

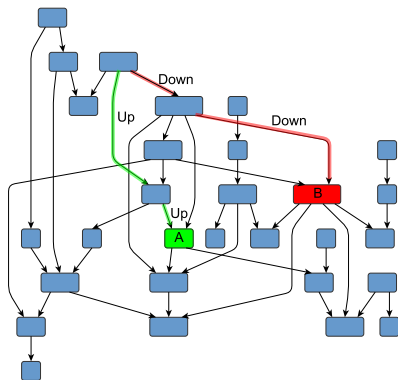
# Evaluation of the Context-Free Path Querying Algorithm Based on Matrix Multiplication

Nikita Mishin, Iaroslav Sokolov, Egor Spirin, Vladimir Kutuev, Egor  
Nemchinov, Sergey Gorbatyuk, **Semyon Grigorev**

JetBrains Research, Programming Languages and Tools Lab  
Saint Petersburg University

June 30, 2019

# Context-Free Path Querying



## Navigation through a graph

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form  $Up^n Down^n$ ?
- Find all paths of form  $Up^n Down^n$  which start from the node A

# Context-Free Path Querying: Relational Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$  — context-free grammar in normal form
  - ▶  $A \rightarrow BC$ , where  $A, B, C \in N$
  - ▶  $A \rightarrow x$ , where  $A \in N, x \in \Sigma$
  - ▶  $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^* \omega\}$

# Context-Free Path Querying: Relational Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$  — context-free grammar in normal form
  - ▶  $A \rightarrow BC$ , where  $A, B, C \in N$
  - ▶  $A \rightarrow x$ , where  $A \in N, x \in \Sigma$
  - ▶  $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^* \omega\}$
- $G = (V, E, L)$  — directed graph
  - ▶  $v \xrightarrow{I} u \in E$
  - ▶  $L \subseteq \Sigma$

# Context-Free Path Querying: Relational Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$  — context-free grammar in normal form
  - ▶  $A \rightarrow BC$ , where  $A, B, C \in N$
  - ▶  $A \rightarrow x$ , where  $A \in N, x \in \Sigma$
  - ▶  $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^* \omega\}$
- $G = (V, E, L)$  — directed graph
  - ▶  $v \xrightarrow{l} u \in E$
  - ▶  $L \subseteq \Sigma$
- $\omega(\pi) = \omega(v_0 \xrightarrow{l_0} v_1 \xrightarrow{l_1} \dots \xrightarrow{l_{n-2}} v_{n-1} \xrightarrow{l_{n-1}} v_n) = l_0 l_1 \dots l_{n-1}$

# Context-Free Path Querying: Relational Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$  — context-free grammar in normal form
  - ▶  $A \rightarrow BC$ , where  $A, B, C \in N$
  - ▶  $A \rightarrow x$ , where  $A \in N, x \in \Sigma$
  - ▶  $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^* \omega\}$
- $G = (V, E, L)$  — directed graph
  - ▶  $v \xrightarrow{l} u \in E$
  - ▶  $L \subseteq \Sigma$
- $\omega(\pi) = \omega(v_0 \xrightarrow{l_0} v_1 \xrightarrow{l_1} \dots \xrightarrow{l_{n-2}} v_{n-1} \xrightarrow{l_{n-1}} v_n) = l_0 l_1 \dots l_{n-1}$
- $R_A = \{(n, m) \mid \exists n\pi m, \text{ such that } \omega(\pi) \in L(\mathbb{G}, A)\}$

# Matrix-Based Algorithm

---

## Algorithm Context-Free Path Querying by Matrix Multiplication

---

```
1: function CONTEXTFREEPATHQUERYING( $D, G$ )
2:    $n \leftarrow$  the number of nodes in  $D$ 
3:    $E \leftarrow$  the directed edge-relation from  $D$ 
4:    $P \leftarrow$  the set of production rules in  $G$ 
5:    $T \leftarrow$  the matrix  $n \times n$  in which each element is  $\emptyset$ 
6:   for all  $(i, x, j) \in E$  do ▷ Matrix initialization
7:      $T_{i,j} \leftarrow T_{i,j} \cup \{A \mid (A \rightarrow x) \in P\}$ 
8:   while matrix  $T$  is changing do
9:      $T \leftarrow T \cup (T \times T)$  ▷ Transitive closure  $T^{cf}$  calculation
10:  return  $T$ 
```

---

# Matrix-Based Algorithm: Technical Details

- $T$  can be represented as set of Boolean matrices: one matrix per nonterminal
- The algorithm can be implemented in terms of Boolean matrices multiplication
- All matrices can be statically allocated in memory



# Research Questions

- Does GPGPUs utilization for CFPQ improve performance in comparison with the CPU version?

