Rytter for CFPQ

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ABSTRACT

Abstract

CCS CONCEPTS

• **Information systems → Graph-based database models**; **Query languages for non-relational engines**; • **Software and its engineering** → *Functional languages*; • **Theory of computation** → *Grammars and context-free languages*;

KEYWORDS

Graph Databases, Language-Constrained Path Problem, Context-Free Path Querying, Parser Combinators, Generalized LL, GLL, Neo4J, Scala

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1 INTRODUCTION

Two steps reduction of CFPQs to Boolean matrix multiplication. First step is reduction of arbitrary CFPQ to Dyck query. Second step is adoptation Rytter's results from [?] for graph.

2 FROM ARBITRARY CFPQ TO DYCK QUERY

This reduction is inspired by the construction described in [1].

Consider a context-free grammar $\mathcal{G} = (\Sigma, N, P, S)$ in BNF where Σ is a terminal alphabet, N is a nonterminal alphabet, P is a set of productions, $S \in N$ is a start nonterminal. Also we denote a directed labeled graph by $G = (V, E, L)$ where $E \subseteq V \times L \times V$ and $L \subseteq \Sigma$.

We should construct new input graph G' and new grammar $\mathcal{G}'$ such that $\mathcal{G}'$ specifies a Dyck language and there is a simple mapping from CFPQ($\mathcal{G}', G'$) to CFPQ($\mathcal{G}, G$). Step-by-step example with description is provided below.

Let the input grammar is

$$S \to a\ S\ b \mid a\ C\ b$$
$$C \to c \mid C\ c$$

The input graph is presented in fig. **??**

(1) Let $\Sigma_{()} = \{t_(, t_) | t \in \Sigma\}$.

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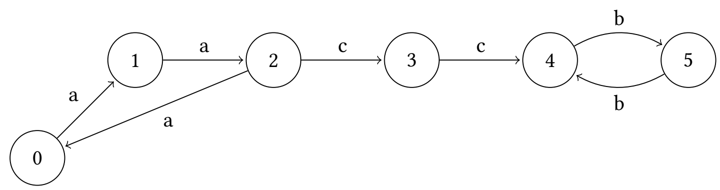
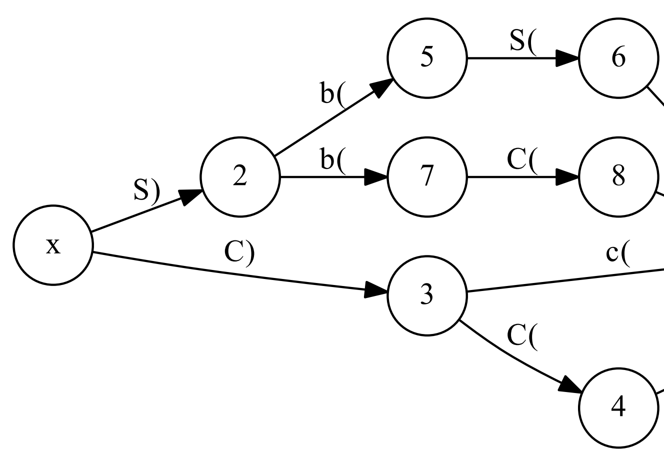
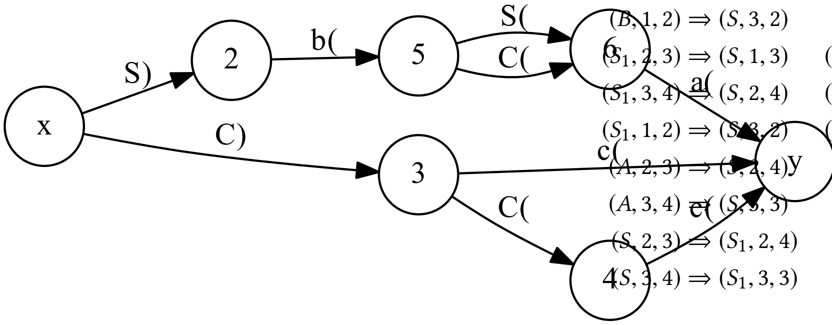


