Context-Free Path Querying with Structural Representation of Result

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ABSTRACT

Graph data model and graph databases are very popular in various areas such as bioinformatics, semantic web, and social networks. One specific problem in the area is a path querying with constraints formulated in terms of formal grammars. There are several solutions to it, but how to provide structural representation of query result which is practical for answer investigation and exploration is still an open problem. In this paper we propose graph parsing technique which allows us to build such representation with respect to given grammar query for arbitrary context-free grammar and graph. Proposed algorithm is based on generalized LL parsing algorithm while previous solutions based mostly on CYK or Earley algorithms.

Categories and Subject Descriptors

H.2.3 [**Information Systems**]: Database Management-Query Languages; D.2.8 [**Software Engineering**]: Metrics—*complexity measures, performance measures*

Keywords

Graph database, path query, graph parsing, context-free grammar, top-down parsing, GLL, LL

1. INTRODUCTION

Graph data model and graph data bases are very popular in various areas such as bioinformatics, semantic web, social networks, etc. Extraction of paths which satisfy specific constraints may be useful for investigation of graph structured data and for detection of relations between data items. One specific problem—path querying with constraints—is usually formulated in terms of formal grammars and is called formal language constrained path problem [4].

Classical parsing techniques can be used to solve formal language constrained path problem. It means that such technique can be used for more common problem—graph parsing. Graph parsing may be required in graph data base querying, formal verification, string-embedded language processing, and another areas where graph structured data is used.

Existing solutions in databases field usually employ such parsing algorithms as CYK or Earley(for example [8], [17]). These algorithms have nonlinear time complexity for unambiguous grammars ($O(n^3)$ and $O(n^2)$ respectively). Moreover, in case of CYK, the input grammar should be transformed to Chomsky normal form (CNF) which leads to grammar size increase. To solve these problems, one can use such parsing algorithms as GLR and GLL which have cubic worst-case time complexity and linear complexity for unambiguous grammars. Also there is no need to transform a grammar to CNF for these algorithms. These facts allow us to improve performance of parsing in some cases.

Despite the fact that there is a set of path querying solutions [17, 8, 3, 12], query result exploration is still a challenge [9], as also a simplification of complex query debugging. Structural representation of query result can be used to solve these problems, and classical parsing techniques provide such representation—derivation tree—which contains exhaustive information about parsed sentence structure in terms of specified grammar.

Graph parsing can also be used to analyze dynamically generated strings or string-embedded languages. String variable in a program may gets multiple values in run time. In order to convey statical analysis, value set of string variable can be over-approximated with regular language which is represented as a finite automaton. Moreover, to check a syntactic correctness of dynamically generated strings, one should check that all generated strings (all paths from start states to final states in the given automaton) are correct with respect to the given context-free grammar. There are solutions to this problem: GLR-based checker of string-embedded SQL queries [2, 6], parser of string-embedded languages [20] based on RNGLR parsing algorithm. RNGLR-based algorithm allows to construct derivation forest (i.e. the set of derivation trees) for all correct paths in the input automaton.

In this paper we propose a graph parsing technique which allows one to construct structural representation of query result with respect to the given grammar. This structure can be useful for query debugging and exploration. Proposed algorithm is based on generalized top-down parsing algorithm—GLL [13]—which has cubic worst-case time complexity and linear time complexity for LL grammars on linear input.

2. PRELIMINARIES

In this work we are focused on the parsing algorithm, and not on the data representation, and we assume that whole input graph can be located in RAM memory in the optimal for our algorithm way.

We start by introduction of necessary definitions.

- Context-free grammar is a quadruple $G = (N, \Sigma, P, S)$, where N is a set of nonterminal symbols, Σ is a set of

terminal symbols, $S \in N$ is a start nonterminal, and P is a set of productions.

- $\mathcal{L}(G)$ denotes a language specified by grammar G, and is a set of terminal strings derived from start nonterminal of G: $L(G) = \{\omega | S \Rightarrow^*_G \omega\}$.
- Directed graph is a triple $M = (V, E, L)$, where V is a set of vertices, $L \subseteq \Sigma$ is a set of labels, and a set of edges $E \subseteq V \times L \times V$. We assume that there are no parallel edges with equal labels: for every $e_1 = (v_1, l_1, v_2) \in E, e_2 = (u_1, l_2, u_2) \in E$ if $v_1 = u_1$ and $v_2 = u_2$ then $l_1 \neq l_2$.
- $tag : E \to L$ is a helper function which allows to get tag of edge.

