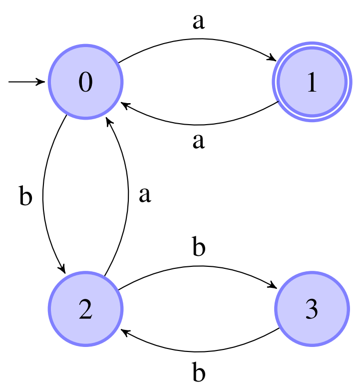
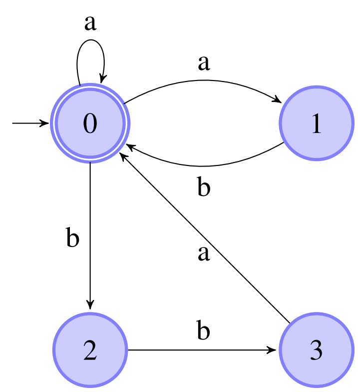
Théorie des Langages – Feuille n° 5

AUTOMATES FINIS ET LANGAGES RÉGULIERS

CORRECTION

Exercice 1 - Soit $\Sigma = \{a, b\}$. Soient les automates M_1 (à gauche) et M_2 (à droite) suivants. En utilisant **le théorème d'Arden**, donnez sous forme d'expressions régulières les langages $\mathcal{L}(M_1)$ et $\mathcal{L}(M_2)$.





Automate M_1

$$\begin{aligned} L_0 &= aL_1 + bL_2 \\ L_1 &= aL_0 + \varepsilon \\ L_2 &= aL_0 + bL_3 \\ L_3 &= bL_2 \end{aligned}$$

On commence par calculer L_3, nécessaire pour calculer $\mathcal{L}(M_1) = L_0$.

$$\begin{aligned} L_3 &= bL_2 \\ &= bbL_3 + baL_0 && \varepsilon \notin bb, \text{ le théorème d'Arden nous donne une solution unique} \\ &= (bb)^*baL_0 \end{aligned}$$

On calcule maintenant $\mathcal{L}(M_1) = L_0$.

$$\begin{aligned} L_0 &= aL_1 + bL_2 \\ &= aaL_0 + a + baL_0 + bbL_3 \\ &= (aa+ba)L_0 + a + bb(bb)^*baL_0 \\ &= (aa+ba+bb(bb)^*ba)L_0 + a && \varepsilon \notin (aa+ba+bb(bb)^*ba), \text{ Arden solution unique} \\ &= (aa+ba+bb(bb)^*ba)^*a && \text{on simplifie} \\ &= (aa+(\varepsilon+bb(bb)^*)ba)^*a \\ &= (aa+(bb)^*ba)^*a \end{aligned}$$

On obtient $\mathcal{L}(M_1) = (aa+(bb)^*ba)^*a$

Automate M_2

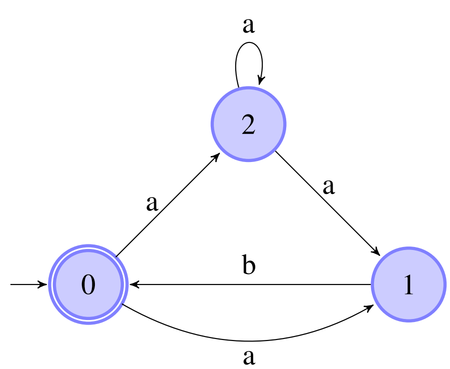
$$\begin{array}{rcl} L_0 & = & a(L_0+L_1)+bL_2+\varepsilon \\ L_1 & = & bL_0 \\ L_2 & = & bL_3 \\ L_3 & = & aL_0 \end{array}$$

On calcule $\mathcal{L}(M_2) = L_0$.

$$\begin{array}{rcll} L_0 & = & a(L_0+L_1)+bL_2+\varepsilon & \\ & = & aL_0+abL_0+bbL_3+\varepsilon & \\ & = & aL_0+abL_0+bbaL_0+\varepsilon & \\ & = & (a+ab+bba)L_0+\varepsilon & \varepsilon \notin (a+ab+bba), \text{Arden solution unique} \\ & = & (a+ab+bba)^* & \end{array}$$

