Evaluation of the Context-Free Path Querying Algorithm Based on Matrix Multiplication

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ABSTRACT

Recently proposed matrix multiplication based algorithm for context-free path querying (CFPQ) offloads the most performance-critical parts onto boolean matrices multiplication. Thus, it is possible to utilize modern parallel hardware and software to achieve high performance of CFPQ easily. In this work, we provide results of empirical performance comparison of different implementations of this algorithm on both real data and synthetic data for the worst cases.

CCS CONCEPTS

• **Information systems → Query languages for non-relational engines**; • **Theory of computation → Grammars and context-free languages**; *Parallel computing models*; • **Computing methodologies → Massively parallel algorithms**; • **Computer systems organization →** *Single instruction, multiple data*;

KEYWORDS

Context-free path querying, transitive closure, graph databases, context-free grammar, GPGPU, CUDA, matrix multiplication, boolean matrix

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1 INTRODUCTION

Language-constrained path querying [5], and particularly Context-Free Path Querying (CFPQ) [14] widely used for graph-structured data analysis in such areas as biological data analysis, RDF, network analysis. Huge amount of the real-world data makes performance of CFPQ evaluation critical for practical tasks, and number of algorithms for CFPQ evaluation proposed ricently [7, 9, 11–13, 15].

One of the most promising algorithms is a matrix-based algorithm, proposed by Rustam Azimov [4]. This algorithm offloads the most critical computations onto boolean matrices multiplication. As a result, it is pretty simple for implementation and allows one to utilize modern massive-parallel hardware for CFPQs evaluation. The implementation provided by authors utilizes GPGPU by using cuSPARSE[1] library which is floating point sparse matrices multiplication library. Even it does not use advanced algorithms for boolean matrices, it outperforms existing algorithms.

It is necessary to investigate the effect of specific algorithms and implementation techniques on the performance of CFPQ. One of the problems is that there is no publically available standard dataset for CFPQ algorithms evaluation which includes both graph-structured data and queries.

In this work, we do an empirical performance comparison of different implementations of matrices multiplication based algorithm for CFPQ on both real data and synthetic data for the worst cases. We make the following contributions in this paper.

(1) We provide a number of implementations of the matrix multiplication based CFPQ algoithm, which utilizes different modern software and hardware. Source code is available on GitHub:!!!
(2) We collect and publish a dataset which contains both real data and syntatic data for wirst cases. This dataset contains data and queries in the simplee textual format, so it can be used for other algorithms evaluation easely. We hope that this dataset can be a base for unified benchmark for CFPQ algorithms.
(3) We provide evaluation results which shows that GPGPU utilization for CFPQ can significantely improve performance,

[1] cuSparse is a library for GPGPU utilization for sparse matrices multiplication. Official documentation:https://docs.nvidia.com/cuda/cusparse/index.html. Access date: 12.03.2019

and that there are many questions for future research in this area.

2 MATRIX-BASED ALGORITHM FOR CFPQ

Matrix-based algorithm for CFPQ was proposed by Rustam Azimov [4]. This algorithm can be expressed in a few lines of code in terms of matrices operations, and it is a sufficient advantage for implementation. It was shown that GPGPU utilization for queries evaluation can significantly improve performance in comparison with other implementations [4] even float matrices used instead of boolean matrices.

Pseudocode of the algorithm is presented in listing 1.

Algorithm 1 Context-free path quering algorithm

```
1:  function contextFreePathQuerying(D, G)
2:      n ← the number of nodes in D
3:      E ← the directed edge-relation from D
4:      P ← the set of production rules in G
5:      T ← the matrix n × n in which each element is ∅
6:      for all (i, x, j) ∈ E do                ▷ Matrix initialization
7:          T_{i,j} ← T_{i,j} ∪ {A | (A → x) ∈ P}
8:      while matrix T is changing do
9:          T ← T ∪ (T × T)                     ▷ Transitive closure calculation
10:     return T
```

Here $D = (V, E)$ be the input graph and $G = (N, \Sigma, P)$ be the input grammar. Each cell of the matrix T contains the set of nonterminals such that $N_k \in T[i,j] \iff \exists p = v_i \dots v_j$—path in D, such that $N_k \overset{*}{\underset{G}{\Rightarrow}} \omega(p)$, where $\omega(p)$ is a word formed by labels along path p. Thus, this algorithm solves reachability problem, or, according to Hellings [8], process CFPQs by using relational query semantics.

