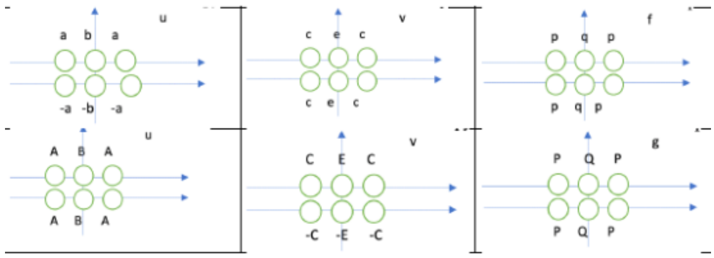
21 April 2022 09:29

$i\partial_t z + \partial_x^2 z + |z|^2 z = 0,$
$z = u + iv$

1) $\partial_t u = -\partial_x^2 v - \left(u^2+v^2\right)v$
2) $\partial_t v = \partial_x^2 u + \left(u^2+v^2\right)u$

1) $\partial_t u + \partial_x^2 v = f(u,v)$
2) $\partial_t v - \partial_x^2 u = g(u,v)$

$f(u,v) = -\left(u^2+v^2\right)v$
$g(u,v) = \left(u^2+v^2\right)u$



$$au_{j-1}^{n+1} + bu_j^{n+1} + au_{j+1}^{n+1} - au_{j-1}^{n} - bu_j^{n} - au_{j+1}^{n} + cv_{j-1}^{n+1} + ev_j^{n+1} + cv_{j+1}^{n+1} + cv_{j-1}^{n} + ev_j^{n} + cv_{j+1}^{n} = pf_{j-1}^{n+1} + qf_j^{n+1} + pf_{j+1}^{n+1} + pf_{j-1}^{n} + qf_j^{n} + pf_{j+1}^{n}$$
$$Au_{j-1}^{n+1} + Bu_j^{n+1} + Au_{j+1}^{n+1} + Au_{j-1}^{n} + Bu_j^{n} + Au_{j+1}^{n} + Cv_{j-1}^{n+1} + Ev_j^{n+1} + Cv_{j+1}^{n+1} - Cv_{j-1}^{n} - Ev_j^{n} - Cv_{j+1}^{n} = Pg_{j-1}^{n+1} + Qg_j^{n+1} + Pg_{j+1}^{n+1} + Pg_{j-1}^{n} + Qg_j^{n} + Pg_{j+1}^{n}$$

u_{test}	v_{test}	f_{test}	g_{test}	Equation
t	0	1	0	$a\tau + b\tau + a\tau + 0 + 0 + 0 + 6\cdot 0 = p + q + p + p + q + p; \tau(2a+b) = 4p + 2q$
$x^2 t$	0	x^2	$-2t$	$2a\tau h^2 = 4ph^2$
0	t	0	1	

Нормировки:
$b = 1$
$E = 1$

b = 1
c =
$\frac{\tau + 2a\tau}{2h^2}$
e =
$-\frac{\tau + 2a\tau}{h^2}$
p =
$\frac{a\tau}{2}$
q =
$\frac{\tau}{2}$

A -
$-\frac{\tau + 2C\tau}{2h^2}$
B -
$\frac{\tau + 2C\tau}{h^2}$
E - 1
P -
$\frac{C\tau}{2}$
Q =
$\frac{\tau}{2}$

Явный шаг по времени.
Схема Маккормака

Предиктор:

$$u_j^{n+\frac{1}{2}} = u_j - \frac{\tau}{h^2}\left(v_{j-1}^n - 2v_j^n + v_{j+1}^n\right) - \tau v_j^n \left(\left(u_j^n\right)^2 + \left(v_j^n\right)^2\right)$$

$$v_j^{n+\frac{1}{2}} = v_j + \frac{\tau}{h^2}\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right) + \tau u_j^n \left(\left(u_j^n\right)^2 + \left(v_j^n\right)^2\right)$$