# Research Questions

- Does GPGPUs utilization for CFPQ improve performance in comparison with the CPU version?
- Is it possible to achieve higher performance by using existing libraries for operations over matrices or do we need to create our own specialized solution?

# Research Questions

- Does GPGPUs utilization for CFPQ improve performance in comparison with the CPU version?
- Is it possible to achieve higher performance by using existing libraries for operations over matrices or do we need to create our own specialized solution?
- Can we achieve high performance with high-level languages?

# Research Questions

- Does GPGPUs utilization for CFPQ improve performance in comparison with the CPU version?
- Is it possible to achieve higher performance by using existing libraries for operations over matrices or do we need to create our own specialized solution?
- Can we achieve high performance with high-level languages?
- Can we improve performance with sparse matrix representation?

[Scipy] Sparse matrices multiplication by using **Scipy** in **Python**

# CPU-Based Implementations

- [Scipy] Sparse matrices multiplication by using **Scipy** in **Python**
- [M4RI] Dense matrices multiplication by using **m4ri** library which implements the Method of Four Russians in **C**

**[GPU4R]** Our own implementation of the Method of Four Russians  
in **CUDA C**

**[GPU4R]** Our own implementation of the Method of Four Russians  
in **CUDA C**

**[GPU\_N]** Our own implementation of the naïve boolean matrix  
multiplication in **CUDA C**



# GPGPU-Based Implementations

**[GPU4R]** Our own implementation of the Method of Four Russians in **CUDA C**

**[GPU\_N]** Our own implementation of the naïve boolean matrix multiplication in **CUDA C**

**[GPU\_Py]** Our own implementation of naïve boolean matrix multiplication in **Python** by using **numba** compiler

- [CuSprs]
- ▶ Rustam Azimov, 2018, “Context-free Path Querying by Matrix Multiplication”
  - ▶ Implementation is based on NVIDIA cuSPARSE library (**CUDA C, GPGPU**)

# Reference Implementations

## [CuSprs]

- ▶ Rustam Azimov, 2018, “Context-free Path Querying by Matrix Multiplication”
- ▶ Implementation is based on NVIDIA cuSPARSE library (**CUDA C, GPGPU**)

## [CYK]

- ▶ X. Zhang et al, 2016, “Context-free path queries on RDF graphs”
- ▶ CYK-based algorithm implemented in **Java** (CPU)

## [RDF]

- ▶ The set of the real-world RDF files (ontologies)

- ▶ Queries:

$$G_4 : s \rightarrow SCOR\ s\ SCO \mid TR\ s\ T \mid SCOR\ SCO \mid TR\ T$$
$$G_5 : s \rightarrow SCOR\ s\ SCO \mid SCO$$

# Dataset

## [RDF]

- ▶ The set of the real-world RDF files (ontologies)

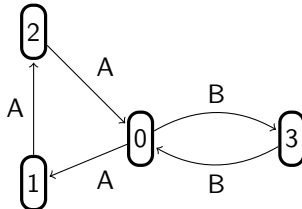
- ▶ Queries:

$G_4 : s \rightarrow SCOR\ s\ SCO \mid TR\ s\ T \mid SCOR\ SCO \mid TR\ T$

$G_5 : s \rightarrow SCOR\ s\ SCO \mid SCO$

## [Worst]

- ▶ The input graph is two cycles of coprime lengths with one shared vertex



- ▶ Query:  $G_1 : s \rightarrow A\ s\ B \mid A\ B$

- [Full]
- ▶ The input graph is sparse, but the result is a full graph
  - ▶ Queries:  
 $G_2 : s \rightarrow s \ s \mid A$   
 $G_3 : s \rightarrow s \ s \ s \mid A$

