Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
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Master Momentum，Master the Match

Summary

In the men’s final of the 2023 Wimbledon Open, Novak Djokovic lost to Al Rakazi after an up-and-down match. In matches, the dominant team sometimes loses due to unexpected fluctuations, often attributed to **"momentum"**. To identify factors that influence momentum, and to determine whether momentum plays a role in a game, we modeled the scoring characteristics of the game and advised coaches and players.

Several models are established: Model I: a player performance evaluation model based on the **entropy weight method and TOPSIS**; Model II: A quantitative model for momentum based on **logistic regression**;

For Model I. First, we extracted the characteristic factors related to the degree of performance of players based on the given data. Then, to realize the purpose of describing the flow of the game and evaluating the performance of the players, the player performance evaluation model was constructed. We processed these characteristic factors, used the entropy weighting method to get the reasonable weights of each factor, and then used TOPSIS to get the player performance score. Finally, we obtained the player performance rating as figure 6.Combined with other figures, we can conclude that momentum can influence a player’s performance, thereby affecting the swings of the game and the runs of success.

For Model II: First, to realize the description of predicting changes in match forms and finding factors reflecting these changes, we constructed a quantitative model of momentum. We constructed a mathematical expression for momentum using a logistic regression model, then used stochastic gradient descent to obtain the values of the parameters in the formula and concluded that factors with larger coefficients are more representative of momentum, i.e., they have a greater degree of influence on changes in the form of the game. we obtained the weight of the factors affecting the winning rate as figure 8. We can see that the variable ’winner’ significantly influences the outcome and the variable ’unf_error’ shows a substantial negative impact on the results.

Finally, The results show that the accuracy of Model II decreases to varying degrees when specific factors are omitted, as evidenced by greater sensitivity to serve and hit factors. Meanwhile, the results of the robustness analysis show that for different random factors, the values obtained do not deviate much, thus indicating that our model has good robustness.

Contents

1 Introduction

1.1 Problem Background

The Wimbledon Tennis Championships is one of the oldest and most prestigious world open tennis tournaments, founded in 1877 by the All England Club and the Lawn Tennis Association of Great Britain, and is one of the four Grand Slams of tennis. Novak Djokovic is one of the most prestigious tennis players, having won 24 Grand Slam titles. In the men's final of the 2023 Wimbledon Open, Djokovic seemed destined to win, however, the match turned out to be an up and down and unexpected affair. After Djokovic won the first set comfortably, the second set was tight between the two, and the third set was an easy win for Al-Rakazi. In the end the match between the young Carlos Alcaraz and Novak Djokovic ended in Alcaraz's favour.

The seemingly dominant side sometimes makes unexpected swings, which is usually attributed to "**momentum**". In sports, a team or a player may feel that they have the "momentum" in a match and are in control of the game. However, such "momentum" seems to be difficult to quantify. In order to measure this phenomenon, we would like to develop a model to evaluate a player's performance at specific times during a game to determine whether momentum exists and how it plays a role in the game.

1.2 Literature Review

Two major problems are discussed in this paper, which are:

- Building a model that reflects the game situation at the time the score occurred and can be used to **evaluate** a player's performance level over time, and to numerically and visually visualise the player's performance level.
- Building a model that **predicts** the occurrence of fluctuations in a match, determines whether momentum played a role in the match, evaluates how the momentum played a role, and generalises the model to other matches.

For the **first** question, tennis is one of the more difficult ball games to predict, and the **Monte Carlo method** used by Krcadinac et al. provides a fairly accurate simulation of the winners and losers of tennis matches [1]. Krcadinac et al. begin their study by confirming that a player who serves has a higher probability of winning than a player who returns the ball and analyzes the change in the probability of winning the next round of service points after "breaking" the opponent's service game. The study by Meier et al. analyzed the win/loss situation of the previous set and the set points (e.g., 6-1, 7-6) to predict the win/loss of the next set [2]. In summary, it can be seen that calculating a player's wins, losses, and scores in past matches, analyzing the required relevant statistics, and **extracting the eigenvalues with a high degree of correlation** are the keys to building an evaluation model.

For the **second** question, based on the first problem, if we assume that "momentum" exists, we believe that the performance of players in a certain period reflects the situation of "momentum", i.e., players who play well in a certain period are affected by the influence of "momentum" in the next game. That is, a player who plays well in a certain period will have a higher scoring rate on the next shot due to the influence of "momentum". Regarding the presence or absence of momentum, Meier et al. found that the probability of winning the next set increases when a returner wins the breakpoint, which **provides evidence for the existence of momentum** [2]. Furthermore, Dietl et al. showed that this effect of momentum is negatively affected by a player's loss of control over the match [3]. This loss of control is an outcome that needs to be taken into account in the study of the "momentum" effect[4]. To sum up, under the evaluation model derived from the first question, we need to comprehensively analyze the performance of players at a certain period and some possible factors affecting the momentum, to **predict** whether the winning rate of the next game will change accordingly, and then determine the impact of "momentum".

1.3 Our work

We do such things as shown in the flowchart below.