As you can see, performance-critical part of this algorithm is matrix multiplication. Note, that the set of nonterminals is finite, we can represent the matrix T as e set of boolean matrices: one for each nonterminal. In this case the matrix updeta operation be $T_{N_i} \leftarrow T_{N_i} + (T_{N_j} \times T_{N_k})$ for each production $N_i \rightarrow N_j\ N_k$ in P. Thus we can reduse CFPQ to boolean matrices multiplication. After such transfromation we can apply the next optimization: we can skip update if there are no changes in the matrices T_{N_j} and T_{N_k} at the previous iteration.

Thus, the most important part is efficient implementation of operations over boolean matrices, and in this work we compare effects of utilization of different approaches to matrices multiplication. All our implemetations are based on the optimized version of the algorithm.

3 IMPLEMENTATION

We implement the matrix-based algorithm for CFPQ by using a number of different programming languages and tools. Our goal is to investigate the effects of the next features of implementation.

- **GPGPU utilization.** It is well-known that GPGPUs are suitable for matrices operations, but the performance of the whole solution depends on task details: overhead on data transferring may negate the effect of parallel computations. Can GPGPUs utilization for CFPQ improve performance in comparison with CPU version?
- **Existing libraries utilization** is a good practice in software engineering. Is it possible to achieve higher performance by using existing libraries for matrices operations or we need to create own solution to get more control?
- **Low-level programming**. GPGPU programming is traditionally low-level programming by using C-based languages (CUDA C, OpenCL C). On the other hand, there is a number of approaches to creating GPGPU-based solution by using such high-level languages as a Python. Can we get a high-performance solution by using such approaches?
- **Sparce matrices.** Real graphs often are sparse, but not always. Is it suitable to use sparse matrix representation for CFPQ?

We provide the next implementations for investigation.

- CPU-based solutions
  [Scipy] Sparse matrices multiplication by using Scipy [10] in Python programming language.
  [M4RI] Dense matrices multiplication by using m4ri[2] [1] library which implements 4 Russian method [3] in C language. This library choosen because it is one of performnat implementation of 4 russian method [2].
- GPGPU-based solutions
  [GPU4R] Manual implementation of 4 Russian method in CUDA C.
  [GPU_N] Manual implementation of naïve boolean matrix multiplication in CUDA C.
  [GPU_Py] Manual implementation of naïve boolean matrix multiplication in Python by using number compiler[3].

As far as a number of matrices and its size can be statically defined at the start, all GPGPU based implementations allocate all required memory on the GPGPU only once, at the start of computations. By this way, it is possible to significantly reduce overhead on data transferring: all input data loads to GPGPU at the start, and result loads from GPGPU to the host at the finish. No active data transferring and memory allocating during query computation.

4 DATASET DESCRIPTION

We create and publish a dataset for CFPQ algorithms evaluation. This dataset contains both the real data and synthetic data for different specific cases, such as the theoretical worst case, or matrices representation specific worst cases.

Our goal is querying algorithms evaluation, not a graph storages or graph databases evaluation, so all data is presented in a text-based format to simplify usage in different environments. Grammars are in Chomsky Normal Form and are stored in the files with `yrd` extension. Each line is a rule in the form of a triple or pair. The example of grammar representation is presented in figure 1

[2] Actually we use pull request which is not merged yet: https://bitbucket.org/malb/m4ri/pull-requests/9/extended-m4ri-to-multiplication-over-the/diff. The original library implements operations over $GF(2)$, and this pull request contains operations over boolean semiring

[3] Numba is a JIT compiler which supports GPGPU for a subset of Python programming. Official page: http://numba.pydata.org/. Access date: 03.05.2019

$$s \rightarrow A\ s\ B$$
$$s \rightarrow A\ B$$

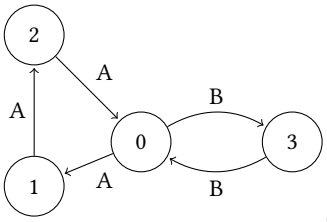
(a) Grammar G_1

```
s a b
s a s1
s1 s b
a A
b B
```

(b) Representation of grammar G_1 in `yrd` file

Figure 1: Example of grammar representation in the `yrd` file

Graphs are represented as a set of triples (edges) and are stored in the files with `txt` extension. Example of graph is presented in figure 2.



(a) Example of input graph D_1

```
0 A 1
1 A 2
2 A 0
0 B 3
3 B 0
```

(b) Representation of the input graph D_1 in `txt` file

Figure 2: Example of graph representation in `txt` file

Each case is a pair of set of graphs and a set of grammars: each query (grammar) should be applied to each graph. Cases are placed in folders with the case-specific name. Grammars and graph are placed in subfolders with names `Grammars` and `Matrices` respectively.