Figure 1: The input graph

(2) Let $N_{()} = \{N_(, N_) | N \in N\}$.
(3) Let $M_{\mathcal{G}} = (V_{\mathcal{G}}, E_{\mathcal{G}}, L_{\mathcal{G}})$ is a directed labeled graph, where $L_{\mathcal{G}} \subseteq (\Sigma_{()} \cup N_{()})$. This graph is created the same manner as described in [1] but we do not require the grammar be in CNF. Let $x \in V_{\mathcal{G}}$ and $y \in V_{\mathcal{G}}$ is "start" and "final" vertices respectively. This graph may be treated as a finite automaton, so it can be minimized and we can compute an ε-closure if the input grammar contains ε productions. The graph $M_{\mathcal{G}}$ for our example is:



The minimized graph:



(4) For each $v \in V$ create $M^v_{\mathcal{G}}$: unique instance of $M_{\mathcal{G}}$.
(5) New graph $G^{'}$ is a graph G where each label t is replaced with $t^i_)$ and some additional edges are created:
- Add an edge $(v', S_(, v)$ for each $v \in V$.
- And the respective $M^v_{\mathcal{G}}$ for each $v \in V$:
  - reattach all edges outgoing from x^v ("start" vertex of $M^v_{\mathcal{G}}$) to v;
  - reattach all edges incoming to y^v ("final" vertex of $M^v_{\mathcal{G}}$) to v.

  New input graph is ready:

(6) New grammar $\mathcal{G}' = (\Sigma^{'}, N', P', S')$ where $\Sigma^{'} = \Sigma_{()} \cup N_{()}$, $N' = \{S'\}$, $P' = \{S' \rightarrow b_(\; S' \; b_); S' \rightarrow b_(\; b_) \mid b_(, b_) \in \Sigma^{'}\} \cup \{S' \rightarrow S' \; S'\}$ is a set of productions, $S' \in N'$ is a start nonterminal.

Now, if CFPQ$(\mathcal{G}', G')$ contains a pair (u'_0, v') such that $e = (u'_0, S_(, u'_1) \in E'$ is an extension edge (step 5, first subitem), then $(u'_1, v') \in$ CFPQ$(\mathcal{G}, G)$. In our example, we can find the following path: $7 \xrightarrow{S_(} 1 \xrightarrow{S_)} 22 \xrightarrow{b_(} 25 \xrightarrow{C_(} 26 \xrightarrow{a_(} 1 \xrightarrow{a_)} 2 \xrightarrow{C_)} 33 \xrightarrow{C_(} 34 \xrightarrow{c_(} 2 \xrightarrow{c_)} 3 \xrightarrow{C_)} 43 \xrightarrow{c_(} 3 \xrightarrow{c_)} 4 \xrightarrow{b_)} 5$. Edge $7 \xrightarrow{S_(} 1$ is the extension, so (1,5) should be in CFPQ$(\mathcal{G}, G)$ and it is true.

3 GRAPH INPUT

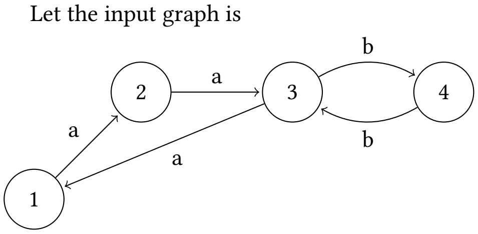
Let the input grammar is

$$S \rightarrow a \; S \; b$$
$$S \rightarrow a \; b$$

The input grammar in CNF is

$$S \rightarrow A \; S_1$$
$$S_1 \rightarrow S \; B$$
$$S \rightarrow A \; B$$
$$A \rightarrow a$$
$$B \rightarrow b$$

