Complex Networks
Centralities

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April 18, 2022

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1 Exercise 1

The formula for the information centrality of each node is displayed bellow.

$$C_I(V) = \frac{1}{C_{VV} + \frac{(T-2R)}{N}}, C = A^{-1} \tag{1}$$

The $C_{VV} + (T - 2R)$ value is inversely proportional to the closeness of vertex V and by dividing this value with N the value is normalized. Thus, the equation (1) actually represents the **closeness centrality** of the node V on a **weighted graph**.

2 Exercise 2

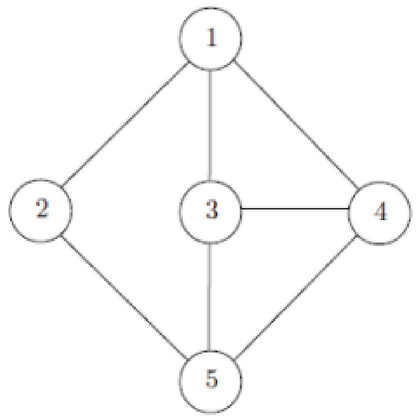
The correlation between **Node Betweenness Centrality** and **Edge Betweenness Centrality** is as follows.

$$C_B(V) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(V)}{\sigma_{st}} \Rightarrow C_B(V) = \frac{1}{2} \sum_{s \neq v \neq t} a_{st} \frac{\sigma_{st}(E)}{\sigma_{st}} \Rightarrow C_B(V) = \frac{1}{2} \sum_{s,t} a_{st} \frac{\sigma_{st}(E)}{\sigma_{st}} - 2(N-1)$$

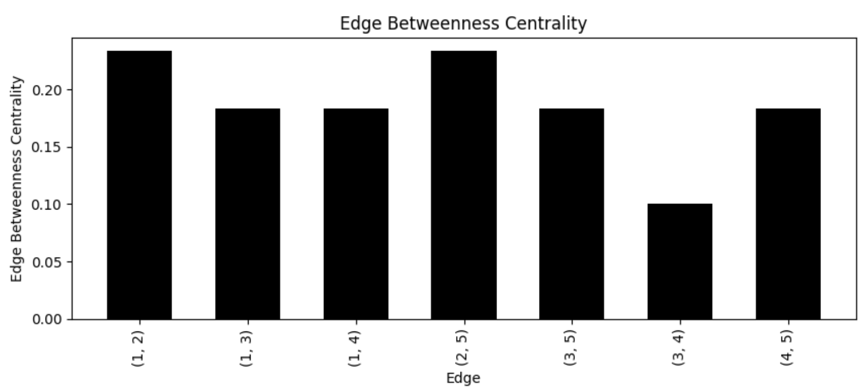
$$\Rightarrow C_B(V) = \frac{1}{2} \sum_{s,t} a_{st} C_B(E) - 2(N-1)$$

3 Exercise 3

The **edge betweenness centrality** of the graph bellow is calculated using NetworkX. All shortest paths are also listed bellow.