On obtient $\mathcal{L}(M_2) = (a+ab+bba)^*$

Exercice 2 - Soit $\Sigma = \{a,b\}$. Soit l'automate M suivant. Montrez, **en utilisant le théorème d'Arden**, que $\mathcal{L}(M) = (aa^*b)^*$



$$\begin{array}{rcl} L_0 & = & aL_1+aL_2+\varepsilon \\ L_1 & = & bL_0 \\ L_2 & = & aL_2+aL_1 \end{array}$$

On commence par calculer L_2, nécessaire pour calculer $\mathcal{L}(M) = L_0$.

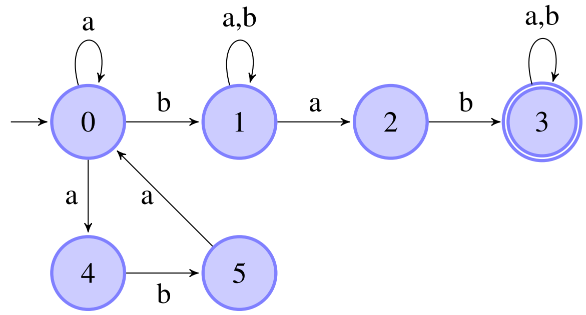
$$\begin{array}{rcll} L_2 & = & aL_2+aL_1 & \varepsilon \notin a, \text{le théorème d'Arden nous donne une solution unique} \\ & = & a^*aL_1 & \end{array}$$

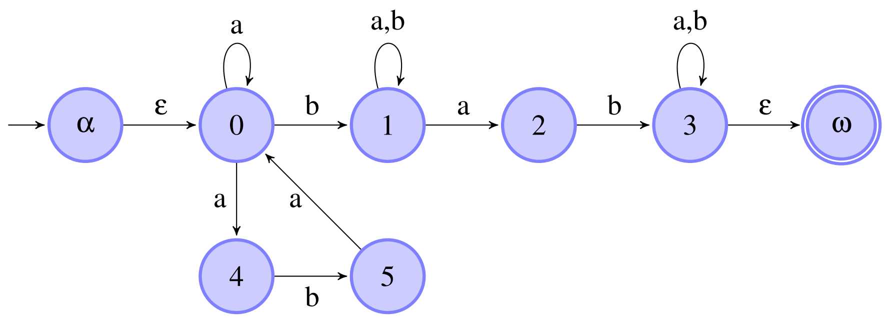
On calcule maintenant $\mathcal{L}(M) = L_0$.

$$\begin{array}{rcll} L_0 & = & aL_1+aL_2+\varepsilon & \\ & = & abL_0+a^*aL_1+\varepsilon & \\ & = & abL_0+a^*abL_0+\varepsilon & \\ & = & (ab+a^*ab)L_0+\varepsilon & \varepsilon \notin (ab+a^*ab), \text{Arden solution unique} \\ & = & (ab+a^*ab)^* & \text{on simplifie} \\ & = & ((\varepsilon+aa^*)ab)^* & \\ & = & (a^*ab)^* = (aa^*b)^* & \end{array}$$

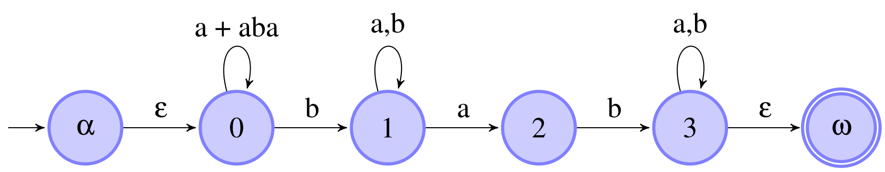
Exercice 3 - Par la méthode **d'élimination des états**, donnez les expressions régulières équivalentes aux automates suivants :