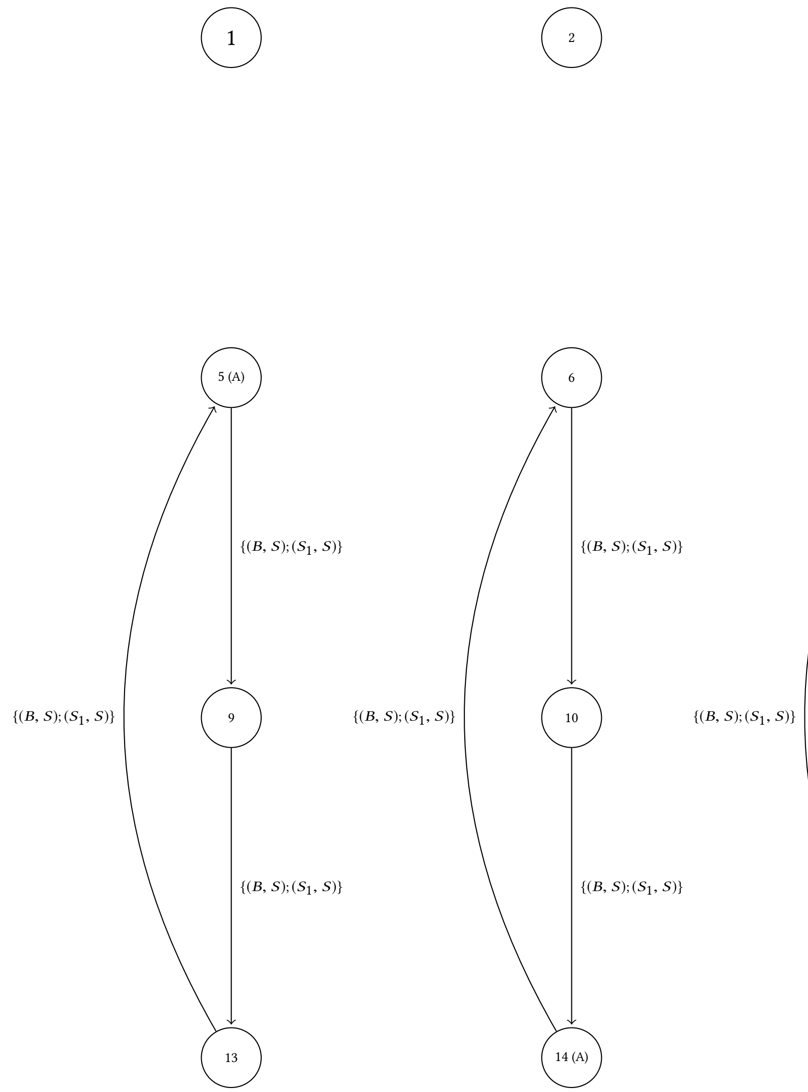
Let the input graph is



The *IMPLIED* relation:

