Analysis of a 3D system

Stable stationary point

Examine the system

$$d_t x = -x - \operatorname{arctg}(z),$$
$$d_t y = x - 2y,$$
$$d_t z = y - 3z.$$

The only stationary point is $\vec{s_0} = (x_0, y_0, z_0) = \vec{0}$.

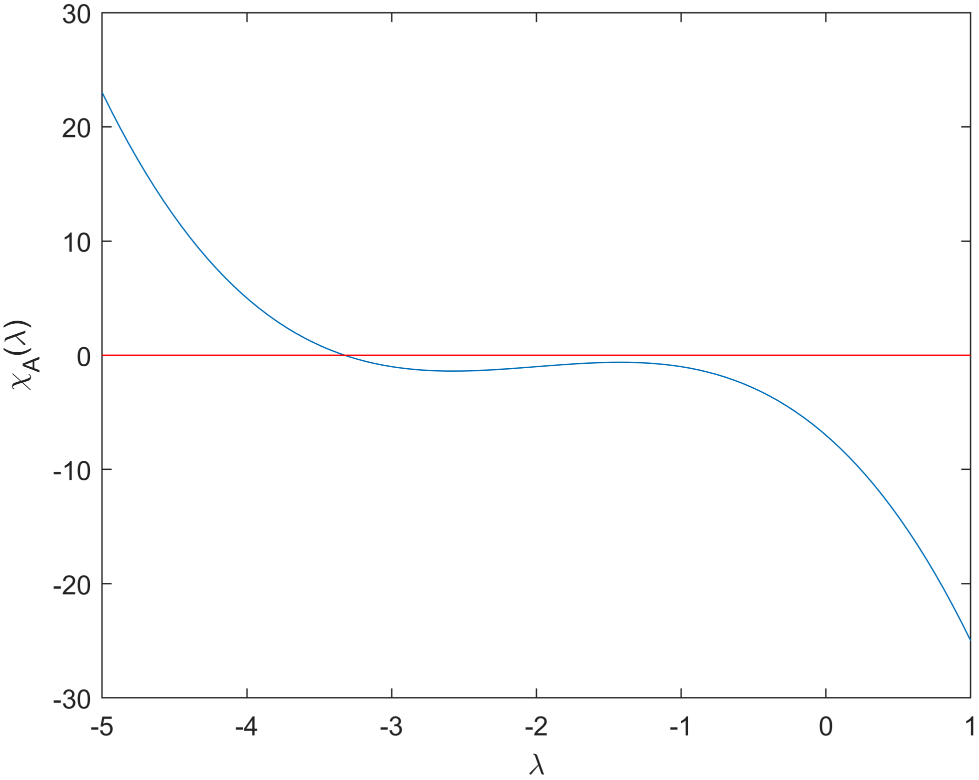
```
A = [-1, 0, -1;
     1, -2, 0;
     0, 1, -3];
x_up = pi/2;
y_up = pi/4;
z_up = pi/12;
```

Characteristic polynomial

```
char_p = -poly(A)
```

```
char_p = 1×4
   -1.0000   -6.0000  -11.0000   -7.0000
```

```
lam = linspace(-5, 1);

figure(1);
plot(lam, polyval(char_p, lam));
line([min(lam), max(lam)], [0, 0], 'Color', 'red');
xlabel('\lambda');
ylabel('\chi_A(\lambda)');
```



