Evaluation of the Context-Free Path Querying Algorithm Based on Matrix Multiplication

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ABSTRACT

Recently proposed matrix multiplication based algorithm for context-free path querying (CFPQ) offloads the most performance-critical parts onto boolean matrices multiplication. Thus, it is possible to utilize modern parallel hardware and software to achieve high performance of CFPQ easily. In this work, we provide results of empirical performance comparison of different implementations of this algorithm on both real data and synthetic data for the worst cases.

CCS CONCEPTS

• **Information systems** → **Query languages for non-relational engines**; • **Theory of computation** → **Grammars and context-free languages**; *Parallel computing models*; • **Computing methodologies** → **Massively parallel algorithms**; • **Computer systems organization** → *Single instruction, multiple data*;

KEYWORDS

Context-free path querying, transitive closure, graph databases, context-free grammar, GPGPU, CUDA, matrix multiplication, boolean matrix

1 INTRODUCTION

Language-constrained path querying [5], and particulary Context-Free Path Querying (CFPQ) [14] widely used for graph-structured data analysis in such areas as biological data analysis, RDF, network analysis. Huge amount of the real-world data makes performance of CFPQ evaluation critical for practical tasks, and number of algorithms for CFPQ evaluation proposed recently [7, 9, 11–13, 15].

One of the most promising algorithm is a matrix-based algorithm, proposed by Rustam Azimov [4]. This algorith offloads the most critical computattions onto boolean matrices multiplication. As a result, it is pretty simple for implementattion and allows one to utilize modern massive-parallel hardware for CFPQs evaluation. Implenetation provided by authors utilizes GPGPU by using cuSPARSE[1] library which is floating point sparse matrices multiplication library. Even it does not use advanced algorithms for boolean matrices, it outperforms existing algorithms.

It is necessary to investigate an effect of specific algorithms and implementation techniques on performance of CFPQ. One of problems is that there is no publically available standartd dataset for CFPQ algorithms evaluation which includes both graph-structured data and queryes.

In this work, we do empirical performance comparison of different implementations of matrices multiplication based algorithm for CFPQ on both real data and synthetic data for the worst cases. We make the following contributions in this paper.

(1) We provide a number of implementations of the matrix multiplication based CFPQ algoithm, which utilizes different modern software and hardware. Source code is available on GitHub:!!!

(2) We collect and publish a dataset which contains both real data and syntatic data for wirst cases. This dataset contains data and queries in the simplee textual format, so it can be used for other algorithms evaluation easely. We hope that this dataset can be a base for unified benchmark for CFPQ algorithms.

(3) We provide evaluation results which shows that !!!

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[1] cuSparse is a library for GPGPU utilization for sparse matrices multiplication. Offical documentation:https://docs.nvidia.com/cuda/cusparse/index.html.Access date: 12.03.2019

2 MATRIX-BASED ALGORITHM FOR CFPQ

Matrix-based algoritm for CFPQ was proposed by Rustam Azimov [4]. This algorithm can be expressed in few lines of code in terms of matrices operations, and it is a sufficient advantage for implementation. It was shown that GPGPU utilization for queryes evaluation can significantly improve performance in comparison with other implementations [4] even float matrices used instead of boolean matrices.

Pseudocode of the algorithm is presented in listing 1.

Algorithm 1 Context-free path quering algorithm

1: **function** contextFreePathQuerying(D, G)
2:     $n \leftarrow$ the number of nodes in D
3:     $E \leftarrow$ the directed edge-relation from D
4:     $P \leftarrow$ the set of production rules in G
5:     $T \leftarrow$ the matrix $n \times n$ in which each element is $\varnothing$
6:     **for all** $(i, x, j) \in E$ **do**   ▷ Matrix initialization
7:         $T_{i,j} \leftarrow T_{i,j} \cup \{A \mid (A \rightarrow x) \in P\}$
8:     **end for**
9:     **while** matrix T is changing **do**
10:         $T \leftarrow T \cup (T \times T)$ ▷ Transitive closure calculation
11:     **end while**
12:     **return** T
13: **end function**

Here $D = (V, E)$ be the input graph and $G = (N, \Sigma, P)$ be the input grammar. Each cell of the matrix T contains the set of nonterminals such that $N_k \in T[i, j] \iff \exists p = v_i \ldots v_j-$path in D, such that $N_k \overset{*}{\underset{G}{\Rightarrow}} \omega(p)$, where $\omega(p)$ is a word formed by labels along path p. Thus, this algorithm solves reachability problem, or, according Hellings [8], process CFPQs by using relational query semantics.