$(B,2,3) \Rightarrow (S,1,3)$	$(B,2,4) \Rightarrow (S,1,4)$	$(B,2,2) \Rightarrow (S,1,2)$	$(B,2,1) \Rightarrow (S$
$(B,3,4) \Rightarrow (S,2,4)$	$(B,3,3) \Rightarrow (S,2,3)$	$(B,3,2) \Rightarrow (S,2,2)$	$(B,3,1) \Rightarrow (S$
$(B,1,2) \Rightarrow (S,3,2)$	$(B,1,3) \Rightarrow (S,3,3)$	$(B,1,4) \Rightarrow (S,3,4)$	$(B,1,1) \Rightarrow (S$
$(S_1,2,3) \Rightarrow (S,1,3)$	$(S_1,2,4) \Rightarrow (S,1,4)$	$(S_1,2,2) \Rightarrow (S,1,2)$	$(S_1,2,1) \Rightarrow (S$
$(S_1,3,4) \Rightarrow (S,2,4)$	$(S_1,3,3) \Rightarrow (S,2,3)$	$(S_1,3,2) \Rightarrow (S,2,2)$	$(S_1,3,1) \Rightarrow (S$
$(S_1,1,2) \Rightarrow (S,3,2)$	$(S_1,1,3) \Rightarrow (S,3,3)$	$(S_1,1,4) \Rightarrow (S,3,4)$	$(S_1,1,1) \Rightarrow (S$
$(A,2,3) \Rightarrow (S,2,4)$	$(A,1,3) \Rightarrow (S,1,4)$	$(A,3,3) \Rightarrow (S,3,4)$	$(A,4,3) \Rightarrow (S$
$(A,3,4) \Rightarrow (S,3,3)$	$(A,4,4) \Rightarrow (S,4,3)$	$(A,2,4) \Rightarrow (S,2,3)$	$(A,1,4) \Rightarrow (S$
$(S,2,3) \Rightarrow (S_1,2,4)$	$(S,1,3) \Rightarrow (S_1,1,4)$	$(S,3,3) \Rightarrow (S_1,3,4)$	$(S,4,3) \Rightarrow (S$
$(S,3,4) \Rightarrow (S_1,3,3)$	$(S,4,4) \Rightarrow (S_1,4,3)$	$(S,2,4) \Rightarrow (S_1,2,3)$	$(S,1,4) \Rightarrow (S$

Grid:



We should introduce the *id* implication such that for every $A \in$ IMPLIED

- $id \times A = A \times id$

In order to compute transitive closure in logarithmic time we add self-loop with weight $\{id\}$ to each vertex.