$$tag(e = (v_1, l, v_2), e \in E) = l$$

- $\oplus : L^+ \times L^+ \to L^+$ denotes a tag concatenation operation.
- Path p in graph M is a list of incident edges:

$$\begin{aligned} p &= e_0, e_1, \ldots, e_{n-1} \\ &= (v_0, l_0, v_1), (v_1, l_1, v_2), \ldots, (v_{n-1}, l_{n-1}, v_n) \end{aligned}$$

  where $v_i \in V$, $e_i \in E$, $e_i = (v_i, l_i, v_{i+1})$, $l_i \in L$, $|p| = n, n \geq 1$.
- P is a set of paths $\{p : p \text{ path in } M\}$, where M is a directed graph.
- $\Omega : P \to L^+$ is a helper function which constructs a string produced by the given path. For every $p \in P$

$$\begin{aligned} &\Omega(p = e_0, e_1, \ldots, e_{n-1}) = \\ &tag(e_0) \oplus \cdots \oplus tag(e_{n-1}). \end{aligned}$$

Using these definitions, we state the context-free language constrained path querying as, given a query in form of grammar G, to construct the set of paths

$$P = \{p | \Omega(p) \in \mathcal{L}(G)\}.$$

Note that, in some cases, P can be an infinite set, and hence it cannot be represented explicitly. In order to solve this problem, in this paper, we construct compact data structure representation which stores all elements of P in finite amount of space and allows to extract any of them.

3. MOTIVATING EXAMPLE

Suppose that you are student in a School of Magic. It is your first day in the School, so navigation in the building is a problem for you. Fortunately, you have a map of the building (fig. 1) and an additional knowledge about building construction:

- there are towers in the school (nodes of the graph in your map);
- towers can be connected by directed galleries (edges in your map);
- galleries have a "magic" property: you can start from any floor, but following each gallery you either end one floor above (edge label is 'a'), either one floor below (edge label is 'b').

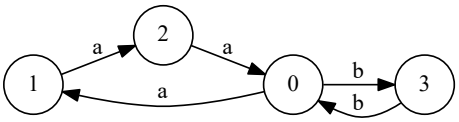


Figure 1: The map of School (input graph M)

You want to find a path from your current position to the same floor in another tower. Map with all such paths can help you. But orienteering is not your forte, so it would be great if the structure of the paths were as simple as possible and all paths had additional checkpoints to control your rout.

It is evident that the simplest structure of required paths is $\{ab, aabb, aaabbb, \ldots\}$. In terms of our definitions, it is necessary to find all paths p such that $\Omega(p) \in \{a^n b^n, n \geq 1\}$ in the graph $M = (\{0; 1; 2; 3\}, E, \{a; b\})$ (figure 1).

Unfortunately, language $\mathcal{L} = \{a^n b^n; n \geq 1\}$ is not regular which restricts the set of tools you can use. Another problem is the infinite size of solution, but you want to get a finite map. Moreover, you want to know a structure of paths in terms of checkpoints.

We are not aware of any existing tools which can help to solve this problem and we create the such tool. Let us to show how to get a map which helps to navigate in this strange School.

Fortunately, the language $\mathcal{L} = \{a^n b^n; n \geq 1\}$ is a context-free language and can be specified with context-free grammar. The fact that one language can be described with more than one grammar allows to add checkpoints: additional nonterminals can mark required parts of a sentence. In our case, a desired checkpoint can be in the middle of the path. As a result, required language can be specified by the grammar G_1 presented in figure 2, where $N = \{s; Middle\}$, $\Sigma = \{a; b\}$, and S is a start nonterminal.

$$\begin{aligned} 0: &\quad S \to a\ S\ b \\ 1: &\quad S \to Middle \\ 2: &\quad Middle \to a\ b \end{aligned}$$

Figure 2: Grammar G_1 for language $L = \{a^n b^n; n \geq 1\}$ with additional marker for a middle of a path

In the next section we present a map construction algorithm, which solves such problems.

4. GRAPH PARSING ALGORITHM

We propose a graph parsing algorithm which allows to construct finite representation of parse forest which contains derivation trees for all matched paths in graph. Finite representation of result set with respect to the specified grammar may be useful not only for results understanding and processing, but also for query debugging.

Our solution is based on generalized LL (GLL) [13, 1] parsing algorithm which allows to process arbitrary (including left-recursive and ambiguous) context-free grammars with worst-case cubic time complexity and linear time complexity for LL grammars on a linear input.