It is known that variants of the *same generation query* **??** are a classical example of queries that are context-free but not regular, so we use this type of queries in our evaluation. The datatset includes data for next cases.

[RDF] The set of real RDF files (ontologies) from [15] and two variants of the same generation query (figures **??**) which describes hierarchy analysis.

[Worst] Theoretical worst case for CFPQ time complexity which is proposed by Hellings [9]: the graph is two cycles of coprime lengths with a single common vertex. The first cycle labeled by an open bracket and the second cycle is labeled by a close bracket. Query is a grammar for A^nB^n languge (grammar G_1, figure 1).

[Full] The case when the input graph is sparse, but the result is a full graph. Such a case may be hard for sparse matrices representation. As an input graph, we use a cycle all edges of which are labeled by the same token. As a query we use two grammars which describe arbitrary repetition of a token: unambiguous and highly ambiguous grammar (figure **??**).

[Sparse] Sparse graphs from [6] which generated by the GTgraph graph generator, and emulates realistic sparse data. Names of these graphs have a form `Gn-p`, where `n` represents the total number of vertices, each pair of vertices is connected by probability `p`. The query is the same generation query.

5 EVALUATION

We evaluate all described implementations on all data and queries from the presented dataset. Also, we provide results for implementation provided in [4] for comparison. Our goal is to compare CFPQ evaluation algorithms, so we exclude time required for load data from files. The time required for data transfer is included.

For evaluation, we use PC with Ubuntu 18.04 installed. It has Intel core i7 8700k 3,7HGz CPU, Ddr4 32Gb RAM, and Geforce 1080Ti GPGPU with 11Gb RAM.

Results of evaluation are presented in the tables below. Time is measured in seconds. Result for each algorithm is an average time of 10 runs. Time is not presented if time limit is expired, or if no memory enough to allocate all necessary data.

First is a **[RDF]** dataset. Results are presented in a table 1. We can see, that in this case running time for all our implementations smaller than time for the reference implementation, and that **[GPU_N]** is faster than other implementations while other implementations demonstrate similar performance. Also, it is obvious that performance improvement in comparison with first implementations is huge and it is necessary to select new significantly biggest RDF files.

Table 2: Worst case evaluation results

#V	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
16	0.032	< 0.001	0.008	0.002	0.027	0.309
32	0.118	0.001	0.034	0.008	0.136	0.441
64	0.476	0.041	0.133	0.032	0.524	0.988
128	2.194	0.226	0.562	0.129	2.751	3.470
256	15.299	1.994	3.088	0.544	11.883	15.317
512	121.287	23.204	13.685	2.499	43.563	102.269
1024	1593.284	528.521	88.064	19.357	217.326	1122.055
2048	-	-	-	325.174	-	-

Results of theoretical worst case (**[Worst]** datatset) is presented in table 2. This case is really hard to process: even for a graph with 1024 vertices query evaluation time greater than 10 seconds even for most performant implementation. Also, we can see, that time grows fast with grows of vertices number.

Table 3: Sparse graphs querying results

Graph	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
G5k-0.001	10.352	0.647	0.113	0.041	0.216	5.729
G10k-0.001	37.286	2.395	0.435	0.215	1.331	35.937
G10k-0.01	97.607	1.455	0.273	0.138	0.763	47.525
G10k-0.1	601.182	1.050	0.223	0.114	0.859	395.393
G20k-0.001	150.774	11.025	1.842	1.274	6.180	-
G40k-0.001	-	97.841	11.663	8.393	37.821	-
G80k-0.001	-	1142.959	88.366	65.886	-	-

Next is a **[Sparse]** datatset. Results are presented in table 3. Evaluation show that for such type of graphs sparsity (value of parameter `p`) is important both for implementattions which use sparse matrices and for implementations which use dense matrices.