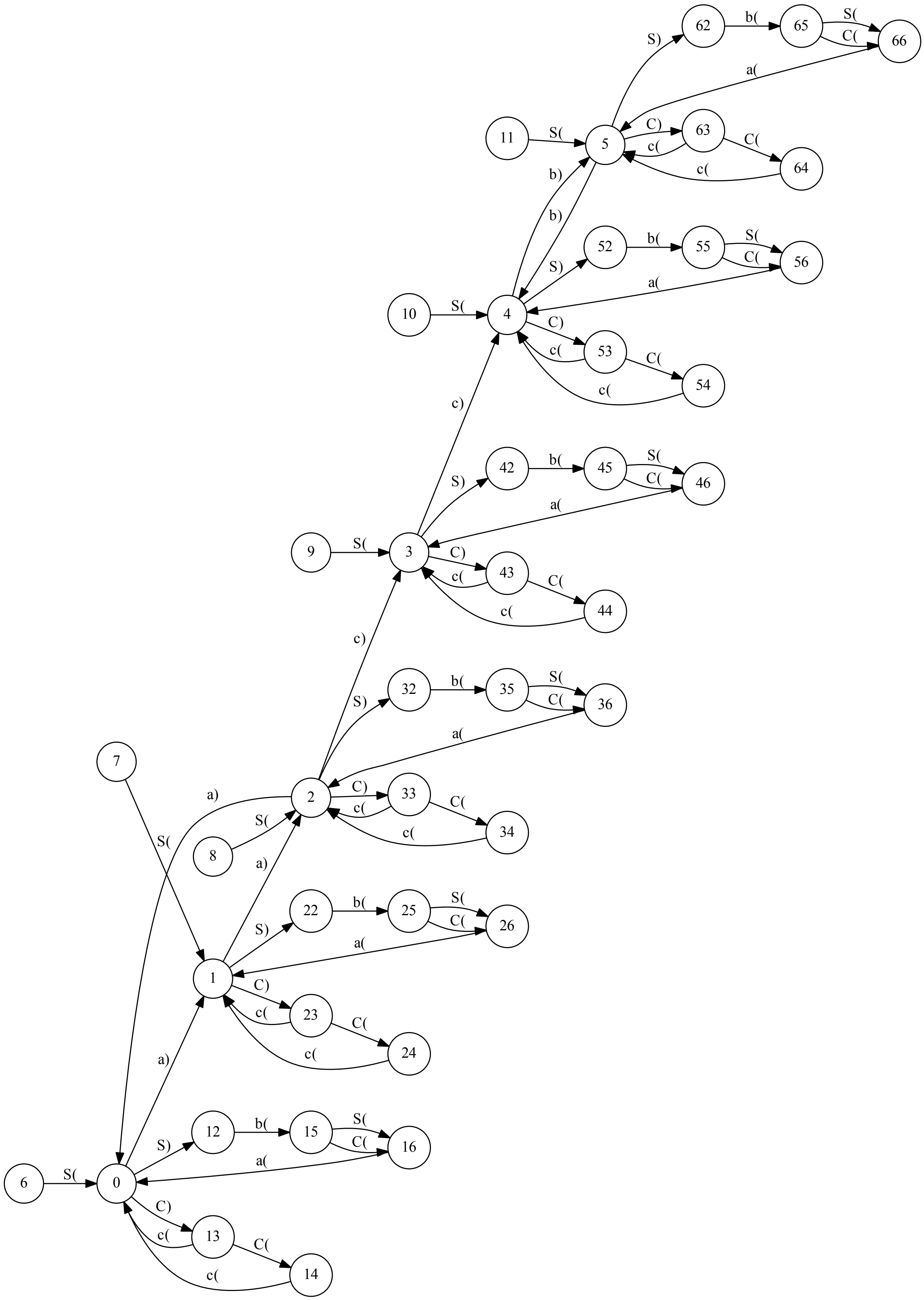
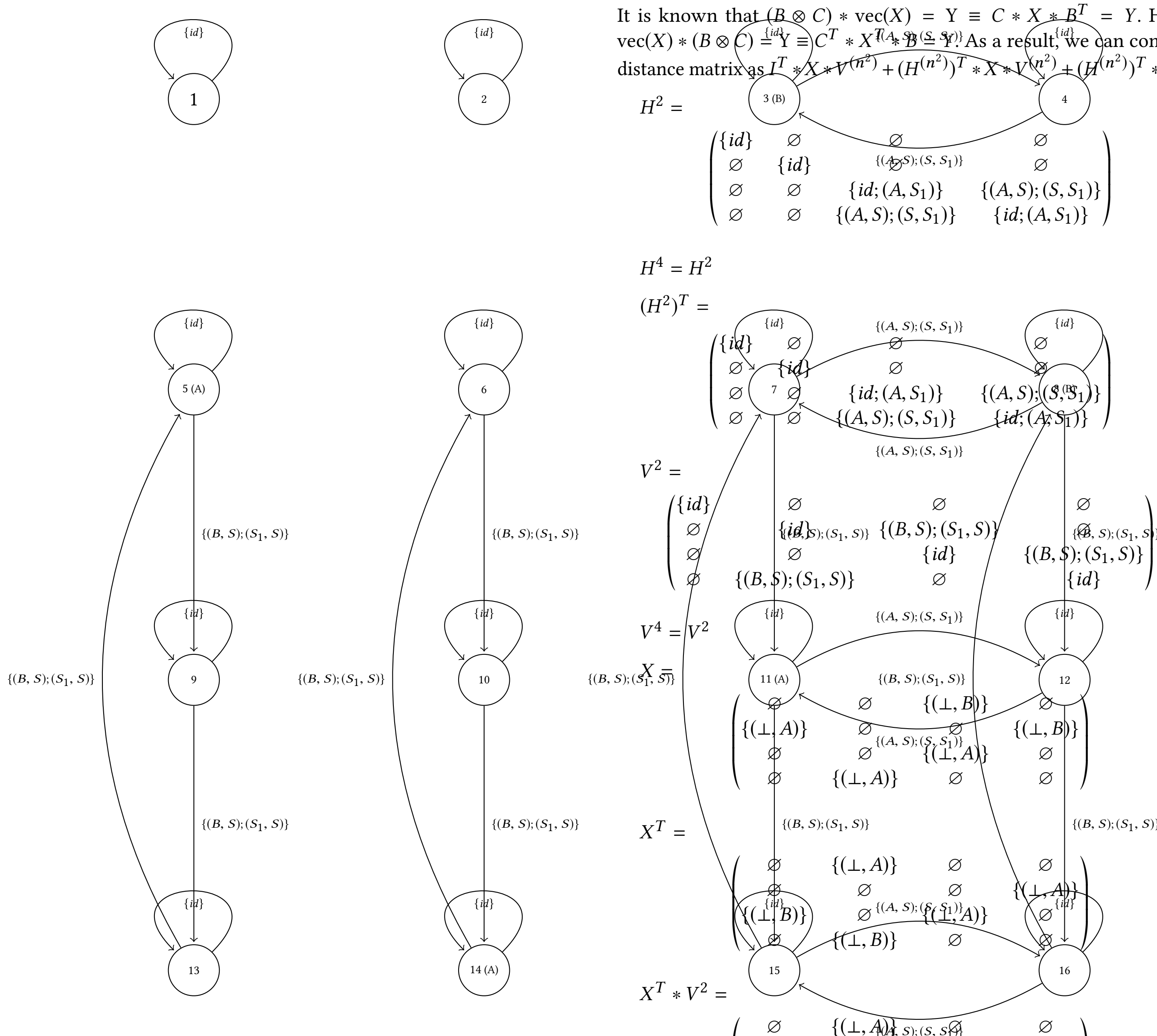