4.1 Generalized LL Parsing Algorithm

Classical LL algorithm operates with a pointer to input (position i) and with a grammar slot—pointer to grammar in form $N \to \alpha \cdot x\beta$. Parsing may be described as a transition of these pointers from the initial position ($i = 0$, $S \to \cdot\beta$, where S is start nonterminal) to the final ($i = input.Length$, $s \to \beta\cdot$). At every step, there are four possible cases in processing of these pointers.

1. $N \to \alpha \cdot x\beta$, when x is a terminal and $x = input[i]$. In this case both pointers should be moved to the right ($i \leftarrow i + 1$, $N \to \alpha x \cdot \beta$).
2. $N \to \alpha \cdot X\beta$, when X is nonterminal. In this case we push return address $N \to \alpha X \cdot \beta$ to stack and move pointer in grammar to position $X \to \cdot\gamma$.
3. $N \to \alpha\cdot$. This case means that processing of nonterminal N is finished. We should pop return address from stack and use it as new slot.
4. $S \to \alpha\cdot$, where S is a start nonterminal of grammar. In this case we should report success if $i = input.Length - 1$ or failure otherwise.

In the second case there can be several slots $X \to \cdot\gamma$, so a strategy on how to choose one of them to continue parsing is needed. In LL(k) algorithm lookahead is used, but this strategy is still not good enough because there are context-free languages for which deterministic choice is impossible even for infinite lookahead [5]. On the contrary to LL(k), generalized LL does not choose at all, handling all possible variants. Note, that instead of immediate processing of all variants, GLL uses descriptors mechanism to store all possible branches and process them sequentially. Descriptor is a quadruple (L, s, j, a) where L is a grammar slot, s is a stack node, j is a position in the input string, and a is a node of derivation tree.

The stack in parsing process is used to store return information for the parser—a name of function which will be called when current function finishes computation. As mentioned before, generalized parsers process all possible derivation branches and parser must store it's own stack for every branch. It leads to an infinite stack growth being done naively. Tomita-style graph structured stack (GSS) [19] combines stacks resolving this problem. Each GSS node contains a pair of position in input and a grammar slot in GLL .

In order to provide termination and correctness, we should avoid duplication of descriptors, and be able to process GSS nodes in arbitrary order. It is necessary to use the following additional sets for this.

- R—working set which contains descriptors to be processed. Algorithm terminates whenever R is empty.
- U—all created descriptors. Each time when we want to add a new descriptor to R, we try to find it in this set first. This way we process each descriptor only once which guarantee termination of parsing.
- P—popped nodes. Allows to process descriptors (and GSS nodes) in arbitrary order.

Instead of explicit code generation used in classical algorithm, we use table version of GLL [7] in order to simplify adaptation to graph processing. As a result, main control function is different from the original one because it should process LL-like table instead of switching between generated parsing functions. Control functions of the table based GLL are presented in Algorithm 1. All other functions are the same as in the original algorithm and their descriptions can be found in the original article [13] or in Appendix A.

Algorithm 1 Control functions of table version of GLL

```
 1: function DISPATCHER( )
 2:   if R.Count ≠ 0 then
 3:     (L, v, i, cN) ← R.Get()
 4:     cR ← dummy
 5:     dispatch ← false
 6:   else
 7:     stop ← true
 8: function PROCESSING( )
 9:   dispatch ← true
10:   switch L do
11:     case (X → α · xβ) where x = input[i + 1])
12:       if cN = dummyAST then
13:         cN ← getNodeT(i)
14:       else
15:         cR ← getNodeT(i)
16:       i ← i + 1
17:       L ← (X → αx · β)
18:       if cR ≠ dummy then
19:         cN ← getNodeP(L, cN, cR)
20:       dispatch ← false
21:     case (X → α · xβ) where x is nonterminal
22:       v ← create((X → αx · β), v, i, cN)
23:       slots ← pTable[x][input[i]]
24:       for all L ∈ slots do
25:         add(L,v,i,dummy)
26:     case (X → α·)
27:       pop(v,i,cN)
28:     case (S → α·) when S is start nonterminal
29:       final result processing and error notification
30: function CONTROL
31:   while not stop do
32:     if dispatch then
33:       DISPATCHER( )
34:     else
35:       PROCESSING( )
```

There can be more than one derivation tree of a string with relation to ambiguous grammar. Generalized LL build all such trees and compact them in a special data structure Shared Packed Parse Forest [11], which will be described in the following section.

4.2 Shared Packed Parse Forest

Binarized Shared Packed Parse Forest (SPPF) [16] compresses derivation trees optimally reusing common nodes and subtrees. Version of GLL which uses this structure for parsing forest representation achieves worst-case cubic space complexity [14].

