

Analysis of a 3D system

Stable stationary point

Examine the system

$$d_t x = -x - \arctg(z),$$

$$d_t y = x - 2y,$$

$$d_t z = y - 3z.$$

The only stationary point is $\vec{s}_0 = (x_0, y_0, z_0) = \vec{0}$.

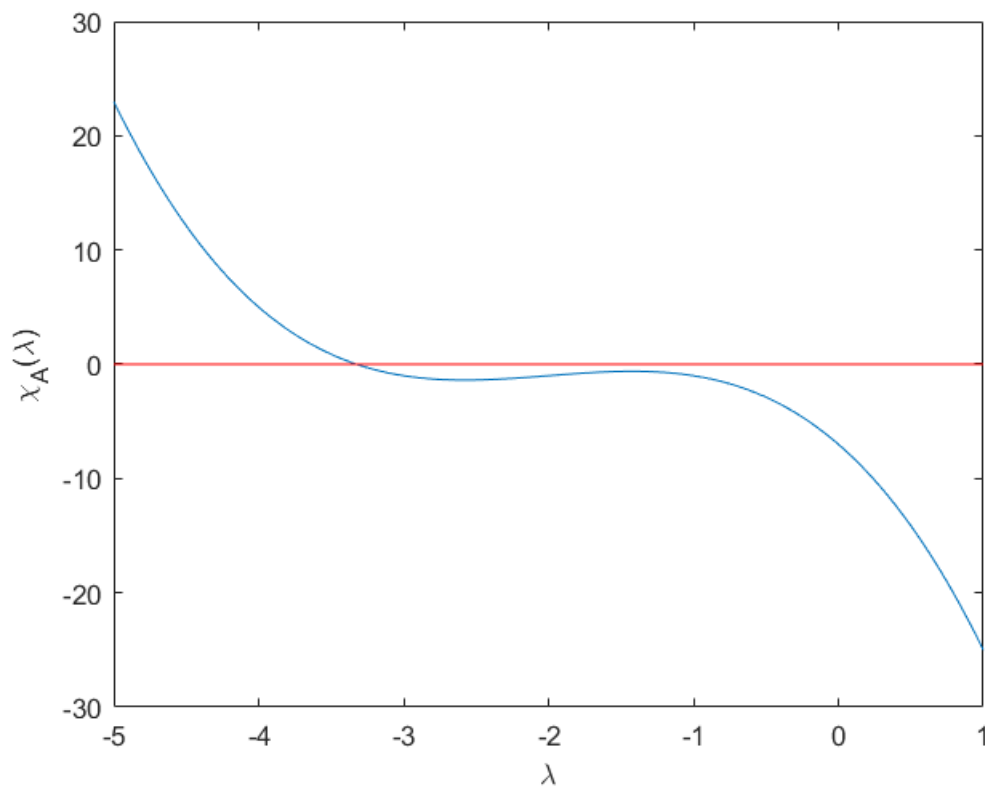
```
A = [-1, 0, -1;  
      1, -2, 0;  
      0, 1, -3];  
x_up = pi/2;  
y_up = pi/4;  
z_up = pi/12;
```

Characteristic polynomial

```
char_p = -poly(A)
```

```
char_p = 1x4  
      -1.0000  -6.0000  -11.0000  -7.0000
```

```
lam = linspace(-5, 1);  
  
figure(1);  
plot(lam, polyval(char_p, lam));  
line([min(lam), max(lam)], [0, 0], 'Color', 'red');  
xlabel('\lambda');  
ylabel('\chi_A(\lambda)');
```



Eigenvalues

```
eig(A)
```

```
ans = 3×1 complex
-1.3376 + 0.5623i
-1.3376 - 0.5623i
-3.3247 + 0.0000i
```

```
% roots(char_p)
```