Figure 2: The same generation query (Query 2) in Meerkat



Note that our graph is a Cartesian product of tho graph H and V with respective matrices.

$$H = \begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(A,S);(S,S_1)\} \\ \varnothing & \varnothing & \{(A,S);(S,S_1)\} & \{id\} \end{pmatrix}$$

$$V = \begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \{(B,S);(S_1,S)\} & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(B,S);(S_1,S)\} \\ \varnothing & \{(B,S);(S_1,S)\} & \varnothing & \{id\} \end{pmatrix}$$

Matrix of $G = V \otimes I + I \otimes H$ where I is identity matrix of size $n \times n$ and $\otimes$ is a Kronecker product.

One step is APSP (or transitive closure) of G. It can be computed as $(V \otimes I + I \otimes H)^{(n^2)}$. It can be "over approximated" as $M = (V^{(n^2)} \otimes I + V^{(n^2)} \otimes H^{(n^2)} + I \otimes H^{(n^2)})$. Now we should check validity of nonterminals. It can be don by multiplication of vector x and M. $x * (V^{(n^2)} \otimes I + V^{(n^2)} \otimes H^{(n^2)} + I \otimes H^{(n^2)}) = x * V^{(n^2)} \otimes I + x * V^{(n^2)} \otimes H^{(n^2)} + x * I \otimes H^{(n^2)}$.

It is known that $(B \otimes C) * \mathrm{vec}(X) = Y \equiv C * X * B^T = Y$. Hence $\mathrm{vec}(X) * (B \otimes C) = Y \equiv C^T * X^T * B = Y$. As a result, we can compute distance matrix as $I^T * X * V^{(n^2)} + (H^{(n^2)})^T * X * V^{(n^2)} + (H^{(n^2)})^T * X * I$.

$$H^2 = \begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \{id;(A,S_1)\} & \{(A,S);(S,S_1)\} \\ \varnothing & \varnothing & \{(A,S);(S,S_1)\} & \{id;(A,S_1)\} \end{pmatrix}$$