Let us present an example of SPPF for the input sentence `"abababab"` and ambiguous grammar G_0 (fig 3).

There are two different leftmost derivations of the given sentence w.r.t. grammar G_0, hence SPPF contains two dif-

$0: \quad S \to \varepsilon$
$1: \quad S \to b\ S\ b$
$2: \quad S \to S\ S$

Figure 3: Grammar G_0

ferent derivation trees. Resulting SPPF(fig. 4a) and two trees extracted from it (fig. 4b and fig. 4c) are presented in the figure 4.

Binarized SPPF can be represented as a graph in which each node has one of four types described below with correspondent graphical notation. Let i and j be the start and the end positions of substring, and let us call a tuple (i, j) an *extension* of node.

- Node of rectangle shape labeled with (i, T, j) is a terminal node.
- Node of oval shape labeled with (i, N, j) is a nonterminal node. This node denotes that there is at least one derivation for substring $\alpha = \omega[i..j-1]$ such that $N \Rightarrow_G^* \alpha, \alpha = \omega[i..j-1]$. All derivation trees for the given substring and nonterminal can be extracted from SPPF by left-to-right top-down graph traversal started from respective node. We use filled shape and label of form ($\lhd\rhd$ (i, N, j)) to denote that there are multiple derivations from nonterminal N for substring $\omega[i..j-1]$.
- Node of rectangle shape labeled with (i, t, j), where t is a grammar slot, is an intermediate node: a special kind of node used for binarization of SPPF.
- Packed node labeled with $(N \to \alpha \cdot \beta, k)$. In our pictures, we use dot shape for these nodes and omit labels because they are important only on SPPF constriction stage. Subgraph with "root" in such node is one variant of derivation from nonterminal N in case when the parent is a nonterminal node labeled with ($\lhd\rhd$ (i, N, j)).

In our examples we remove redundant intermediate and packed nodes from the SPPF to simplify it and to decrease the size of structure.

4.3 GLL-based Graph Parsing

In this section we present such modification of GLL algorithm, that for input graph M, set of start vertices $V_s \subseteq V$, set of final vertices $V_f \subseteq V$, and grammar G_1, it returns SPPF which contains all derivation trees for all paths p in M, such that $\Omega(p) \in L(G_1)$, and $p.start \in V_s,\ p.end \in V_f$. In other words, we propose GLL-based algorithm which can solve language constrained path problem.

First of all, notice that an input string for classical parser can be represented as a linear graph, and positions in the input are vertices of this graph. This observation can be generalized to arbitrary graph with remark that for a position there is a set of labels of all outgoing edges for given vertex instead of just one next symbol. Thus, in order to use GLL for graph parsing we need to use graph vertices as positions in input and modify **Processing** function to process multiple "next symbols". Required modifications are presented in the Algorithm 2 (line **5** and **17**). Small modification is also required for initialization of R set: it is necessary to add not only one initial descriptor but the set of descriptors for all vertices in V_s. All other functions are reused from original algorithm without any changes.

Algorithm 2 Processing function modified in order to process arbitrary directed graph

```
 1: function PROCESSING( )
 2:   dispatch ← true
 3:   switch L do
 4:     case (X → α · xβ) where x is terminal
 5:       for all {e|e ∈ input.outEdges(i), tag(e) = x} do
 6:         new_cN ← cN
 7:         if new_cN = dummyAST then
 8:           new_cN ← GETNODET(e)
 9:         else
10:           new_cR ← GETNODET(e)
11:         L ← (X → αx · β)
12:         if new_cR ≠ dummy then
13:           new_cN ← GETNODEP(L, new_cN, new_cR)
14:         ADD(L,v,target(e),new_cN)
15:     case (X → α · xβ) where x is nonterminal
16:       v ← CREATE((X → αx · β), v, i, cN)
17:       slots ← ⋃_{e∈input.OutEdges(i)} pTable[x][e.Token]
18:       for all L ∈ slots do
19:         ADD(L,v,i,dummy)
20:     case (X → α·)
21:       POP(v,i,cN)
22:     case _
23:       final result processing and error notification
```

Note that our solution handles arbitrary numbers of start and final vertices, which allows one to solve different kinds of problems arising in the field, namely all paths in graph, all paths from specified vertex, all paths between specified vertices. Also SPPF represents a structure of paths in terms of grammar which provides exhaustive information about result.

Note that termination of proposed algorithm is inherited from the basic GLL algorithm. We process finite graphs, hence the set of positions is finite, and tree construction has not been changed. As a result, the total number of descriptors is finite, and each of them is added in R only once, thus main loop is finite.

