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Context-Free Path Querying by Kronecker Product

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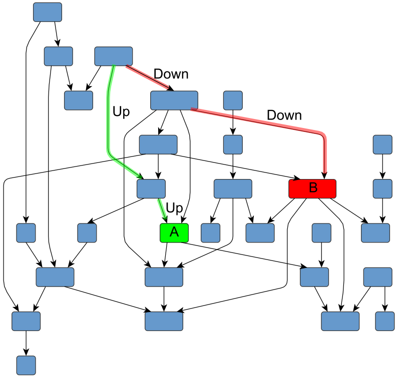
Context-Free Path Querying



Navigation through a graph

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form **Up**^{n} **Down**^{n}?
- Find all paths of form **Up**^{n} **Down**^{n} which start from the node A

CFPQ: Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$ — context-free grammar in normal form
  - $A \rightarrow BC$, where $A, B, C \in N$
  - $A \rightarrow x$, where $A \in N, x \in \Sigma \cup \{\varepsilon\}$
  - $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^{*} \omega\}$

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- $\omega(\pi) = \omega(v_0 \xrightarrow{l_0} v_1 \xrightarrow{l_1} \cdots \xrightarrow{l_{n-2}} v_{n-1} \xrightarrow{l_{n-1}} v_n) = l_0 l_1 \cdots l_{n-1}$

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- $R_A = \{(n, m) \mid \exists n \pi m$, such that $\omega(\pi) \in L(\mathbb{G}, A)\}$

CFPQ: Original Matrix-Based Algorithm

Algorithm Context-free path querying algorithm

function $\textsc{evalCFPQ}(D = (V, E, L), G = (\Sigma, N, P))$
  $n \leftarrow |V|$
  $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i}$ is a matrix $n \times n$, $T^{A_i}_{k,l} \leftarrow \texttt{false}\}$
  for all $(i, x, j) \in E$, $A_k \mid A_k \to x \in P$ **do** $T^{A_k}_{i,j} \leftarrow \texttt{true}$
  for all $A_k \mid A_k \to \varepsilon \in P$ **do**
    for all $i \in \{0, \ldots, n-1\}$ **do** $T^{A_k}_{i,i} \leftarrow \texttt{true}$
  while any matrix in T is changing **do**
    for $A_i \to A_j A_k \in P$ **do** $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$
  return T

Context-Free Path Querying: Grammar Transformation

- $\mathbb{G} = (\Sigma, N, P)$ — context-free grammar in general form
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- Every context-free grammar can be transformed to binary normal form
- The transformation takes time and can lead to a significant grammar size increase

Research Questions

- Can we create the matrix-based CFPQ algorithm that does not require grammar transformation?
- What matrix operations should be used?
- Does using matrix optimizations still significantly increases performance?

Recursive State Machines (RSM)

- RSM behaves as a set of finite state machines (FSM)
- Each FSM (box) works almost the same as a classical FSM, but it also handles additional recursive calls and employs an implicit call stack to call one component from another and then return execution flow back
- Any CFG can be easily encoded by an RSM with one box per nonterminal

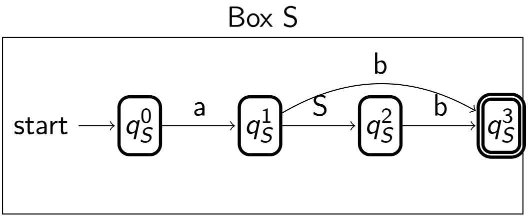

Figure: The RSM for grammar with rules $S \to aSb \mid ab$

Kronecker Product

- Kronecker product $A \otimes B$ for matrix A of size $m \times n$ and matrix B of size $p \times q$
  - Multiply each element of A and the matrix B
  - As a result we have $pm \times qn$ block matrix
- We need to intersect the FSMs from RSM generated by the context-free grammar and FSM generated by the graph
- Kronecker product can be used for constructing such intersections

Kronecker Product Based CFPQ Algorithm

Algorithm Kronecker product based CFPQ

```
function contextFreePathQuerying(G, 𝒢)
    R ← Recursive automata for G
    M1, M2 ← Adjacency matrices for R and 𝒢
    for s ∈ 0..dim(M1) − 1 do
        for i ∈ 0..dim(M2) − 1 do
            M2[i, i] ← M2[i, i] ∪ getNonterminals(R, s, s)
    while Matrix M2 is changing do
        M3 ← M1 ⊗ M2                                  ▷ Evaluate Kronecker product
        C3 ← transitiveClosure(M3)
        n ← dim(M3)                                   ▷ Matrix M3 size = n × n
        for i ∈ 0..n − 1 do
            for j ∈ 0..n − 1 do
                if C3[i, j] then
                    s, f ← getStates(C3, i, j)
                    if getNonterminals(R, s, f) ≠ ∅ then
                        x, y ← getCoordinates(C3, i, j)
                        M2[x, y] ← M2[x, y] ∪ getNonterminals(R, s, f)
    return M2
```

Kronecker Product Based CFPQ Algorithm: Technical Details

- We can use the sparse and block nature of the obtained matrices to apply wide class of optimizations
- We can operate over Boolean matrices
- We still can use existing high-performance math libraries for intersection if they provide a satisfying operation of element-wise multiplication

Implementations

- **Kron** — implementation of the proposed algorithm using **SuiteSparse** C implementation of **GraphBLAS** API, which provides a set of sparse matrix operations

Implementations

- **Kron** — implementation of the proposed algorithm using **SuiteSparse** C implementation of **GraphBLAS** API, which provides a set of sparse matrix operations
- We compare our implementation with **Orig** — the best CPU implementations of the original matrix-based algorithm using M4RI library with sparse matrix representation

Evaluation

- OS: Ubuntu 18.04
- CPU: Intel(R) Core(TM) i7-4790 CPU 3.60GHz
- RAM: DDR4 32 Gb

Evaluation results[1] [2]

	Graph	#V	#E	*Kron*	*Orig*		Graph	#V	#E	*Kron*	*Orig*
RDF	generations	129	351	0.04	0.03	RDF	core	1323	8684	0.28	0.12
	travel	131	397	0.05	0.05		pways	6238	37196	4.88	0.18
	skos	144	323	0.02	0.04	Worst case	WC_1	64	65	0.03	0.04
	unv-bnch	179	413	0.05	0.04		WC_2	128	129	0.16	0.23
	foaf	256	815	0.07	0.02		WC_3	256	257	0.96	1.99
	atm-prim	291	685	0.24	0.02		WC_4	512	513	7.14	23.21
	ppl_pets	337	834	0.18	0.03		WC_5	1024	1025	121.99	528.52
	biomed	341	711	0.24	0.05	Full	F_1	100	100	0.17	0.02
	pizza	671	2604	1.14	0.08		F_2	200	200	1.04	0.03
	wine	733	2450	1.71	0.06		F_3	500	500	18.86	0.03
	funding	778	1480	0.43	0.07		F_4	1000	1000	554.22	0.07

[1] Queries are based on the context-free grammars for nested parentheses

[2] Time is measured in seconds

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- We still can use existing high-performance libraries for matrix operations
- The idea of the proposed algorithm looks viable
- Dataset is published: both graphs and queries
  - Link: `https://github.com/JetBrains-Research/CFPQ_Data`

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- Extend our algorithm to single-path and all-path query semantics

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Thanks!