

MCV4U

CALCULUS & VECTORS

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1 Vectors

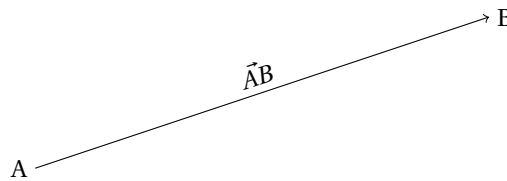
Vectors are mathematical entities that extend our understanding beyond the one-dimensional quantities.

Unlike scalar values that only have magnitude, vectors incorporate both magnitude and direction, offering a versatile toolkit for describing dynamic systems. Below are some examples of vectors and scalar quantities.

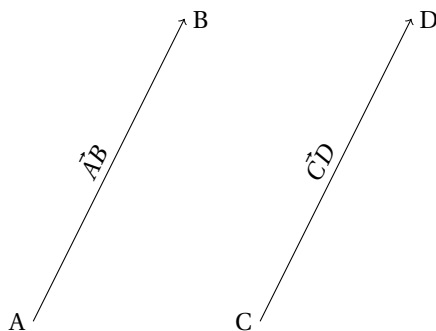
- **Scalar Quantities:** Mass, Temperature, Time, Distance, Speed, Energy, Work, Power, Pressure, Volume, Density
- **Vector Quantities:** Displacement, Velocity, Acceleration, Force, Momentum, Weight

When written in mathematical equations, vectors are usually represented via a symbol with a vector indicator (i.e. \vec{v}) or via a jointure of the two points (i.e. \vec{AB} is a vector from point A to point B). Vectors can also be represented in many other ways, but the most common ways are: algebraically, numerically, and geometrically. Below are examples of each:

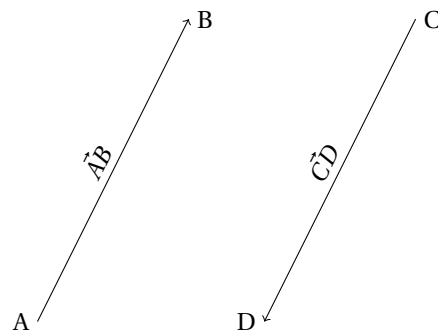
- **Algebraically:** $\vec{a} = \langle x, y \rangle$
- **Numerically:** $\vec{a} = [x, y, z]^1$
- **Geometrically:**



Vectors can be equal (or equivalent) to each other. For two vectors to be equal (or equivalent) they must have the same magnitude and direction. Vectors can also be opposite to each other; to be opposite vectors must have the same magnitude but opposite directions (i.e. $\vec{v} = -\vec{v}$).



(a) Equivalent Vectors



(b) Opposite Vectors

¹Numerical vectors can also be written as column matrices

1.1 Vector Addition and Subtraction

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4.5 The Chain, Product, and Quotient Rules**4.6 Higher Order Derivatives****5 Curve Sketching**