR4 32Gb RAM, and Geforce GTX 1070 GPGPU with 8Gb RAM.

The results of the evaluation are summarized in the tables below. We provide results only for part of the collected data set because of the page limit. Running time is measured in seconds, RAM memory consumption is measured in megabytes unless specified otherwise. Note that for all implementations except our own we provide results from the related paper. The cell is left blank if the time limit is exceeded, or if there is not enough memory to allocate the data.

The results of the first dataset **[RDF]** are presented in table 1. We can see that the running time of both CPU and GPGPU versions is small even for graphs with a big number of vertices and edges. The relatively small number of edges of interest may be the reason for such behavior. It is necessary to extend the dataset with new queries which involve more different types of edges.

[7] CFPQ_Data data set GitHub repository: https://github.com/JetBrains-Research/CFPQ_Data. Access date: 12.11.2019.

[8] https://old.datahub.io/dataset/geospecies. Access date: 12.11.2019.

[9] Protein sequences data base: https://www.uniprot.org/. RDFs with data are avalable here: ftp://ftp.uniprot.org/pub/databases/uniprot/current_release/rdf. Access date: 12.11.2019

[10] eClassOWL project: http://www.heppnetz.de/projects/eclassowl/. eclass-514en file is available here: http://www.ebusiness-unibw.org/ontologies/eclass/5.1.4/eclass_514en.owl. Access date: 12.11.2019.

Table 1: RDFs querying results

RDF					Query G_1			Query G_2		
Name	#V	#E	#type	#subClassOf	RG_CPU	RG_M4RI	RG_CUSP	RG_CPU	RG_M4RI	RG_CUSP
funding	778	1480	304	90	0.01	<0.01	0.02	< 0.01	< 0.01	< 0.01
pizza	671	2604	365	259	0.01	<0.01	0.02	< 0.01	< 0.01	< 0.01
wine	733	2450	485	126	0.01	<0.01	0.02	< 0.01	< 0.01	< 0.01
core	1323	8684	1412	178	< 0.01	0.12	0.02	< 0.01	< 0.01	< 0.01
pathways	6238	37196	3118	3117	0.01	0.18	0.03	< 0.01	0.06	< 0.01
go-hierarchy	45007	1960436	0	490109	0.09	-	1.50	< 0.01	-	0.55
enzyme	48815	219390	14989	8163	0.02	61.23	0.10	< 0.01	6.97	0.02
eclass_514en	239111	1047454	72517	90962	0.06	-	0.39	0.01	-	0.10
go	272770	1068622	58483	90512	0.49	-	0.83	0.01	-	0.11

Also, we can see, that *m4ri* version which uses dense bit matrices requires more memory. Thus we recommend to use sparse matrices on GPGPU.

Geospecies dataset currently can be processed only by using CPU version and we compare our matrix-based CPU implementation with the result form [14] for $AnnGram_{rel}$ algorithm[11]. Fortunately, both algorithms calculate queries under relational semantics. The result is provided in the table 2.

Table 2: Evaluation results on geospecies data

RG_CPU		Neo4j_AnnGram$_{rel}$	
Time	Memory (Gb)	Time	Memory (Gb)
6.8	6.83	6 953.9	29.17

As we can see, that the matrix-based algorithm implemented for RedisGraph is more than 1000 times faster than based on annotated grammar implemented for Neo4j and use more than 4 times less memory. Thus we can conclude that the matrix-based algorithm is better than other CFPQ algorithms for query evaluation under a relational semantics for real-world data processing. CFPQ evaluation under other semantics (single path, all paths, etc) by using a matrix-based algorithm is a direction for future research.

The next is the **[FreeScale]** datatset. We compare our implementations with two implementations form [14] which evaluate queries under relatonal semantics: *Neo4j_AnnGram$_{rel}$* and *Neo4j_Matrix*. The results are presented in table 3. The evaluation shows that sparsity of graphs (value of parameter p) is important both for implementations which use sparse matrices and for implementations which use dense matrices. Note that the results for implementations for Neo4j are restored from graphics provided in [14]. So, values are not precize, but it is possible to compare implementations.

Evaluation shows that our CPU version is comparable with *Neo4j_AnnGram$_{rel}$* and for relatively dense graphs (each vertex has 10 connections) our implementation is faster. Moreover, while *Neo4j_Matrix* is out of limits on the biggest graph, our implementation works fine. So, it is important to use appropriate libraries for matrix-based algorithm implementation. Also, we can see, that GPGPU version which utilizes sparse matrices is significantly faster than the other implementations. Note, that for GPGPU versions we include time requred for data transferring and formans convertion.

