Extended Context-Free Grammars Parsing with Generalized LL

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Abstract. Parsing plays an important role in static program analysis: during this step a structural representation of code is created upon which further analysis is performed. Parser generator tools, being provided with syntax specification, automate parser development. Language documentation often acts as such specification. Documentation usually takes form of ambiguous grammar in Extended Backus-Naur Form which most parser generators fail to process. Automatic grammar transformation generally leads to parsing performance decrease. Some approaches support EBNF grammars natively, but they all fail to handle ambiguous grammars. On the other hand, Generalized LL parsing algorithm admits arbitrary context-free grammars and achieves good performance, but cannot handle EBNF grammars. The main contribution of this paper is a modification of GLL algorithm which can process grammars in a form which is closely related to EBNF (Extended Context-Free Grammar). We also show that the modification improves parsing performance as compared to grammar transformation based approach.

Keywords: Parsing, Generalized Parsing, Extended Context-Free Grammar, GLL, SPPF, EBNF, ECFG, RRPG, Recursive Automata

1   Introduction

Static program analysis is usually performed over a structural representation of code and parsing is a classical way to get such representation. Parser generators are often used to automate parser creation: these tools derives parser from grammar. It decreases amount of effort required for syntax analyzer development and maintenance.

Extended Backus-Naur Form [23] is a metasyntax for expressing context-free grammars. In addition to the Backus-Naur Form syntax it uses the following constructions: alternation |, optional symbols [...], repetition { ... }, and grouping (...).

This form is widely used for grammar specification in technical documentation because expressive power of EBNF makes syntax specification much more

compact and human-readable. Because documentation is one of the main sources of information for parsers developers, it would be helpful to have a parser generator which supports grammar in EBNF. Note, that EBNF is a standardized notation for *extended context-free grammars* [11] which can be defined as follows.

Definition 1 *An **extended context-free grammar** (ECFG) [11] is a tuple (N, Σ, P, S), where N and Σ are finite sets of nonterminals and terminals respectively, $S \in N$ is the start symbol, and P (the productions) is a map from N to regular expressions over alphabet $N \cup \Sigma$.*

ECFG is widely used as an input format for parser generators, but classical parsing algorithms often require CFG, and, as a result, parser generators usually require conversion to CFG. It is possible to transform ECFG to CFG [10], but this transformation leads to grammar size increase and change in grammar structure: new nonterminals are added during transformation. As a result, parser constructs derivation tree with respect to the transformed grammar, making it harder for a language developer to debug grammar and use parsing result later.

There is a wide range of parsing techniques and algorithms [3, 4, 6, 9–11, 13, 14] which are able to process grammar in ECFG. Detailed review of results and problems in ECFG processing area is provided in the paper "Towards a Taxonomy for ECFG and RRPG Parsing" [11]. We only note that most of algorithms are based on classical LL [9, 3, 5] and LR [14, 13, 4] techniques, and they can handle only appropriate subclasses of ECFG. As a result, parsing techniques for ECFG is an actual problem, and there is no solution for handling arbitrary (including ambiguous) ECFGs.

The LL-based parsing algorithms are more intuitive than LR-based and can provide better error diagnostic. Currently LL(1) seems is be the most practical algorithm. Unfortunately, some languages are not LL(k) for any k , and left recursive grammars are a problem for LL-based tools. Another restriction for LL parsers is ambiguities in grammar which, being combined with previous flaws, complicates industrial parsers creation. Generalized LL, proposed in [17],solves all these problems: it handles arbitrary CFGs, including ambiguous and left recursive. Worst-case time and space complexity of GLL is cubic in terms of input size and, for LL(1) grammars, it demonstrates linear time and space complexity.

In order to improve performance of GLL algorithm, modification for left factorized grammars processing was introduced in [19]. Factorization transforms grammar so that there are no two productions with same prefixes (see fig 1 for example). It is shown, that factorization can reduce memory usage and increase performance by reusing common parts of rules for one nonterminal. Similar idea can be applied to ECFGs processing (module some details).

To summarize, if it were possible to handle ECFG specification with tools based on generalized parsing algorithm, it would greatly simplify language development. In this work we present a modification of generalized LL parsing algorithm which handles arbitrary ECFGs without any transformations. Also we demonstrate that proposed modifications improve parsing performance and memory usage.