As you can see, performance-critical part of this algorithm is a matrix multiplication. Note, that the set of nonterminals is finite, we can represent the matrix T as e set of boolean matrices: one for each nonterminal. In this case the matrix updeta operation be $T_{N_i} \leftarrow T_{N_i} + (T_{N_j} \times T_{N_k})$ for each production $N_i \rightarrow N_j N_k$ in P. Thus we can reduse CFPQ to boolean matrices multiplication. After such transfromation we can apply the next optimization: we can skip update if there are no changes in the matrices T_{N_j} and T_{N_k} at the previous iteration.

Thus, the most important part is efficient implementation of operations over boolean matrices, and in this work we compare effects of utilization of different approaches to matrices multiplication. All our implemetations are based on the optimized version of the algorithm.

3 IMPLEMENTATION

We implement matrix-based algorithm for CFPQ by using a number of different programming languages and tools. Our goal is to investigate effects of the next features of implmention.

- **GPGPU utilization.** It is mell-known that GPGPUs are sutable for matrices operations, but performance of whole solution depends on task details: overhead on data transferring may negate effect of parallel computations. Can GPGPUs

utilization for CFPQ improve performance in comparison with CPU version?
- **Existing libraries utilization** is a good practice in software engeneering. Is it possible to achaive highe performance by using existing libraries for matrices operations or we need to create own solution to get more control?
- **Low-level programming.** GPGPU programming is traditionally low-level programming by using C-based languages (CUDA C, OpenCL C). On the other hand, there are number of approaches to create GPGPU-based solution by ysing such high-level languages as a Python. Can we get high-performance solution by using such approaches?
- **Sparce matrices.** Real graphs often are sparse, but not always. Is it sutable to use sparse matrix representation for CFPQ?

We provide next implementations for investigation.

- CPU-based solutions
  [Scipy] Saprse matrices multiplication by using Scipy [10] in Python programming language.
  [M4RI] Dense matrices multiplication by using m4ri[2] [1] library which implements 4 russian method [3] in C language. This library choosen because it is one of performnat implementation of 4 russian method [2].
- GPGPU-based solutions
  [GPU4R] Manual implemenattion of 4 russian metod in CUDA C.
  [GPU_N] Manual implementation of naïve boolean matrix multiplication in CUDA C.
  [GPU_Py] Manual implementation of naïve boolean matrix multiplication in Pyton by using numba compiler[3].

As far as number of matrices and its size can be statically defined at he start, all GPGPU based implementattions allocate all requred memory on the GPGPU only onece, at the start of computattions. By this way it is possible to significante reduce overhead on data transferring: all input data loads to GPGPU at the start, and result loads from GPGPU to the host at the finish. No active data transferring and memory allocating duting query computation.

4 DATASET DESCRIPTION

We create and publish a dataset for CFPQ algorithms evaluation. This dataset contains both the real data and syntetic data for different cpecific cases, such as theoretical worst case, or matrices representation specific worst cases.

Our goal is querying algorithms evaluation, not a graph storages or graph databases evaluation, so all data is presented in text-based format to simplify usage in different environments. Grammars are in Chomscy Normal Form and are stored in the files with yrd extension. Each line is a rule in form of triple or pair. The example of grammar representation is presented in figure 1

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[2]Actually we use pull request which is not merged yet: https://bitbucket.org/malb/m4ri/pull-requests/9/extended-m4ri-to-multiplication-over-the/diff. The original library implements operations over $GF(2)$, and this pull request conteains operations over boolean semiring
[3]Numba is a JIT compiler which supports GPGPU for subset of Python programming. Offical page: http://numba.pydata.org/. Access date: 03.05.2019

```
                         s a b
                         s a s1
                         s1 s b
                         a A
                         b B
   s   →   A s B
   s   →   A B
                      (b) Representation of grammar $G_1$
       (a) Grammar $G_1$    in yrd file
```
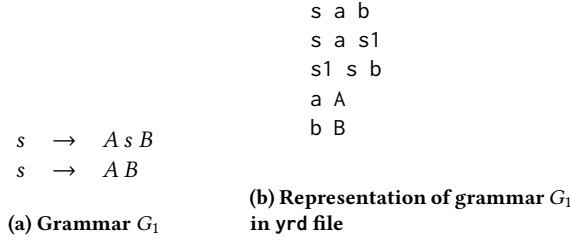
Figure 1: Example of grammar representation in the yrd file