Eigenvalues

```
eig(A)
```

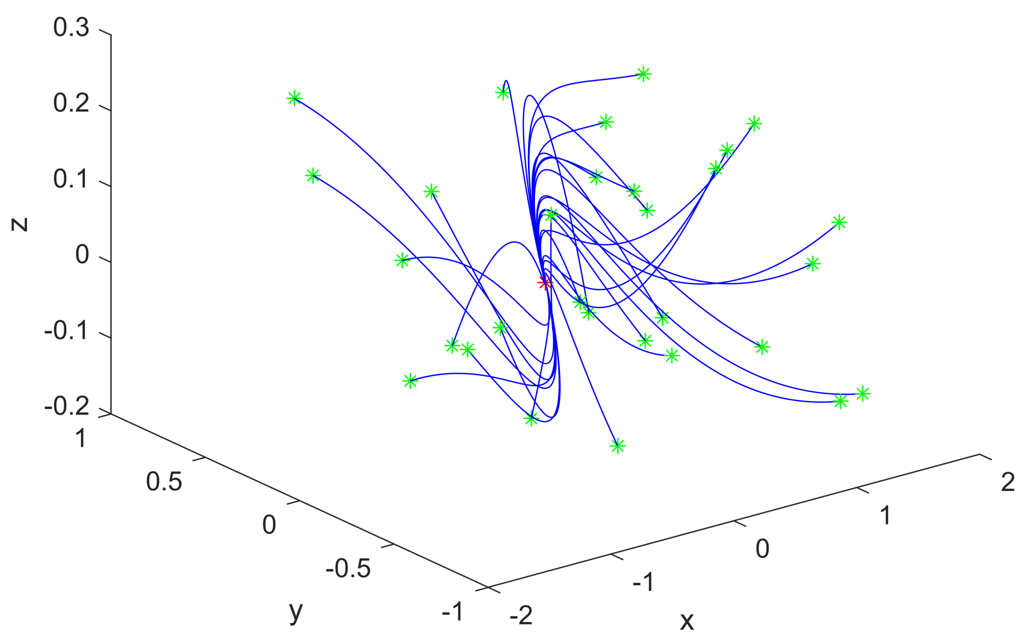
```
ans = 3×1 complex
  -1.3376 + 0.5623i
  -1.3376 - 0.5623i
  -3.3247 + 0.0000i
```

```
% roots(char_p)
```

Streamslice from some random points

```
x_up = pi/2;
y_up = pi/4;
z_up = pi/12;

[x, y, z] = meshgrid(...
    linspace(-x_up, x_up, 101),...
    linspace(-y_up, y_up, 101),...
    linspace(-z_up, z_up, 101));

dx = -x - atan(z);
dy = x -2*y;
dz = y -3*z;

Nstart = 30;
rng('default');
```

```
sx = (rand(Nstart, 1) - .5) * 2*x_up;
sy = (rand(Nstart, 1) - .5) * 2*y_up;
sz = (rand(Nstart, 1) - .5) * 2*z_up;

figure(2);
streamline(stream3(x, y, z, dx, dy, dz, sx, sy, sz));
view(3);
xlabel('x');
ylabel('y');
zlabel('z');
hold on;
plot3(0, 0, 0, '*r');
plot3(sx, sy, sz, '*g');
hold off;
```



Unstable stationary point

Now consider a system

$$d_t x = -0.01x - \text{arctg}(z),$$
$$d_t y = x - 0.02y,$$
$$d_t z = y - 3z,$$

with one stationary point $\vec{s_0} = (x_0, y_0, z_0) = \vec{0}$.

Use Runge-Kutta method to integrate the system.

First, redefine the parameters

```
A = [-.01, 0, -1;
    1, -.02, 0;
    0, 1, -3];
x_up = pi/.02;
y_up = pi/.0004;
z_up = pi/.0012;

rng('default');
Nstart = 10;
sx = normrnd(0, 10, Nstart, 1);
sy = normrnd(0, 10, Nstart, 1);
sz = normrnd(0, 10, Nstart, 1);
```