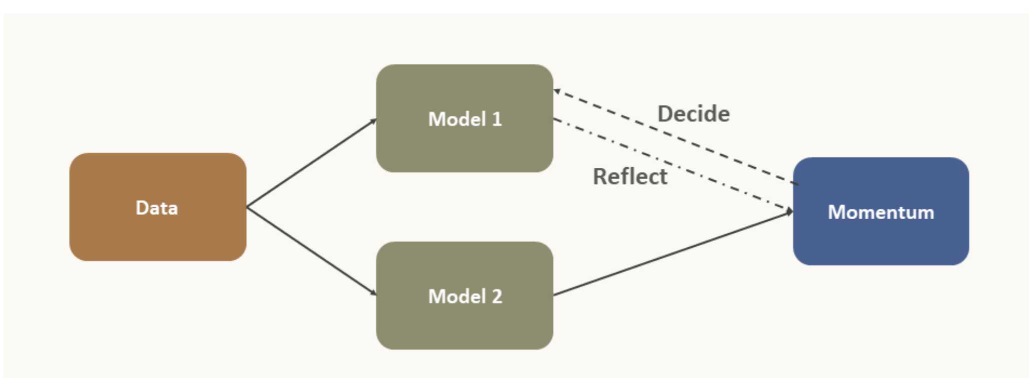

Figure 1: Flowchart

1. Using the **entropy weight method and TOPSIS** to establish a model to measure the performance of players in a certain period. By capturing the scores of both players in the course of the game as well as some other possible relevant characteristic factors, determining the weights of each factor by **entropy weighting method**, and finally establishing a model that can quantitatively measure the degree of performance of a certain player at a specific period in the game by **TOPSIS**.

2. Using the **logistic regression model** to quantitatively measure the win rate of a player, and then determine the point in time when the game fluctuates. Whether to serve direct score, break the opponent's serve score and other indicators reflecting the "momentum" as a variable to establish a logistic regression model, the use of **maximum likelihood estimation** and **stochastic gradient descent** to determine the value of the parameters in the **logistic regression model**, and then determine the degree of influence of the indicators on the win rate, to get the player at a certain time The quantitative value of the player's "momentum" at a certain moment.

3. Using the above model, test the accuracy of the model's prediction of the fluctuation of the winners and losers in the match with one or more other matches, and make suggestions on the players' matchups according to the fluctuation of the "momentum". At the same time, the model will be extended to other sports, such as women's tennis and table tennis, to evaluate the generalizability of the model.

2 Preparation of the Models

2.1 Assumptions and Explanations

To simplify the problem, we made the following assumptions, each of which has a corresponding reasonable explanation.

- ***Assumption 1:*** The time intervals between different scores have almost the same effect on momentum, so we can reduce the game timeline to points of equal time intervals.
  $\hookrightarrow$ ***Explanation:***Relative to the time intervals between different sets, the time intervals between different scores are smaller and the time interval gaps are smaller and are considered to be equally spaced time points for simplicity of modeling.
- ***Assumption 2:***At a given moment, a player's momentum is equal to the derivative of the player's overall performance score at that moment.
  $\hookrightarrow$ ***Explanation:***We consider momentum as a factor influencing the player's overall performance, which meeans it can directly determine whether the player's performance improves or deteriorates. Therefore, we assume momentum is the first derivative of the player's overall performance score at the current moment.

Additional assumptions are made to simplify analysis for individual sections. These assumptions will be discussed at the appropriate locations.

2.2 Data

2.2.1 Data Collection

Websites, where we collect data, are listed in Table 1.

Table 1: Data used in this paper

Data	**Websites**
Wimbledon	given
US Open(women)	https://www.365scores.com
Roland-Garros(men)	https://www.365scores.com

2.2.2 Data Processing

The given data is first preprocessed, and the methods are listed in Table 2.

Table 2: Data Processing

Original Data Names	Data Processing Methods	Processed Data Names
elapsed_time	Convert its units to minutes	elapsed_time
p1_score, p2_score	15, 30, 40, and AD are mapped to 1, 2, 3, 4 respectively	p1_score, p2_score
p1_score, p2_score	game_score_lead = p1_score -p2_score	game_score_lead
p1_points_won, p2_points_won	set_score_lead = p1_points_won - p2_points_won	set_score_lead
point_victor	2 is mapped to -1	point_victor
game_victor	2 is mapped to -1	game_victor
set_victor	2 is mapped to -1	set_victor
p1_points_won, p2_points_won	scores of two players in the past 5 matches.	score_sum_p1, score_sum_p2
point_victor	Record the consecutive wins of two players.	p1_streak, p2_streak
p1_streak, p2_streak	streak = p1_streak - p2_streak	streak
server	2 is mapped to -1	server
serve_no	serve_no = server * serve_no	serve_no
serve_depth	CTL is mapped to 1 and NCTL is mapped to -1	serve_depth
return_depth	return_depth = return_depth * server	return_depth
serve_width	serve_width = serve_width * server	serve_width

speed_mph, return_depth	Delete rows with missing data.	speed_mph, return_depth
p1_ace, p2_ace, p1_winner, p2_winner, p1_double_fault, p2_double_fault, p1_unf_err, p2_unf_err, p1_net_pt, p2_net_pt, p1_net_pt_won, p2_net_pt_won, p1_break_pt, p2_break_pt, p1_break_pt_won, p2_break_pt_won, p1_break_pt_missed, p2_break_pt_missed	Accumulate within each set	p1_ace, p2_ace, p1_winner, p2_winner, p1_double_fault, p2_double_fault, p1_unf_err, p2_unf_err, p1_net_pt, p2_net_pt, p1_net_pt_won, p2_net_pt_won, p1_break_pt, p2_break_pt, p1_break_pt_won, p2_break_pt_won, p1_break_pt_missed, p2_break_pt_missed
p1_ace, p2_ace, p1_winner, p2_winner, p1_double_fault, p2_double_fault, p1_unf_err, p2_unf_err, p1_net_pt, p2_net_pt, p1_net_pt_won, p2_net_pt_won, p1_break_pt, p2_break_pt, p1_break_pt_won, p2_break_pt_won, p1_break_pt_missed, p2_break_pt_missed	Take the difference	ace, winner, double_fault, unf_err, net_pt, net_pt_won, break_pt, break_pt_won, break_pt_missed, distance_run

3 Model 1

For the first problem, a model is needed to capture the flow of the game each time a score occurs. We developed **a player performance evaluation model based on the entropy weight method and TOPSIS**. First, based solely on the timeline distribution of game scores, a score-time plot can be made for the entire game, as shown below.