2   ECFG Handling with Generalized LL Algorithm

The purpose of generalized parsing algorithms is to provide arbitrary context-free grammars handling. Generalized LL algorithm (GLL) [17] inherits properties of classical LL algorithms: it is more intuitive and provides better syntax error diagnostic than generalized LR algorithms. Also, our experience shows that GLR-based solutions are more complex rather then GLL-based, which agrees with the observation in [11] that LR-based ECFG parsers are very complex. Thus, we choose GLL as a base for our solution. In this section we present GLL-style parser for arbitrary ECFG processing.

2.1   Generalized LL Parsing Algorithm

An idea of the GLL algorithm is based on descriptors which can uniquely define state of parsing process. Descriptor is a four-element tuple (L, i, T, S) where elements are defined as follows:

- L is a grammar slot — pointer to position in grammar of the form $(S \rightarrow \alpha \cdot \beta)$;
- i — position in the input;
- T — already built node of parse forest;
- S — current Graph Structured Stack (GSS) [1] node.

A GLL move through grammar and input simultaneously, creating multiple descriptors for the case of ambiguity, and using queue for descriptors processing control. In the initial state there is only one descriptor which consists of start positions in grammar $(L = (S \rightarrow \cdot \beta))$ and input $(i = 0)$, dummy tree node and the bottom of GSS. At each step, the algorithm dequeues the next descriptor and acts depending on the grammar and input. If there is an ambiguity, then algorithm queues descriptors for all cases to handle them later. To achieve cubic time complexity it is important that descriptor can be enqueued only if it has not been created before. To check it global storage of all created descriptors is used.

There is a table based approach [15] which generates only tables for given grammar instead of full parser code. The idea is similar to the one in the original paper and uses the same tree construction and stack processing routines. Pseudocode illustrating this approach can be found in appendix A. Note, that we do not include check for first/follow sets in this paper.

2.2   Grammar Factorization

In order to improve performance of GLL Elizabeth Scott and Adrian Johnstone offered a support for left-factorized grammars in this parsing algorithm [19].

From basic GLL algorithm description you can see, that it is possible to decrease parse time and the amount of required memory by reducing descriptors to process. One of the way to do it is to reduce the number of grammar slots, and it can be achieved by grammar factorization. An example of factorization

is provided in fig. 1: production P_0 transforms to P_0' during factorization. This example is discussed in the paper [19], and it is shown, that such transformation can give significant performance improvement on some grammars, because factorized grammar produces less slots.

$$S ::= a\ a\ B\ c\ d \mid a\ a\ c\ d \mid a\ a\ c\ e \mid a\ a \qquad S ::= a\ a\ (B\ c\ d \mid c\ (d \mid e) \mid \varepsilon\ )$$

       (a) The original grammar P_0 (b) The factorized grammar P_0'

Fig. 1. Example of grammar factorization

We can evolve this idea to support ECFG, and in the next section we will show how to do it.

2.3   Recursive Automata and ECFGs

In order to ease adoption of ideas of grammar factorization for handling ECFGs with GLL we use recursive automaton (RA) [21] for ECFG representation. We use the following definition of RA.

Definition 2 *Recursive automaton (RA) R is a tuple $(\Sigma, Q, S, F, \delta)$, where Σ is a set of terminals, Q — set of states, $S \in Q$ — start state, $F \subseteq Q$ — set of final states, $\delta : Q \times (\Sigma \cup Q) \to Q$ — transition function.*

The only difference between Recursive Automaton and Finite State Automaton (FSA) is that transitions in RA are labeled either by terminal (Σ) or by state (Q). Further in this paper, we call transitions by elements from Q as *nonterminal transitions* and by terminal as *terminal transitions*.

Note that grammar factorization leads to partial minimization of automata in right-hand side of productions. Also we can note that grammar slots are equivalent to states of automata which built from right-hand side of productions. Right parts of ECFG productions are regular expressions over the union alphabet of terminals and nonterminals. So, our goal is to build RA with minimal number of states for given ECFG, which can be done by following steps.

