MCV4U

CALCULUS & VECTORS

January 2, 2024

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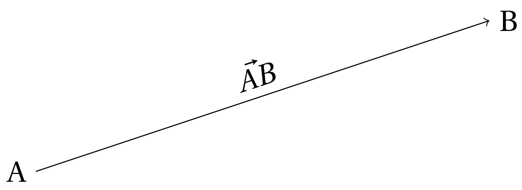
1 Vectors

Vectors are mathematical entities that extend our understanding beyond the one-dimensional quantities. Unlike scalar values that only have magnituide, vectors incorporate both magnitude and direction, offering a versatile toolkit for describing dynamic sytstems. Below are some examples of vectors and scalar quantites.

- **Scalar Quantities:** Mass, Temperature, Time, Distance, Speed, Energy, Work, Power, Pressure, Volume, Density
- **Vector Quantities:** Displacement, Velocity, Acceleration, Force, Momentum, Weight

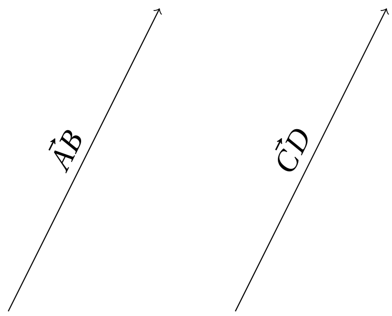
When written in mathematical equations, vectors are usually represented via a a symbol with a vector indicator (i.e $\vec{v}$) or via a jointery of the two points (i.e $\vec{AB}$ is a vector from point A to point B) Vectors can also be represented in many other ways, but the most common ways are: algebraically, numerically, and geometrically. Below are examples of each:

- **Algebraically:** $\vec{a} = \langle x, y \rangle$
- **Numerically:** $\vec{a} = [a, b, c]$ (Can also be written as a column matrix)
- **Geometrically:**

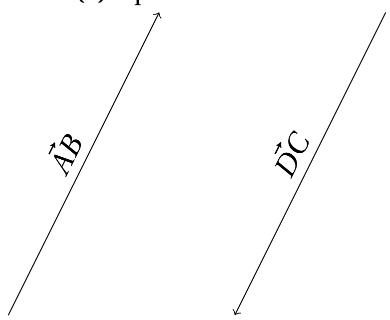


Vectors can be equal (or equivalent) to each other. For two vectors to be equal (or equivalent) they must have the same magnitude and direction. Vectors can also be opposite to each other; to be opposing vectors must have the same magnitude but opposite directions (i.e $\vec{v} = -\vec{v}$).

1.1 Vector Addition and Substraction



(a) Equivalent Vectors



(b) Equivalent Vectors

1.2 Scalar Multiplication

1.3 Properties of Vectors

1.4 Vectors as Forces

1.5 Vectors as Velocity

1.6 Vectors in R2

1.7 Algebraic Vectors in R3

1.8 Dot Product and Cross Product

1.9 Application of Dot and Cross Product

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