

# Context-Free Path Querying with Structural Representation of Result

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**Abstract.** Graph data model and graph databases are very popular in various areas such as bioinformatics, semantic web, and social networks. One specific problem in the area is a path querying with constraints formulated in terms of formal grammars. The query in this approach is written as grammar, and paths querying is graph parsing with respect to given grammar. There are several solutions to it, but how to provide structural representation of query result which is practical for answer processing and debugging is still an open problem. In this paper we propose a graph parsing technique which allows one to build such representation with respect to given grammar in polynomial time and space for arbitrary context-free grammar and graph. Proposed algorithm is based on generalized LL parsing algorithm, while previous solutions are based mostly on CYK or Earley algorithms, which reduces time complexity in some cases

**Keywords:** Graph database, path query, graph parsing, context-free grammar, top-down parsing, GLL, LL

## 1 Introduction

Graph data model and graph data bases are very popular in various areas such as bioinformatics, semantic web, social networks, etc. Extraction of paths which satisfy specific constraints may be useful for investigation of graph structured data and for detection of relations between data items. One specific problem—path querying with constraints—is usually formulated in terms of formal grammars and is called formal language constrained path problem [5].

Classical parsing techniques can be used to solve formal language constrained path problem. It means that such technique can be used for more common problem—graph parsing. Graph parsing may be required in graph data base querying, formal verification, string-embedded language processing, and another areas where graph structured data is used.

Existing solutions in databases field usually employ such parsing algorithms as CYK or Earley (for example [8], [17]). These algorithms have nonlinear time complexity for unambiguous grammars ( $O(n^3)$  and  $O(n^2)$  respectively). Moreover, in case of CYK, the input grammar should be transformed to Chomsky

normal form (CNF) which leads to grammar size increase. To solve these problems, one can use such parsing algorithms as GLR and GLL which have cubic worst-case time complexity and linear complexity for unambiguous grammars. Also there is no need to transform a grammar to CNF for these algorithms. These facts allow us to improve performance of parsing in some cases.

Despite the fact that there is a set of path querying solutions [17,8,4,12], query result exploration is still a challenge [10], as also a simplification of complex query debugging. Structural representation of query result can be used to solve these problems, and classical parsing techniques provide such representation—derivation tree—which contains exhaustive information about parsed sentence structure in terms of specified grammar.

Graph parsing can also be used to analyze dynamically generated strings or string-embedded languages. There are solutions to this problem: GLR-based checker of string-embedded SQL queries [3,7], parser of string-embedded languages [20] based on RNGLR parsing algorithm. RNGLR-based algorithm allows to construct derivation forest (i.e. the set of derivation trees) for all correct paths in the input automaton.

Contribution of this paper

1. Proposed algorithm is based on generalized top-down parsing algorithm—GLL [13]—yes we can use advanced techniques. GLL is topdown, so yes, we can [9]
2. we propose a graph parsing technique which allows one to construct structural representation of query result with respect to the given grammar. This structure can be useful for query debugging and exploration.
3. In evaluation we show that advanced techniques allow to increase performance in comparison with CYK-based implementation

## 2 Preliminaries

In this work we are focused on the parsing algorithm, and not on the data representation, and we assume that whole input graph can be located in RAM memory in the optimal for our algorithm way.

We start by introduction of necessary definitions.

- Context-free grammar is a quadruple  $G = (N, \Sigma, P, S)$ , where  $N$  is a set of nonterminal symbols,  $\Sigma$  is a set of terminal symbols,  $S \in N$  is a start nonterminal, and  $P$  is a set of productions.
- $\mathcal{L}(G)$  denotes a language specified by grammar  $G$ , and is a set of terminal strings derived from start nonterminal of  $G$ :  $L(G) = \{\omega | S \Rightarrow_G^* \omega\}$ .
- Directed graph is a triple  $M = (V, E, L)$ , where  $V$  is a set of vertices,  $L \subseteq \Sigma$  is a set of labels, and a set of edges  $E \subseteq V \times L \times V$ . We assume that there are no parallel edges with equal labels: for every  $e_1 = (v_1, l_1, v_2) \in E$ ,  $e_2 = (u_1, l_2, u_2) \in E$  if  $v_1 = u_1$  and  $v_2 = u_2$  then  $l_1 \neq l_2$ .
- $tag : E \rightarrow L$  is a helper function which allows to get tag of edge.