1. For each right-hand side of productions build a FSA using Thompson's method [22].
2. Create a map M from nonterminal to corresponded start state. This map should kept consistent during all follows steps.
3. Convert FSAs from previous step to deterministic FSAs without ε-transitions using the algorithm described in [2].
4. Minimize DFSAs by using John Hopcroft's algorithm [12].
5. Replace transitions by nonterminals with transitions labeled by start states by using map M. Result of this step is an required RA. We also use map M to define function $\Delta : Q \to N$ where N is nonterminal name.

An example of ECFG to RA transformation is presented in fig. 2, where state 0 is start state of result RA.

$S ::= a^+ S \ b? \mid c$

(a) Grammar G_0 (b) FSA for G_0 (c) RA R_0 for G_0
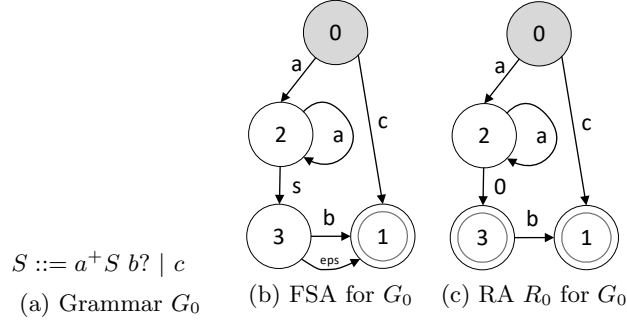
Fig. 2. Grammar to RA transformation

2.4   Input Processing

In this section we describe changes required in control functions of basic GLL algorithm to handle ECFG. At each step the main function **parse** dequeues next descriptor to be processed. Suppose that current descriptor is a tuple (C_S, C_U, C_i, C_N), where C_S — state of RA, C_U — GSS node, C_i — position in input string ω, and C_N — SPPF node. It is possible to get the following nonexclusive cases during this descriptor processing.

– C_S is a final state. Pop, because processing of nonterminal finished.
– There is a terminal transition $C_S \xrightarrow{\omega.[C_i]} q$. Create descriptor with state q and position $(C_i + 1)$. Queue it without checking its existence.
– There are nonterminal transitions. Call **create** function for each such transition. It performs necessary operations with GSS and checks if there are already built SPPF for current input position and nonterminal.

All required functions presented below. Function **add** queues descriptor if it has not already been created, and this function has not been changed.

> **function** CREATE($S_{call}, S_{next}, u, i, w$)
> $A \leftarrow \Delta(S_{call})$
>   **if** ($\exists$ GSS node labeled (A, i)) **then**
> $v \leftarrow$ GSS node labeled (A, i)
>   **if** (there is no GSS edge from v to u labeled (S_{next}, w)) **then**
>   add a GSS edge from v to u labeled (S_{next}, w)
>   **for** $((v, z) \in \mathcal{P})$ **do**
> $(y, N) \leftarrow$ **getNodes**$(S_{next}, u.nonterm, w, z)$
>   **if** $N \neq \$$ **then**
> $(\_, \_, h) \leftarrow N$
>   **pop**(u, h, N)
> $(\_, \_, h) \leftarrow y$
>   **add**(S_{next}, u, h, y)
>   **else**

$v \leftarrow$ **new** GSS node labeled (A, i)
create a GSS edge from v to u labeled (S_{next}, w)
add$(S_{call}, v, i, \$)$
 return v
function POP(u, i, z)
 if $((u, z) \notin \mathcal{P})$ **then**
  $\mathcal{P}.add(u, z)$
  for all GSS edges (u, S, w, v) **do**
   $(y, N) \leftarrow$ **getNodes**$(S, v.nonterm, w, z)$
   if $N \neq \$$ **then**
    pop(v, i, N)
   if $y \neq \$$ **then**
    add(S, v, i, y)