3.1 Edge Betweenness Centrality



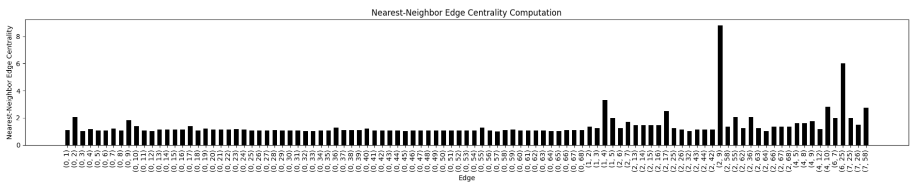
3.2 Shortest Paths

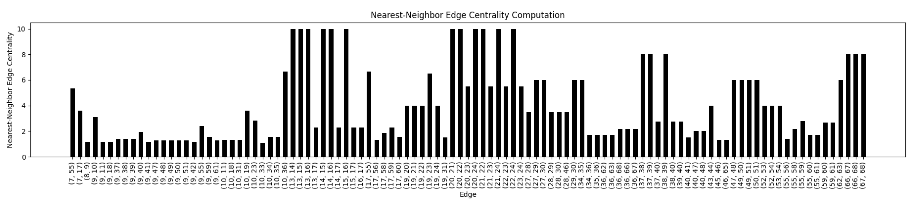
- Shortest Paths from 1 to 2: [[1, 2]]
- Shortest Paths from 1 to 3: [[1, 3]]
- Shortest Paths from 1 to 4: [[1, 4]]
- Shortest Paths from 1 to 5: [[1, 2, 5], [1, 3, 5], [1, 4, 5]]
- Shortest Paths from 2 to 1: [[2, 1]]
- Shortest Paths from 2 to 3: [[2, 1, 3], [2, 5, 3]]
- Shortest Paths from 2 to 4: [[2, 1, 4], [2, 5, 4]]
- Shortest Paths from 2 to 5: [[2, 5]]
- Shortest Paths from 3 to 1: [[3, 1]]
- Shortest Paths from 3 to 2: [[3, 1, 2], [3, 5, 2]]
- Shortest Paths from 3 to 4: [[3, 4]]
- Shortest Paths from 3 to 5: [[3, 5]]
- Shortest Paths from 4 to 1: [[4, 1]]
- Shortest Paths from 4 to 2: [[4, 1, 2], [4, 5, 2]]
- Shortest Paths from 4 to 3: [[4, 3]]
- Shortest Paths from 4 to 5: [[4, 5]]
- Shortest Paths from 5 to 1: [[5, 2, 1], [5, 3, 1], [5, 4, 1]]
- Shortest Paths from 5 to 2: [[5, 2]]
- Shortest Paths from 5 to 3: [[5, 3]]
- Shortest Paths from 5 to 4: [[5, 4]]

4 Exercise 4

The **nearest-neighbor edge centrality** of each edge is computed with the formula bellow and can be observed in the provided bar charts. A CSV file is also submitted to show more accurate measurements.

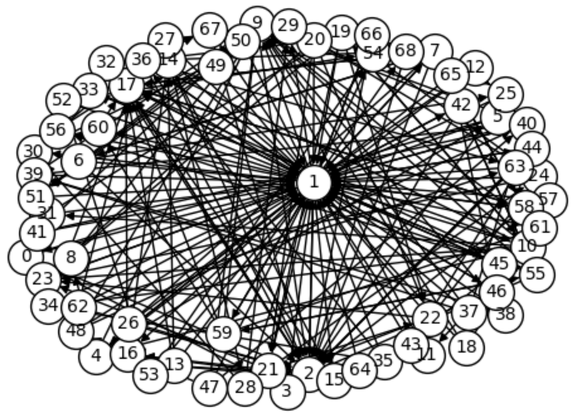
$$DC((a,b)) = \frac{DC(a)+DC(b)-2}{|DC(a)-DC(b)|+1}$$

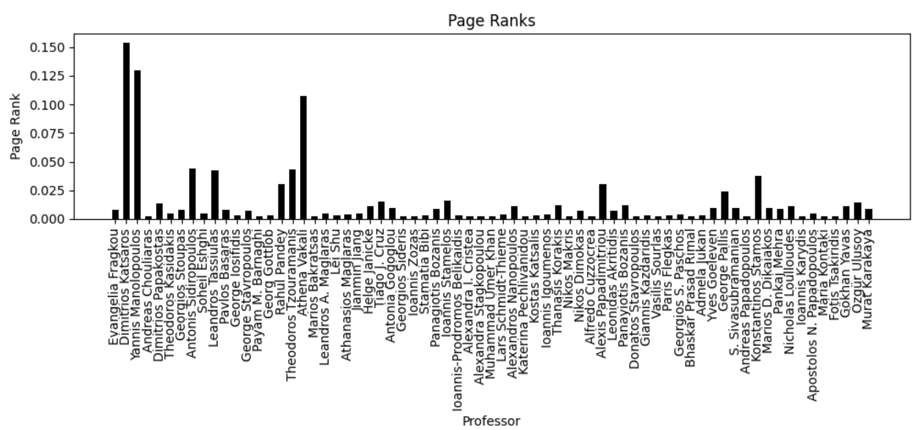




5 Exercise 5

The **Page Rank** for each node of the graph below was calulated using NetworkX

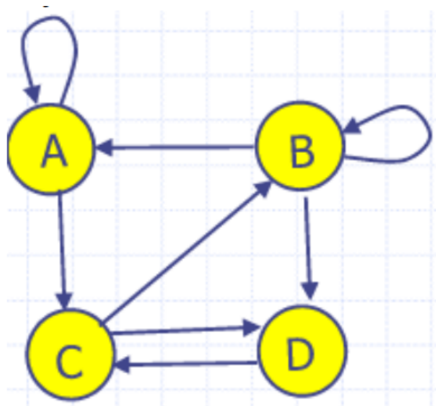




A CSV file containing the accurate measurements was also submitted.

6 Exercise 6

Calculate the **Page Rank** of each node of the graph bellow by hand.



No damping factor is needed since the graph is both stochastic and primitive (aperiodic and irreducible). Thus, the system bellow is solved.

