Q1. (a) as $R(t) \in C[0,1] \cap C^1(0,1)$ then

$$\begin{aligned} R'\left(\frac{1}{2}\right) &= \frac{R(1) - R(0)}{1 - 0} \\ &= \frac{11 - 0}{1} \\ &= 11 \end{aligned}$$

$\therefore R'\left(\frac{1}{2}\right) = 11$

(b)

$$\begin{aligned} \int_0^1 R(t)dt &= (\frac{1}{3})(3) + (\frac{1}{3})(3) + (\frac{1}{3})(5) \\ &= 1 + 1 + \frac{5}{3} \\ &= \frac{6}{3} + \frac{5}{3} \\ &= \frac{11}{3} \end{aligned}$$

(c)

$$\begin{aligned} \int_0^{\frac{1}{3}} R'(t)dt &= R\left(\frac{1}{3}\right) - R\left(0\right) \\ &= 11 - 8 \\ &= 3 \end{aligned}$$

(d)

$$\sum_{k=1}^{n} R\left(\frac{1}{4} + \frac{k}{2n}\right) \cdot \frac{1}{2n} = \int_{\frac{1}{4}}^{\frac{3}{4}} R(t)dt$$