function PARSE
 $R.add(StartState, newGSSnode(StartNonterminal, 0), 0, \$)$
 while $R \neq \varnothing$ **do**
  $(C_S, C_U, C_i, C_N) \leftarrow R.Get()$
  $C_R \leftarrow \$$
  if $(C_N = \$)\&(C_S$ is final state$)$ **then**
   $eps \leftarrow$ **getNodeT**(ε, C_i)
   $(\_, N) \leftarrow$ **getNodes**$(C_S, C_U.nonterm, \$, eps)$
   pop(C_U, C_i, N)
  for each $transition(C_S, label, S_{next})$ **do**
   switch $label$ **do**
    case $Terminal(x)$ where $(x = input[i])$
     $R \leftarrow$ **getNodeT**(x, C_i)
     $(y, N) \leftarrow$ **getNodes**$(S_{next}, C_U.nonterm, C_N, R)$
     if $N \neq \$$ **then**
      pop$(C_U, i + 1, N)$
     $R.add(S_{next}, C_U, i + 1, y)$
    case $Nonterminal(S_{call})$
     create$(S_{call}, S_{next}, C_U, C_i, C_N)$
 if exists SPPF node $(StartNonterminal, 0, input.length)$ **then**
  return this node
 else
  report failure

2.5   Parse Forest Construction

Result of the parsing process is structural representation of input — a derivation tree, or parse forest for the case of many derivation variants.

First, we should define derivation tree for recursive automaton: it is an ordered tree whose root is labeled with start state, leaf nodes are labeled with terminals or ε and interior nodes are labeled with nonterminals N and have a

sequence of children that corresponds to transition labels of path in automaton that starts from the state q_i, where $\Delta(q_i) = N$.

Definition 3 *Derivation tree of sentence α for the recursive automaton $R = (\Sigma, Q, S, F, \delta)$:*

- *Ordered rooted tree. Root is labeled with $\Delta(S)$*
- *Leaves are terminals $a \in \Sigma$*
- *Nodes are nonterminals $A \in \Delta(Q)$*
- *Node with label $N_i \in \Delta(q_i)$ has children $l_0 \ldots l_n (l_i \in \Sigma \cup \Delta(Q))$ iff there exists a path $q_i \xrightarrow{l_0} q_{i+1} \xrightarrow{l_1} \ldots \xrightarrow{l_n} q_m,\ q_m \in F$.*

For arbitrary grammars RA can be ambiguous in terms of acceptance paths, and, as a result, it is possible to get more than one derivation tree for input string. Shared Packed Parse Forest (SPPF) [16] can be used for compact representation of all possible derivation trees. We use the binarized version of SPPF, which proposed in [20] in order to decrease memory usage. Binarized SPPF can be used in GLL [18] and contains the following types of nodes (here i and j are start and end of derived substring in terms of positions in input string):

- Packed nodes are of the form (S, k), where S is a state of automaton, k — start of derived substring of right child. Packed node necessarily has right child — symbol node, and optional left child — symbol or intermediate node.
- Symbol nodes have labels (X, i, j) where $X \in \Sigma \cup \Delta(Q) \cup \{\varepsilon\}$. Terminal symbol nodes ($X \in \Sigma \cup \{\varepsilon\}$) are leaves. Nonterminal nodes ($X \in \Delta(Q)$) may have several packed children.
- Intermediate nodes have labels (S, i, j), where S is a state of automaton, and may have several packed children.

Let us describe modifications of original SPPF construction functions. The function **getNodeT**(x, i) which creates terminal nodes is reused without modification from basic algorithm. To handle nondeterminism in states we defined function **getNodes** which checks if the next state of RA is final and for that case constructs nonterminal nodes in addition to intermediate. It uses modified function **getNodeP**, instead of grammar slot it takes separately a state of RA and symbol for new SPPF node: current nonterminal or next RA state.

    function GETNODES(S, A, w, z)
        if (S is final state) **then**
            $x \leftarrow$ **getNodeP**(S, A, w, z)
        else
            $x \leftarrow \$$
        if ($w = \$$)& not (z is nonterminal node and it's extents are equal) **then**
            $y \leftarrow z$
        else
            $y \leftarrow$ **getNodeP**(S, S, w, z)
        return (y, x)

function GETNODEP(S, L, w, z)
  $(\_, k, i) \leftarrow z$
  if $(w \neq \$)$ **then**
    $(\_, j, k) \leftarrow w$
    $y \leftarrow$ find or create SPPF node labelled (L, j, i)
    if $(\nexists$ child of y labelled $(S, k))$ **then**
      $y\prime \leftarrow$ **new** $packedNode(S, k)$
      $y\prime.addLeftChild(w)$
      $y\prime.addRightChild(z)$
      $y.addChild(y\prime)$
  else
    $y \leftarrow$ find or create SPPF node labelled (L, k, i)
    if $(\nexists$ child of y labelled $(S, k))$ **then**
      $y\prime \leftarrow$ **new** $packedNode(S, k)$
      $y\prime.addRightChild(z)$
      $y.addChild(y\prime)$
  return y

3 Evaluation

We have implemented parser generator for proposed algorithm, and in this section we show SPPF example and performance comparison with the parser built on factorized grammar.