$H^4 = H^2$

$$(H^2)^T = \begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \{id;(A,S_1)\} & \{(A,S);(S,S_1)\} \\ \varnothing & \varnothing & \{(A,S);(S,S_1)\} & \{id;(A,S_1)\} \end{pmatrix}$$

$$V^2 = \begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \{(B,S);(S_1,S)\} & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(B,S);(S_1,S)\} \\ \varnothing & \{(B,S);(S_1,S)\} & \varnothing & \{id\} \end{pmatrix}$$

$V^4 = V^2$

$$X = \begin{pmatrix} \varnothing & \varnothing & \{(\perp,B)\} & \varnothing \\ \{(\perp,A)\} & \varnothing & \varnothing & \{(\perp,B)\} \\ \varnothing & \varnothing & \{(\perp,A)\} & \varnothing \\ \varnothing & \{(\perp,A)\} & \varnothing & \varnothing \end{pmatrix}$$

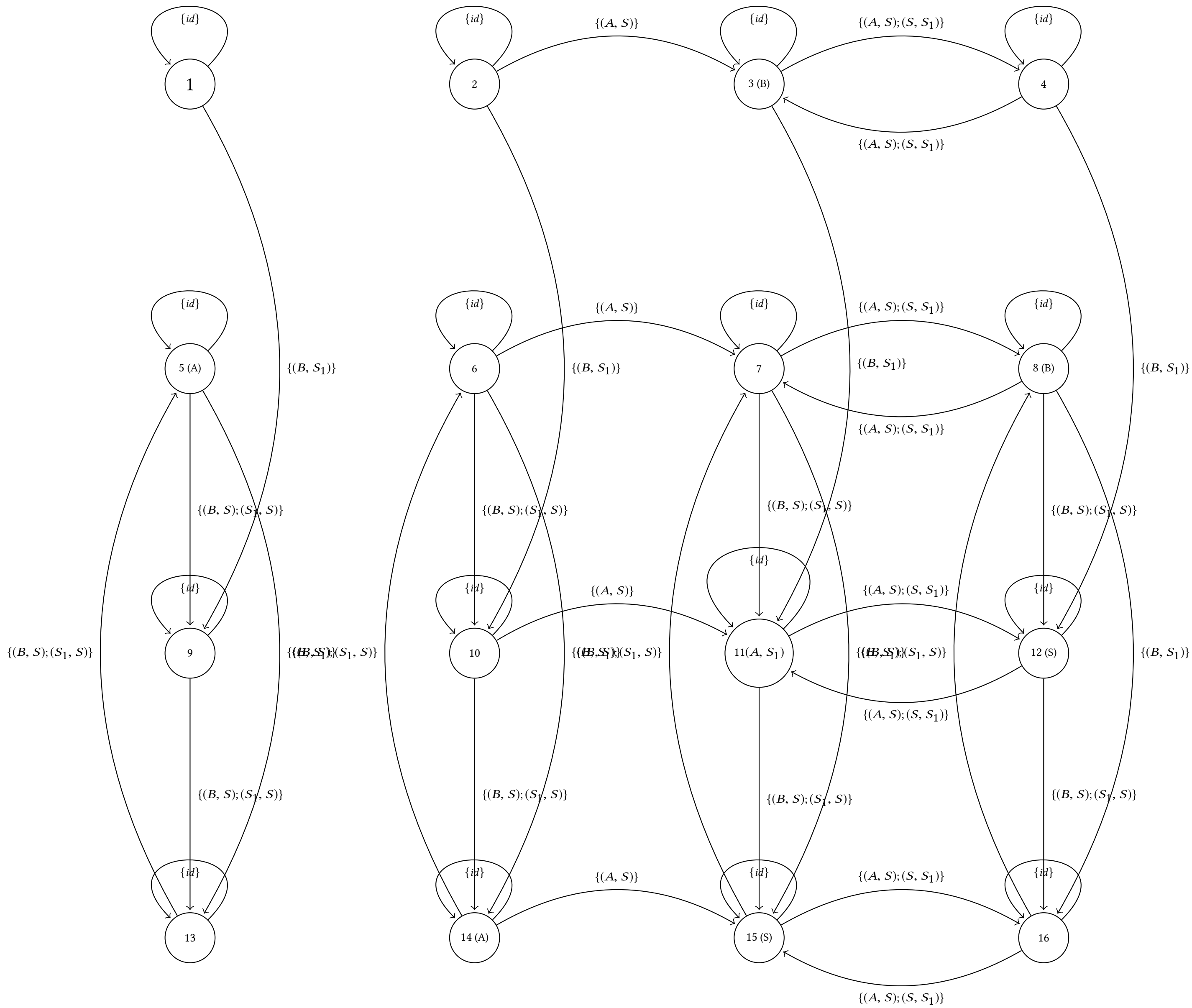
$$X^T = \begin{pmatrix} \varnothing & \{(\perp,A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp,A)\} \\ \{(\perp,B)\} & \varnothing & \{(\perp,A)\} & \varnothing \\ \varnothing & \{(\perp,B)\} & \varnothing & \varnothing \end{pmatrix}$$