$$tag(e = (v_1, l, v_2), e \in E) = l$$

- $\oplus : L^+ \times L^+ \rightarrow L^+$  denotes a tag concatenation operation.
- Path  $p$  in graph  $M$  is a list of incident edges:

$$\begin{aligned} p &= e_0, e_1, \dots, e_{n-1} \\ &= (v_0, l_0, v_1), (v_1, l_1, v_2), \dots, (v_{n-1}, l_{n-1}, v_n) \end{aligned}$$

where  $v_i \in V$ ,  $e_i \in E$ ,  $e_i = (v_i, l_i, v_{i+1})$ ,  $l_i \in L$ ,  $|p| = n$ ,  $n \geq 1$ .

- $P$  is a set of paths  $\{p : p \text{ path in } M\}$ , where  $M$  is a directed graph.
- $\Omega : P \rightarrow L^+$  is a helper function which constructs a string produced by the given path. For every  $p \in P$

$$\begin{aligned} \Omega(p = e_0, e_1, \dots, e_{n-1}) &= \\ \text{tag}(e_0) \oplus \dots \oplus \text{tag}(e_{n-1}). \end{aligned}$$

Using these definitions, we state the context-free language constrained path querying as, given a query in form of grammar  $G$ , to construct the set of paths

$$P = \{p | \Omega(p) \in \mathcal{L}(G)\}.$$

Note that, in some cases,  $P$  can be an infinite set, and hence it cannot be represented explicitly. In order to solve this problem, in this paper, we construct compact data structure representation which stores all elements of  $P$  in finite amount of space and allows to extract any of them.

## 2.1 Generalized LL Parsing Algorithm

Classical LL algorithm operates with a pointer to input (position  $i$ ) and with a grammar slot—pointer to grammar in form  $N \rightarrow \alpha \cdot x \beta$ . Parsing may be described as a transition of these pointers from the initial position ( $i = 0$ ,  $S \rightarrow \cdot \beta$ , where  $S$  is start nonterminal) to the final ( $i = \text{input.Length}$ ,  $s \rightarrow \beta \cdot$ ). At every step, there are four possible cases in processing of these pointers.

1.  $N \rightarrow \alpha \cdot x \beta$ , when  $x$  is a terminal and  $x = \text{input}[i]$ . In this case both pointers should be moved to the right ( $i \leftarrow i + 1$ ,  $N \rightarrow \alpha x \cdot \beta$ ).
2.  $N \rightarrow \alpha \cdot X \beta$ , when  $X$  is nonterminal. In this case we push return address  $N \rightarrow \alpha X \cdot \beta$  to stack and move pointer in grammar to position  $X \rightarrow \cdot \gamma$ .
3.  $N \rightarrow \alpha \cdot$ . This case means that processing of nonterminal  $N$  is finished. We should pop return address from stack and use it as new slot.
4.  $S \rightarrow \alpha \cdot$ , where  $S$  is a start nonterminal of grammar. In this case we should report success if  $i = \text{input.Length} - 1$  or failure otherwise.

In the second case there can be several slots  $X \rightarrow \cdot \gamma$ , so a strategy on how to choose one of them to continue parsing is needed. In LL(k) algorithm lookahead is used, but this strategy is still not good enough because there are context-free languages for which deterministic choice is impossible even for infinite lookahead [6]. On the contrary to LL(k), generalized LL does not choose at all, handling all possible variants. Note, that instead of immediate processing

of all variants, GLL uses descriptors mechanism to store all possible branches and process them sequentially. Descriptor is a quadruple  $(L, s, j, a)$  where  $L$  is a grammar slot,  $s$  is a stack node,  $j$  is a position in the input string, and  $a$  is a node of derivation tree.