Table 1: RDFs querying results

RDF			Query 1						Query 2					
Name	#V	#E	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
atom-primitive	291	685	0.003	0.002	0.002	0.001	0.005	0.269	0.001	< 0.001	0.001	< 0.001	0.002	0.267
biomed.-mesure-primitive	341	711	0.003	0.005	0.002	0.001	0.005	0.283	0.004	< 0.001	0.001	< 0.001	0.005	0.280
foaf	256	815	0.002	0.009	0.002	< 0.001	0.005	0.270	0.001	< 0.001	0.001	< 0.001	0.002	0.263
funding	778	1480	0.004	0.007	0.004	0.001	0.005	0.279	0.002	< 0.001	0.003	< 0.001	0.004	0.274
generations	129	351	0.003	0.003	0.002	< 0.001	0.005	0.273	0.001	< 0.001	0.001	< 0.001	0.002	0.263
people_pets	337	834	0.003	0.003	0.003	0.001	0.007	0.284	0.001	< 0.001	0.001	< 0.001	0.003	0.277
pizza	671	2604	0.006	0.008	0.003	0.001	0.006	0.292	0.002	< 0.001	0.002	< 0.001	0.005	0.278
skos	144	323	0.002	0.004	0.002	< 0.001	0.005	0.273	< 0.001	< 0.001	0.001	< 0.001	0.002	0.265
travel	131	397	0.003	0.005	0.002	< 0.001	0.006	0.268	0.001	< 0.001	0.001	< 0.001	0.003	0.271
univ-bench	179	413	0.002	0.004	0.002	< 0.001	0.005	0.266	0.001	< 0.001	0.001	< 0.001	0.003	0.266
wine	733	2450	0.007	0.006	0.004	0.001	0.007	0.294	0.001	< 0.001	0.003	< 0.001	0.003	0.281

Note that behaveour of sparse matrices based implementation is as expected, but for dense matrices we can see, that more sparse graphs processed faster. Reasons of such behaviour shoud be invesigated. Note that we estimate only query execution time, so it is hard to compare our results with results presented in [6]. But it would be interesting to do such a comparison in the future because the running time of our **[GPU_N]** implementation is significantly smaller than the provided in [6].

The last dataset is a **[Full]**, and results a shown in table 4

As we expect, this case is very hard for sparse metrices based implementations: running time grows too fast. Also we can see, that grammar size is important. Both queries specify the same restriction, but grammar for Query 2 contains more rules, and as a result, the running time for big graphs differs more than 2 times.

Finally, we can cocnlude that GPGPU utilization for CFPQ can significantly improve performance, but it should be done more research on abvanced optimization techniqes. That meens that low-level programming is necessary for high-performance solution. On the other hand, high-level implementation (**[GPU_Py]**) are comparable with other GPGPU-based implementations. So, it may be a way to find balance between implementation complexity and performance. Highly optimized existinng libraries can be useful: implementation based on m4ri is faster then reference implementation and other CPU-based implementation. Moreover it is comparable with some GPGPU-based implementations in come cases. Sparse matrices utilization should be investigated more. The main question is can ew create efficient implementattion for sparse boolena matrices multiplication.

6 CONCLUSION AND FUTURE WORK

We provide a number of implementations of the matrix-based algorithm for context-free path querying, collect a dataset for evaluation and provide results of evaluation of our implementation on the collected dataset. Our evaluation shows that GPGPU utilization for boolean matrices multiplication can significantly increase the performance of CFPQs evaluation, but requires more research on implementation details.

The first direction for future research is a more detailed CFPQ algorithms investigation. We should do more evaluation on sparse matrices on GPGPUs and investigate technics for high-performance GPGPU code creation. Also, it is necessary to implement and evaluate solutions for graphs which are not fit in RAM, and for big queries which disallow to allocate all required matrices on single GPGPU. We hope that it is possible to utilize existing technics for huge matrices multiplication for this problem.

Another direction is dataset improvement. First of all, it is necessary to collect more data, and more grammars/queries. Especially it would be interesting to add to dataset more real graphs and more real queries. Secondly, it is necessary to discuss and fix the data format to be able to evaluate different algorithms. We think that it is necessary to create a public dataset for CFPQ algorithms evaluation, and collaboration with the community is required.

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Table 4: Full querying results

#V	Query 1						Query 2					
	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
100	0.007	0.002	0.002	< 0.001	0.003	0.278	0.023	0.076	0.005	0.001	0.007	0.290
200	0.040	0.003	0.002	0.001	0.004	0.279	0.105	0.098	0.004	0.001	0.007	0.296
500	0.480	0.003	0.003	0.001	0.004	0.329	1.636	0.094	0.007	0.001	0.010	0.382
1000	3.741	0.007	0.005	0.001	0.006	0.571	13.071	0.106	0.009	0.001	0.009	0.839
2000	40.309	0.063	0.019	0.003	0.017	1.949	93.676	0.108	0.030	0.005	0.026	3.740
5000	651.343	0.366	0.125	0.038	0.150	99.651	1205.421	0.851	0.195	0.075	0.239	201.151
10000	-	1.932	0.552	0.315	0.840	1029.042	-	4.690	1.055	0.648	1.838	-
25000	-	33.236	7.252	5.314	15.521	-	-	70.823	15.240	10.961	36.495	-
50000	-	360.035	58.751	44.611	129.641	-	-	775.765	130.203	91.579	226.834	-
80000	-	1292.817	256.579	190.343	641.260	-	-	-	531.694	376.691	-	-

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