

Context-Free Path Querying Can be Fast if Cooked Properly

Arseniy Terekhov
simpletondl@yandex.ru
Saint Petersburg State University
St. Petersburg, Russia

Artyom Khoroshev
!!!
!!!
St. Petersburg, Russia

Semyon Grigorev
s.v.grigoriev@spbu.ru
semen.grigorev@jetbrains.com
Saint Petersburg State University
St. Petersburg, Russia
JetBrains Research
St. Petersburg, Russia

ABSTRACT

Recently proposed matrix multiplication based algorithm for context-free path querying (CFPQ) offloads the most performance-critical parts onto boolean matrices multiplication. Thus, it is possible to achieve high performance of CFPQ by means of modern parallel hardware and software. In this paper, we provide results of empirical performance comparison of different implementations of this algorithm on both real-world data and synthetic data for the worst cases.

KEYWORDS

Context-free path querying, transitive closure, graph databases, linear algebra, context-free grammar, GPGPU, CUDA, boolean matrix, matrix multiplication

1 INTRODUCTION

Formal language constrained path querying, or formal language constrained path problem [3], is a graph analysis problem in which formal languages are used as a constraints for navigational path queries. In this approach a path is viewed as a word constructed by concatenation of edge labels. Paths of interest are constrained with some formal language: a query should find only paths labeled by words from the language. The class of language constraints which is most widely spread is regular: it is used in various graph query languages and engines. While being more expressive context-free path querying (CFPQ) [23] is still at the early stage of development. Context-free constraints allow one to express such important class of queries as *same generation queries* [1] which can not be expressed in terms of regular constraints.

Several algorithms for CFPQ based on such parsing techniques as (G)LL, (G)LR, and CYK are proposed recently [4, 5, 9, 11, 15, 17, 19, 21, 24]. But recent research by Jochem Kuijpers et.al. [14] shows that existing solutions are not applicable for real-world graph analysis because of significant running time and memory consumption. At the same time, Nikita Mishin et.al in [16] show that the matrix-based CFPQ algorithm demonstrates good performance on real-world data. Matrix-based algorithm is proposed by Rustam Azimov [2] and offloads the most critical computations onto boolean matrices multiplication. This algorithm is easy to implement, and employ modern massive-parallel hardware for CFPQ. The paper measures the performance of the algorithm, not integrating it with a graph storage, while J. Kuijpers provides an evaluation of algorithms which are integrated with Neo4j¹

¹Neo4j graph database web page: <https://neo4j.com/>. Access date: 12.11.2019.

graph database. Also, in [14] matrix-based algorithm is evaluated in simple single-thread Java implementation, while N. Mishin shows that the most efficient implementation should utilize high-performance matrix multiplication libraries which are highly parallel or better utilize GPGPU. Thus, evaluation of the matrix-based algorithm which is integrated with a graph storage and implemented in the appropriate way is required.

In this work we show that CFPQ in relational semantics (according to Hellings [10]) can be fast enough to be applicable to real-world graph analysis. We use RedisGraph² [6] graph database as a storage. This database uses adjacency matrices as a representation of a graph and GraphBLAS [13] for matrices manipulation. These facts allow us to integrate matrix-based CFPQ algorithm to RedisGraph with minimal effort. We make the following contributions in this paper.

- (1) We provide a number of implementations of the matrix multiplication based CFPQ algorithm which uses RedisGraph as graph storage. The first implementation is CPU-based and utilizes SuteSparse³ [7] implementation of GraphBLAS API for matrices manipulation. The second implementation is GPGPU-based and includes both the existing implementation from [16] and our own CUSP⁴-based implementation. The source code is available on GitHub⁵.
- (2) We extend the dataset presented in [16] with new real-world and synthetic cases of CFPQ⁶.
- (3) We provide evaluation which shows that matrix-based CFPQ implementation for RedisGraph database is fast enough for real-world data analysis.

2 MATRIX-BASED ALGORITHM FOR CFPQ

The matrix-based algorithm for CFPQ was proposed by Rustam Azimov [2]. This algorithm can be expressed in terms of operations over boolean matrices (see listing 1) which is an advantage for implementation.

Here $D = (V, E)$ is the input graph and $G = (N, \Sigma, P)$ is the input grammar. For each matrix T^{A_k} , $T^{A_k}[i, j] = \text{true} \iff \exists \pi = v_i \dots v_j \text{--path in } D, \text{ such that } A_k \xRightarrow{*}_G \omega(\pi)$, where $\omega(\pi)$ is a word formed by the labels along the path π . Thus, this algorithm solves the reachability problem, or, according to Hellings [10], implements relational query semantics.