Graphs are represented as a set of triples (edges) and are stored in the files with txt extension. Example of graph is presented in figure 2.

```
                         0 A 1
                         1 A 2
                         2 A 0
                         0 B 3
                         3 B 0
                      (b) Representation of the input
   (a) Example of input graph $D_1$   graph $D_1$ in txt file
```
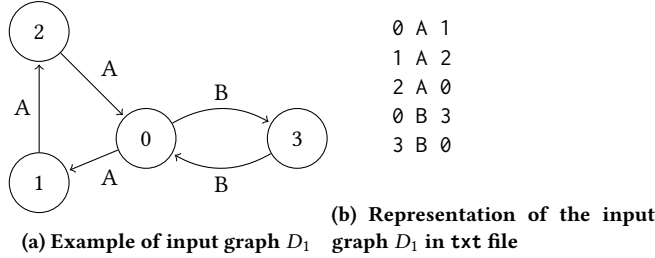
Figure 2: Example of graph representation in txt file

Each case is a pair of set of graphs and set of grammars: each query (grammar) should be applied to each graph. Cases are placed in folders with case-specific name. Grammars and graph are placed in subfolders with names Grammars and Matrices respectively.

It is known that varians of the *same generation query* ?? are classical example of queryes that are context-free but not regular, so we use this type of queryes in our evaluation. The datatset includes data for next cases.

[RDF] The set of real RDF files (ontologies) from [15] and two variants of the same generation query (figures ??) which describes hyrachy analysis.

[Worst] Theoretical worst case for CFPQ time complexity which is proposed by Hellings [9]: graph is a two cycles of coprime lengths with single common vertex. First cycle labelled by open bracket and the second cysle is labelled by close bracked. Query is a grammar for $A^n B^n$ languge (grammar G_1, figure 1).

[Full] The case when input graph is sparce, but result is a full graph. Such case may be a hard for sparse matrices representation. As an input graph we use a cycle all edges of which is labelled by the same token. As a queryes we use two grammars which describe arbitrary repetition of a token: unambiguous and highly ambiguous grammar (figure ??).

[Sparse] Sparse graphs from [6] which generated by the GT-graph graph generator, and emulates realistic sparse data. Names of these graphs have a form Gn-p, where n represents the total number of vertices, each pair of vertices is connected by probability p. Query is a same generation query.

5 EVALUATION

We evaluate all described implementattions on all data and queries from presented dataset. Our goal is to comare CFPQ evaluation algorithms, so we exclude time required for load data from files. Time requred for data transferring is included.

For evaluation we use PC with the next characteristics.

- OS
- CPU
- RAM
- GPU
  - Name
  - GHz
  - RAM
  - Threads
  - ...

Results of evaluation are presented in tables below. Time is mesured in milliseconds. Time for each algorithm is an averege time of 10 runs.

First is a **[RDF]** dataset. Results are presented in a table 1.

We can see, that in this case running time for all our implementations smaller than time for reference implementattion, and that **[GPU_N]** is faster than other implementations while other implementations demonsrtate similar performance. Also it is obvious that performance improvement in comarison with first implementations is huge and it is necessary to select new significantly biggest RDF files.

Results of theoretical worst case (**[Worst]** datatset) is presented in table 2.

Table 2: Worst case evaluation results

#V	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
16	0.032	< 0.001	0.008	0.002	!!!	!!!
32	0.118	0.001	0.034	0.008	!!!	!!!
64	0.476	0.041	0.133	0.032	!!!	!!!
128	2.194	0.226	0.562	0.129	2.751	!!!
256	15.299	1.994	3.088	0.544	11.883	!!!
512	121.287	23.204	13.685	2.499	43.563	!!!
1024	1593.284	528.521	88.064	19.357	217.326	!!!
2048	-	-	-	325.174	-	!!!

In this case !!!!! In this case !!!!!
Next is a **[Sparse]** datatset. Results are presented in table 3.

Table 3: Sparse graphs querying results

Graph	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
G5k-0.001	10.352	0.647	0.113	0.041	0.216	!!!
G10k-0.001	37.286	2.395	0.435	0.215	!!!	!!!
G10k-0.01	97.607	1.455	0.273	0.138	0.763	!!!
G10k-0.1	!!!	1.050	0.223	0.114	0.859	!!!
G20k-0.001	150.774	11.025	1.842	1.274	6.180	!!!
G40k-0.001	-	97.841	11.663	8.393	37.821	!!!
G80k-0.001	-	1142.959	88.366	65.886	-	!!!