3.1 Feature Extraction

In order to achieve the assessment of player performance during a certain period of time, it is necessary to capture some relevant characteristic factors of both players at the time when scoring occurs in order to measure player performance. The general idea is shown in the Figure 2.

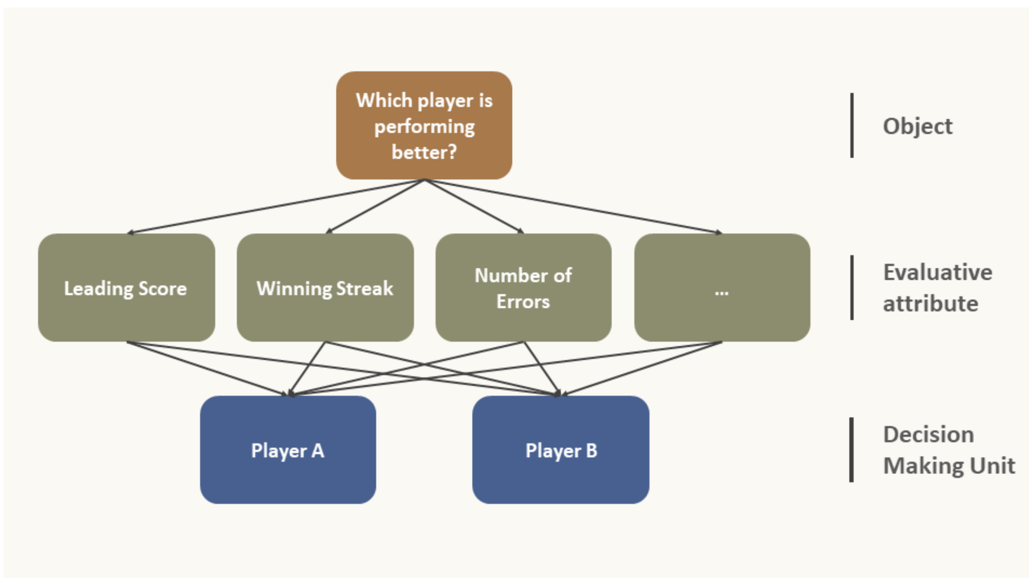

Figure 2: Feature Extraction

In the preprocessed dataset, the features shown in the table below are extracted.

3.2 TOPSIS Evaluation Model Based on Entropy Weight Method

The TOPSIS method, or the Distance to Superior and Inferior Solutions method, is a commonly used comprehensive evaluation method that can fully utilize the information of the original data, and its results can accurately reflect the gap between the samples. To evaluate and rank each sample, the ideal optimal solution and the worst solution in a system composed of all samples can be constructed based on these sample data. The core idea of TOPSIS is to evaluate the combined distance of any of the solutions in the system of solutions from the ideal optimal solution and the worst solution through certain calculations. If a sample is closer to the ideal optimal solution and further away from the worst solution, it is reasonable to assume that this sample is better (high evaluation score). The ideal optimal solution is the ideal solution of each indicator value taken to the optimal value of the evaluation indicators in the system, the worst solution is the ideal solution of each indicator value taken to the worst value of the evaluation indicators in the system. It should be noted that the data in the ideal optimal solution are the data in the sample and do not choose the data that are not in the sample, and the ideal worst solution is the same.

The specific realization steps are as follows.

3.2.1 Data standardisation

The types of metrics in the dataset are categorized into two types of metrics: very large metrics and very small metrics. For example, the number of aces played by a player in a set is considered that the more the number of aces the better the player's performance, then this is a positive indicator; the number of times a player serves double faults in a set, it is considered that the more the number of double faults the worse the player's performance is, so this is a negative indicator. Data standardization refers to the treatment of all data in a formula to eliminate the effect of different scales. Here, the following formulas are used to standardize the positive and negative indicators respectively.

For positive indicator, it is standardized by formula:

$$x_{ij} = 0.998 \frac{x_{ij} - min(x_{1j}, ..., x_{nj})}{max(x_{1j}, ..., x_{nj}) - min(x_{1j}, ..., x_{nj})} + 0.002 \tag{1}$$

For negative indicator, it is standardized by formula:

$$x_{ij} = 0.998 \frac{max(x_{1j}, ..., x_{nj}) - x_{ij}}{max(x_{1j}, ..., x_{nj}) - min(x_{1j}, ..., x_{nj})} + 0.002 \tag{2}$$

3.2.2 Entropy Weighting Method for Determining Weights

Individual indicators in a single sample may have different degrees of influence on the overall evaluation of the sample, and entropy weighting is a method to determine the weights of different indicators.