²RedisGraph is a graph database which is based on Property Graph Model. Project web page: <https://oss.redislabs.com/redisgraph/>. Access date: 12.11.2019.

³SuteSparse is a sparse matrix software which includes GraphBLAS API implementation. Project web page: <http://faculty.cse.tamu.edu/davis/suitesparse.html>. Access date: 12.11.2019.

⁴CUSP is an open source library for sparse matrix multiplication on GPGPU. Project site: <https://cusplibrary.github.io/>. Access date: 12.11.2019.

⁵Sources of matrix-based CFPQ algorithm for RedisGraph database: <https://github.com/YaccConstructor/RedisGraph>. Access date: 12.11.2019.

⁶The CFPQ_Data data set fro CFPQ algorithms evaluation and comparison. GitHub page: https://github.com/JetBrains-Research/CFPQ_Data. Access date: 12.11.2019.

Listing 1 Context-free path querying algorithm

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1: function CONTEXTFREEPATHQUERYING( $D, G$ )
2:    $n \leftarrow$  the number of nodes in  $D$ 
3:    $E \leftarrow$  the directed edge-relation from  $D$ 
4:    $P \leftarrow$  the set of production rules in  $G$ 
5:    $N \leftarrow$  the set of nonterminals in  $G$ 
6:    $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i} \text{ is a matrix } n \times n \text{ in which each}$ 
    $\text{element is false}\}$ 
7:   for all  $(i, x, j) \in E$  do ▷ Matrix initialization
8:     for  $A_k \mid A_k \rightarrow x \in P$  do
9:        $T_{i,j}^{A_k} \leftarrow \text{true}$ 
10:  for  $A_k \mid A_k \rightarrow \varepsilon \in P$  do
11:     $T_{i,i}^{A_k} \leftarrow \text{true}$ 
12:  while any matrix in  $T$  is changing do ▷ Transitive
  closure calculation
13:    for  $A_i \rightarrow A_j A_k \in P$  do
14:       $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$ 
15:  return  $T$ 

```

The performance-critical part of the algorithm is boolean matrix multiplication. Note, that if the matrices T_{N_j} and T_{N_k} have not been changed at the previous iteration, then we can skip update operation. Such optimization can improve performance. Also, it is important for applications that real-world data is often sparse, so it should be a better solution to use libraries which manipulate with sparse matrices.

3 IMPLEMENTATION

As we can see, CFPQ can be naturally reduced to linear algebra. Linear algebra for graph problems is an actively developed area. One of the most important results is a GraphBLAS API which provides a way to operate over matrices and vectors over user-defined semirings.

Previous works show [2, 16] that existing linear algebra libraries utilization is the right way to get high-performance CFPQ implementation in minimal effort. But none of these works do not provide an evaluation with data storage, only pure time of algorithm execution was measured.

We provide a number of implementations of matrix-based CFPQ algorithm. All of them are based on RedisGraph — we use RedisGraph as a storage and implement CFPQ as an extension by using provided mechanism. Note that currently, we do not provide full integration with querying mechanism: one can not use Cypher, which uses in RedisGraph as a query language. Instead, in the current implementation query is provided explicitly as a file with grammar in Chomsky normal form. So, we can evaluate querying algorithms, but we should improve integration to make our solution applicable.

CPU-based implementation uses SuteSparse implementation of GraphBLAS, which is used in RedisGraph, and predefined boolean semiring. Thus we avoid data format problems: we use native RedisGraph representation of the adjacency matrix in our algorithm.

GPGPU-based implementation is provided in two versions. The first one uses m4ri method implemented in [16], and the second one utilizes a modified CUSP library for matrix operations. Both these implementations require matrix format conversion.

4 DATASET DESCRIPTION

In our evaluation we use combined dataset which contains the following parts.

- CFPQ_Data dataset which provided in⁷ [16] and contains both synthetic and real-world graphs and queries. Real-world data includes RDFs, syntactic cases include theoretical worst-case and random graphs.
- Dataset which provided in [14]. We integrate both Geospecies — RDF which contains information about biological hierarchy⁸ and same generation query over *broaderTransitive* relation, and Synthetic — the set of graph generated by using the Barabási-Albert model of scale-free networks and same generation query, data into CFPQ_Data and use it in our evaluation.