[11]Only *AnnGram* works correctly and fits limits, other implementations are faster, but return an incorrect result, or do not fit memory limits.

Finally, we can conclude that the matrix-based algorithm paired with a suitable database and employing appropriate libraries for linear algebra is a promising way to make CFPQ applicable for real-world data analysis. We show that SuiteSparse-based CPU implementation is performant enough to be comparable with GPGPU-based implementations on real-world data. It means that we can handle more complex data. Also, we can see, that more complex queries should be added to the dataset to make it more representable.

6 CONCLUSION AND FUTURE WORK

We implemented a CPU and GPGPU based context-free path querying for RedisGraph and showed that CFPQ can be performant enough to analyze real-world data. However, our implementations are prototypes and we plan to provide full integration of CFPQ to RedisGraph. First of all, it is necessary to extend Cypher graph query language, which is used in RedisGraph, to support syntax for specification of context-free constraints. There is a proposal which describes such syntax extension[12] and we plan to support proposed the syntax in libcypher-parser[13] which is used in RedisGraph.

Current version uses CUSP matrix multiplication library for GPGPU utilization, but it may be better to use GraphBLAST[14] [22] – Gunrock[15] [20] based implementation of GraphBLAS API for GPGPU. First of all, we plan to evaluate GraphBLAST based implementation of CFPQ. Also, we plan to investigate how multi-GPU support for GraphBLAST influences the performance of CFPQ in the case of processing huge real-world data.

Our implementations compute relational semantics of a query, but some problems require to find a path which satifies the constraints. Best to our knowledge, there is no matrix-based algorithm for single path or all path semantics, thus we see it as a direction for future research.

Another important open question is how to update the query results dynamically when data changes. The mechanism for result updating allows one to recalculate query faster and use the result as an index for other queries.

[12]Proposal with path pattern syntax for openCypher: https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc. It is shown that context-free constraints are expressible in proposed syntax. Access date: 12.11.2019

[13]Web page of libcypher-parser project: http://cleishm.github.io/libcypher-parser/. Access date: 12.11.2019

[14]GraphBLAST project: https://github.com/gunrock/graphblast. Access date: 12.11.2019.

[15]Gunrock project web page: https://gunrock.github.io/docs/. Access date: 12.11.2019.

Table 3: Free scale graphs querying results

Graph	RG_CPU		RG_m4ri		RG_CUSP		Neo4j_AnnGram$_{rel}$		Neo4j_Matrix	
	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem
G(100,1)	< 0.01	< 0.01	< 0.01	0.10	0.01	2.00	< 0.02	0.08	0.20	0.03
G(100,3)	< 0.01	< 0.01	< 0.01	0.10	0.04	2.00	0.02	0.15	0.40	0.03
G(100,5)	< 0.01	< 0.01	< 0.01	0.10	0.05	2.00	0.03	0.21	0.40	0.03
G(100,10)	< 0.01	< 0.01	0.01	0.10	0.07	2.00	0.09	0.60	0.60	0.03
G(500,1)	< 0.01	< 0.01	< 0.01	2.00	0.01	2.00	< 0.02	0.20	20.00	0.60
G(500,3)	< 0.01	< 0.01	< 0.01	2.00	0.07	2.00	0.03	0.50	40.00	0.60
G(500,5)	< 0.01	0.17	< 0.01	2.00	0.10	2.00	0.10	1.10	50.00	0.60
G(500,10)	1.24	0.78	0.01	2.00	0.11	4.00	0.50	4.00	55.00	0.60
G(2500,1)	< 0.01	0.11	0.07	30.00	0.03	2.00	0.03	0.70	0.023	14.00
G(2500,3)	0.01	0.11	0.11	30.00	0.10	2.00	0.15	2.50	0.105	14.00
G(2500,5)	2.06	0.11	0.11	30.00	0.12	4.00	0.70	8.00	1.636	14.00
G(2500,10)	3.25	3.77	0.13	30.00	0.31	31.20	5.00	20.00	13.071	14.00
G(10000,1)	< 0.01	0.47	1.55	200.00	0.04	2.0	0.10	2.50	-	-
G(10000,3)	5.439	1.15	3.60	200.00	0.20	3.20	0.40	10.00	-	-
G(10000,5)	7.978	2.64	3.32	200.00	0.25	13.20	3.00	35.00	-	-
G(10000,10)	13.180	21.08	3.60	200.00	1.23	198.00	40.00	240.00	-	-

Also, further improvements of the dataset are required. For example, it is necessary to include real-world cases from the area of static code analysis [12, 18, 25].

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