Université de Paris

Algorithmie Avancée

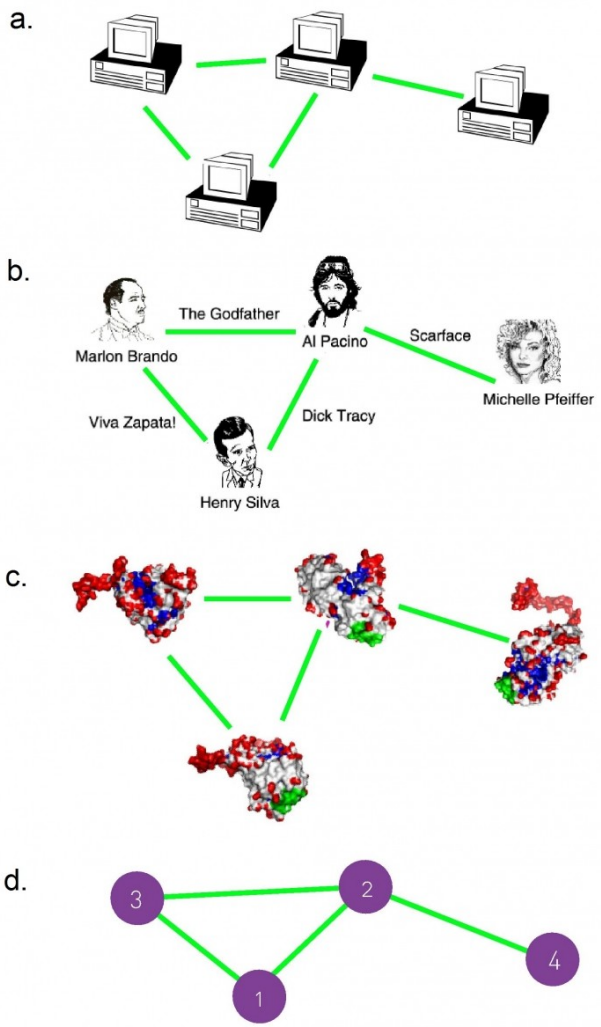
Mise en Contexte / Mise en Oeuvre

Année 2020-2021 par Prof. Nicolas Loménie
Sur la base du cours de Prof. Etienne Birmelé (2016-2020)

Etre capable de comprendre et d'analyser un monde interconnecté
De la théorie des graphes vers la théorie des réseaux

(From Graph Theory to Network Science ; The Network Science by Prof. Barasabi)

Mise en Contexte



Different Networks, Same Graph
The figure shows a small subset of (a) the Internet, where routers (specialized computers) are connected to each other; (b) the Hollywood actor network, where two actors are connected if they played in the same movie; (c) a protein-protein interaction network, where two proteins are connected if there is experimental evidence that they can bind to each other in the cell. While the nature of the nodes and the links differs, these networks have the same graph representation, consisting of N = 4 nodes and L = 4 links, shown in (d).

http://networksciencebook.com/chapter/2#networks-graphs

Small World Concept

Network	Nodes	Links	Directed / Undirected	N	L	‹K›
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.34
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Mobile-Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorships	Undirected	23,133	93,437	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Papers	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

Table 2.1

Canonical Network Maps

The basic characteristics of ten networks used throughout this book to illustrate the tools of network science. The table lists the nature of their nodes and links, indicating if links are directed or undirected, the number of nodes (N) and links (L), and the average degree for each network. For directed networks the average degree shown is the average in- or out-degrees $\langle k \rangle = \langle k_{in} \rangle = \langle k_{out} \rangle$ (see Equation (2.5)).

http://networksciencebook.com/chapter/2#networks-graphs

Théorie des Graphes 5

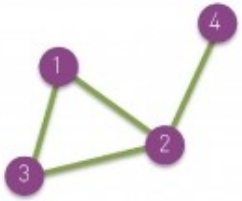
- AlgoAvanceeParE_Birmele.pdf

  Support de cours de Prof. Etienne Birmelé

  Planche 84 à 104 (MST Prim, PCC Dijkstra)

Graphologie

a. Undirected

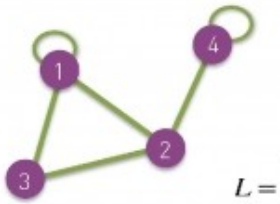


$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$A_{ii} = 0 \qquad A_{ij} = A_{ji}$