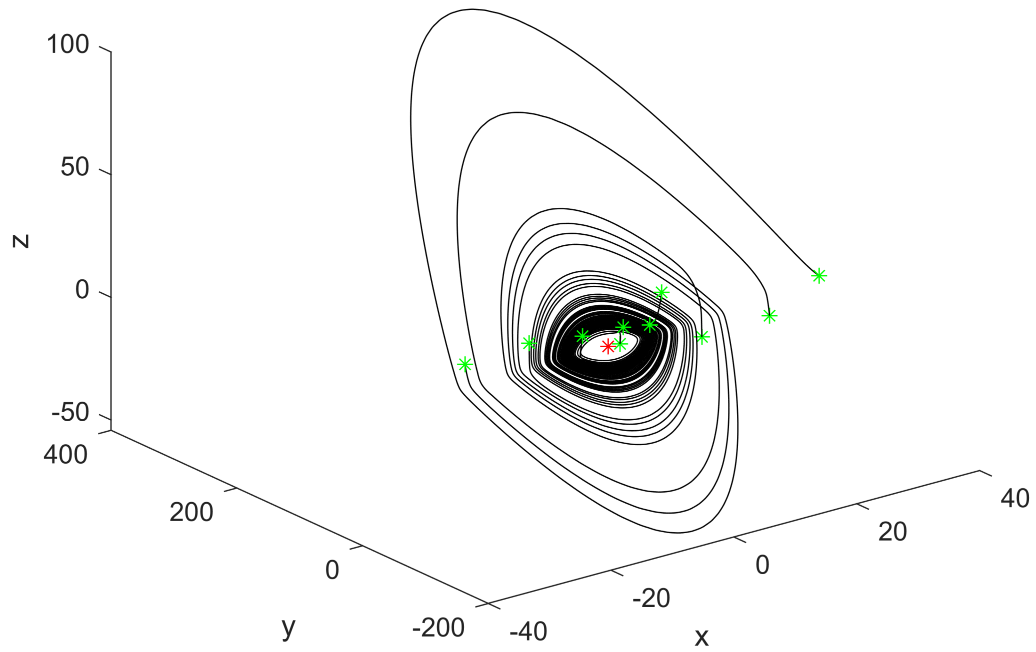
There are eigenvalues with positive real part, and therefore the stationary point $\vec{s_0} = (x_0, y_0, z_0) = \vec{0}$ is unstable.

```
eig(A)
```

```
ans = 3×1 complex
   0.0374 + 0.5665i
   0.0374 - 0.5665i
  -3.1048 + 0.0000i
```

Now integrate the system

```
T = 200;
tau = .01;
tspan = 0:tau:T;
figure(3)
for k = 1 : Nstart
    [t, S] = ode45(@ode_system, tspan, [sx(k), sy(k), sz(k)]);
    tl = length(t);
    plot3(S(:, 1), S(:, 2), S(:, 3), '-k'); hold on;
%     plot3(S(round(tl/4):end, 1), S(round(tl/4):end, 2), S(round(tl/4):end, 3), '-k');
    hold on;
end
plot3(0, 0, 0, '*r');
plot3(sx, sy, sz, '*g');
hold off;
xlabel('x');
ylabel('y');
zlabel('z');
```



Poincare Method

Define the starting point $S_0 = (0,\ 20.2,\ 6.7)$. Make two steps: in the direction of the second basis vector $\overrightarrow{v_2} = (0, 1, 0)$ and in the direction of the third basis vector $\overrightarrow{v_3} = (0, 0, 1)$.

```
S_0 = [0, 20.2, 6.7];
S_p = [S_0 + [0, 1, 0]; S_0 + [0, 0, 1]];
```

Initialize the matrix $A$of the linear operator and integrate the system starting from S_p points.

```
A_p = zeros(2);

for k = 1 : 2
    [t, S] = ode45(@ode_system, tspan, S_p(k, :));
    tl = length(t);

    half_ind = find(S(:, 1) > 0, 1);
    S = S(half_ind:end, :);

    one_ind = find(S(:, 1) < 0, 1);
    A_p(:, k) = S(one_ind, 2:3) - S_0(2:3);
end

eig_vals = eig(A_p);
abs(eig_vals)
```

```
ans = 2×1
    0.6263
    0.0296
```

The absolute values of both eigenvales are less than 1. Therefore, there is a local convergence of the system near the limiting cycle.

Modified system with nonstationary point

```
function ds = ode_system(t, s)
ds = zeros(3, 1);
ds(1) = -.01*s(1) - atan(s(3));
ds(2) = s(1) - .02*s(2);
ds(3) = s(2) - 3*s(3);
end
```