MSF 526
Illinois Institute of Technology
Homework 3
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Numerical Methods for Option Pricing

Problem 1. Implementing a binomial tree function for pricing American options in the Black-Scholes framework.

The output of implemented function is shown below. Binomial tree pricing methods are relatively simple in implementation; they do not carry weight of excessive computations; and they provide for relatively stable and accurate results.

Parameters:
S0 = 40, Ks = [30, 35, 40, 45, 50], r = 0.05, T = 1,
Sigma = 0.15, q = 0.01, M = 12

Expected results from matlab:

```
# binprice(S0, K, r, T, dt=1/12, sigma, callput, q)
# K = [30, 35, 40, 45, 50]
# Calls = [11.0887, 6.6616, 3.1301, 1.2023, 0.3478]
# Puts = [0.0241, 0.3715, 1.7746, 5.1145, 10.0000]
```

Calculated prices for call options:
[11.200171   6.7285585   3.1615443   1.2144072   0.35130224]

Calculated prices for put options:
[0.0243059   0.37458628 1.7841662   5.1260157 10.   ]

Problem 2. Implementing a finite difference scheme for pricing European options in the Black-Scholes framework using an explicit scheme.

Explicit method is based on discretization of PDE based on relationship between one value of the option at time j Δt and three different values of the option at time (j+1) Δt. The advantage of the method is relatively simple implementation and reduced computational effort. The disadvantage of the method is its instability. It can produce wrong results as it is shown in the examples below where the results are a negative option value or extremely large value with the certain parameters of the computational grid.

Call option:
Parameters:
S0 = 40, K = 30, r = 0.05, T = 1,
Sigma = 0.15, q = 0.01, M = 100, N = 1000, S_max = 100

Calculated option value = 11.090625776844401

Call option:
Parameters:
S0 = 40, K = 30, r = 0.05, T = 1,
Sigma = 0.15, q = 0.01, M = 100, N = 100, S_max = 100

Calculated option value = 2560.921839027467

Put option:
Parameters:
S0 = 45, K = 40, r = 0.05, T = 1,
Sigma = 0.4, q = 0.01, M = 100, N = 1000, S_max = 100

Calculated option value = 1.2386629726502104e+192

Put option:
Parameters:
S0 = 45, K = 40, r = 0.05, T = 1,
Sigma = 0.4, q = 0.01, M = 30, N = 50, S_max = 100

Calculated option value = -1554787.0984461748

Problem 3. Implementing a finite difference scheme for pricing American options in the Black-Scholes framework using Gauss-Seidel method to solve a Crank-Nicolson formulation of the finite difference scheme.

Crank-Nicolson method uses a combination of explicit and implicit methods of finite differences. It does provide for much better stability (convergence) than explicit method and yet it is easier to implement than extremely robust but more complex implicit method. Below are the results of the work of implemented method for the same inputs we had experienced instability while using explicit method. We can clearly see that the results are very stable.

Call option:
Parameters:
S0 = 40, K = 30, r = 0.05, T = 1,
Sigma = 0.15, q = 0.01, M = 100, N = 1000, S_max = 100

Calculated option value = 11.098310964068975

Call option:
Parameters:
S0 = 40, K = 30, r = 0.05, T = 1,
Sigma = 0.15, q = 0.01, M = 100, N = 100, S_max = 100

Calculated option value = 11.090358448628667

Put option:
Parameters:
S0 = 45, K = 40, r = 0.05, T = 1,
Sigma = 0.4, q = 0.01, M = 100, N = 1000, S_max = 100

Calculated option value = 3.899252290182546

Put option:
Parameters:
S0 = 45, K = 40, r = 0.05, T = 1,
Sigma = 0.4, q = 0.01, M = 30, N = 50, S_max = 100

Calculated option value = 3.891840742868172