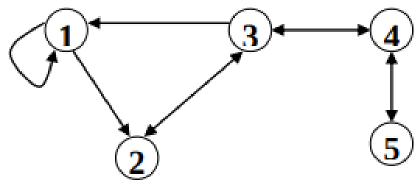
$$\begin{cases} PR(A) = \frac{PR(A)}{2} + \frac{PR(B)}{3} \\ PR(B) = \frac{PR(B)}{3} + \frac{PR(C)}{2} \\ PR(C) = \frac{PR(A)}{2} + PR(D) \\ PR(D) = \frac{PR(B)}{3} + \frac{PR(C)}{2} \\ PR(A) + PR(B) + PR(C) + PR(D) = 1 \end{cases}$$

(1) $\Rightarrow PR(A) = \frac{1}{6}, PR(B) = \frac{1}{4}, PR(C) = \frac{1}{3}, PR(D) = \frac{1}{4}$

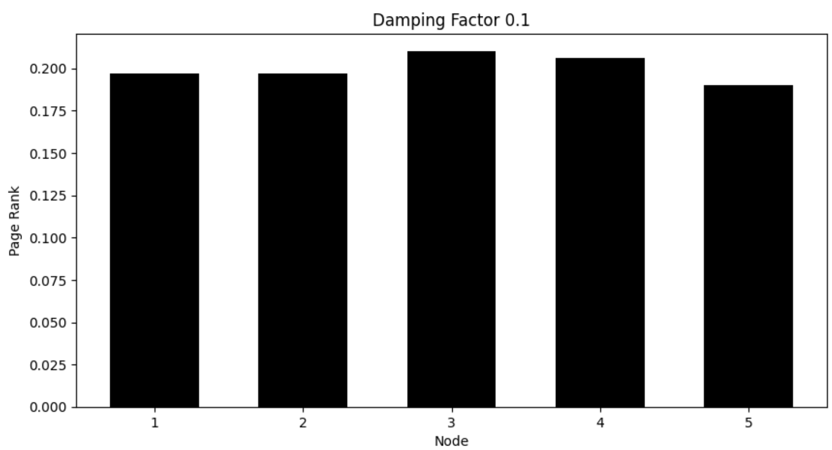
A more descriptive solution of the system above is provided in the submission files.

7 Exercise 7

Calculating the **Page Rank** of each node of the graph bellow for different damping factor values.