3.1 SPPF example

For the SPPF example we have taken ECFG grammar G_0(fig. 2a). It contains constructions(option and repetition) that should be converted with use of extra nonterminals to build regular GLL parser. Our generator constructs recursive automaton R_0(fig. 2c) and parser for it. Possible trees for input *aacb* are shown in fig. 3a. SPPF build by parser(fig. 3b) combines all of them.
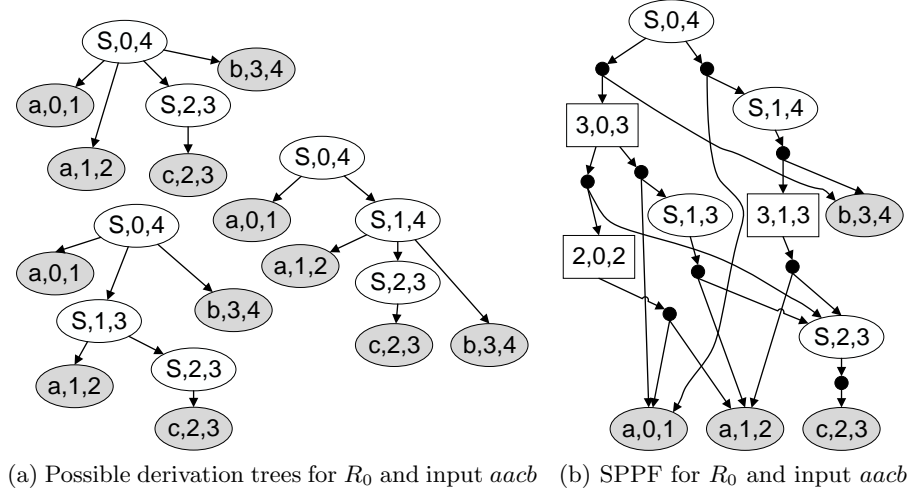
3.2 Performance measure

We have compared our parsers built on factorized grammar and on minimized recursive automaton. Grammar G_1(fig. 4a) was used for the tests, it has long tails in alternatives which are not unified with factorization. FSA built for this grammar presented in fig. 4b.

  For this grammar parser for RA should create less GSS edges because the tails of alternatives in producions are represented by the only path in RA. This fact leads to decrease of SPPF nodes and descriptors.

  Experiments were performed on inputs of different length and are presented in fig. 5. Exact values for the input a^{40} shown in table 1.

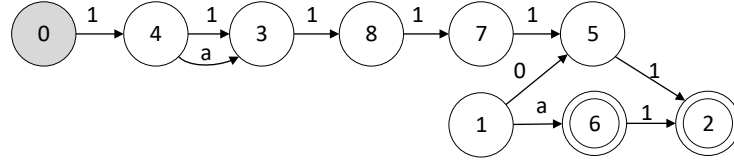 All tests were run on a PC with the following characteristics:

(a) Possible derivation trees for R_0 and input *aacb*   (b) SPPF for R_0 and input *aacb*

Fig. 3. Example for input *aacb*

$$S ::= K \ (K \ K \ K \ K \ K \mid a \ K \ K \ K \ K)$$
$$K ::= S \ K \mid a \ K \mid a$$

(a) Grammar G_1

(b) RA for grammar G_1

Fig. 4. Grammar G_1 and RA for it

- OS: Microsoft Windows 10 Pro x64
- CPU: Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz, 3601 Mhz, 4 Cores, 4
  Logical Processors
- RAM: 32 GB

Results of performed experiments approve the fact that on some grammars or approach show better results then parsers built on factorized grammars. With grammar G_1 in general minimized RA version works 33% faster, uses 27% less descriptors, 29% less GSS edges and 33% less SPPF nodes.

4   Conclusion and Future Work

Described algorithm and parser generator based on it implemented in F# as part of the YaccConstructor project. Source code available here: `https://github.com/YaccConstructor/YaccConstructor`.

