AP Calculus AB: Free-Response Question

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Introduction

A Calculator is not allowed for this question, and, all AP guidlines (as stated below) are to be followed.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Questions

$t_{\text{(hours)}}$	1	3	5	6	9
$s(t)_{\text{(sales per hour)}}$	2	11	8	5	2

The rate at which cars are sold at a dealership is modeled by $s(t)$ where $s(t)$ is continous, twice-differentiable function. S is measured in sales per hour and t is measured in hours where $0 \leq t \leq 11$ such that $t = 0$ is 8am. Values are given in the above table for selected values of t.

(a) Using the Fundamental Theorem of Calculus, determine the definite inegral notation for $s(t)$ over the interval $[a, b]$.

(b) Use a right Riemann sum with the four subinvtervals indicated by the data in the table to approximate $\int_3^6 R(t)dt$. Indicate units of measure.

(c) Is the approximation in part (b) an overestimate or underestimate of $\int_3^6$?

(d) The sum $\sum_{k=1}^{n} S\left(3 + \frac{2(k-1)}{n}\right)\frac{2}{n}$ is a left Riemann sum with n subintervals of equal length. The limit of this sum as n goes to ∞ can be stated as a definite integrals. Express the limit as a definite integral.

Solutions

(a) **[1 pt]** To receive the first point, the student must understand correctly write out the integral over the interval $[a, b]$. In this case, the student must write out the integral as follows:

$$\int_a^b s(t)dt$$

[1 pt] Another point is then awarded if the student is able to express in mathematical notation that the definite integral can be evaluated at the end points and then those end points can be substracted:

$$\int_a^b s(t)dt = S(b) - S(a)$$

Note: The student is allowed to used the following notation in their answer, but, only this notation and the one that preceeds it are allowed.

$$\int_a^b s(t)dt = S(t)|_a^b$$

[1 pt] A final point is awarded if the student is able to correcly state the Fundamental Theorem of Calulus. In this case, stating that the above is true as $S(t)$ is the antiderivative of $s(t)$

(b) **[1pt]** The student is awarded a point for recognizing the right Riemann Sum and plugging in the correct values as shown below

$$(6)(5) + (5)(8)$$

[1pt] Another point is then awarded for returning a correct answer of 70, and for stating the correct units of measurement, in this case, *sales*

(c) **[1pt]** A point is awarded for recognising and stating that in the interval $[3, 6]$ the function is decreasing and concave down.

[1 pt] Another point is awarded for stating that due to the above reasons, the approximation from the right Riemann sum is an overestimate of the actual value of the integral.

(d) **[1 pt]** The student is awarded a point of recognising the correct upper and lower bounds (where a is the lower bound and b is the upper bound) of the integral, in this case, $a = 3$ and $b = 5$

[1 pt] Another point is awarded for the student correctly recognizing the intergrand and correctly stating all the above factors in the form of a definite integral, as shown below

$$\int_3^5 s(t)dt$$