- In [16] was shown that matrix-based algorithm is performed enough to handle bigger RDFs than used in initial data sets, such as [24]. So, we add a number of big RDFs to CFPQ_Data and use them in our evaluation. New RDFs: *go-hierarchy*, *go*, *enzyme*, *core*, *pathways* — parts of UniProt database⁹, and *eclass-514en* that given from eClassOWL project¹⁰.

The variants of the *same generation query* [1] is used in almost all cases because it is an important example of real-world queries that are context-free but not regular. So, variations of the same generation query is used in our evaluation. All queries are added to the CFPQ_Data data set.

For RDFs querying we use two queries over *subClassOf* and *type* relations. The first query is the grammar G_1 :

$$\begin{aligned}
 s &\rightarrow \text{subClassOf}^{-1} s \text{ subClassOf} & s &\rightarrow \text{type}^{-1} s \text{ type} \\
 s &\rightarrow \text{subClassOf}^{-1} \text{subClassOf} & s &\rightarrow \text{type}^{-1} \text{type}
 \end{aligned}$$

The second one is the grammar G_2 :

$$s \rightarrow \text{subClassOf}^{-1} s \text{ subClassOf} \mid \text{subClassOf}.$$

5 EVALUATION AND DISCUSSION

We evaluate all the described implementations on all the datasets and the queries presented. We compare our implementations with [16] and [14]. We measure full time of query execution, so all overhead on data preparing is included. Thus we can estimate applicability of matrix-based algorithm for real-world solutions.

For evaluation, we use a PC with Ubuntu 18.04 installed. It has Intel core i7-6700 CPU, 3.4GHz, DDR4 32Gb RAM, and Geforce GTX 1070 GPGPU with 8Gb RAM.

The results of the evaluation are summarized in the tables below. We provide results only for part of the collected data set because of the page limit. Time is measured in seconds unless specified otherwise. Note that for all implementations except our own we provide results from related paper. The cell is left blank if the time limit is exceeded, or if there is not enough memory to allocate the data.

The results of the first dataset [RDF] are presented in table 1. We can see, that in this case the running time of all our implementations is smaller than of the reference implementation, and all

⁷CFPQ_Data data set GitHub repository: https://github.com/JetBrains-Research/CFPQ_Data. Access date: 12.11.2019.

⁸<https://old.datahub.io/dataset/geospecies>. Access date: 12.11.2019.

⁹Protein sequences data base: <https://www.uniprot.org/>. RDFs with data are available here: ftp://ftp.uniprot.org/pub/databases/uniprot/current_release/rdf. Access date: 12.11.2019

¹⁰eClassOWL project: <http://www.heppnetz.de/projects/eclassowl/>. eclass-514en file is available here: http://www.ebusiness-unibw.org/ontologies/eclass/5.1.4/eclass_514en.owl. Access date: 12.11.2019.

Table 1: RDFs querying results (time in milliseconds)

RDF			Query G_1						Query G_2					
Name	#V	#E	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
funding	778	1480	4	7	4	1	5	279	2	< 1	3	< 1	4	274
pizza	671	2604	6	8	3	1	6	292	2	< 1	2	< 1	5	278
wine	733	2450	7	6	4	1	7	294	1	< 1	3	< 1	3	281
core	1323	8684	2	4	2	< 1	5	273	< 1	< 1	1	< 1	2	265
pathways	6238	37196	3	5	2	< 1	6	268	1	< 1	1	< 1	3	271
go-hierarchy	45007	1960436	2	4	2	< 1	5	266	1	< 1	1	< 1	3	266
enzyme	48815	219390	2	4	2	< 1	5	273	< 1	< 1	1	< 1	2	265
eclass_514er	239111	1047454	3	5	2	< 1	6	268	1	< 1	1	< 1	3	271
go	272770	1068622	2	4	2	< 1	5	266	1	< 1	1	< 1	3	266

implementations but **[CuSprs]** demonstrate similar performance. It is obvious that performance improvement in comparison with the first implementation is huge and it is necessary to extend the dataset with new RDFs of the significantly bigger size.

Geospecies dataset currently can be processed only by using CPU version. So, we can compare our matrix-based CPU implementation with the best result form [14]. Result is provided in the table 2.

Table 2: Evaluation results geospecies data

d	CPU	Neo4j	d
Time	Mem	Time	Mem
16	0.032	0.008	0.002

Table 3: Evaluation results for the worst case

#V	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
16	0.032	< 1	0.008	0.002	0.027	0.309
32	0.118	0.001	0.034	0.008	0.136	0.441
64	0.476	0.041	0.133	0.032	0.524	0.988
128	2.194	0.226	0.562	0.129	2.751	3.470
256	15.299	1.994	3.088	0.544	11.883	15.317
512	121.287	23.204	13.685	2.499	43.563	102.269
1024	1593.284	528.521	88.064	19.357	217.326	1122.055
2048	-	-	-	325.174	-	-

Results of the theoretical worst case (**[Worst]** dataset) are presented in table 3. This case is really hard to process: even for a graph of 1024 vertices, the query evaluation time is greater than 10 seconds even for the most performant implementation. We can see, that the running time grows too fast with the number of vertices.

