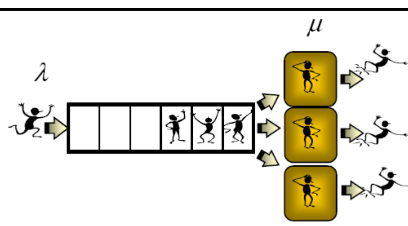
Réseaux, Performances et Sécurité

Cours 10: *M/M/s*

Osman SALEM
osman.salem@parisdescartes.fr
Maître de Conférences - HDR

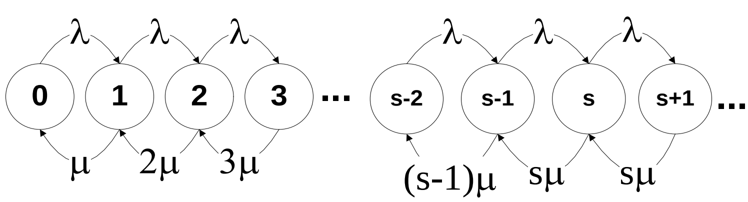
UNIVERSITÉ PARIS DESCARTES
UFR DE MATHÉMATIQUES ET INFORMATIQUE

M/M/s (s > 1)



- s serveurs identiques $\rho = \frac{\lambda}{\mu}$
- λ et μ sont les taux d'arrivées et de service
- ρ est le taux d'utilisation d'un serveur ρ<1

$$\lambda_n = \lambda, \text{ pour } n = 0, 1, 2, \ldots..$$
$$\mu_n = n\mu, \text{ pour } n = 1, 2, \ldots, s$$
$$= s\mu, \text{ pour } n = s, s+1, \ldots$$



M/M/s (cont.)

états	entrant = sortant
0	$\mu P_1 = \lambda P_0$
1	$2\mu P_2 + \lambda P_0 = (\lambda + \mu)\, P_1$
2	$3\mu P_3 + \lambda P_1 = (\lambda + 2\mu)\, P_2$
....	....
s-1	$s\mu P_s + \lambda P_{s-2} = \{\lambda + (s-1)\mu\} P_{s-1}$
s	$s\mu P_{s+1} + \lambda P_{s-1} = (\lambda + s\mu)\, P_s$
s+1	$s\mu P_{s+2} + \lambda P_s = (\lambda + s\mu)\, P_{s+1}$
....	....

M/M/s (cont.)

- P_1, P_2, P_3 ... En fonction de P_0

  $P_1 = (\lambda / \mu)\, P_0$

  $P_2 = (\lambda / 2\mu)\, P_1 = (1/2!) \times (\lambda / \mu)^2\, P_0$

  $P_3 = (\lambda / 3\mu)\, P_2 = (1/3!) \times (\lambda / \mu)^3\, P_0$

  ....

  $P_s = (1/s!) \times (\lambda / \mu)^s\, P_0$

  $P_{s+1} = (1/s) \times (\lambda / \mu)\, P_s = \dfrac{(\lambda/\mu)^s}{s!} \times \dfrac{\lambda}{s\mu} P_0$

M/M/s (cont.)

$$P_{s+2} = (1/s)\cdot(\lambda/\mu)P_{s+1} = \frac{(\lambda/\mu)^s}{s!}\left(\frac{\lambda}{s\mu}\right)^2 P_0$$

$\ldots$

$$P_{s+j} = (1/s)\cdot(\lambda/\mu)P_{s+j-1} = \frac{(\lambda/\mu)^s}{s!}\left(\frac{\lambda}{s\mu}\right)^j P_0$$

$\ldots$

M/M/s (cont.)

$$P_n = \frac{(\lambda/\mu)^n}{n!} P_0$$ pour n = 1, 2, …., s

$$P_n = \frac{(\lambda/\mu)^s}{s!}\left(\frac{\lambda}{s\mu}\right)^{n-s} P_0$$ pour n = s+1,s+2,...

$$\sum_{n=0}^{\infty} P_n = 1 = \sum_{n=0}^{c} P_n + \sum_{n=c}^{\infty} P_n$$

$$= \sum_{n=0}^{c} \frac{(\lambda/\mu)^n}{n!} P_0 + \sum_{n=c}^{\infty} \frac{(\lambda/\mu)^s}{s!}\left(\frac{\lambda}{s\mu}\right)^{n-s} P_0 = 1$$

M/M/s (cont.)

$$P_n = \frac{(\lambda/\mu)^n}{n!} P_0, \qquad \text{Si } 0 \le n \le s$$

$$= \frac{(\lambda/\mu)^n}{s!s^{n-s}} P_0, \qquad \text{Si } n \ge s$$

Si $\lambda < s\mu$ =>

$$P_0 = \left\{ \sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \sum_{n=s}^{\infty} (\lambda/s\mu)^{n-s} \right\}^{-1}$$

$$= \left\{ \sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \frac{1}{1-(\lambda/s\mu)} \right\}^{-1}$$

M/M/s (cont.)

L_q: avec $\rho = \lambda / \mu$

$$L_q = \sum_{n=0}^{s-1} (n-s) P_n =$$

$$= \sum_{j=0}^{\infty} j P_{s+j} \qquad avec\ n = s + j$$

$$= \sum_{j=0}^{\infty} j \frac{(\lambda/\mu)^{s+j}}{s!s^j} P_0 = \frac{(\lambda/\mu)^s}{s!} P_0 \sum_{j=0}^{\infty} j \frac{(\lambda/\mu)^j}{s^j} =$$

$$= \frac{(\lambda/\mu)^s \rho_s}{s!(1-\rho_s)^2} P_0$$

M/M/s (cont.)

$$W_q = \frac{L_q}{\lambda} = \frac{\left(\frac{\lambda}{\mu}\right)^s \rho_s}{\lambda . s!(1-\rho_s)^2} P_0$$

$$W_s = W_q + \frac{1}{\mu} = \frac{\left(\frac{\lambda}{\mu}\right)^s \rho_s}{\lambda . s!(1-\rho_s)^2} P_0 + \frac{1}{\mu}$$

$$L_s = \lambda\left(W_q + 1/\mu\right) = L_q + \frac{\lambda}{\mu} = \frac{\left(\frac{\lambda}{\mu}\right)^s \rho_s}{.s!(1-\rho_s)^2} P_0 + \frac{\lambda}{\mu}$$