4.4 Complexity

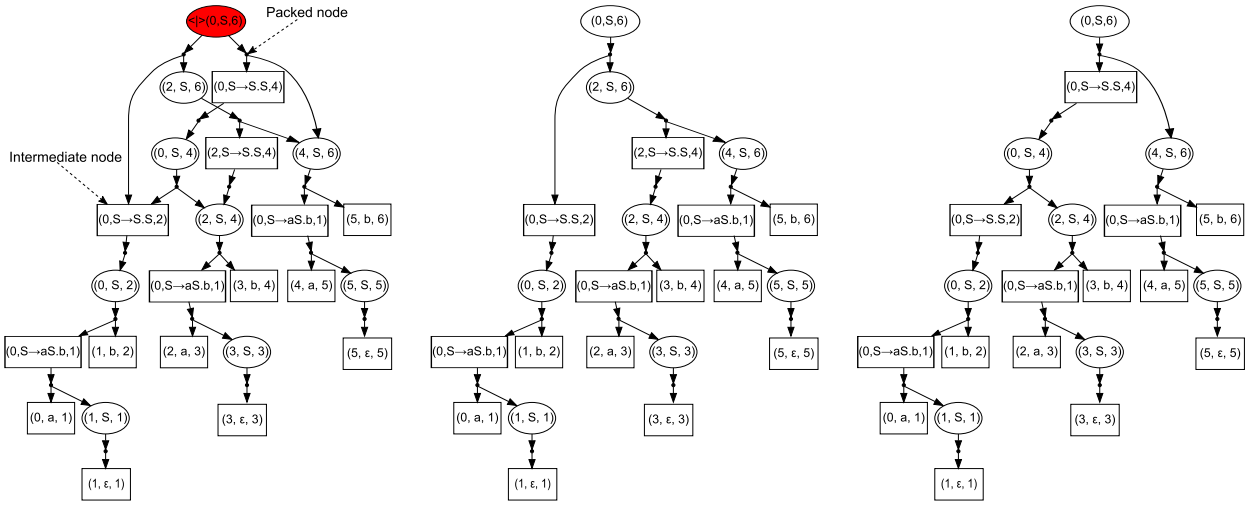
Time complexity estimation in terms of input graph and grammar size is quite similar to the estimation of GLL complexity provided in [14].

LEMMA 1. *For any descriptor (L, u, i, w) either $w = \$$ or w has extension (j, i) where u has index j.*

PROOF. Proof of this lemma is the same as provided for original GLL in [14] because main function used for descriptors creation has not been changed. □

THEOREM 1. *The GSS generated by GLL-based graph parsing algorithm for grammar G and input graph $M = (V, E, L)$ has at most $O(|V|)$ vertices and $O(|V|^2)$ edges.*

PROOF. Proof is the same as the proof of **Theorem 2** from [14] because structure of GSS has not been changed.



(a) SPPF (b) First derivation tree (c) Second derivation tree

Figure 4: SPPF for sentence `"ababab"` and grammar G_0

□

THEOREM 2. *The SPPF generated by GLL-based graph parsing algorithm on input graph $M = (V, E, L)$ has at most $O(|V|^3 + |E|)$ vertices and edges.*

PROOF. Let us estimate the number of nodes of each type.

- **Terminal nodes** are labeled with (v_0, T, v_1), and such label can only be created if there is such $e \in E$ that $e = (v_0, T, v_1)$. Note, that there are no duplicate edges. Hence there are at most $|E|$ terminal nodes.
- **ε-nodes** are labeled with (v, ε, v), hence there are at most $|V|$ of them.
- **Nonterminal nodes** have labels of form (v_0, N, v_1), so there are at most $O(|V|^2)$ of them.
- **Intermediate nodes** have labels of form (v_0, t, v_1), where t is a grammar slot, so there are at most $O(|V|^2)$ of them.
- **Packed nodes** are children either of intermediate or nonterminal nodes and have label of form $(N \to \alpha \cdot \beta, v)$. There are at most $O(|V|^2)$ parents for packed nodes and each of them can have at most $O(|V|)$ children.

As a result, there are at most $O(|V|^3+|E|)$ nodes in SPPF. The packed nodes have at most two children so there are at most $O(|V|^3 + |E|)$ edges which source is packed node. Nonterminal and intermediate nodes have at most $O(|V|)$ children and all of them are packed nodes. Thus there are at most $O(|V|^3)$ edges with source in nonterminal or intermediate nodes. As a result there are at most $O(|V|^3 + |E|)$ edges in SPPF.

