Lotka-Volterra system and quality of ode45

Lotka-Voltera system is defined as a system of nonlinear ordinary differential equations:

$\dot{x} = x(\alpha - \beta y)$
$\dot{y} = y(-\gamma + \delta x)$

Define the parameters;

```
alpha =4;
beta =3;
gamma =2;
delta =1;
```

Stationary points are $z_0^* = (0,0)$ and $z_1^* = \left(\frac{\gamma}{\delta}, \frac{\alpha}{\beta}\right)$.

```
z_st0 = [0; 0];
z_st1 = [gamma/delta; alpha/beta];
```

First integral of the system is $V(x,y) = \delta x + \beta y - \log(x^\gamma y^\alpha)$:

```
V = @(x, y) delta*x + beta*y - log(x.^gamma .* y.^alpha);
```

Define initial conditions for ode45. Assume automatic time-step selection.

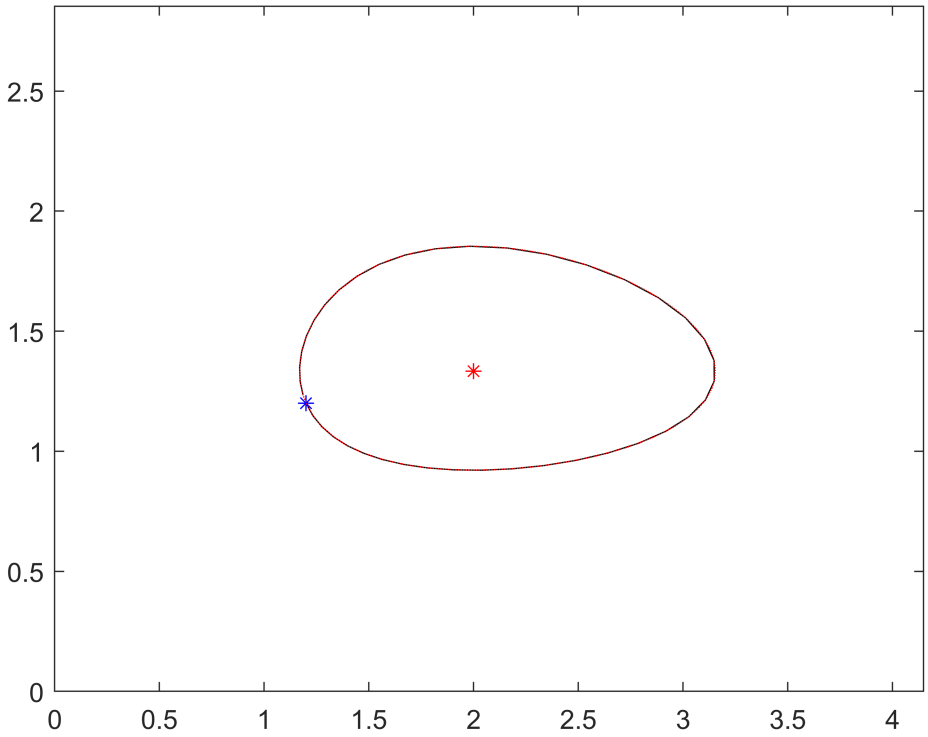
```
z_0 = [1.2, 1.2];
% T = 1;
T = 2*pi/sqrt(alpha*gamma);
tspan = [0, T];
V_0 = V(z_0(1), z_0(2));
```

Integrate the system:

```
[t, z] = ode45(@(t, z) lotka_volterra(t, z, [alpha, beta, gamma, delta]), ...
    tspan, z_0);
```

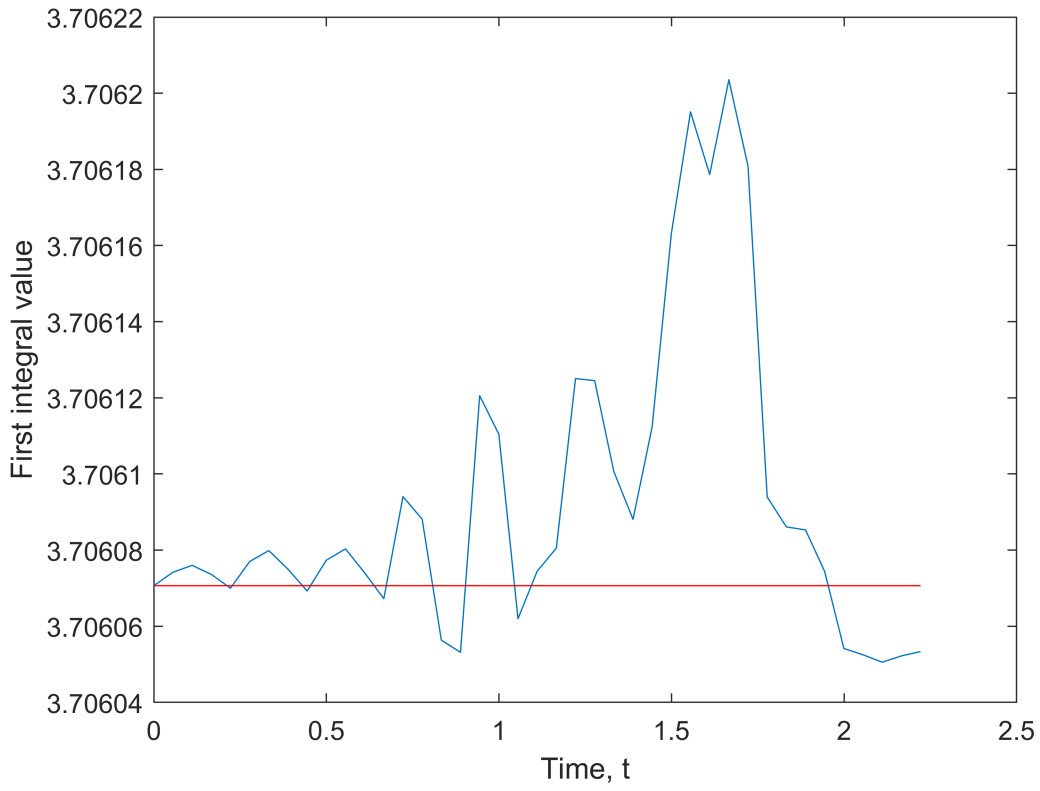
Plot the trajectory:

```
figure;
plot(z(:, 1), z(:, 2), '-k');
hold on;
plot(z_st1(1), z_st1(2), '*r')
plot(z_0(1), z_0(2), '*b')
fcontour(V, 'LevelList', V_0, 'LineColor', 'red', 'LineStyle', ':')
axis([0, max(z(:, 1)) + 1, 0, max(z(:, 2)) + 1]);
hold off;
```