$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{2L}{N}$

b. Self-loops

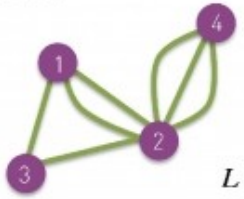


$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$\exists i,\ A_{ii} \neq 0 \qquad A_{ij} = A_{ji}$

$L = \frac{1}{2}\sum_{i,j=1, i\neq j}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii} \qquad ?$

c. Multigraph
(undirected)

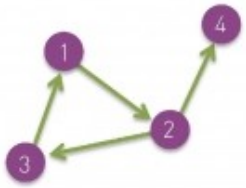


$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

$A_{ii} = 0 \qquad A_{ij} = A_{ji}$

$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{2L}{N}$

d. Directed

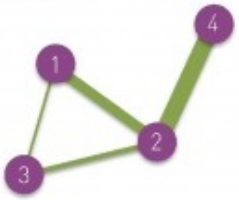


$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$A_{ij} \neq A_{ji}$

$L = \sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{L}{N}$

e. Weighted
(undirected)

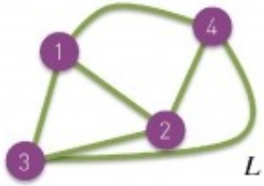


$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

$A_{ii} = 0 \qquad A_{ij} = A_{ji}$

$< k >= \frac{2L}{N}$

f. Complete Graph
(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$A_{ii} = 0 \qquad A_{i\neq j} = 1$

$L = L_{\max} = \frac{N(N-1)}{2} \qquad < k >= N - 1$

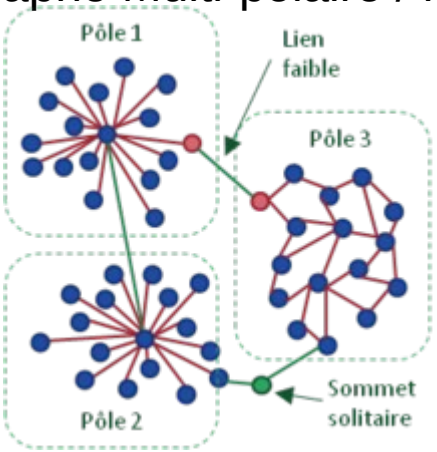
In network science we often distinguish networks by some elementary property of the underlying graph. Here we summarize the most commonly encountered network types. We also list real systems that share the particular property. Note that many real networks combine several of these elementary network characteristics. For example the WWW is a directed multi-graph with self-interactions; the mobile call network is directed and weighted, without self-loops

http://networksciencebook.com/chapter/2#networks-graphs

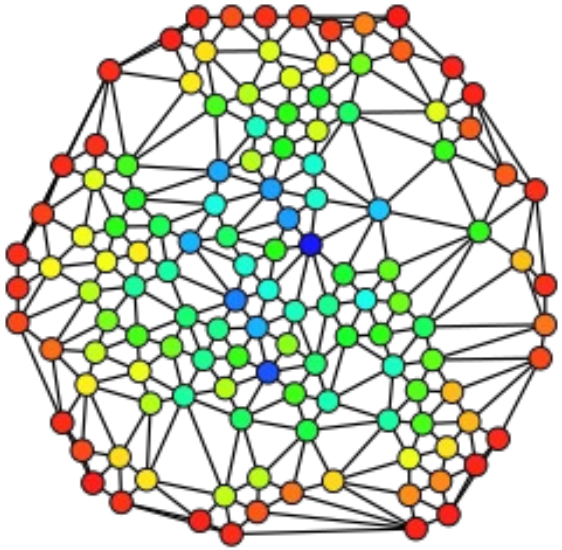
Graphologie

Génération aléatoire de grands graphes via matrice d'adjacence ou par récursion parfois
Plus le problème de la visualisation ergonomique

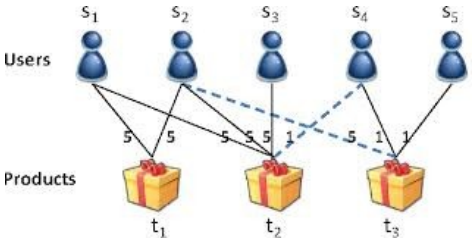
Graphe Multi-polaire / réseaux de neurones