□

THEOREM 3. *The worst-case space complexity of GLL-based graph parsing algorithm for graph $M = (V, E, L)$ is $O(|V|^3 + |E|)$.*

Immediately follows from theorems 1 and 2.

THEOREM 4. *The worst-case runtime complexity of GLL-based graph parsing algorithm for graph $M = (V, E, L)$ is*

$$O\left(|V|^3 * \max_{v \in V}\left(deg^+(v)\right)\right).$$

PROOF. From Lemma 1, there are at most $O(|V|^2)$ descriptors. Complexity of all functions which were used in algorithm is the same as in proof of **Theorem 4** from [14] except **Processing** function in which not a single next input token, but the whole set of outgoing edges, should be processed. Thus, for each descriptor at most

$$\max_{v \in V}\left(deg^+(v)\right)$$

edges are processed, where $deg^+(v)$ is outdegree of vertex v.

Thus, worst-case complexity of proposed algorithm is

$$O\left(V^3 * \max_{v \in V}\left(deg^+(v)\right)\right).$$

□

We can get estimations for linear input from theorem 4. For any $v \in V$, $deg^+(v) \leq 1$, thus $\max_{v \in V}(deg^+(v)) = 1$ and worst-case time complexity $O(|V|^3)$, as expected. For LL grammars and linear input complexity should be $O(|V|)$ for the same reason as for original GLL.

As discussed in [10], special data structures, which are required for the basic algorithm, can be not rational for practical implementation, and it is necessary to find balance between performance, software complexity, and hardware resources. As a result, we can get slightly worse performance than theoretical estimation in practice.

Note that result SPPF contains only paths matched specified query, so result SPPF size is $O(|V'|^3 + |E'|)$ where $M' = (V', E', L')$ is a subgraph of input graph M which contains only matched paths. Also note that each specific path can be explored by linear SPPF traversal.

4.5 Example

Let us present a solution for the problem stated in motivating example section (3): grammar G_1 is a query and we want to find all paths in graph M (presented in picture 1) which match this query. Result SPPF for this input is presented in figure 5. Note that presented version does not contains redundant nodes. Each terminal node corresponds to the edge in the input graph: for each node with label (v_0, T, v_1) there is $e \in E : e = (v_0, T, v_1)$. We duplicate terminal nodes only for figure simplification.

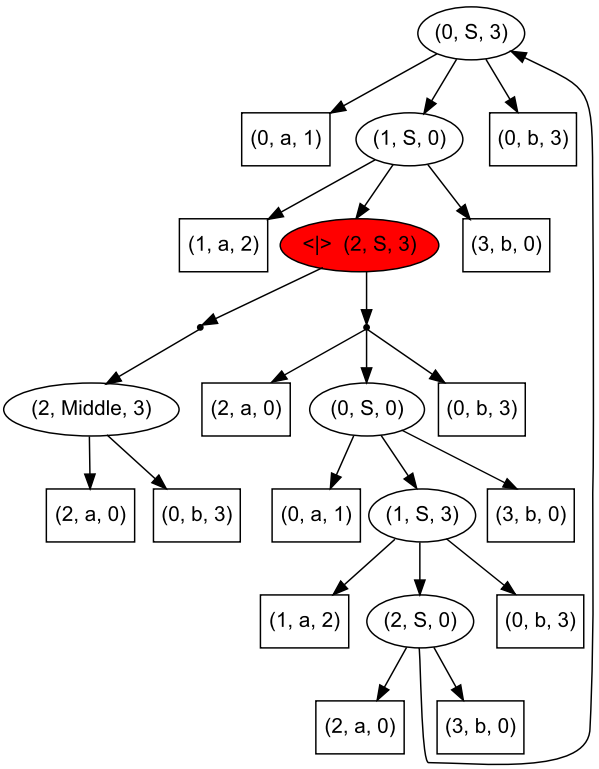


Figure 5: Result SPPF for input graph M(fig. 1) and query G_1(fig. 2)

As an example of derivation structure usage, we can find a middle of any path in example simply by finding correspondent nonterminal *Middle* in SPPF. So we can find out that there is only one (common) middle for all results, and it is a vertex with $id = 0$.

Extensions stored in nodes allow us to check whether path from u to v exists and to extract it. We need only to traverse SPPF which can be done in polinomial time (in terms of SPPF size) to extract any path .