Firstly, the concept of information entropy is briefly introduced. The inverse of the probability p of a random variable X taking a certain value $\frac{1}{p}$ is the amount of information I. A large probability event has less information and a small probability event has more information. The information entropy is the expectation of the amount of information, and for a random variable X, the information entropy can be expressed by the following equation:

$$H(X) = \sum_{i=1}^{n} p_i ln(\frac{1}{p_i}) \tag{3}$$

The basic idea of the entropy weight method is to determine the objective weights according to the magnitude of the variability of the indicators. Generally speaking, if the information entropy of an indicator is smaller, it indicates that the degree of variability of the indicator value is larger, the amount of information provided is larger, and the role it can play in the comprehensive evaluation is also larger, and its weight is also larger. On the contrary, the larger the information entropy of an indicator is, the smaller the degree of variation of the indicator value is, the smaller the amount of information provided, the smaller the role it can play in the comprehensive evaluation, and the smaller its weight is. More detailed information about the entropy weighting method can be found in reference[5].

The standardised data were processed to calculate the weight p_{ij} accounted for by the value of the i-th sample indicator under the j-th indicator x_j:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{n} x_{ij}} (j = 1, 2, ..., m) \tag{4}$$

Next, the information entropy e_j is calculated for the j^{th} indicator:

$$e_j = -\frac{1}{ln(n)} \sum_{i=1}^{n} p_{ij} ln p_{ij} \tag{5}$$

Calculate the information entropy redundancy:

$$g_j = 1 - e_j \tag{6}$$

Finally, the weights of the indicators are obtained:

$$w_j = \frac{g_j}{\sum_{j=1}^{m} g_j} \tag{7}$$

The weights of the indicators were obtained as shown in Table 3:

Table 3: Entropy Weight

Criteria	**Entropy_Weight**
server	0.122204
winner	0.006681
p1_points_won	0.039702
p1_net_pt_won	0.090990
p1_double_fault	0.193320
serve_depth	0.117945
p1_distance_run	0.056452
rally_count	0.102394
p1_ace	0.002051
p1_score	0.002882
p1_streak	0.001376
serve_no	0.057858
serve_width	0.055773
p1_unf_err	0.002724
p1_net_pt	0.002862
p1_break_pt	0.003047
p1_break_pt_won	0.004864
speed_mph	0.016380
return_depth	0.120492

In order to clearly identify the magnitude of the individual weights, the above data were sorted and plotted to get Figure 3:

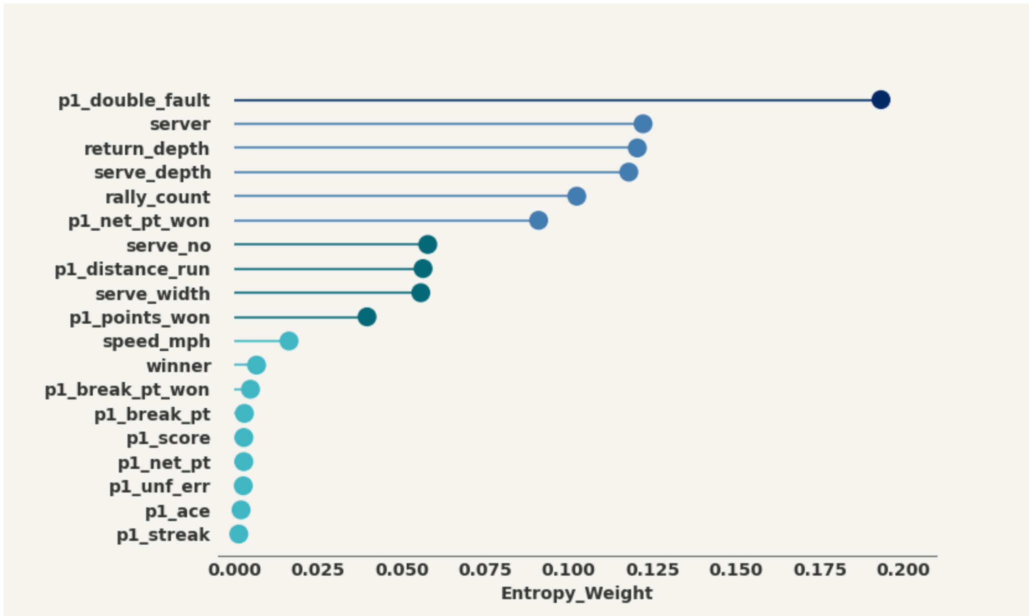

Figure 3: Entropy Weight

The pseudo-code is shown below:

Algorithm 1: Entropy Weight Method

```
#Normalization
if column[x] is positive feature then
    x=0.998*(x-min{x})/(max{x}-min{x})+0.002;
else
    x=0.998*(max{x}-x)/(max{x}-min{x})+0.002;

#Calculate weights
matrix+=epsilon;
p=matrix/sum_column{matrix};
entropy=(-1/log(matrix_len))*sum_columnp*log(p);
weights=1-entropy;
weights/=sumweights
```

3.2.3 Completion of Evaluation Scoring

Find the positive ideal solution and the negative ideal solution in the standardised data, i.e. the solution consisting of the maximum and minimum values of all the features of the sample, using the following formula:

$$Z^{+} = (Z_1^{+}, Z_2^{+}, ..., Z_n^{+}) = \max_{1 \leq i \leq n} (Z_{ij}) j = 1, 2, ...m \tag{8}$$

$$Z^{-} = (Z_1^{-}, Z_2^{-}, ..., Z_n^{-}) = \min_{1 \leq i \leq n} (Z_{ij}) j = 1, 2, ...m \tag{9}$$

Based on the weights w_j calculated by the entropy weighting method, the distance of each sample from the positive ideal solution and the negative ideal solution (Euclidean distance) is calculated with the following formula.