The stack in parsing process is used to store return information for the parser—a name of function which will be called when current function finishes computation. As mentioned before, generalized parsers process all possible derivation branches and parser must store its own stack for every branch. It leads to an infinite stack growth being done naively. Tomita-style graph structured stack (GSS) [19] combines stacks resolving this problem. Each GSS node contains a pair of position in input and a grammar slot in GLL.

In order to provide termination and correctness, we should avoid duplication of descriptors, and be able to process GSS nodes in arbitrary order. It is necessary to use the following additional sets for this.

- $R$ —working set which contains descriptors to be processed. Algorithm terminates whenever  $R$  is empty.
- $U$ —all created descriptors. Each time when we want to add a new descriptor to  $R$ , we try to find it in this set first. This way we process each descriptor only once which guarantee termination of parsing.
- $P$ —popped nodes. Allows to process descriptors (and GSS nodes) in arbitrary order.

There can be more than one derivation tree of a string with relation to ambiguous grammar. Generalized LL build all such trees and compact them in a special data structure Shared Packed Parse Forest [11], which will be described in the following section.

## 2.2 Shared Packed Parse Forest

Binarized Shared Packed Parse Forest (SPPF) [16] compresses derivation trees optimally reusing common nodes and subtrees. Version of GLL which uses this structure for parsing forest representation achieves worst-case cubic space complexity [14].

Binarized SPPF can be represented as a graph in which each node has one of four types described below with correspondent graphical notation. Let  $i$  and  $j$  be the start and the end positions of substring, and let us call a tuple  $(i, j)$  an *extension* of node.

- Node of rectangle shape labeled with  $(i, T, j)$  is a terminal node.
- Node of oval shape labeled with  $(i, N, j)$  is a nonterminal node. This node denotes that there is at least one derivation for substring  $\alpha = \omega[i..j-1]$  such that  $N \Rightarrow_G^* \alpha, \alpha = \omega[i..j-1]$ . All derivation trees for the given substring and nonterminal can be extracted from SPPF by left-to-right top-down graph traversal started from respective node. We use filled shape and label of form  $(\blacklozenge (i, N, j))$  to denote that there are multiple derivations from nonterminal  $N$  for substring  $\omega[i..j-1]$ .

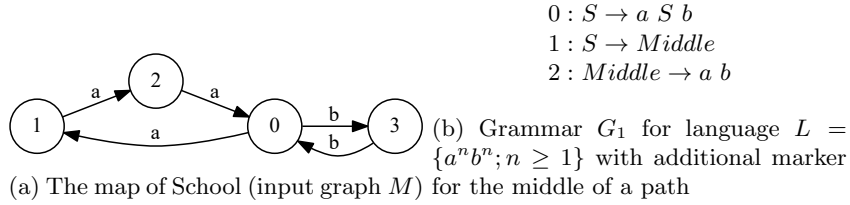
- Node of rectangle shape labeled with  $(i, t, j)$ , where  $t$  is a grammar slot, is an intermediate node: a special kind of node used for binarization of SPPF.
- Packed node labeled with  $(N \rightarrow \alpha \cdot \beta, k)$ . In our pictures, we use dot shape for these nodes and omit labels because they are important only on SPPF construction stage. Subgraph with “root” in such node is one variant of derivation from nonterminal  $N$  in case when the parent is a nonterminal node labeled with  $(\langle \triangleright (i, N, j))$ .

In our examples we remove redundant intermediate and packed nodes from the SPPF to simplify it and to decrease the size of structure.