Lets find paths p_i such that $S \underset{G_1}{\Longrightarrow}^* \Omega(p_i)$ and p_i starts from the vertex 0. To do this, we should find vertices with label $(0, S, \_)$ in SPPF. (There are two vertices with such labels: $(0, S, 0)$ and $(0, S, 3)$.) Then let us to extract corresponded paths from SPPF. There is a cycle in SPPF in our example, so there are **at least** two different paths:

$$p_0 = \{(0,a,1);(1,a,2);(2,a,0);(0,b,3);(3,b,0);(0,b,3)\}$$

and

$$p_1 = \{(0,a,1);(1,a,2);(2,a,0);(0,a,1);(1,a,2);(2,a,0);\\ (0,b,3);(3,b,0);(0,b,3);(3,b,0);(0,b,3);(3,b,0)\}.$$

We demonstrate that SPPF which was constructed by described algorithm can be useful for query result investigation. But in some cases explicit representation of matched subgraph is preferable, and required subgraph may be extracted from SPPF trivially by its traversal.

5. EVALUATION

In this section we show that performance of implemented algorithm is in good agreement with theoretical estimations, and that the worst-case of time and space complexity can be achieved.

We use two grammars for balanced brackets — ambiguous grammar G_0(fig. 3) and unambiguous grammar G_2(fig. 6) — in order to investigate performance and grammar ambiguity correlation.

$$\begin{aligned} 0: &\quad S \to a\ S\ b\ S \\ 1: &\quad S \to \varepsilon \end{aligned}$$

Figure 6: Unambiguous grammar G_2 for balanced brackets

As input we use complete graphs in which for each terminal symbol there is an edge labeled with it between every two vertices. Note that we use only terminal symbols for edges labels. The task we solve in our experiments is to find all paths from all vertices to all vertices satisfied specified query. Such designed input looks hard for querying in terms of required resources because there is a correct path between any two vertices and result set is infinite.

For complete graph $M = (V, E, L)$

$$\max_{v \in V} \left(deg^+(v)\right) = (|V| - 1) * |\Sigma|$$

, where Σ is terminals of input grammar, hence we should get time complexity $O(|V|^4)$ and space complexity $O(|V|^3)$.

All tests were run on a PC with the following characteristics:

- OS: Microsoft Windows 10 Pro
- System Type: x64-based PC
- CPU: Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz, 3601 Mhz, 4 Core(s), 4 Logical Processor(s)
- RAM: 32 GB

Performance measurement results are presented in figure 7. For time measurement results we have that all two curves can be fit with polynomial function of degree 4 to a high level of confidence with R^2.

Also we present SPPF size in terms of nodes for both G_0 and G_2 grammars (fig. 8). As was expected, all two curves are cubic to a high level of confidence with $R^2 = 1$.

6. CONCLUSION AND FUTURE WORK

We propose GLL-based algorithm for context-free path querying which constructs finite structural representation

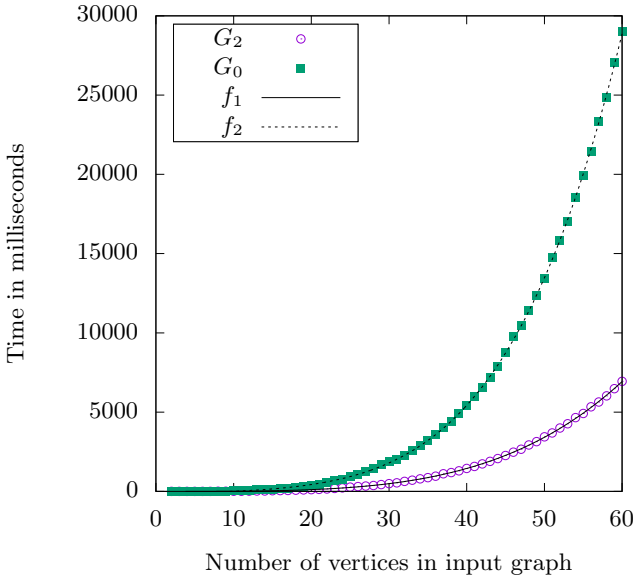


Figure 7: Performance on complete graphs for grammar G_0 and G_2
$f_1(x) = 0.000496 * x^4 + 0.001252 * x^3 + 0.068492 * x^2 - 0.306749 * x$; $R^2 = 0.99996$
$f_2(x) = 0.003369 * x^4 - 0.114919 * x^3 + 3.161793 * x^2 - 22.54949 * x$; $R^2 = 0.99995$

