ADBIS 2020

Context-Free Path Querying by Kronecker Product

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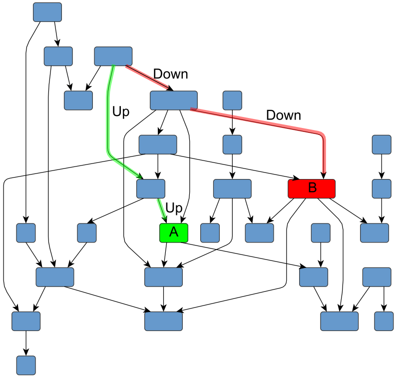
August 26, 2020

Context-Free Path Querying



Navigation through a graph

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form **Up**^{n} **Down**^{n}?
- Find all paths of form **Up**^{n} **Down**^{n} which start from the node A

CFPQ: Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$ — context-free grammar in normal form
  - $A \to BC$, where $A, B, C \in N$
  - $A \to x$, where $A \in N, x \in \Sigma \cup \{\varepsilon\}$
  - $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^* \omega\}$

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- $\omega(\pi) = \omega(v_0 \xrightarrow{l_0} v_1 \xrightarrow{l_1} \cdots \xrightarrow{l_{n-2}} v_{n-1} \xrightarrow{l_{n-1}} v_n) = l_0 l_1 \cdots l_{n-1}$

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- $R_A = \{(n, m) \mid \exists n \pi m$, such that $\omega(\pi) \in L(\mathbb{G}, A)\}$

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- Solutions based on different parsing techniques (CYK, LL, LR, etc.)
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- All existing solutions works only with context-free grammar in normal form (CNF, BNF)
- The transformation takes time and can lead to a significant grammar size increase

Recursive State Machines (RSM)

- RSM behaves as a set of finite state machines (FSM) with additional recursive calls
- Any CFG can be easily encoded by an RSM with one box per nonterminal

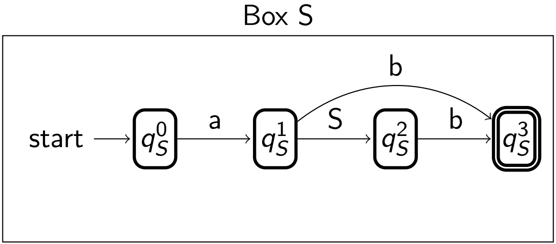
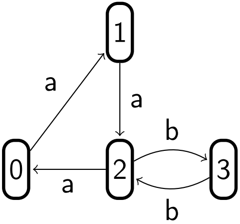


Figure: The RSM for grammar with rules $S \to aSb \mid ab$

CFPQ Algorithm Iteration 1



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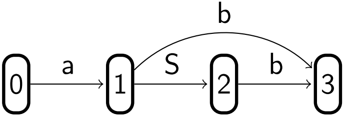

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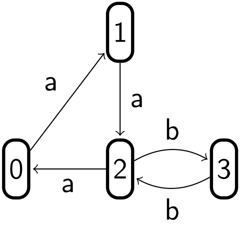

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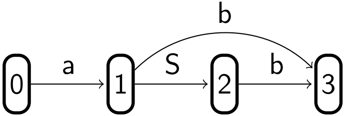

$=$

$$
\begin{array}{lllll}
0,0 & \xrightarrow{a} & 1,1 & & \\
\underline{\mathbf{1}},0 & \xrightarrow{\mathbf{a}} & 2,1 & \xrightarrow{\mathbf{b}} & \underline{\mathbf{3}},3 \\
2,0 & \xrightarrow{a} & 0,1 & & \\
2,2 & \xrightarrow{b} & 3,3 & & \\
3,2 & \xrightarrow{b} & 2,3 & & \\
3,1 & \xrightarrow{b} & 2,3 & &
\end{array}
$$

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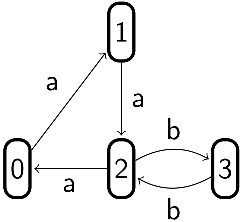

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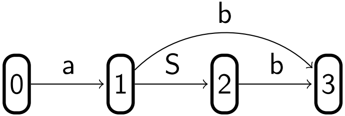

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$$2,0 \xrightarrow{a} 0,1$$

$$2,2 \xrightarrow{b} 3,3$$

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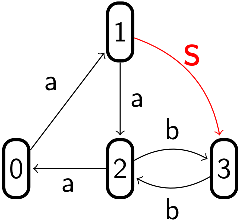
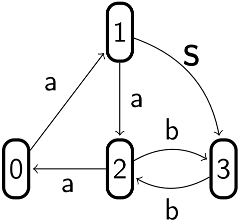
$\longrightarrow$



CFPQ Algorithm Iteration 2



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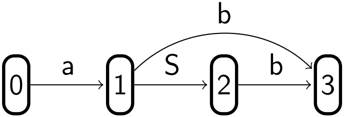

