



Context-Free Path Querying via Matrix Equations

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Context-Free Path Querying (CFPQ)

- Context-free grammar $G = (N, \Sigma, R)$
 $\mathcal{L}(G_S) = \{\omega \mid S \Rightarrow_G^* \omega\}, S \in N$
- Directed graph $D = (V, E, \sigma)$, $\sigma \subseteq \Sigma$, $E \subseteq V \times \sigma \times V$
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- Both rich theoretical foundations and constantly improving modifications/implementations
- Parallel techniques and GPGPU
- Approximate computational methods to accelerate CFPQs processing

Main goal

- To propose a new way to reduce CFPQ to a problem with available high-performance solutions — to solving the systems of equations over real numbers \mathbb{R}

Reduction from Solving Boolean Matrix Equations

$$S \rightarrow aSb \mid ab$$

$$T_E \in \mathbb{M}^{|V| \times |V|} : (T_E)_{ij} = 1 \iff (i, j) \in R_E \quad \forall E \in (N \cup \Sigma)$$

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$$\Downarrow$$

$$\{T_S^k\} : \begin{aligned} T_S^0 &= \mathbf{0} \\ T_S^{k+1} &= T_a T_S^k T_b + T_a T_b \end{aligned}$$

$$T_S^\infty - \text{least solution } T_S = T_a T_S T_b + T_a T_b$$

Reduction from Solving Matrix Equations over \mathbb{R}

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$$\{\mathcal{T}_S^k\} : \begin{array}{l} \mathcal{T}_S^0 = \mathbf{0} \\ \mathcal{T}_S^{k+1} = \epsilon(T_a \mathcal{T}_S^k T_b + T_a T_b) \end{array}$$

\mathcal{T}_S^∞ – least solution $\mathcal{T}_S = \epsilon(T_a \mathcal{T}_S T_b + T_a T_b)$,
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$$(\mathcal{T}_S^{k+1})_{ij} > 0 \iff (\mathcal{T}_S^{k+1})_{ij} = 1$$

$$\text{ceil}(\mathcal{T}_S^\infty) = \mathcal{T}_S^\infty$$

Methods for Solving Equations

- Linear equations
 - ▶ Sylvester equations: $AXB + CXD = F$
 - ▶ Linear systems $Ax = b$
- Nonlinear equations
 - ▶ Newton's method

$$X = G(X) \Rightarrow F(X) = X - G(X) = \mathbf{0}$$

$$X_{i+1} = X_i - (F'(X_i))^{-1}F(X_i) \iff \begin{cases} F'(X_i)H_i = -F(X_i) \\ X_{i+1} = X_i + H_i \end{cases}$$

First implementation

- *SciPy*
 - ▶ sSLV — to solve as a sparse linear system
 - ▶ dNWT — to find a root of a function with Newton's method
- Comparative analysis of matrix-based approach and equation-based approach (in ms)

Ontology	V	dNWT	sSLV	dGPU	sCPU	sGPU
bio-meas	341	284	35	276	91	24
people-pets	337	73	49	144	38	6
funding	778	502	184	1246	344	27
wine	733	791	171	722	179	6
pizza	671	334	161	943	256	23

- Equation-based approach for CFPQ was proposed
- The feasibility of using both accurate and approximate methods of computational mathematics was assessed
- The evaluation on a set of conventional benchmarks showed that our approach comparable with the matrix-based approach and applicable for real-world data processing
- **ACM SIGMOD 2020 Student Research Competition:**
Yuliya Susanina. Context-Free Path Querying via Matrix Equations.
(accepted to participate in the second round)

- Employ high-performance solvers which utilize GPGPU and distributed computations
- Determine the subclasses of (system of) polynomial equations the solution of which can be reduced to CFPQ
- Try to construct a bidirectional reduction between CFPQ and these subclasses, thereby finding efficient solutions for both these problems