Define the distance of the i-th evaluation object from the positive ideal solution as:

$$D_i^{+} = \sqrt{\sum_{j=1}^{m} w_j (Z_J^{+} - Z_{ij})^2} \tag{10}$$

Define the distance of the ith evaluation object from the negative ideal solution as:

$$D_i^{-} = \sqrt{\sum_{j=1}^{m} w_j (Z_J^{-} - Z_{ij})^2} \tag{11}$$

The formula for calculating the relative proximity of each sample to the positive ideal solution is as follows:

$$S_i = \frac{D_i^{-}}{D_i^{+} + D_i^{-}} \tag{12}$$

S_i is a value between 0 and 1. The larger it is, the closer the sample is to the ideal optimal solution and the higher the corresponding evaluation score.

3.3 Results of Model 1

As a result, we obtained a model that evaluates a player's performance in that period based on the data of the player's characteristic metrics at the time of the occurrence of the score. After applying the model to multiple games in the dataset, we obtained a graph of player performance over time for each game. In this case, the visualization of the first game is shown in Figure 4.

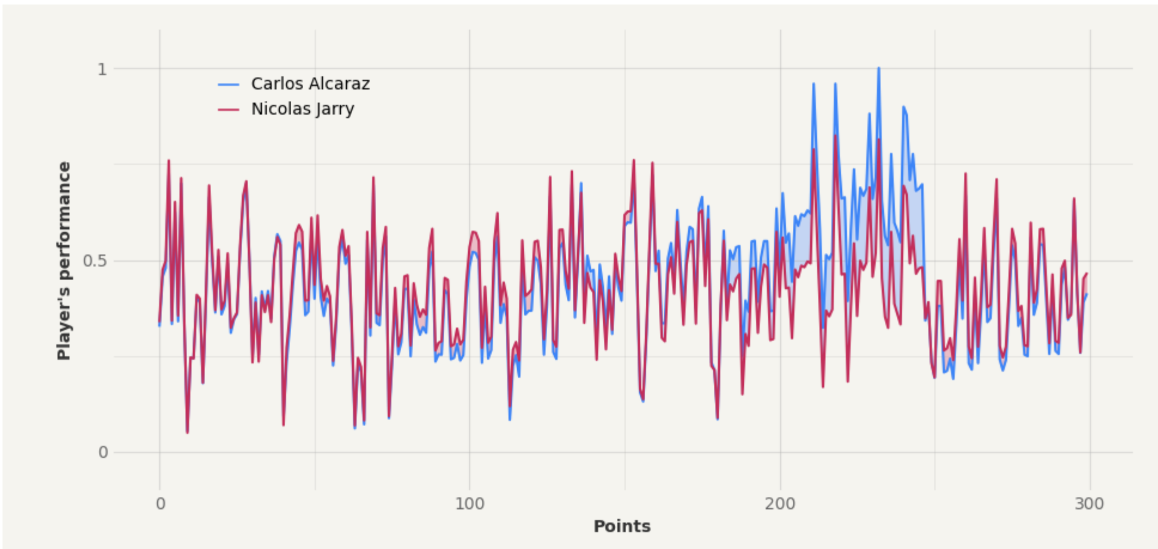

Figure 4: Players' Performance in the First Game

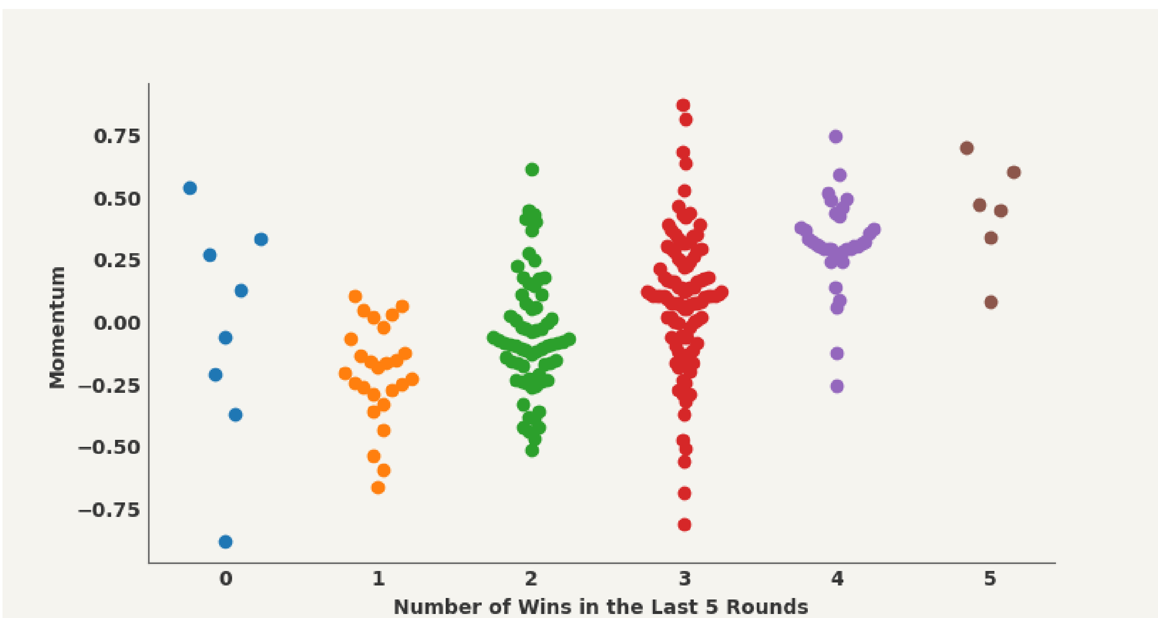

Figure 5: Momentum

Based on the assumption that the fitted performance scores over time, i.e., the derivatives of the S_i-t plot, can characterize the "momentum", the derivatives of the performances of the two players of the match over time are made as figure 6. From Figure 4, we can