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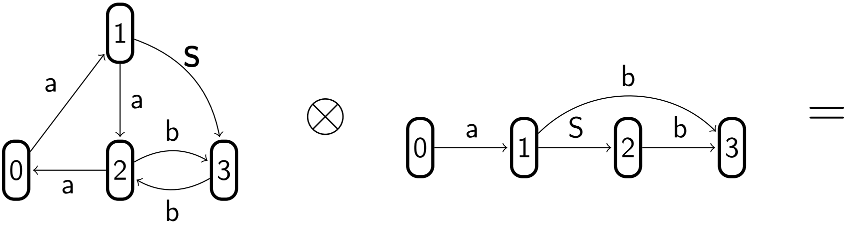

$$\begin{array}{lclclclc}
\underline{\mathbf{0}}, 0 & \xrightarrow{\mathbf{a}} & 1, 1 & \xrightarrow{\mathbf{S}} & 3, 2 & \xrightarrow{\mathbf{b}} & \underline{\mathbf{2}}, 3 \\
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\end{array}$$

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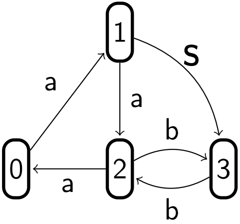

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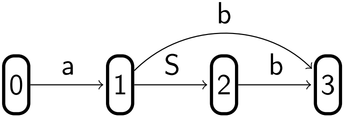

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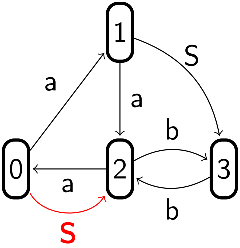

CFPQ Algorithm: Kronecker Product

Automaton intersection is a **Kronecker product** of adjacency matrices for $\mathcal{G}$ and $\mathcal{G}_{RSM}$

$$
\begin{pmatrix}
. & \{a\} & . & . \\
. & . & \{S\} & \{b\} \\
. & . & . & \{b\} \\
. & . & . & .
\end{pmatrix}
\otimes
\begin{pmatrix}
. & \{a\} & . & . \\
. & . & \{a\} & . \\
\{a\} & . & . & \{b\} \\
. & . & \{b\} & .
\end{pmatrix}
=
$$

$$
\left(\begin{array}{cccc|cccc|cccc|cccc}
. & . & . & . & . & \{a\} & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & \colorbox{red}{$\{a\}$} & . & . & . & . & . & . & . & . & . \\
. & . & . & . & \{a\} & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
\hline
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \colorbox{red}{$\{b\}$} \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & \{b\} & . \\
\hline
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & \{b\} \\
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\hline
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
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Implementations

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- **Kron** — implementation of the proposed algorithm using **SuiteSparse** C implementation of **GraphBLAS** API, which provides a set of sparse matrix operations
- We compare our implementation with **Orig** — the best CPU implementations of the original matrix-based algorithm using M4RI library

Evaluation

- OS: Ubuntu 18.04
- CPU: Intel(R) Core(TM) i7-4790 CPU 3.60GHz
- RAM: DDR4 32 Gb

Evaluation results[1] [2]

	Graph	#V	#E	*Kron*	*Orig*
RDF	generations	129	351	0.04	0.03
	travel	131	397	0.05	0.05
	skos	144	323	0.02	0.04
	unv-bnch	179	413	0.05	0.04
	foaf	256	815	0.07	0.02
	atm-prim	291	685	0.24	0.02
	ppl_pets	337	834	0.18	0.03
	biomed	341	711	0.24	0.05
	pizza	671	2604	1.14	0.08
	wine	733	2450	1.71	0.06
	funding	778	1480	0.43	0.07

	Graph	#V	#E	*Kron*	*Orig*
RDF	core	1323	8684	0.28	0.12
	pways	6238	37196	4.88	0.18
Worst case	WC_1	64	65	0.03	0.04
	WC_2	128	129	0.16	0.23
	WC_3	256	257	0.96	1.99
	WC_4	512	513	7.14	23.21
	WC_5	1024	1025	121.99	528.52
Full	F_1	100	100	0.17	0.02
	F_2	200	200	1.04	0.03
	F_3	500	500	18.86	0.03
	F_4	1000	1000	554.22	0.07

[1] Queries are based on the context-free grammars for nested parentheses

[2] Time is measured in seconds

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- Extend our algorithm to single-path and all-path query semantics

Contact Information

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  - Rustam.Azimov@jetbrains.com
- Egor Orachev: egor.orachev@gmail.com
- Ilya Epelbaum: iliyepelbaun@gmail.com

- Dataset: https://github.com/JetBrains-Research/CFPQ_Data
- Algorithm implementations: https://github.com/YaccConstructor/RedisGraph

Thanks!