$$X^T * V^2 = \begin{pmatrix} \varnothing & \{(\perp,A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp,A)\} \\ \{(\perp,B)\} & \varnothing & \{(\perp,A)\} & \varnothing \\ \varnothing & \{(\perp,B)\} & \{(\perp,S)\} & \varnothing \end{pmatrix}$$

$$(H^2)^T * X^T = \begin{pmatrix} \varnothing & \{(\perp,A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp,A)\} \\ \{(\perp,B)\} & \varnothing & \{(\perp,A);(\perp,S_1)\} & \varnothing \\ \varnothing & \{(\perp,B)\} & \{(\perp,S)\} & \varnothing \end{pmatrix}$$

$$(H^2)^T * X^T * V^2 = \begin{pmatrix} \varnothing & \{(\perp,A)\} & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{(\perp,A)\} \\ \{(\perp,B)\} & \varnothing & \{(\perp,A);(\perp,S_1)\} & \{(\perp,S)\} \\ \varnothing & \{(\perp,B)\} & \{(\perp,S)\} & \varnothing \end{pmatrix}$$

$$(X^T * V^2 + (H^2)^T * X^T * V^2 + (H^2)^T * X^T)^T = \begin{pmatrix} \varnothing & \varnothing & \{(\perp,B)\} & \varnothing \\ \{(\perp,A)\} & \varnothing & \varnothing & \{(\perp,B)\} \\ \varnothing & \varnothing & \{(\perp,A);(\perp,S_1)\} & \{(\perp,S)\} \\ \varnothing & \{(\perp,A)\} & \{(\perp,S)\} & \varnothing \end{pmatrix}$$



$H =$

$$\begin{pmatrix} \{id\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \{id\} & \{(A,S)\} & \varnothing \\ \varnothing & \varnothing & \{id\} & \{(A,S);(S,S_1)\} \\ \varnothing & \varnothing & \{(A,S);(S,S_1)\} & \{id\} \end{pmatrix}$$

$V =$

$$\begin{pmatrix} \{id\} & \varnothing & \{(B,S_1)\} & \varnothing \\ \varnothing & \{id\} & \{(B,S);(S_1,S)\} & \{(B,S_1)\} \\ \varnothing & \varnothing & \{id\} & \{(B,S);(S_1,S)\} \\ \varnothing & \{(B,S);(S_1,S)\} & \varnothing & \{id\} \end{pmatrix}$$

REFERENCES

[1] Krishnendu Chatterjee, Bhavya Choudhary, and Andreas Pavlogiannis. 2017. Optimal Dyck Reachability for Data-dependence and Alias Analysis. *Proc. ACM Program. Lang.* 2, POPL, Article 30 (Dec. 2017), 30 pages. https://doi.org/10.1145/3158118