### 3 Motivating Example

Suppose that you are student in a School of Magic. It is your first day at School, so navigation in the building is a problem for you. Fortunately, you have a map of the building (fig. 1a) and additional knowledge about building construction:

- there are towers in the school (depicted as nodes of the graph in your map);
- towers can be connected by one-way galleries (represented as edges in your map);
- galleries have a “magic” property: you can start from any floor, but by following each gallery you either end up one floor above (edge label is ‘a’), or one floor below (edge label is ‘b’).



(a) The map of School (input graph  $M$ ) for the middle of a path

You want to find a path from your current position to the same floor in another tower. Map with all such paths can help you. But orienteering is not your forte, so it would be great if the structure of the paths were as simple as possible and all paths had additional checkpoints to control your rout.

It is evident that the simplest structure of required paths is  $\{ab, aabb, aaabbb, \dots\}$ . In terms of our definitions, it is necessary to find all paths  $p$  such that  $\Omega(p) \in \{a^n b^n, n \geq 1\}$  in the graph  $M = (\{0; 1; 2; 3\}, E, \{a; b\})$  (figure 1a).

Unfortunately, language  $\mathcal{L} = \{a^n b^n; n \geq 1\}$  is not regular which restricts the set of tools you can use. Another problem is the infinite size of solution, but,

being incapable to comprehend an infinite set of paths, you want to get a finite map. Moreover, you want to know structure of paths in terms of checkpoints.

We are not aware of any existing tools which can solve this problem, thus we have created such tool. Let us show how to get a map which helps to navigate in this strange School.

Fortunately, the language  $\mathcal{L} = \{a^n b^n; n \geq 1\}$  is a context-free language and it can be specified with context-free grammar. The fact that one language can be described with multiple grammars allows to add checkpoints: additional nonterminals can mark required parts of sentences. In our case, desired checkpoint can be in the middle of the path. As a result, required language can be specified by the grammar  $G_1$  presented in figure 1b, where  $N = \{s; Middle\}$ ,  $\Sigma = \{a; b\}$ , and  $S$  is a start nonterminal.

Let us present a solution for this problem. Result SPPF for this input is presented in figure 2a. Note that presented version does not contains redundant nodes. Each terminal node corresponds to the edge in the input graph: for each node with label  $(v_0, T, v_1)$  there is  $e \in E : e = (v_0, T, v_1)$ . We duplicate terminal nodes only for figure simplification.

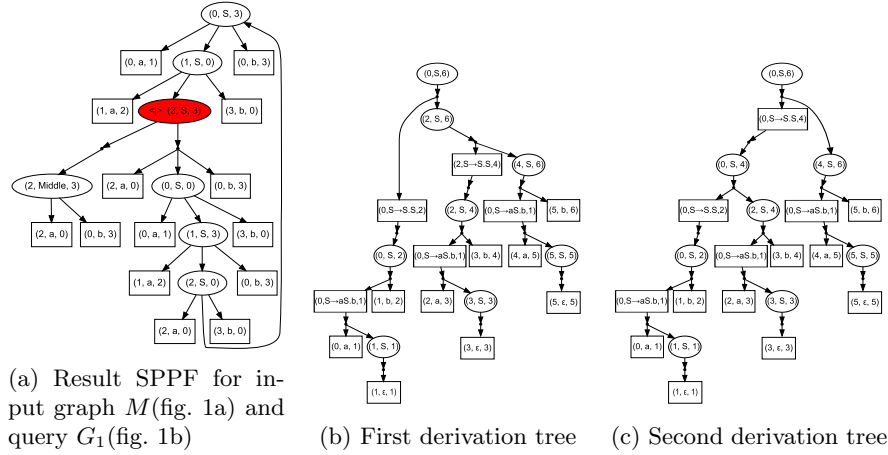


Fig. 2: SPPF for sentence "ababab" and grammar  $G_0$

As an example of derivation structure usage, we can find a middle of any path in example simply by finding correspondent nonterminal *Middle* in SPPF. So we can find out that there is only one (common) middle for all results, and it is a vertex with  $id = 0$ .