	Time, s	Descriptors	GSS Edges	GSS Nodes	SPPF Nodes
Factorized grammar	81.814	7940	6974	80	111127244
Minimized RA	54.637	5830	4234	80	74292078

Table 1. Experiments results for input a^{40}

As we show in evaluation, proposed modification not only increase performance, but also decrease memory usage. It is critical for big input processing. For example, Anastasia Ragozina in her master's thesis [15] shows that GLL can be used for graph parsing. In some areas graphs can be really huge, for example metagenomic assemblies in bioinfomatics, and social graphs. We hope that proposed modification can improve performance not only in case of classical parsing, but in graph parsing too. We perform some tests that demonstrate performance increasing in metagenomic analysis, but full integration with graph parsing and formal algorithm description is required.

One of way to specify any useful manipulations on derivation tree (or semantic of language) is an attributed grammars, but it is not supported in the algorithm which presented in this article. There is number of works on subclasses of attributed ECFGs (for example [3]), however still no solution for arbitrary ECFGs. Thus, arbitrary attributed ECFGs and semantic calculation support is a future work.

Yet another question is possibility of unification our results with tree languages: our definition of derivation tree for ECFG is quite similar to unranked tree and SPPF is similar to automata for unranked trees [8]. Theory of tree languages seems more mature than theory of general SPPF manipulations and relations between tree languages and SPPF investigation may get interesting results. For example, relations between tree languages and ECFG discussed in the paper [7], but mainly in context of XML manipulation.

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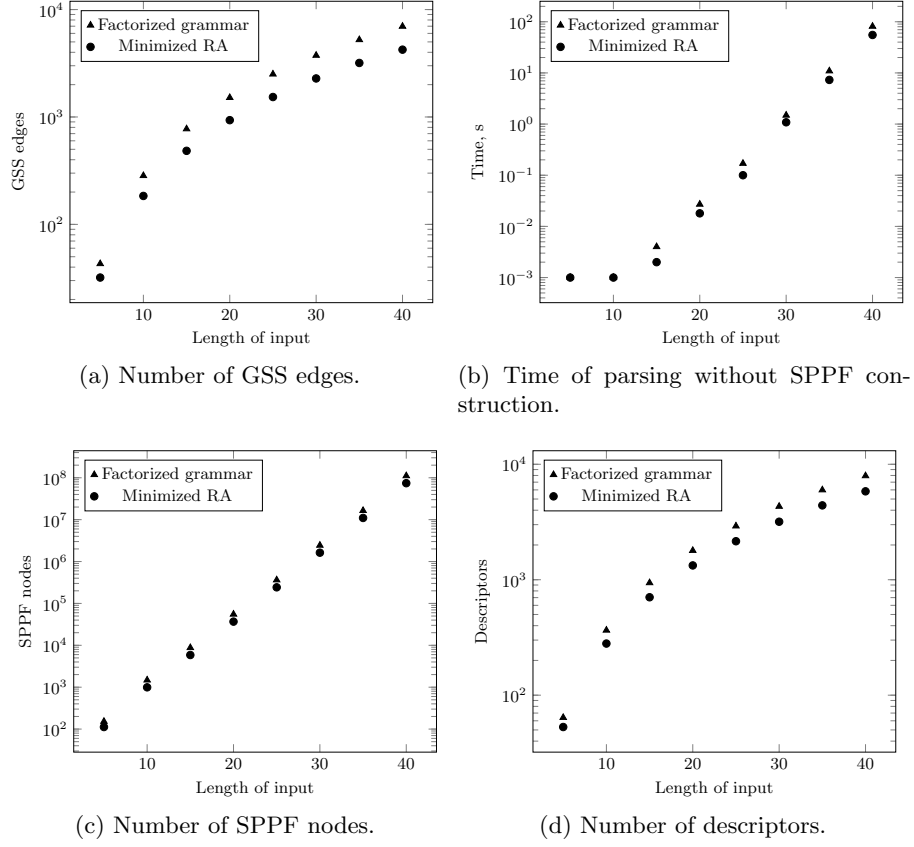
(a) Number of GSS edges.

(b) Time of parsing without SPPF construction.

(c) Number of SPPF nodes.

(d) Number of descriptors.