Table 1: RDFs querying results

RDF			Query 1						Query 2					
Name	#V	#E	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
atom-primitive			0.003	0.002	0.002	0.001	0.005	!!!	0.001	< 0.001	0.001	< 0.001	0.002	!!!
biomedical-mesure-primitive			0.003	0.005	0.002	0.001	0.005	!!!	0.004	< 0.001	0.001	< 0.001	0.005	!!!
foaf			0.002	0.009	0.002	< 0.001	0.005	!!!	0.001	< 0.001	0.001	< 0.001	0.002	!!!
funding			0.004	0.007	0.004	0.001	0.005	!!!	0.002	< 0.001	0.003	< 0.001	0.004	!!!
generations			0.003	0.003	0.002	< 0.001	0.005	!!!	0.001	< 0.001	0.001	< 0.001	0.002	!!!
people_pets			0.003	0.003	0.003	0.001	0.007	!!!	0.001	< 0.001	0.001	< 0.001	0.003	!!!
pizza			0.006	0.008	0.003	0.001	0.006	!!!	0.002	< 0.001	0.002	< 0.001	0.005	!!!
skos			0.002	0.004	0.002	< 0.001	0.005	!!!	< 0.001	< 0.001	0.001	< 0.001	0.002	!!!
travel			0.003	0.005	0.002	< 0.001	0.006	!!!	0.001	< 0.001	0.001	< 0.001	0.003	!!!
univ-bench			0.002	0.004	0.002	< 0.001	0.005	!!!	0.001	< 0.001	0.001	< 0.001	0.003	!!!
wine			0.007	0.006	0.004	0.001	0.007	!!!	0.001	< 0.001	0.003	< 0.001	0.003	!!!

For such type of graphs !!!! Note that we estimate only query execution time, so it is hard to compare our results with results presented in [6]. But it would be interesting to do such comaprison in future because running time of our **[GPU_N]** implementation is significantly smaller than provided in [6].

The last dataset is a **[Full]**, and results a shown in table 4

Finally, we can cocnlude that

- On GPU utilization
- On Existinng libraries
- On Low-level programming
- On sparse matrices

6 CONCLUSION AND FUTURE WORK

We provide a number of implementations of matrix-based algorithm for context-free path querying, collect a dataset for evaluation and provide results of evaluation of our implementation on collected dataset. Our evaluation shows that GPGPU utilization for boolean matrices multiplication can significantely increase perfprmance of CFPQs evaluation, but requires more research on implementattion details.

First direction for future research is a more detailed CFPQ algorithms investigation. We should do more evaluation on sparse matrices on GPGPUs and investigate technics for high-performance GPGPU code creation.

Also it is nesessary to implement and evaluate solutions for graphs which is not fit in RAM, and for big queryes which disallow to allocate all requred matrices on single GPGPU. We hope that it is possible to utilize existing technics for huge matrices multiplication for this problem.

Another direcion is a dataset improvement. First of all, it is necessary to collect more data, and more grammars/queries. Especially it would be interesting to add to dataset more real graphs and more real queryes. Secondly, it is nesessary to discuss and fix data format to be able to evaluate different algorithms. We think that it is necessary to create public dataset for CFPQ algorithms evaluation, and collaboration with community is requred.

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Table 4: Full querying results

#V	Query 1						Query 2					
	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
100	0.007	!!!	0.002	< 0.001	0.003	!!!	0.023	!!!	0.005	0.001	0.007	!!!
200	0.040	!!!	0.002	0.001	0.004	!!!	0.105	!!!	0.004	0.001	0.007	!!!
500	0.480	!!!	0.003	0.001	0.004	!!!	1.636	!!!	0.007	0.001	0.010	!!!
1000	3.741	!!!	0.005	0.001	0.006	!!!	13.071	!!!	0.009	0.001	0.009	!!!
2000	40.309	!!!	0.019	0.003	0.017	!!!	93.676	!!!	0.030	0.005	0.026	!!!
5000	651.343	!!!	0.125	0.038	0.150	!!!	!!!	!!!	0.195	0.075	0.239	!!!
10000	-	!!!	0.552	0.315	0.840	!!!	!!!	!!!	1.055	0.648	1.838	!!!
25000	-	!!!	7.252	5.314	15.521	!!!	-	!!!	15.240	10.961	36.495	!!!
50000	-	!!!	58.751	44.611	129.641	!!!	-	!!!	130.203	91.579	!!!	!!!
80000	-	!!!	256.579	190.343	641.260	!!!	-	!!!	531.694	376.691	!!!	!!!

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