Extensions stored in nodes allow us to check whether path from  $u$  to  $v$  exists and to extract it. We need only to traverse SPPF which can be done in polynomial time (in terms of SPPF size) to extract any path.

Lets find paths  $p_i$  such that  $S \xRightarrow{G_1}^* \Omega(p_i)$  and  $p_i$  starts from the vertex 0. To do this, we should find vertices with label  $(0, S, -)$  in SPPF. (There are two vertices with such labels:  $(0, S, 0)$  and  $(0, S, 3)$ .) Then let us to extract corresponded paths from SPPF. There is a cycle in SPPF in our example, so there are **at least** two different paths:

$$p_0 = \{(0, a, 1); (1, a, 2); (2, a, 0); (0, b, 3); (3, b, 0); (0, b, 3)\}$$

and

$$p_1 = \{(0, a, 1); (1, a, 2); (2, a, 0); (0, a, 1); (1, a, 2); (2, a, 0); (0, b, 3); (3, b, 0); (0, b, 3); (3, b, 0); (0, b, 3); (3, b, 0)\}.$$

We demonstrate that SPPF which was constructed by described algorithm can be useful for query result investigation. But in some cases explicit representation of matched subgraph is preferable, and required subgraph may be extracted from SPPF trivially by its traversal.

In the next section, we present a graph parsing algorithm which can be applicable to this kind of problems.

## 4 GLL-based Graph Parsing Algorithm

We propose a graph parsing algorithm which allows to construct finite representation of parse forest which contains derivation trees for all matched paths in graph. Finite representation of result set with respect to the specified grammar may be useful not only for results understanding and processing, but also for query debugging.

Our solution is based on generalized LL (GLL) [13,2] parsing algorithm which allows to process arbitrary (including left-recursive and ambiguous) context-free grammars with worst-case cubic time complexity and linear time complexity for LL grammars on a linear input.

In this section we present such modification of GLL algorithm, that for input graph  $M$ , set of start vertices  $V_s \subseteq V$ , set of final vertices  $V_f \subseteq V$ , and grammar  $G_1$ , it returns SPPF which contains all derivation trees for all paths  $p$  in  $M$ , such that  $\Omega(p) \in L(G_1)$ , and  $p.start \in V_s$ ,  $p.end \in V_f$ . In other words, we propose GLL-based algorithm which can solve language constrained path problem.

First of all, notice that an input string for classical parser can be represented as a linear graph, and positions in the input are vertices of this graph. This observation can be generalized to arbitrary graph with remark that for a position there is a set of labels of all outgoing edges for given vertex instead of just one next symbol. Thus, in order to use GLL for graph parsing we need to use graph vertices as positions in input and modify step 1 to process multiple “next symbols”. Small modification is also required for initialization of  $R$  set: it is necessary to add not only one initial descriptor but the set of descriptors for all vertices in  $V_s$ . All other functions are reused from original algorithm without any changes.

Note that our solution handles arbitrary numbers of start and final vertices, which allows one to solve different kinds of problems arising in the field, namely all paths in graph, all paths from specified vertex, all paths between specified vertices. Also SPPF represents a structure of paths in terms of grammar which provides exhaustive information about result.

Note that termination of proposed algorithm is inherited from the basic GLL algorithm. We process finite graphs, hence the set of positions is finite, and tree construction has not been changed. As a result, the total number of descriptors is finite, and each of them is added in  $R$  only once, thus main loop is finite.

The worst-case space complexity of proposed algorithm for graph  $M = (V, E, L)$  is  $O(|V|^3 + |E|)$ . The worst-case runtime complexity of proposed algorithm for graph  $M = (V, E, L)$  is  $O\left(|V|^3 * \max_{v \in V} (deg^+(v))\right)$ .

Note that result SPPF contains only paths matched specified query, so result SPPF size is  $O(|V'|^3 + |E'|)$  where  $M' = (V', E', L')$  is a subgraph of input graph  $M$  which contains only matched paths.