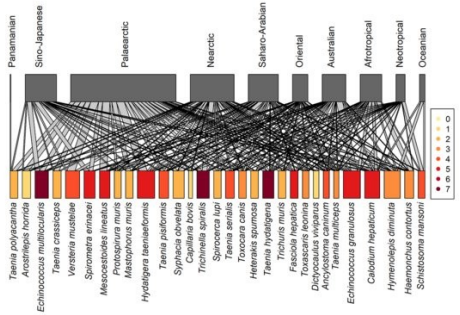


$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$



Graphe bi-partite : Web of Life





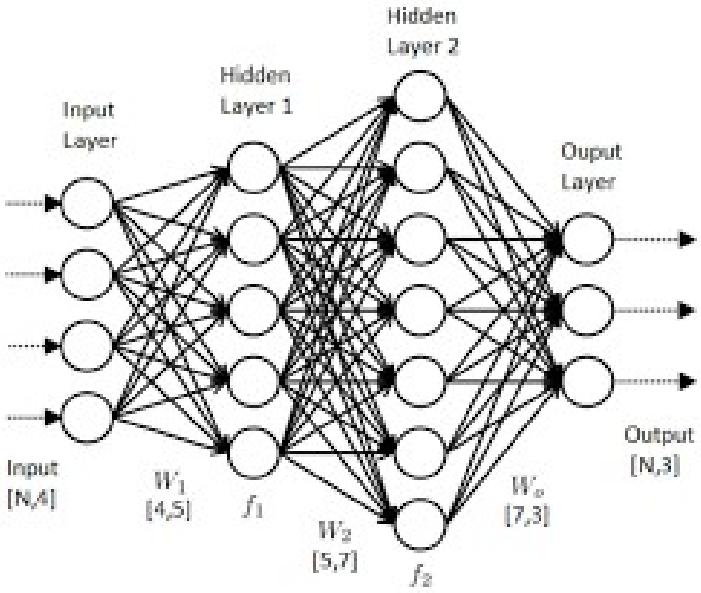
Problématiques algorithmiques d'implémentation,
de visualisation, de manipulation etc. ,
toujours d'actualité : package R, etc.

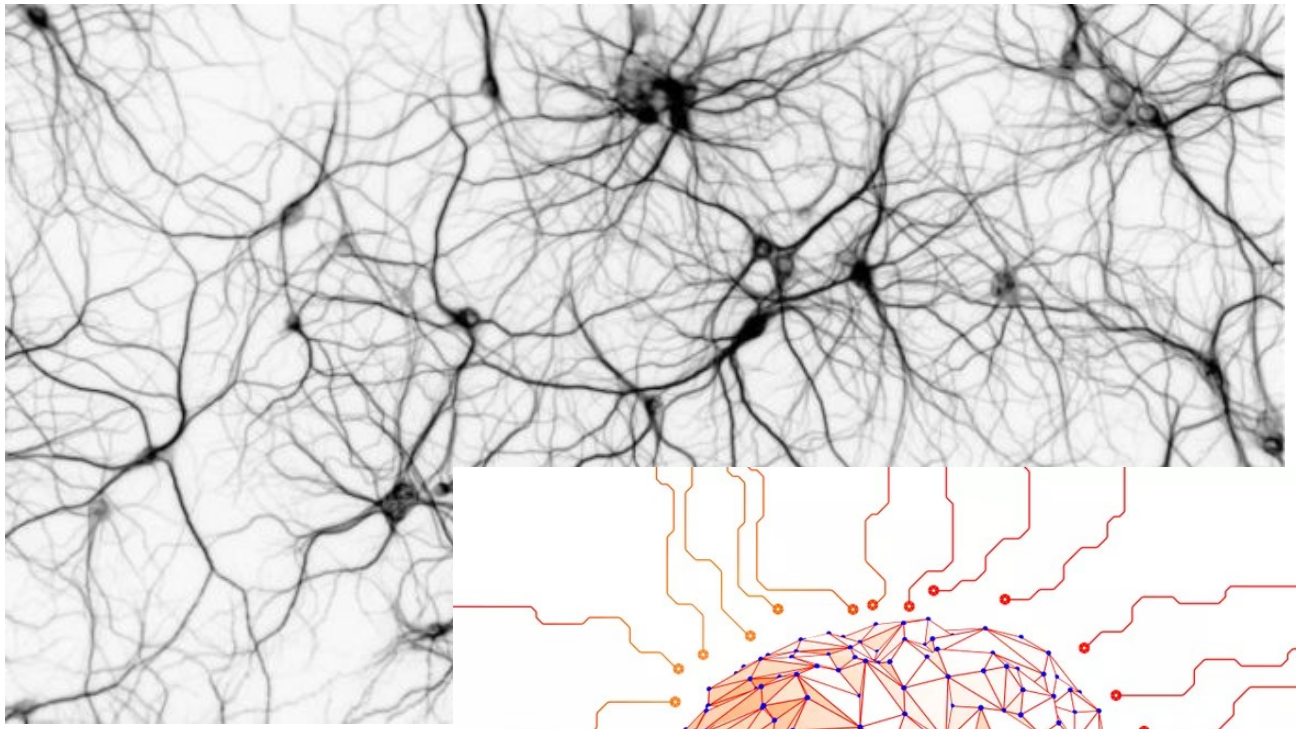
https://fr.wikipedia.org/wiki/Th%C3%A9orie_des_graphes