## [Full]

- ▶ The input graph is sparse, but the result is a full graph

- ▶ Queries:

$$G_2 : s \rightarrow s \ s \mid A$$

$$G_3 : s \rightarrow s \ s \ s \mid A$$

## [Sparse]

- ▶ Sparse graphs are generated by GTgraph

- ▶ Query:  $G_1 : s \rightarrow A \ s \ B \mid A \ B$

- OS: Ubuntu 18.04
- CPU: Intel core i7 8700k 3,7GHz
- RAM: DDR4 32 Gb
- GPGPU: NVIDIA GeForce 1080Ti (11Gb RAM)



# Evaluation: [RDF]<sup>2</sup>

RDF			Query $G_4$						
Name	#V	#E	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs	CYK <sup>1</sup>
atm-prim	291	685	3	2	2	1	5	269	515285
biomed	341	711	3	5	2	1	5	283	420604
foaf	256	815	2	9	2	< 1	5	270	5027
funding	778	1480	4	7	4	1	5	279	499
generations	129	351	3	3	2	< 1	5	273	6091
people_pets	337	834	3	3	3	1	7	284	82081
pizza	671	2604	6	8	3	1	6	292	3233587
skos	144	323	2	4	2	< 1	5	273	1044
travel	131	397	3	5	2	< 1	6	268	13971
unv-bnch	179	413	2	4	2	< 1	5	266	20981
wine	733	2450	7	6	4	1	7	294	4075319

<sup>1</sup>Results from X. Zhang et al, 2016, "Context-Free Path Queries on RDF Graphs"

<sup>2</sup>Time in milliseconds

## Evaluation: [Worst]<sup>3</sup>

#V	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
16	0.032	< 0.001	0.008	0.002	0.027	0.309
32	0.118	0.001	0.034	0.008	0.136	0.441
64	0.476	0.041	0.133	0.032	0.524	0.988
128	2.194	0.226	0.562	0.129	2.751	3.470
256	15.299	1.994	3.088	0.544	11.883	15.317
512	121.287	23.204	13.685	2.499	43.563	102.269
1024	1593.284	528.521	88.064	19.357	217.326	1122.055
2048	-	-	-	325.174	-	-

---

<sup>3</sup>Time in seconds

## Evaluation: [Worst]<sup>3</sup>

#V	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
16	0.032	< 0.001	0.008	0.002	0.027	0.309
32	0.118	0.001	0.034	0.008	0.136	0.441
64	0.476	0.041	0.133	0.032	0.524	0.988
128	2.194	0.226	0.562	0.129	2.751	3.470
256	15.299	1.994	3.088	0.544	11.883	15.317
512	121.287	23.204	13.685	2.499	43.563	102.269
1024	1593.284	528.521	88.064	19.357	217.326	1122.055
2048	-	-	-	325.174	-	-

<sup>3</sup>Time in seconds

## Evaluation: [Sparse]<sup>4</sup>

Graph	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
G5k-0.001	10.352	0.647	0.113	0.041	0.216	5.729
G10k-0.001	37.286	2.395	0.435	0.215	1.331	35.937
G10k-0.01	97.607	1.455	0.273	0.138	0.763	47.525
G10k-0.1	601.182	1.050	0.223	0.114	0.859	395.393
G20k-0.001	150.774	11.025	1.842	1.274	6.180	-
G40k-0.001	-	97.841	11.663	8.393	37.821	-
G80k-0.001	-	1142.959	88.366	65.886	-	-

---

<sup>4</sup>Time in seconds

## Evaluation: [Sparse]<sup>4</sup>

Graph	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
G5k-0.001	10.352	0.647	0.113	0.041	0.216	5.729
G10k-0.001	37.286	2.395	0.435	0.215	1.331	35.937
G10k-0.01	97.607	1.455	0.273	0.138	0.763	47.525
G10k-0.1	601.182	1.050	0.223	0.114	0.859	395.393
G20k-0.001	150.774	11.025	1.842	1.274	6.180	-
G40k-0.001	-	97.841	11.663	8.393	37.821	-
G80k-0.001	-	1142.959	88.366	65.886	-	-

