

# Complex Networks

## Infection Spreading

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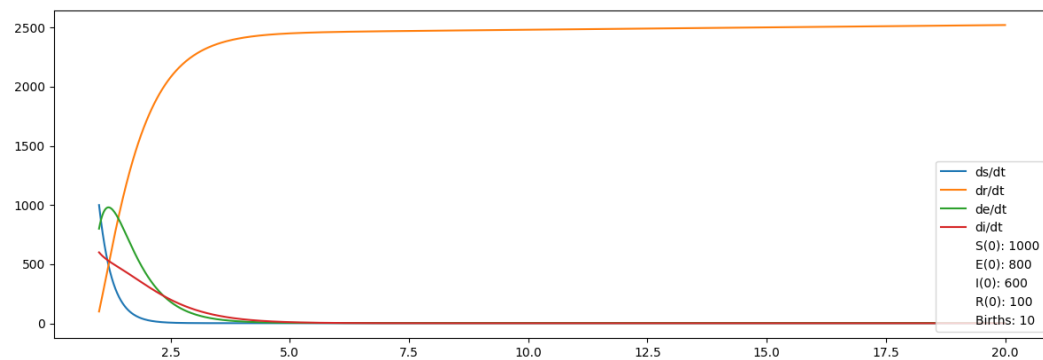
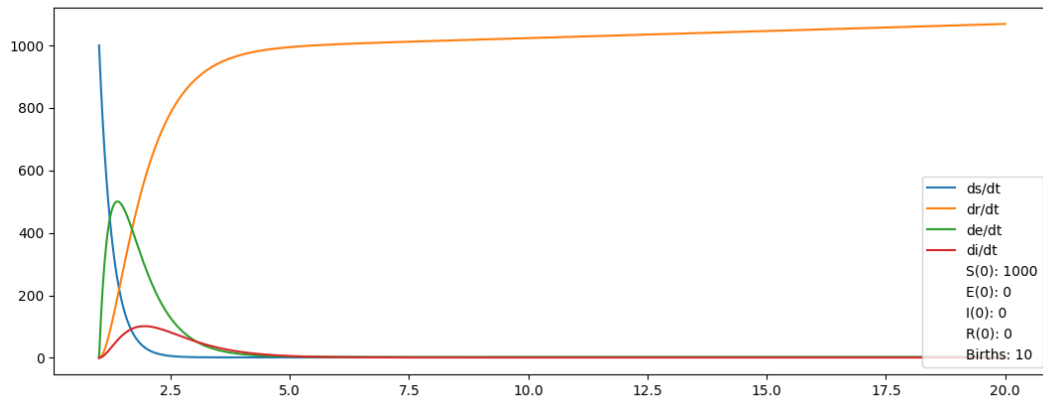
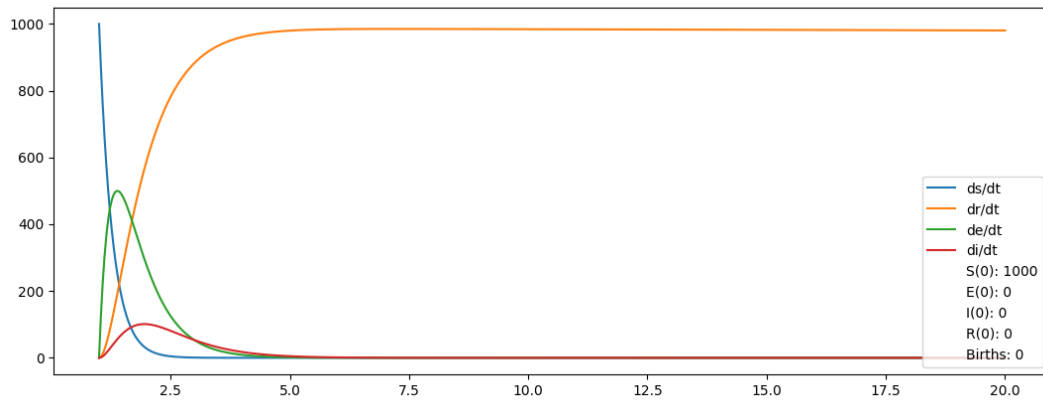
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# 1 Exercise 1