Streamslice from some random points

```
x_up = pi/2;
y_up = pi/4;
z_up = pi/12;

[x, y, z] = meshgrid(...
    linspace(-x_up, x_up, 101),...
    linspace(-y_up, y_up, 101),...
    linspace(-z_up, z_up, 101));

dx = -x - atan(z);
dy = x - 2*y;
dz = y - 3*z;

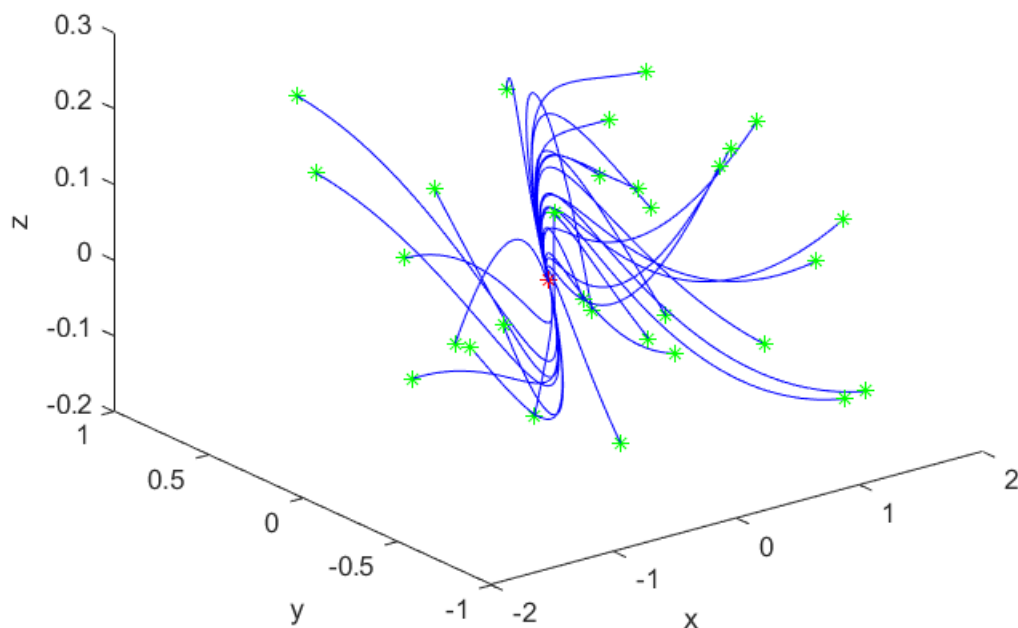
Nstart = 30;
rng('default');
```

```

sx = (rand(Nstart, 1) - .5) * 2*x_up;
sy = (rand(Nstart, 1) - .5) * 2*y_up;
sz = (rand(Nstart, 1) - .5) * 2*z_up;

figure(2);
streamline(stream3(x, y, z, dx, dy, dz, sx, sy, sz));
view(3);
xlabel('x');
ylabel('y');
zlabel('z');
hold on;
plot3(0, 0, 0, '*r');
plot3(sx, sy, sz, '*g');
hold off;

```



Unstable stationary point

Now consider a system

$$d_t x = -0.01x - \arctg(z),$$

$$d_t y = x - 0.02y,$$

$$d_t z = y - 3z,$$

with one stationary point $\vec{s}_0 = (x_0, y_0, z_0) = \vec{0}$.

Use Runge-Kutta method to integrate the system.

First, redefine the parameters

```
A = [-.01, 0, -1;  
     1, -.02, 0;  
     0, 1, -3];  
x_up = pi/.02;  
y_up = pi/.0004;  
z_up = pi/.0012;  
  
rng('default');  
Nstart = 10;  
sx = normrnd(0, 10, Nstart, 1);  
sy = normrnd(0, 10, Nstart, 1);  
sz = normrnd(0, 10, Nstart, 1);
```