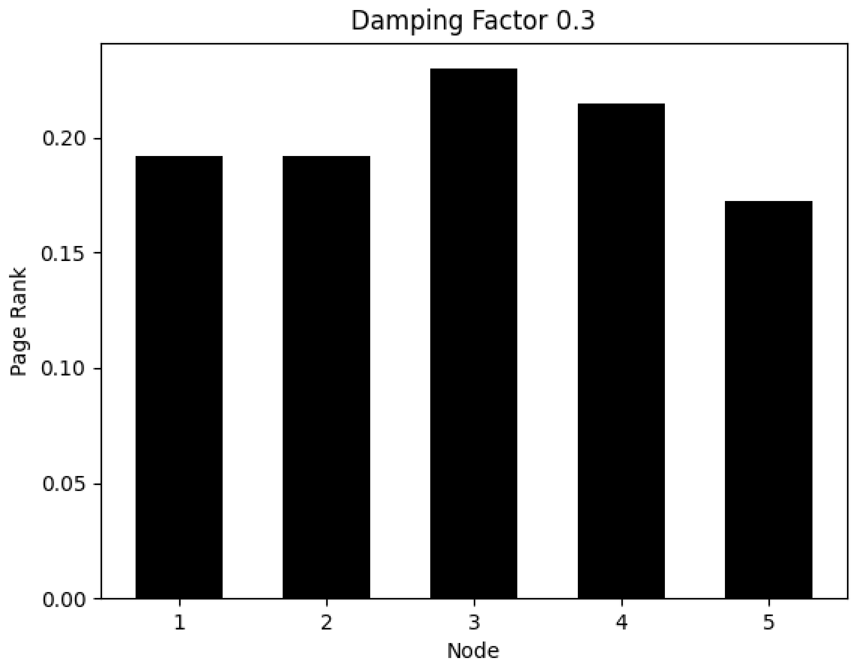


7.1 Damping Factor: 0.1



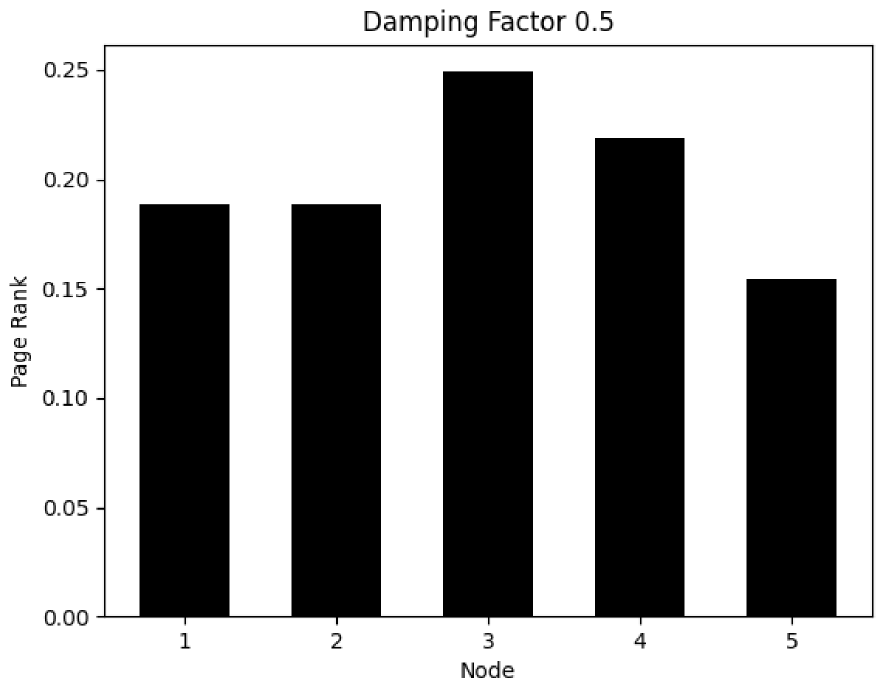
$$\begin{cases} PR(1) = 0.19684160416666668 \\ PR(2) = 0.19684160416666668 \\ PR(3) = 0.20998562500000004 \\ PR(4) = 0.20602969444444447 \\ PR(5) = 0.19030147222222224 \end{cases}$$

7.2 Damping Factor: 0.3



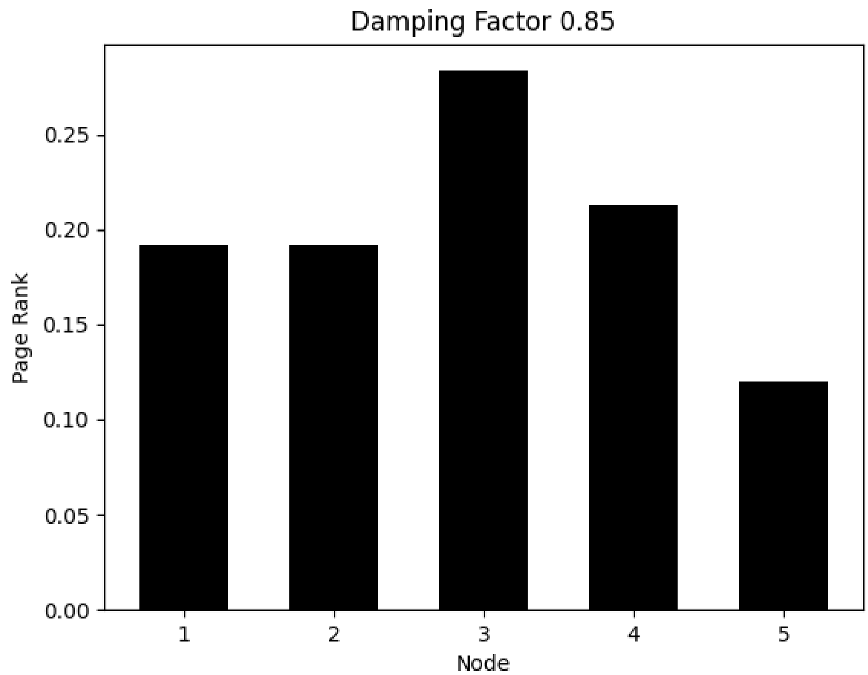
$$\begin{cases} PR(1) = 0.19173097283593749 \\ PR(2) = 0.19173097283593749 \\ PR(3) = 0.22971395539062495 \\ PR(4) = 0.21462949178124996 \\ PR(5) = 0.17219460715624998 \end{cases}$$

7.3 Damping Factor: 0.5



$$\begin{cases} PR(1) = 0.188679300475513 \\ PR(2) = 0.188679300475513 \\ PR(3) = 0.24905635729248143 \\ PR(4) = 0.21886822966391167 \\ PR(5) = 0.15471681209258092 \end{cases}$$

7.4 Damping Factor: 0.85



$$\begin{cases} PR(1) = 0.19182193316290375 \\ PR(2) = 0.19182193316290375 \\ PR(3) = 0.28340244242002904 \\ PR(4) = 0.21259959667728376 \\ PR(5) = 0.12035409457687965 \end{cases}$$