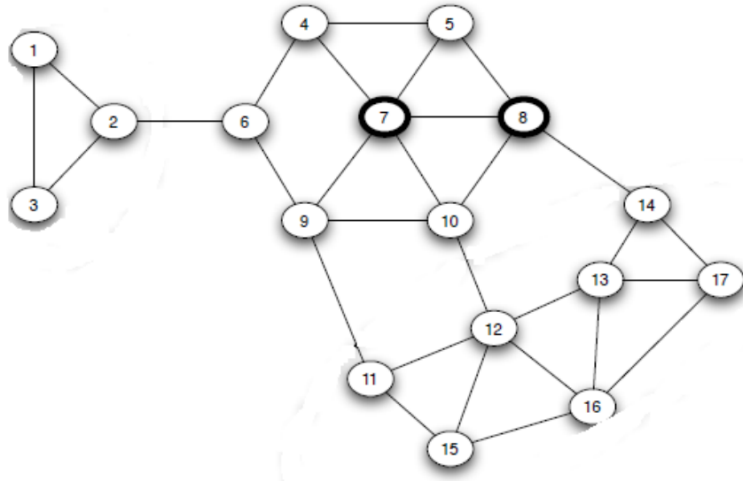
## 1.1 Graphs



## 1.2 Notes

- The whole population is recovering since the infection weights are not that strong.
- A small fraction of the exposed population is actually infected.
- When births are included into the computation, and the number of deaths are exceeded by the number of births, the final population size is increased.

## 2 Exercise 2



### 2.1 Infection Spreading

With nodes 7 and 8 already infected, the nodes that are also infected are listed below.

- 5, 10  $\leftarrow$  7, 8
- 4  $\leftarrow$  5, 7
- 9  $\leftarrow$  7, 10
- 6  $\leftarrow$  9, 4

Thus, the nodes infected by 7 and 8 are the nodes 4, 5, 6, 9 and 10

### 2.2 Cluster of Nodes that Blocks Spreading

$$\begin{cases} d(2) = 2/3 \\ d(11) = 2/3 \\ d(12) = 4/5 \\ d(14) = 3/4 \end{cases} \implies \min[d(\text{node}_i)] = 2/3 > 1 - q = 1/2 \quad (1)$$

Thus, the nodes selected that block spreading are **2, 11, 12** and **14**.

The clusters that block spreading are  $\{1,2,3\}$  and  $\{11,12,13,14,15,16\}$

### 2.3 Additional Nodes that Increase Spreading

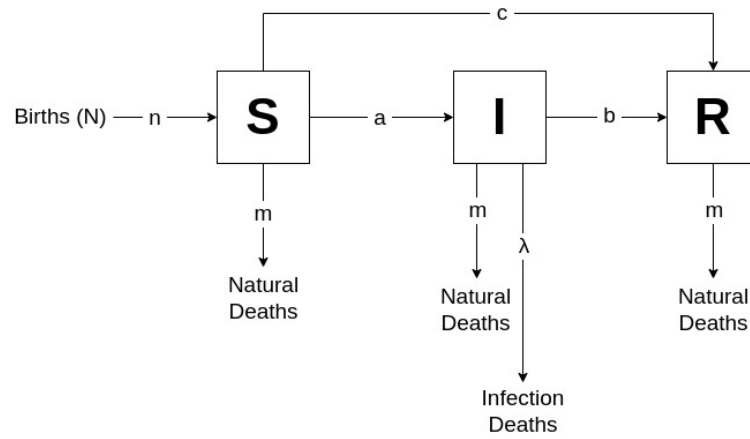
We can add one of the nodes below to enable spreading on cluster 11,12,13,14,16,17.

- 11:  $11 \rightarrow 12 \rightarrow 15 \rightarrow 16 \rightarrow 13 \rightarrow 17 \rightarrow 14$
- 12:  $12 \rightarrow 11 \rightarrow 15 \rightarrow 16 \rightarrow 13 \rightarrow 17 \rightarrow 14$
- 13:  $13 \rightarrow 14 \rightarrow 17 \rightarrow 12 \rightarrow 16 \rightarrow 15 \rightarrow 11$
- 14:  $14 \rightarrow 17 \rightarrow 13 \rightarrow 12 \rightarrow 16 \rightarrow 11 \rightarrow 15$
- 16:  $16 \rightarrow 12 \rightarrow 11 \rightarrow 15 \rightarrow 13 \rightarrow 17 \rightarrow 14$
- 17:  $17 \rightarrow 14 \rightarrow 13 \rightarrow 16 \rightarrow 12 \rightarrow 11 \rightarrow 15$

With an updated threshold of  $q=2/5$  there is no single node that enables spreading the infection to the whole network. This is attributed to the fact that at least two nodes are needed to spread both on clusters  $\{1,2,3\}$  and on  $\{11,12,13,14,15,16\}$ .

### 3 Exercise 3

#### 3.1 SIR Model



#### 3.2 Differential Equations

$$\begin{cases} \frac{dS(t)}{dt} = -aS(t)I(t) + nN - mS \\ \frac{dI(t)}{dt} = aS(t)I(t) - (c + m + \lambda)I(t) \\ \frac{dR(t)}{dt} = cI(t) - mR(t) \end{cases} \quad (2)$$