Plot the difference between the actual first integral and the calculated first integral

```
V_T = V(z(:, 1), z(:, 2));

figure;
plot(t, V_T);
line([0 t(end)], [V_0 V_0], 'Color', 'red')
xlabel('Time, t');
ylabel('First integral value');
```



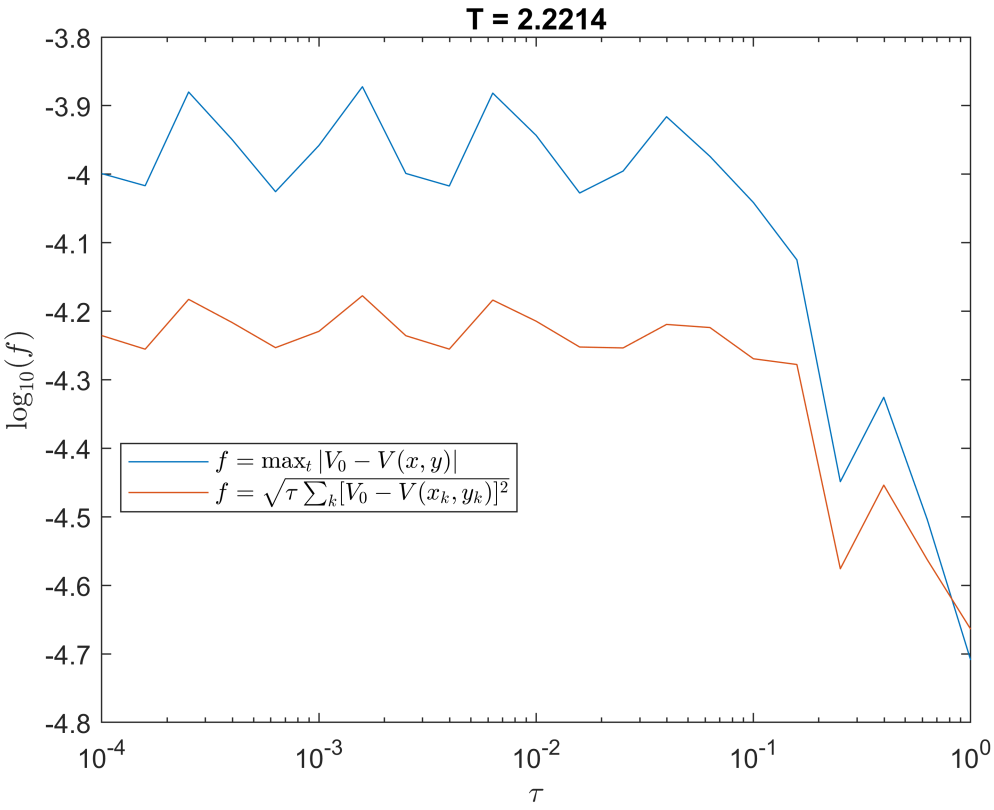
Now consider a fixed time step τ .

```
tau = logspace(-4, 0, 21).';

V_tau_max = zeros(size(tau));
V_tau_int = zeros(size(tau));
for k = 1 : length(tau)
    tspan = 0 : tau(k) : T;
    [t, z] = ode45(@(t, z) lotka_volterra(t, z, [alpha, beta, gamma, delta]), ...
        tspan, z_0);

    V_tau_max(k) = max(abs(V(z(:, 1), z(:, 2)) - V_0)); % C norm
    V_tau_int(k) = sqrt(tau(k)*sum((V(z(:, 1), z(:, 2)) - V_0).^2)); % L^2 norm
end
```

Plot the resulting error in bilog scale

```
figure;
% loglog(tau, V_tau_max);
semilogx(tau, log10(abs(V_tau_max)));
hold on;
% loglog(tau, V_tau_int);
semilogx(tau, log10(abs(V_tau_int)));
hold off;
xlabel('$\tau$', 'Interpreter', 'latex');
ylabel('$\log_{10}(f)$', 'Interpreter', 'latex');
legend('$f = \max_t |V_0 - V(x, y)|$', ...
    '$f = \sqrt{\tau \sum_k [V_0 - V(x_k, y_k)]^2}$', ...
```

```
    'Interpreter', 'latex', 'Location', 'best');
title(['T = ' num2str(T)])
```



Define function to solve Lotka-Volterra using ode45

```
function dz = lotka_volterra(~, z, par)
alpha = par(1);
beta = par(2);
gamma = par(3);
delta = par(4);

dz = zeros(2, 1);
dz(1) = z(1) * (alpha - beta*z(2));
dz(2) = z(2) * (-gamma + delta*z(1));
end
```