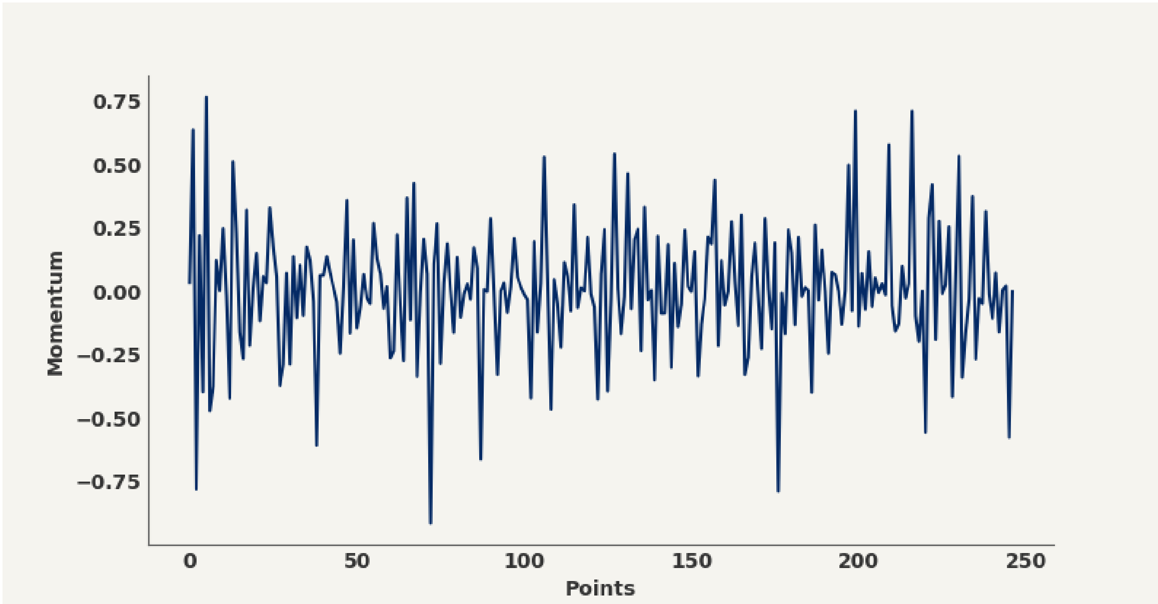

Figure 6: Momentum in the First Game

observe that the performances of the two players are initially comparable. However, at a certain point, Carlos Alcaraz's performance starts to noticeably improve. Upon examining the momentum figure (Figure 6), it is unsurprisingly observed that Carlos's momentum is also relatively high during this period(According to Assumption 2, momentum is the derivative of performance), which means the momentum does not change randomly during the game. Furthermore, from figure 5 it is evident that when the momentum is high, the performance is much better. Consequently, we can conclude that momentum can influence a player's performance, thereby affecting the swings of the game and the runs of success.

3.4 Test the Model

We calculated the TOPSIS scores for each sample. It can be observed that over 70% of samples have TOPSIS scores greater than 0.5 and quite a large number of them are close to 1. This indicates that, overall, the samples exhibit relatively better comprehensive performance compared to the ideal solution across multiple attributes. Furthermore, it suggests the effectiveness of the entropy weight model in addressing this problem.

4 Model 2

For Problem 3, a model is desired to predict shifts in momentum during a game. **A quantitative model for momentum based on logistic regression** is considered here. Logistic regression is mainly used in binary classification problems, i.e., the dataset is divided into two categories. Here, the quantification of "momentum" is transformed into the quantification of whether or not a score is scored at that point in time (no score is scored as 0, and a score is scored as 1), and then the problem is a dichotomous problem, which is in line with the objective of the logistic regression model.

4.1 Logistic Regression

The principle of binary logistic regression is shown in Figure 7:

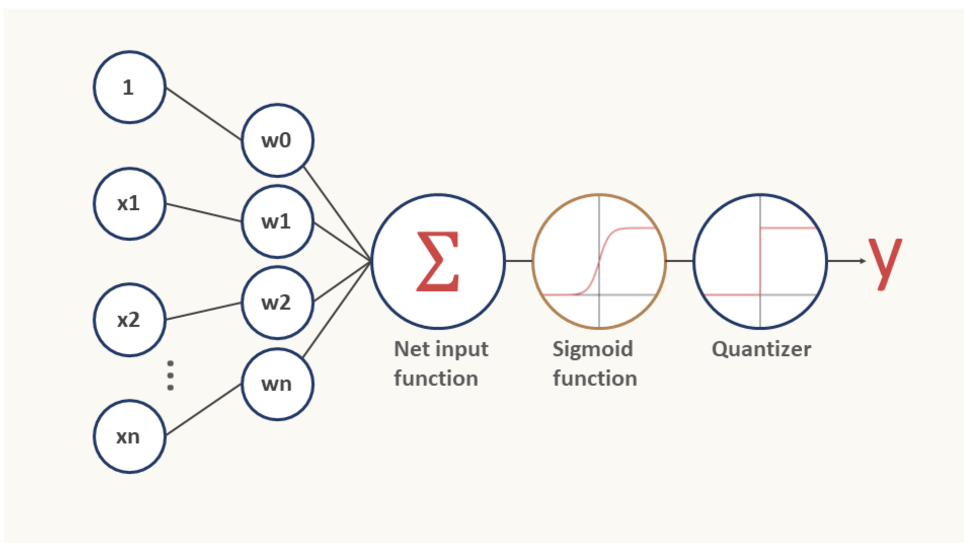

Figure 7: Binary Logistic Regression

For binary logistic regression, it is assumed that for the given dataset there exists a certain straight line that can complete the linear classification of the data. The decision boundary can be expressed as

$$\theta_0 + \theta_1 x_1 + ... + \theta_n x_n = 0 \tag{13}$$

where $x_1, x_2, ..., x_n$ are the n eigenvalues of a certain sample of the extracted dataset, $\theta_0, \theta_1, ..., \theta_n$ are the parameters to be determined.