1. Automate M_1

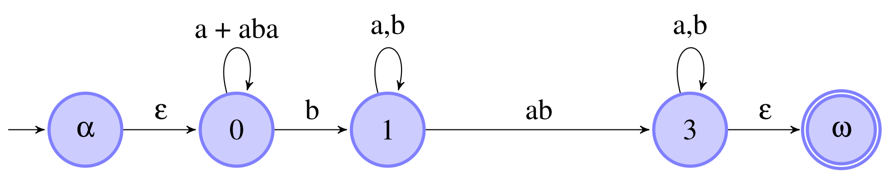




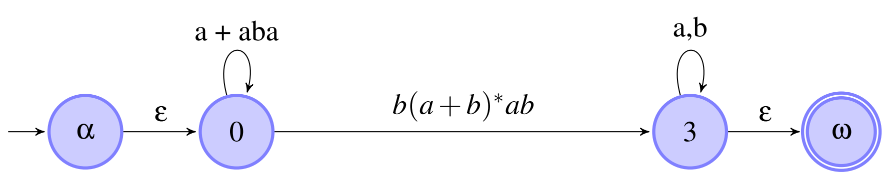
Elimination des états 4 et 5 :



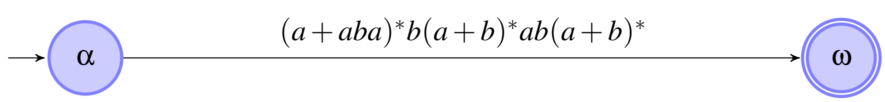
Elimination de l'état 2 :



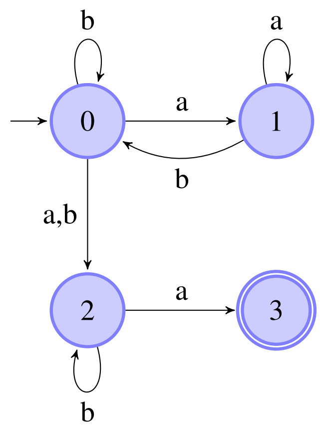
Elimination de l'état 1 :

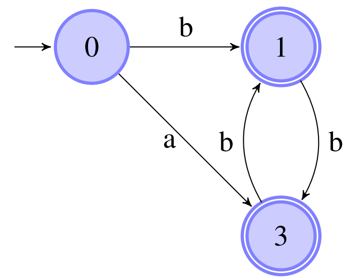


Elimination des états 0 et 3 :