## 5 Evaluation

One of classical graph querying problems is a navigation queries for ontologies, and we apply our algorithm to this problem in order to estimate its practical value. We used dataset from paper [21]. Our algorithm is aimed to process graphs, so RDF files were converted to edge-labeled directed graph. For each triple  $(o, p, s)$  from RDF we added two edges:  $(o, p, s)$  and  $(s, p^{-1}, o)$ .

We perform two classical *same-generation queries* [1]. Such queries are part of BIOPROBLEM!!!!

**Query 1** is based on the grammar for retrieving concepts on the same layer (presented in figure 3a). For this query our algorithm demonstrates up to 1000 times better performance and provides identical results as compared to the presented in [21] for  $Q_1$ .

**Query 2** is based on the grammar for retrieving concepts on the adjacent layers (presented in figure 3b). Note that this query differs from the original query  $Q_2$  from article [21] in the following details. First of all, we count only triples for nonterminal  $S$  because only paths derived from it correspond to paths between concepts on adjacent layers. Algorithm which is presented in [21] returns triples for all nonterminals. Moreover, grammar  $\mathcal{G}_2$ , which is presented in [21], describes paths not only between concepts on adjacent layers. For example, path “*subClassOf subClassOf<sup>-1</sup>*” can be derived in  $\mathcal{G}_2$ , but it is a path between concepts on the same layer, not adjacent. We changed the grammar to fit a query to a description provided in paper [21]. Thus results of our query is different from results for  $Q_2$  which provided in paper [21].

All tests were run on a x64-based PC with Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz and 32 GB RAM. Results of both queries are presented in table 1, where #triples is a number of  $(o, p, s)$  triples in RDF file, and #results is a number



of triples of form  $(S, v_1, v_2)$ . In our approach result triples can be founded by filtering out all SPPF nonterminal nodes labeled by  $(v_1, S, v_2)$ .

$0 : S \rightarrow \text{subClassOf}^{-1} S \text{ subClassOf}$	$0 : S \rightarrow B \text{ subClassOf}$
$1 : S \rightarrow \text{type}^{-1} S \text{ type}$	$1 : B \rightarrow \text{subClassOf}^{-1} B \text{ subClassOf}$
$2 : S \rightarrow \text{subClassOf}^{-1} \text{subClassOf}$	$2 : B \rightarrow \text{subClassOf}^{-1} \text{subClassOf}$
$3 : S \rightarrow \text{type}^{-1} \text{type}$	

(a) Grammar for query 1
(b) Grammar for query 2

Fig. 3: Grammars for evaluation

Table 1: Evaluation results for Query 1 and Query 2

Ontology	#triples	Query 1		Query 2	
		time(ms)	#results	time(ms)	#results
skos	252	10	810	1	1
generations	273	19	2164	1	0
travel	277	24	2499	1	63
univ-bench	293	25	2540	11	81
foaf	631	39	4118	2	10
people-pets	640	89	9472	3	37
funding	1086	212	17634	23	1158
atom-primitive	425	255	15454	66	122
biomedical-measure-primitive	459	261	15156	45	2871
pizza	1980	697	56195	29	1262
wine	1839	819	66572	8	133

As a result, we conclude that our algorithm is fast enough to be applicable to some real-world problems.

## 6 Conclusion and Future Work

We propose GLL-based algorithm for context-free path querying which constructs finite structural representation of all paths satisfying given constraint. Provided data structure can be useful for result investigation and processing, and for query debugging. Presented algorithm has been implemented in F# programming language [18] and is available on GitHub:<https://github.com/YaccConstructor/YaccConstructor>.

We are working on performance improvement by implementation of recently proposed modifications in original GLL algorithm [15,2]. One direction of our

research is generalization of grammar factorization proposed in [15] which may be useful for the processing of regular queries which are common in real world application.

## Acknowledgments

Jelle Hellings  
Ekaterina Verbitskaia  
JetBrains  
etal

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