

MCV4U

CALCULUS & VECTORS

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1 Vectors

Vectors are mathematical entities that extend our understanding beyond the one-dimensional quantities.

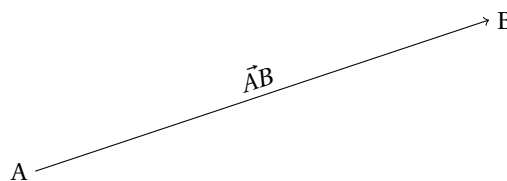
Unlike scalar values that only have magnitude, vectors incorporate both magnitude and direction, offering a versatile toolkit for describing dynamic systems.

Ex. Scalar Vs. Vector Quantities

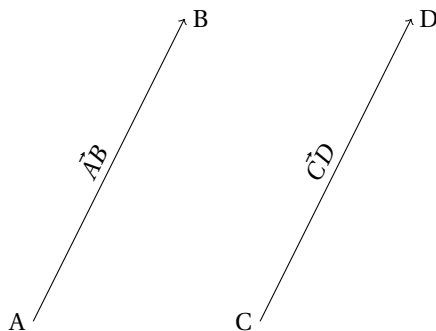
- **Scalar Quantities:** Mass, Temperature, Time, Distance, Speed, Energy, Work, Power, etc.
- **Vector Quantities:** Displacement, Velocity, Acceleration, Force, Momentum, etc.

When written in mathematical equations, vectors are usually represented via a symbol with a vector indicator (i.e. \vec{v}) or via a jointure of the two points (i.e. \vec{AB} is a vector from point A to point B). Vectors can also be represented in many other ways, but the most common ways are: algebraically, numerically, and geometrically. Below are examples of each:

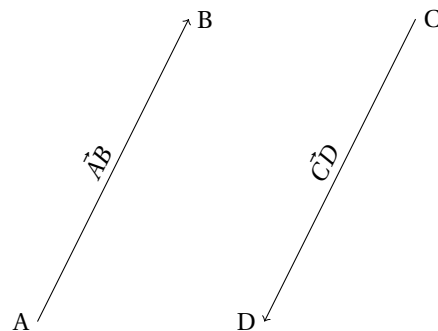
- **Algebraically:** $\vec{a} = \langle x, y \rangle$
- **Numerically:** $\vec{a} = [x, y, z]^1$
- **Geometrically:**



Vectors can be equal (or equivalent) to each other. For two vectors to be equal (or equivalent) they must have the same magnitude and direction. Vectors can also be opposite to each other; to be opposing vectors must have the same magnitude but opposite directions (i.e. $\vec{v} = -\vec{v}$).



(a) Equivalent Vectors



(b) Opposite Vectors

¹Numerical vectors can also be written as column matrices

1.1 Vector Addition and Subtraction

1.2 Scalar Multiplication**1.3 Properties of Vectors****1.4 Vectors as Forces****1.5 Vectors as Velocity****1.6 Vectors in \mathbb{R}^2** **1.7 Algebraic Vectors in \mathbb{R}^3** **1.8 Dot Product and Cross Product****1.9 Application of Dot and Cross Product****1.10 Scalar and Vector Projections****2 Lines and Planes****2.1 Vector, Parametric, and Symmetric Equations of a Line****2.2 Vector and Parametric Equations of a Plane****2.3 Cartesian (Scalar) Equation of a Plane****2.4 Intersection of a Lines and Planes****2.5 Intersection of Two Planes****2.6 Intersection of Three Planes****3 Limits and Continuity****3.1 Introduction to Limits****3.2 Special Limits with Trigonometric Functions****3.3 Asymptotes and Holes****3.4 Continuity****4 Derivatives****4.1 Slope of a Curved Line****4.2 The Derivative Function****4.3 Differentiability****4.4 Increasing/Decreasing Functions**

4.5 The Chain, Product, and Quotient Rules**4.6 Higher Order Derivatives****5 Curve Sketching**