AP Calculus AP:

Notes, Formulas, Examples

Alexandru Stan

Start date: February 2, 2024
End date: May 13, 2024

Sections based off of Colleged Board units and Mrs. Cooper's Lessons.

Formatting may vary and be of differ in quality

Units

---

1 Limits and Continuity

The limit is when a given value approaches, or gets *really close* (infinitely) to another value. The standard limit notation is:

$$\lim_{x \to c} f(x)$$

represents when x can approach c from either left $(-)$ or the right $(+)$. By adding a sign superscript to the c, it means that x can only approach from that direction:

$$\lim_{x \to c^+} f(x)$$

Right hand limit, x approaches c from values greater than c

$$\lim_{x \to c^-} f(x)$$

Left hand limit, x approaches c from values lower than c

1.1 Limits to Infinity

If a degree (biggest exponent) of a polynomial is greater than or equal to 1, its limit as x approaches $\pm\infty$ will also be $\pm\infty$. This depends on the sign of the leading coefficient and the degree of polynomial

Example:

$$f(x) = 3x^3 - 7x^2 + 2$$

$$\lim_{x \to \infty} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

The degree of $f(x)$ is 3, and the leading coefficient is positive. The graph goes down to up from left to right.

With Fractions, just find whether the highest deree is the numerator or the denominator. Numerator means ∞, denominator means 0

1.2 Asympotes

Functions can have asymptotes, either vertical or horizontal. In the case of vertical asymptotes, the limit would be *unbounded* as it approaches that x value.

Example:

$$f(x) = \frac{2x-4}{x-3}$$
$$\lim_{x \to 3} f(x) = \text{undef}$$
$$\lim_{x \to 3^-} f(x) = -\infty$$
$$\lim_{x \to 3^+} f(x) = \infty$$

As with vertical asymptotes, as x approaches c (in this case $\pm\infty$), the limit would approach the horizontal asymptote. Although the y-value never actually touches the asymptote, the limit gets really close to the value, from both below and above

1.3 Limit Properties

The limits of combined functions can be found by finding the limit of each of the individual functions, then applying the operations.

- **Addition/Substraction**

  When taking the limit of the sum or difference of multiple functions, it's the same thing as taking the sum of difference of each of the seperate limits of each function

$$\lim_{x\to c}[f(x)+g(x)] \implies \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$$

$$\lim_{x\to c}[f(x)-g(x)] \implies \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$$

Note that when the limit of either function is *undefined* the combined limit would also be undefined

- **Multiplication**

  Multiplication of the limits of functions is quite straightforward

$$lim_{x\to c}[f(x)\cdot g(x)] \implies \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$$

  The same exception applies when one of the limits is *undefined.* This just makes the entire combined limit undefined

- **Division**

  Division is basically the same as the other basic operations except if the denominator is 0

$$\lim_{x\to c}\frac{f(x)}{g(x)} \implies \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$$

- **Composite Functions**

  When working with composite functions, it's the same thing as taking the limit of the inner function, then evaluating the outer function normally

$$\lim_{x\to c} f\Bigg(g(x)\Bigg) \implies f\Bigg(\lim_{x\to c} g(x)\Bigg)$$

1.4 Solving Limits

The first thing to always try to do when solving limits is **direct substitution**. If this is not possible (undefined limit), then algebraic manipulation (factoring) is the next step

$$
\begin{aligned}
& lim_{x\to c}\frac{x^4+3x^3-10x^2}{x^2-2x} \\
&= \lim_{x\to can}\frac{x^2(x^2+3x-10)}{x(x-2)} \\
&= \lim_{x\to c}\frac{x^2(x+5)(x-2)}{x(x-2)} \\
&= \lim_{x\to c} x^2(x+5)
\end{aligned}
$$

When encountering radicals, conjugates can be used.

$$
\begin{aligned}
& \lim_{x\to c}\frac{x+4}{\sqrt{3x+13}-1} \\
&= \lim_{x\to c}\frac{x+4}{\sqrt{3x+13}-1}\cdot\frac{\sqrt{3x+13}+1}{\sqrt{3x+13}+1} \\
&= \lim_{x\to c}\frac{(x+4)(\sqrt{3x+13}+1)}{3x+12} \\
&= \lim_{x\to c}\frac{(x+4)(\sqrt{3x+13}+1)}{3(x+4)} \\
&= \lim_{x\to c}\frac{\sqrt{3x+13}+1}{3}
\end{aligned}
$$

When dealing with trigonometric equations, trig identities can be used (assuming direct substitution doesn't work)

$$
\begin{aligned}
&\lim_{x \to c} \frac{\cot^2(x)}{1 - \sin(x)} \\
&= \lim_{x \to c} \frac{\cos^2(x)}{(\sin^2(x))(1 - \sin(x))} \\
&= \lim_{x \to c} \frac{1 - \sin^2(x)}{(\sin^2(x))(1 - \sin(x))} \\
&= \lim_{x \to c} \frac{(1 + \sin(x))(1 - \sin(x))}{(\sin^2(x))(1 - \sin(x))} \\
&= \lim_{x \to c} \frac{1 + \sin(x)}{\sin^2(x)}, \text{for x} \neq (2k+1)\frac{\pi}{2}
\end{aligned}
$$

However, functions can not always be factored, so in that case they will just be undefined

$$
\begin{aligned}
&\lim_{x \to 1} \frac{2x}{x^2 - 7x + 6} \\
&= \lim_{x \to 1} \frac{2x}{(x-6)(x-1)} \\
&= \frac{2}{0} \\
&= \text{undef}
\end{aligned}
$$

1.5 Continuity