The next is the **[Sparse]** dataset presented in table ?? . The evaluation shows that sparsity of graphs (value of parameter p) is important both for implementations which use sparse matrices and for implementations which use dense matrices. Note that the behavior of the sparse matrices based implementation is as expected, but for dense matrices we can see, that more sparse graphs are processed faster. Reasons for such behavior demand further investigation. Note that we estimate only the query execution time, so it is hard to compare our results with the results presented in [8]. Nevertheless, the running time of our **[GPU_N]**

implementation is significantly smaller than the one provided in [8].

Finally, we can conclude that GPGPU utilization for CFPQ can significantly improve performance, but more research on advanced optimization techniques should be done. On the other hand, the high-level implementation (**[GPU_Py]**) is comparable with other GPGPU-based implementations. So, it may be a balance between implementation complexity and performance. Highly optimized existing libraries can be of some use: the implementation based on m4ri is faster than the reference implementation and the other CPU-based implementation. Moreover, it is comparable with some GPGPU-based implementations in some cases. Sparse matrices utilization demands more thorough investigation. The main question is if we can create an efficient implementation for sparse boolean matrices multiplication.

6 CONCLUSION AND FUTURE WORK

We implemented a CPU and GPGPU based context-free path querying for RedisGraph and showed that CFPQ can be performant enough to analyze real-world data. However, our implementations are prototypes and we plan to provide full integration of CFPQ to RedisGraph. First of all, it is necessary to extend Cypher graph query language, which is used in RedisGraph, to support syntax for specification of context-free constraints. There is a proposal which describes such syntax extension¹¹ and we plan to support proposed the syntax in libcypher-parser¹² which is used in RedisGraph.

Current version uses CUSP matrix multiplication library for GPGPU utilization, but it may be better to use GraphBLAST¹³ [22] — Gunrock¹⁴ [20] based implementation of GraphBLAS API for GPGPU. First of all, we plan to evaluate GraphBLAST based implementation of CFPQ. Also, we plan to investigate how multi-GPU support for GraphBLAST influences the performance of CFPQ in the case of processing huge real-world data.

¹¹Proposal with path pattern syntax for openCypher: <https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc>. It is shown that context-free constraints are expressible in proposed syntax. Access date: 12.11.2019

¹²Web page of libcypher-parser project: <http://cleishm.github.io/libcypher-parser/>. Access date: 12.11.2019

¹³GraphBLAST project: <https://github.com/gunrock/graphblast>. Access date: 12.11.2019.

¹⁴Gunrock project web page: <https://gunrock.github.io/docs/>. Access date: 12.11.2019.

Table 4: Free scale graphs querying results

Graph	CPU		m4ri		CUSP		neo4j	
	Time	Mem	Time	Mem	Time	Mem	Time	Mem
G(100,1)	< 1	< 1	0.002	< 1	0.003	0.278	0.023	0.076
G(100,3)	< 1	< 1	0.002	0.001	0.004	0.279	0.105	0.098
G(100,5)	< 1	< 1	0.003	0.001	0.004	0.329	1.636	0.094
G(100,10)	< 1	< 1	0.005	0.001	0.006	0.571	13.071	0.106
G(500,1)	< 1	< 1	0.019	0.003	0.017	1.949	93.676	0.108
G(500,3)	0.003	< 1	0.125	0.038	0.150	99.651	1205.421	0.851
G(500,5)	0.005	< 1	0.552	0.315	0.840	1029.042	-	4.690
G(500,10)	1.239	7202	7.252	5.314	15.521	-	-	70.823
G(2500,1)	40.309	0.063	0.019	0.003	0.017	1.949	93.676	0.108
G(2500,3)	651.343	0.366	0.125	0.038	0.150	99.651	1205.421	0.851
G(2500,5)	-	1.932	0.552	0.315	0.840	1029.042	-	4.690
G(2500,10)	-	360.035	58.751	44.611	129.641	-	-	775.765
G(10000,1)	0.009	1024	0.019	0.003	0.017	1.949	93.676	0.108
G(10000,3)	5.439	4353	0.125	0.038	0.150	99.651	1205.421	0.851
G(10000,5)	7.978	8193	0.552	0.315	0.840	1029.042	-	4.690
G(10000,10)	13.180	47362	256.579	190.343	641.260	-	-	-

Our implementations compute relational semantics of a query, but some problems require to find a path which satisfies the constraints. Best to our knowledge, there is no matrix-based algorithm for single path or all path semantics, thus we see it as a direction for future research.

Another important open question is how to update the query results dynamically when data changes. The mechanism for result updating allows one to recalculate query faster and use the result as an index for other queries.

Also, further improvements of the dataset are required. For example, it is necessary to include real-world cases from the area of static code analysis [12, 18, 25].

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