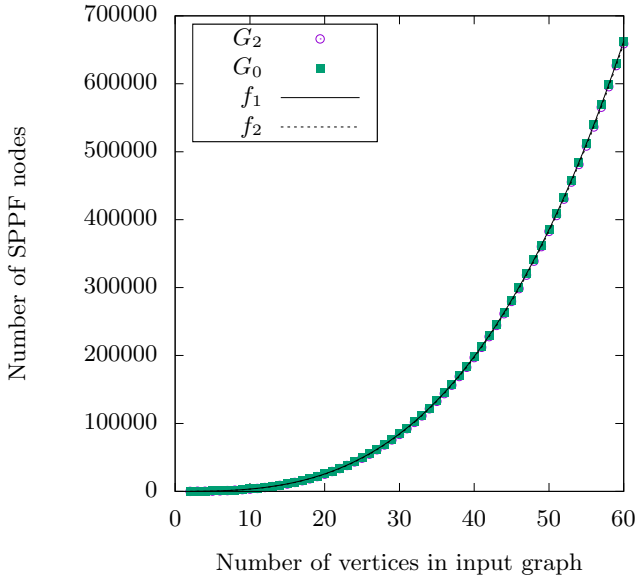


Figure 8: SPPF size on complete graph for grammar G_0 and G_2 a complete graphs
$f_1(x) = 3.000047 * x^3 + 3.994579 * x^2 + 4.191568 * x$; $R^2 = 1$
$f_2(x) = 3.000050 * x^3 + 2.994338 * x^2 + 4.196472 * x$; $R^2 = 1$

of all paths satisfying given constraint. Provided data structure can be useful for result investigation and processing, and for query debugging. Presented algorithm has been implemented in F# programming language [18] and is available on GitHub:https://github.com/YaccConstructor/YaccConstructor.

In order to estimate practical value of proposed algorithm, we should perform evaluation on a real dataset and real queries. One possible application of our algorithm is metagenomical assembly querying, and we are currently working on this topic.

We are also working on performance improvement by implementation of recently proposed modifications in original GLL algorithm [15, 1]. One direction of our research is generalization of grammar factorization proposed in [15] which may be useful for the processing of regular queries which are common in real world application.

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APPENDIX

A. GLL PSEUDOCODE

Main functions of GLL parsing algorithms:

- stack and descriptors manipulation functions 3;
- SPPF construction functions 4.
- R—working set which contains descriptors to process.
- U—all descriptors was created.
- P—popped nodes.

Algorithm 3 Stack and descriptors manipulation functions

```
1: function ADD(L, v, i, a)
2:   if (L, v, i, a) ∉ U then
3:     U.add(L, v, i, a)
4:     R.add(L, v, i, a)
5: function POP(v, i, z)
6:   if v ≠ v0 then
7:     P.add(v, z)
8:     for all (a, u) ∈ v.outEdges do
9:       y ← getNodeP(v.L, a, z)
10:      ADD(v.L, u, i, y)
11: function CREATE(L, v, i, a)
12:   if (L, i) ∉ GSS.nodes then
13:     GSS.nodes.add(L, i)
14:   u ← GSS.NODES.GET(L, i)
15:   if (u, a, v) ∉ GSS.edges then
16:     GSS.edges.add(u, a, v)
17:     for all (u, z) ∈ P do
18:       y ← getNodeP(L, a, z)
19:       (_, _, k) ← z.lbl
20:       ADD(L, v, k, y)
      return u
```

Algorithm 4 SPPF construction functions

```
1: function getNodeT(x, i)
2:   if x = ε then
3:     h ← i
4:   else
5:     h ← i + 1
6:   if (x, i, h) ∉ SPPF.nodes then
7:     SPPF.nodes.add(x, i, h)
     return SPPF.nodes.get(x, i, h)
8: function getNodeP((X → ω1 · ω2), a, z)
9:   if ω1 is terminal or non-nullable nonterminal and ω2 ≠
   ε then return z
10:  else
11:    if ω2 = ε then
12:      t ← X
13:    else
14:      h ← (→ ω1 · ω2)
15:    (q, k, i) ← z.lbl
16:    if a ≠ dummy then
17:      (s, j, k) ← a.lbl
18:      y ← findOrCreate SPPF.nodes (n.lbl = (t, i, j))
19:      if y does not have a child with label (X → ω1 · ω2)
   then
20:        y′ ← newPackedNode(a, z)
21:        y.chld.add y′
22:        return y
23:      else
24:        y ← findOrCreate SPPF.nodes (n.lbl = (t, k, i))
25:        if y does not have a child with label (X → ω1 · ω2)
   then
26:          y′ ← newPackedNode(z)
27:          y.chld.add y′
28:          return y
     return SPPF.nodes.get(x, i, h)
```