Fig. 5. Experiments results.

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A   GLL pseudocode

function ADD(L, u, i, w)
    if $(L, u, i, w) \notin U$ **then**
        $U.add(L, u, i, w)$
        $R.add(L, u, i, w)$

function CREATE(L, u, i, w)
    $(X ::= \alpha A \cdot \beta) \leftarrow L$
    if ($\exists$ GSS node labeled (A, i)) **then**
        $v \leftarrow$ GSS node labeled (A, i)
        if (there is no GSS edge from v to u labeled (L, w)) **then**
            add a GSS edge from v to u labeled (L, w)
            for $((v, z) \in \mathcal{P})$ **do**
                $y \leftarrow$ **getNodeP**(L, w, z)
                add(L, u, h, y) where h is the right extent of y
    else
        $v \leftarrow$ **new** GSS node labeled (A, i)
        create a GSS edge from v to u labeled (L, w)
        for each alternative α_k **of** A **do**
            add$(\alpha_k, v, i, \$)$
    return v

function POP(u, i, z)
    if $((u, z) \notin \mathcal{P})$ **then**
        $\mathcal{P}.add(u, z)$
        for all GSS edges (u, L, w, v) **do**
            $y \leftarrow$ **getNodeP**(L, w, z)
            add(L, v, i, y)

function GETNODET(x, i)
    if $(x = \varepsilon)$ **then**
        $h \leftarrow i$
    else
        $h \leftarrow i + 1$
    $y \leftarrow$ find or create SPPF node labelled (x, i, h)
     return y

function GETNODEP$(X ::= \alpha \cdot \beta, w, z)$
    if $(\alpha$ is a terminal or a non-nullable nontermial$)$ & $(\beta \neq \varepsilon)$ **then**
        return z
    else
        if $(\beta = \varepsilon)$ **then**
            $L \leftarrow X$
        else
            $L \leftarrow (X ::= \alpha \cdot \beta)$
        $(\_, k, i) \leftarrow z$
        if $(w \neq \$)$ **then**
            $(\_, j, k) \leftarrow w$
            $y \leftarrow$ find or create SPPF node labelled (L, j, i)
            if $(\nexists$ child of y labelled $(X ::= \alpha \cdot \beta, k))$ **then**
                $y\prime \leftarrow$ **new** $packedNode(X ::= \alpha \cdot \beta, k)$
                $y\prime.addLeftChild(w)$
                $y\prime.addRightChild(z)$
                $y.addChild(y\prime)$
        else
            $y \leftarrow$ find or create SPPF node labelled (L, k, i)
            if $(\nexists$ child of y labelled $(X ::= \alpha \cdot \beta, k))$ **then**
                $y\prime \leftarrow$ **new** $packedNode(X ::= \alpha \cdot \beta, k)$
                $y\prime.addRightChild(z)$
                $y.addChild(y\prime)$
        return y

function DISPATCHER
    if $R \neq \varnothing$ **then**
        $(C_L, C_u, C_i, C_N) \leftarrow R.Get()$
        $C_R \leftarrow \$$
        $dispatch \leftarrow false$
    else
        $stop \leftarrow true$

function PROCESSING
    $dispatch \leftarrow true$
    switch C_L **do**
        case $(X \rightarrow \alpha \cdot x\beta)$ where $(x = input[C_i] \parallel x = \varepsilon)$
            $C_R \leftarrow$ **getNodeT**(x, C_i)
            if $x \neq \varepsilon$ **then**

$$C_i \leftarrow C_i + 1$$
$$C_L \leftarrow (X \rightarrow \alpha x \cdot \beta)$$
$$C_N \leftarrow \textbf{getNodeP}(C_L, C_N, C_R)$$
$$dispatch \leftarrow false$$

    case $(X \rightarrow \alpha \cdot A\beta)$ where A is nonterminal
       create$((X \rightarrow \alpha A \cdot \beta), C_u, C_i, C_N)$

    case $(X \rightarrow \alpha \cdot)$
       pop(C_u, C_i, C_N)

function PARSE
    while not *stop* **do**
       if *dispatch* **then**
          dispatcher()
       else
          processing()
    if exists SPPF node (S,0,n), where S - start nonterminal, n - length of input **then**
       return this node
    else
       report failure