---

<sup>4</sup>Time in seconds

## Evaluation: [Sparse]<sup>4</sup>

Graph	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
G5k-0.001	10.352	0.647	0.113	0.041	0.216	5.729
G10k-0.001	37.286	2.395	0.435	0.215	1.331	35.937
G10k-0.01	97.607	1.455	0.273	0.138	0.763	47.525
G10k-0.1	601.182	1.050	0.223	0.114	0.859	395.393
G20k-0.001	150.774	11.025	1.842	1.274	6.180	-
G40k-0.001	-	97.841	11.663	8.393	37.821	-
G80k-0.001	-	1142.959	88.366	65.886	-	-

---

<sup>4</sup>Time in seconds

## Evaluation: [Full]<sup>5</sup>

#V	Query G <sub>2</sub>					
	Scipy	M4RI	GPU4R	GPU_N	GPU_Py	CuSprs
100	0.007	0.002	0.002	< 0.001	0.003	0.278
200	0.040	0.003	0.002	0.001	0.004	0.279
500	0.480	0.003	0.003	0.001	0.004	0.329
1000	3.741	0.007	0.005	0.001	0.006	0.571
2000	40.309	0.063	0.019	0.003	0.017	1.949
5000	651.343	0.366	0.125	0.038	0.150	99.651
10000	-	1.932	0.552	0.315	0.840	1029.042
25000	-	33.236	7.252	5.314	15.521	-
50000	-	360.035	58.751	44.611	129.641	-
80000	-	1292.817	256.579	190.343	641.260	-

---

<sup>5</sup>Time in seconds

# Conclusion

- GPGPUs utilization significantly increases the performance of CFPQ



# Conclusion

- GPGPUs utilization significantly increases the performance of CFPQ
- High performance libraries utilization is a good idea
  - ▶ But not always: M4RI (CPU) is better then cuSPARSE (GPGPU)

# Conclusion

- GPGPUs utilization significantly increases the performance of CFPQ
- High performance libraries utilization is a good idea
  - ▶ But not always: M4RI (CPU) is better then cuSPARSE (GPGPU)
- Automatic translation from a high-level language to GPGPU language provides a good balance between performance and implementation complexity

# Conclusion

- GPGPUs utilization significantly increases the performance of CFPQ
- High performance libraries utilization is a good idea
  - ▶ But not always: M4RI (CPU) is better then cuSPARSE (GPGPU)
- Automatic translation from a high-level language to GPGPU language provides a good balance between performance and implementation complexity
- Sparse matrix representation is important for performance

# Conclusion

- GPGPUs utilization significantly increases the performance of CFPQ
- High performance libraries utilization is a good idea
  - ▶ But not always: M4RI (CPU) is better then cuSPARSE (GPGPU)
- Automatic translation from a high-level language to GPGPU language provides a good balance between performance and implementation complexity
- Sparse matrix representation is important for performance
- Dataset is published: both graphs and queries
- Implementations are available on GitHub
- Link: <https://github.com/SokolovYaroslav/CFPQ-on-GPGPU>

- Investigate implemented algorithms to explain nontrivial behaviors
- Create open extensible platform for CFPQ algorithms comparison
- Evaluate other CFPQ algorithms
  - ▶ Sparse matrices
  - ▶ Distributed matrix multiplication
  - ▶ LL- and LR-based algorithms
- Add new data and queries to the dataset
  - ▶ Bigger RDFs
  - ▶ Static code analysis

# Contact Information

- Semyon Grigorev:
  - ▶ s.v.grigoriev@spbu.ru
  - ▶ Semen.Grigorev@jetbrains.com
- Nikita Mishin: mishinnikitam@gmail.com
- Iaroslav Sokolov: sokolov.yas@gmail.com
- Egor Spirin: egor@spirin.tech
- Vladimir Kutuev: vladimir.kutuev@gmail.com
- Egor Nemchinov: nemchegor@gmail.com
- Sergey Gorbatyuk: sergeygorbatyuk171@gmail.com
  
- Dataset and algorithm implementations:  
<https://github.com/SokolovYaroslav/CFPQ-on-GPGPU>

# Thanks!