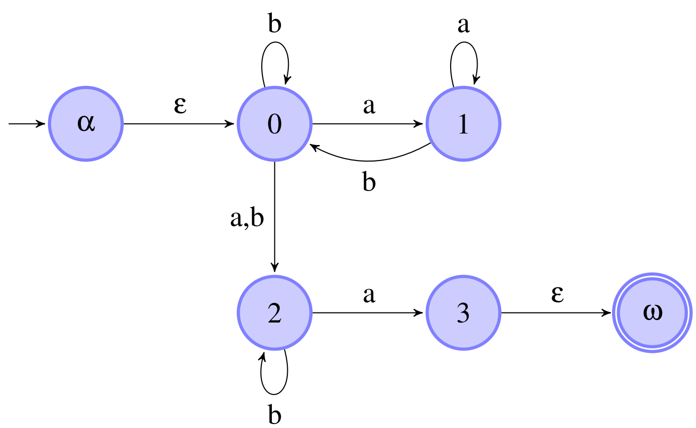


2. Automate M_2 (à gauche) et automate M_3 (à droite)

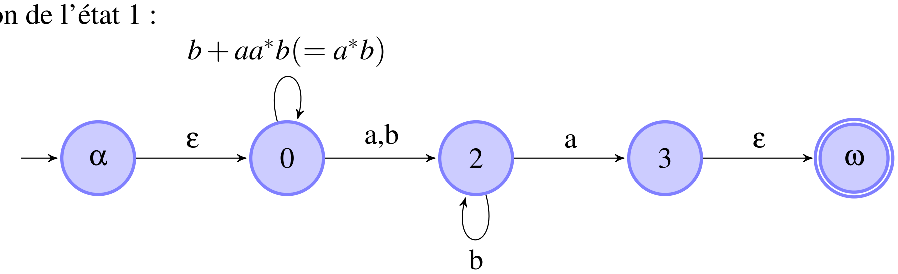




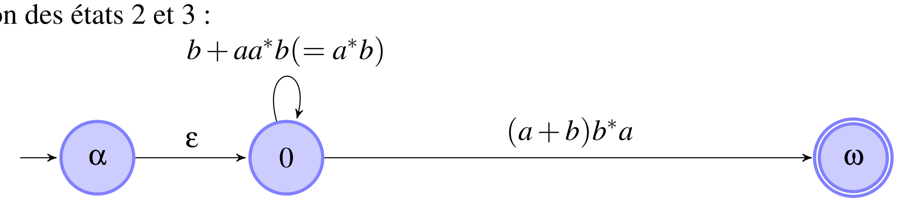
Automate M_2



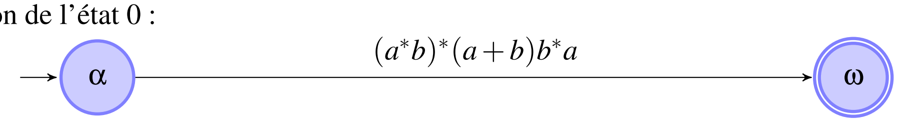
Elimination de l'état 1 :



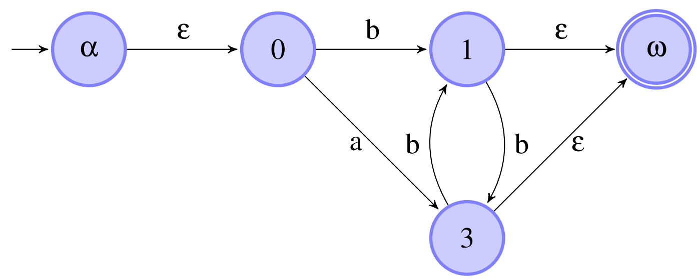
Elimination des états 2 et 3 :



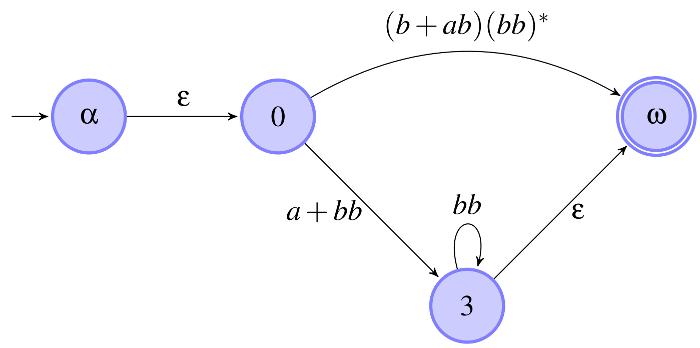
Elimination de l'état 0 :



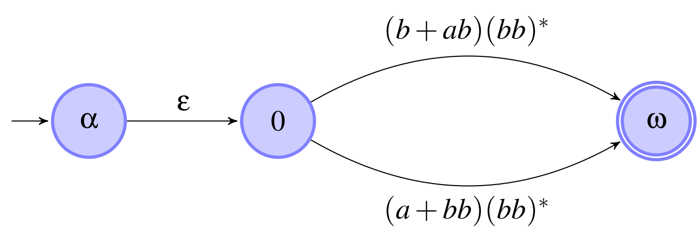
Automate M_3



Elimination de l'état 1 :



Elimination de l'état 3 :



$$
\begin{aligned}
\mathcal{L}(M_3) &= (b+ab)(bb)^* + (a+bb)(bb)^* \\
&= (b+a+bb+ab)(bb)^* \\
&= (b+a)(\varepsilon+b)(bb)^* \\
&= (b+a)((bb)^* + b(bb)^*) \\
&= (b+a)b^*
\end{aligned}
$$