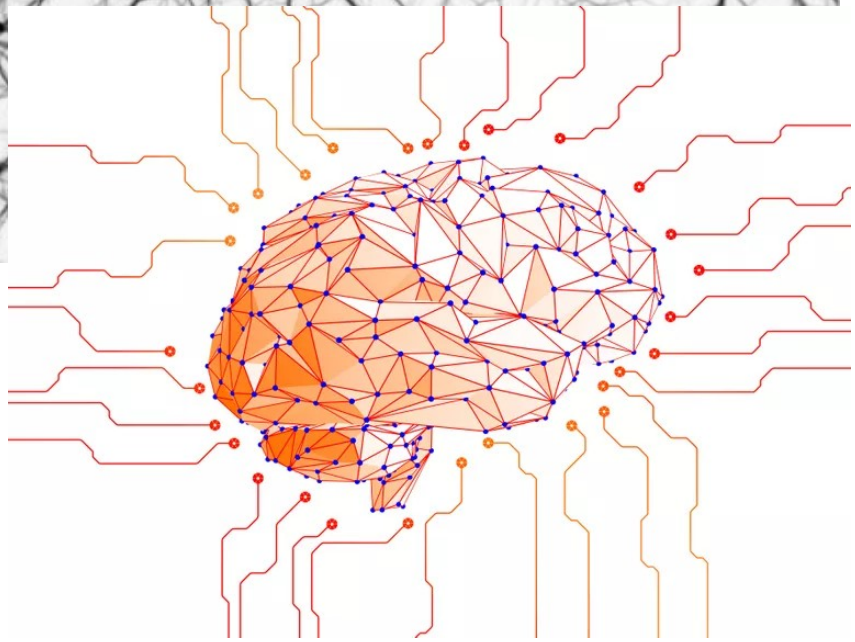
http://www.web-of-life.es/

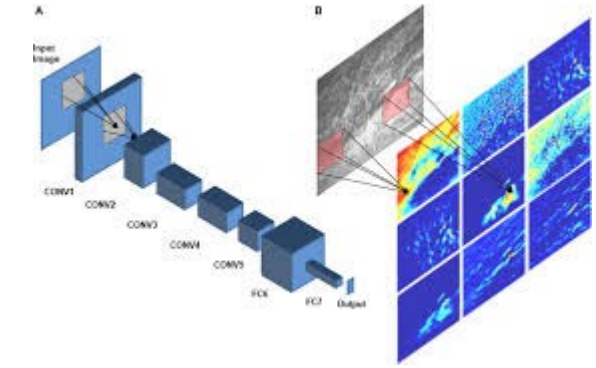
https://fr.wikipedia.org/wiki/Centralit%C3%A9

Graphologie









https://towardsdatascience.com/a-comprehensive-guide-to-convolutional-neural-networks-the-eli5-way-3bd2b1164a53