Корректор:

$$u_j^{n+1} = \frac{u_j^n + u_j^{n+\frac{1}{2}}}{2} - \frac{\tau}{2h^2}\left(v_{j-1}^n - 2v_j^n + v_{j+1}^n\right) - \frac{\tau}{2} v_j^n \left(\left(u_j^n\right)^2 + \left(v_j^n\right)^2\right)$$

$$v_j^{n+1} = \frac{v_j^n + v_j^{n+\frac{1}{2}}}{2} + \frac{\tau}{2h^2}\left(u_{j-1}^n - 2u_j^n + u_{j+1}^n\right) + \frac{\tau}{2} u_j^n \left(\left(u_j^n\right)^2 + \left(v_j^n\right)^2\right)$$

Алгоритм:

1) Делаем явный шаг по времени по схеме, например, Маккормака. Обозначим получившиеся решения как $\tilde{u}$, $\tilde{v}$.
2) Представим точное решение как
   $u_j = \tilde{u}_j + \varepsilon_j,\ v_j = \tilde{v}_j + \delta_j$
3) Подставляем данные представления в компактную схему и линеаризуем ее.

$$a\left(u_{j-1}^{n+1} + u_{j+1}^{n+1}\right) + bu_j^{n+1} - a\left(u_{j-1}^n + u_{j+1}^n\right) - bu_j^n + c\left(v_{j-1}^{n+1} + v_{j+1}^{n+1}\right) + ev_j^{n+1} + c\left(v_{j-1}^n + v_{j+1}^n\right)$$
$$+ ev_j^n = p\left(f_{j-1}^{n+1} + f_{j+1}^{n+1}\right) + qf_j^{n+1} + p\left(f_{j-1}^n + f_{j+1}^n\right) + qf_j^n$$

$$f(u,v) = -\left(u^2 + v^2\right)v$$
$$g(u,v) = \left(u^2 + v^2\right)u$$

$$a\left(\tilde{u}_{j-1}^{n+1} + \varepsilon_{j-1} + \tilde{u}_{j+1}^{n+1} + \varepsilon_{j+1}\right) + b\left(\tilde{u}_j^{n+1} + \varepsilon_j\right) - a\left(u_{j-1}^n + u_{j+1}^n\right) - bu_j^n$$
$$+c\left(\tilde{v}_{j-1}^{n+1} + \delta_{j-1} + \tilde{v}_{j+1}^{n+1} + \delta_{j+1}\right) + e\left(\tilde{v}_j^{n+1} + \delta_j\right) + c\left(v_{j-1}^n + v_{j+1}^n\right) + ev_j^n$$
$$= p\left(-\left(\tilde{v}_{j-1}^{n+1} + \delta_{j-1}\right)\cdot\left(\left(\tilde{u}_{j-1}^{n+1} + \varepsilon_{j-1}\right)^2 + \left(\tilde{v}_{j-1}^{n+1} + \delta_{j-1}\right)^2\right) - \left(\tilde{v}_{j+1}^{n+1} + \delta_{j+1}\right)\cdot\left(\left(\tilde{u}_{j+1}^{n+1} + \varepsilon_{j-1}\right)^2 + \left(\tilde{v}_{j+1}^{n+1} + \delta_{j-1}\right)^2\right)\right) - q\left(\tilde{v}_j^{n+1} + \delta_j\right)$$
$$\cdot\left(\left(\tilde{u}_j^{n+1} + \varepsilon_j\right)^2 + \left(\tilde{v}_j^{n+1} + \delta_j\right)^2\right) + p\left(-v_{j-1}^n\left(\left(u_{j-1}^n\right)^2 + \left(v_{j-1}^n\right)^2\right) + -v_{j+1}^n\left(\left(u_{j+1}^n\right)^2 + \left(v_{j+1}^n\right)^2\right)\right) - qv_j^n\left(\left(u_j^n\right)^2 + \left(v_j^n\right)^2\right)$$