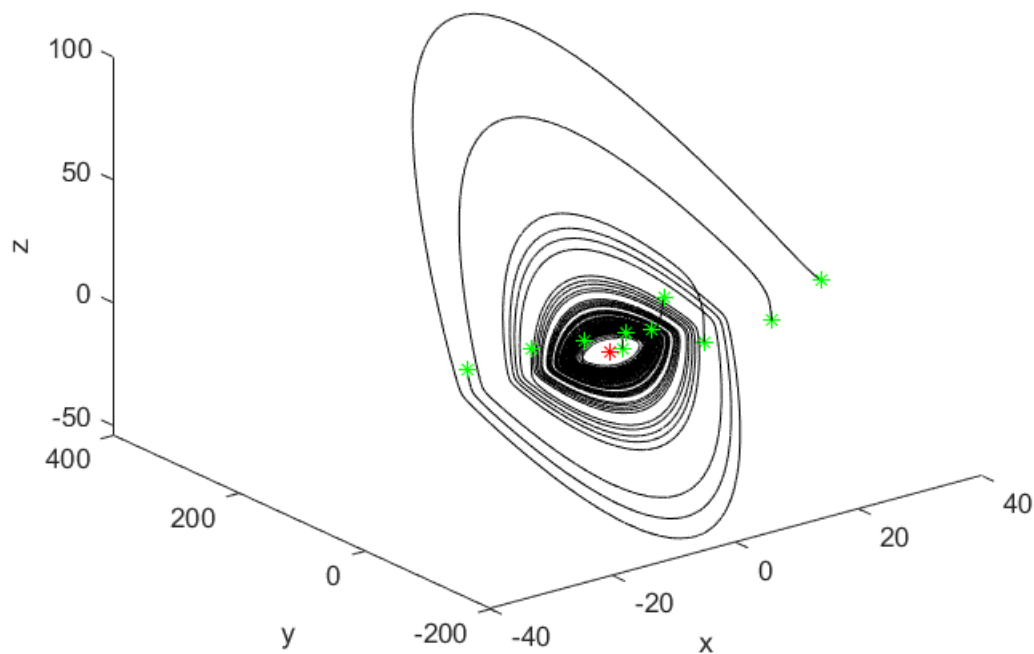
There are eigenvalues with positive real part, and therefore the stationary point $\vec{s}_0 = (x_0, y_0, z_0) = \vec{0}$ is unstable.

```
eig(A)
```

```
ans = 3x1 complex  
    0.0374 + 0.5665i  
    0.0374 - 0.5665i  
   -3.1048 + 0.0000i
```

Now integrate the system

```
T = 200;  
tau = .01;  
tspan = 0:tau:T;  
figure(3)  
for k = 1 : Nstart  
    [t, S] = ode45(@ode_system, tspan, [sx(k), sy(k), sz(k)]);  
    tl = length(t);  
    plot3(S(:, 1), S(:, 2), S(:, 3), '-k'); hold on;  
    % plot3(S(round(tl/4):end, 1), S(round(tl/4):end, 2), S(round(tl/4):end, 3), '-k');  
    hold on;  
end  
plot3(0, 0, 0, '*r');  
plot3(sx, sy, sz, '*g');  
hold off;  
xlabel('x');  
ylabel('y');  
zlabel('z');
```



Poincare Method

Define the starting point $S_0 = (0, 20.2, 6.7)$. Make two steps: in the direction of the second basis vector

$\vec{v}_2 = (0, 1, 0)$ and in the direction of the third basis vector $\vec{v}_3 = (0, 0, 1)$.

```
S_0 = [0, 20.2, 6.7];
S_p = [S_0 + [0, 1, 0]; S_0 + [0, 0, 1]];
```

Initialize the matrix A of the linear operator and integrate the system starting from S_p points.

```
A_p = zeros(2);

for k = 1 : 2
    [t, S] = ode45(@ode_system, tspan, S_p(k, :));
    t1 = length(t);

    half_ind = find(S(:, 1) > 0, 1);
    S = S(half_ind:end, :);

    one_ind = find(S(:, 1) < 0, 1);
    A_p(:, k) = S(one_ind, 2:3) - S_0(2:3);
end

eig_vals = eig(A_p);
abs(eig_vals)
```

```
ans = 2×1  
    0.6263  
    0.0296
```

The absolute values of both eigenvalues are less than 1. Therefore, there is a local convergence of the system near the limiting cycle.

Modified system with nonstationary point

```
function ds = ode_system(~, s)  
ds = zeros(3, 1);  
ds(1) = -.01*s(1) - atan(s(3));  
ds(2) = s(1) - .02*s(2);  
ds(3) = s(2) - 3*s(3);  
end
```