Remember that $\theta^T x = \theta_0 + \theta_1 x_1 + ... + \theta_n x_n$. If $\theta^T x > 0$, then it can be determined that the sample point belongs to category 1.

Logistic regression further finds a direct relationship between the classification probability p(Y=1) and the input vector x. For the above $\theta^T x$, it takes continuous values and therefore cannot fit discrete variables. Consider fitting the conditional probability p(Y=1 | x) with its taken values. Since the conditional probability takes values from 0 to 1, consider acting on $\theta^T x$ with a certain function that takes values from 0 to 1. Here the sigmoid function is used:

$$y = \frac{1}{1 + e^{-\theta^T x}} \tag{14}$$

The sigmoid function is a monotonically increasing function with domain of definition R and domain of values (0,1), which is applied to $\theta^T x$ to obtain a fitted value of the conditional probability. It is then obtained:

$$ln\frac{y}{1-y} = \theta^T x \tag{15}$$

Considering y as the probability that x is a positive example, 1-y is the probability that x is its negative example. The ratio of the two is called odds (odds) and refers to the ratio of the probability of the event occurring to the probability of it not occurring if the event occurs with probability p. The log odds:

$$ln(odds) = ln\frac{y}{1-y} \tag{16}$$

Considering y as a class of a posteriori probability estimates, rewriting the formula has:

$$\theta^T x = ln\frac{P(Y=1|x)}{1 - P(Y=1|x)} \tag{17}$$

$$P(Y=1|x) = \frac{1}{1 + e^{-\theta^T x}} \tag{18}$$

That is, the log odds of the output Y=1 are modeled by a linear function of the input x. This is the logistic regression model. This models the relationship between the eigenvalues of the sample and the probability of classification, i.e., the odds of winning the next ball.

4.2 Determination of Parameters

After determining the mathematical form of the logistic regression model, it is necessary to determine by some method the values of the parameters in $\theta_0 + \theta_1 x_1 + ... + \theta_n x_n$ in the values of the parameters. Here, the great likelihood estimation and stochastic gradient descent methods are used to solve the problem.

The core of the method of maximum likelihood estimation is to find a set of parameters such that the likelihood (probability) of the sample is maximized under this set of parameters. This is realized as follows.

Let P(Y=1 | x)=p(x), P(Y=0 | x)=p(x) then the likelihood function can be obtained as:

$$L(\theta) = \prod [p(x_i)]^{y_i} [1 - p(x_i)]^{1-y_i} \tag{19}$$

For ease of solution, taking logarithms on both sides of the equation yields the log-likelihood function

$$lnL(\theta) = \sum [y_i lnp(x_i) + (1 - y_i) ln(1 - p(x_i))] \tag{20}$$

In machine learning, there is the concept of the loss function, which measures the degree to which the model is wrong in its predictions. If we take the average log-likelihood loss over the entire dataset, we get

$$J(\theta) = -\frac{1}{n} lnL(\theta) \tag{21}$$

By maximizing the likelihood function or minimizing the likelihood loss function, near-optimal parameter values can be found. In the process of parameter optimization, the main goal is to find a direction in which the parameters can be moved so that the value of the loss function can be reduced, and this direction is often obtained by various combinations of first-order partial derivatives or second-order partial derivatives. The loss function for logistic regression is:

$$J(\theta) = -\frac{1}{n} (\sum [y_i lnp(x_i) + (1 - y_i) ln(1 - p(x_i))]) \tag{22}$$

Gradient descent is used to find the direction of descent by the first-order derivative of $J(\theta)$ with respect to θ and to update the parameters in an iterative manner with the update:

$$g_i = \frac{\partial J(\theta)}{\partial \theta_i} = p(x_i) - y_i \tag{23}$$

$$\theta_i^{k+1} = \theta_i^k - \alpha g_i \tag{24}$$

where α is the learning rate. The algorithm is described in more detail in reference[6].

4.3 Conclusion of Model 2

With a random factor of 42, our model achieved an accuracy rate of 67.56%, and the final obtained weight parameters were as shown in Figure 8:

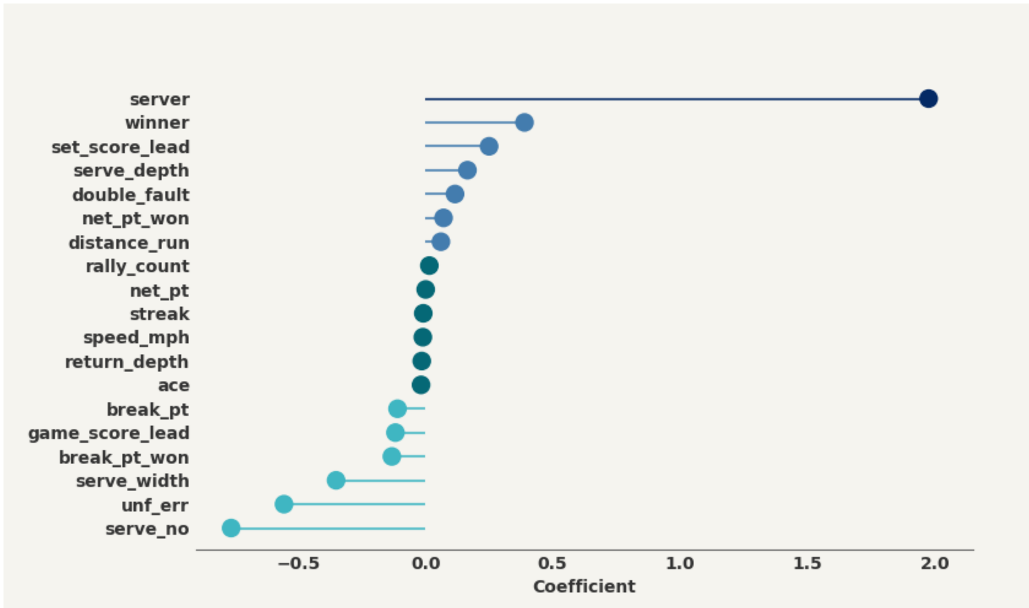

Figure 8: Entropy Weight

Figure 8 manifests that, aside from factors related to serving, the variable 'winner' significantly influences the outcome. We hypothesize that this could be atributed to substantial boost in the player's confidence due to his untouchable winning shot. Conversely, the variable 'unf_error' shows a substantial negative impact on the results, possibly due to a significant blow to a player's confidence when committing unforced errors. These fluctuations in mentality are likely to have an effect on the likelihood of scoring.

4.4 Test the Model

4.4.1 Sensitivity Analysis

To assess the sensitivity of the model, we conoducted ablation experiments. We partitioned the factors into four parts and systematically removed each part.Our original accuracy is 67.56% The accuracy is presented as table 4.

the results indicate a varying degree of accuracy reduction when specific factors are omitted, which manifest a higher sensitivity to serving and hitting factors.

Table 4: Abation study

Factor Name	Constituents of Factors	Accuracy after Removal of a Factor
Score factor	set_score_lead, game_score_lead, streak	66.77%
Serve factor	server, serve_no serve_width, serve_depth	59.76%
Hitting factor	ace, winner, double_fault, unf_err, net_pt, net_pt_won, break_pt, break_pt_won	65.98%
Other factor	distance_run, speed_mph, rally_count, return_depth	67.13%

4.4.2 Robustness Analysis

In assessing the robustness of our model, we introduced variability by setting random factors to different values during training on diverse datasets. From table 5, it is easy to notice that for different random factors, the fluctuation of the accuracy are minimal, which indicate that our model possesses good robustness.

Table 5: Robustness Analysis

Random Factor	Accuracy
20	68.17%
30	67.80%
40	67.07%

Additionally, we extracted a small amount of data from tennis matches outside of Wimbledon and from other types of ball games for prediction purposes, and achieved satisfactory results.

5 Conclusion

5.1 Summary of Results

Model 1 is a player performance evaluation model based on the entropy weight method and TOPSIS. Based on the extraction of the relevant characteristic factors of the player at the point in time when the scoring occurs in a game, the weight of each factor is calculated using the entropy weighting method, and finally, the evaluation of the player's performance at that point in time is completed using TOPSIS. Using Model 1, the visualization of player performance over time in a game is achieved, which in turn enables the observation of the flow of the game.

Model 2 is a logistic regression-based model for predicting the fluctuation of players' "momentum". After analyzing the relevant features of the player's scoring time point, we extract the features that affect the "momentum", establish a logistic regression model, use the stochastic gradient descent method to get the parameters of the model, and finally realize the prediction of whether the player scores at the next time point and then predict the fluctuation of the "momentum" in the game. "fluctuations in the game.

5.2 Strengths

- The advantage of **model 1** is that we have considered the weights of the indicators with the entropy weighting method, instead of thinking of giving weights to the indicators. The weights obtained by the entropy weighting method are more reasonable.
- The advantage of **model** 2 is that we considered as many relevant factors as possible with the logistic regression model, reducing the characteristic factors ignored because of subjective reasons; on top of that, we used the machine learning method of stochastic gradient descent to get the model parameters, and considered that the factor with a large parameter had a large degree of correlation with the momentum, and got the factor that could reflect the momentum more fully.

5.3 Weaknesses and Improvements

- The deficiency of **model 1** is that too many factors are considered, and some of the factors may be correlated (not independent of each other), then the weights obtained by the entropy weighting method may not be optimal. The improvement method is that after extracting the relevant feature values, the data are downsized by factor analysis, and the factors with similar effects are merged, so that fewer factors need to be analyzed by entropy weighting method afterwards.
- The shortcoming of **model 2** is that the stochastic gradient descent method of determining the logistic regression parameters may not always obtain the optimal parameter values. The improvement method is that machine learning methods including genetic algorithms, simulated annealing algorithms, cross-evolutionary algorithms and other machine learning methods can be selected to determine the parameters, and then evaluate the correctness of the formula obtained by each method to predict the momentum fluctuations, and select the method with the best prediction results.

Memorandum

To: The Coach
From: Team 2429429
Date: February 5th, 2024
Subject: Master Momentum, Master the Match

In this memo, we focus on proving to coaches that momentum plays a role in the game and advise players based on the factors that mainly affect momentum.

First, according to model 1, we prove that the change in the form of winning and losing in a match is not random, i.e. momentum plays a role in it. According to the hypothesis, momentum reflects the changes in players' performance. Then we derive the player performance score obtained from model 1 to get a new curve and observe the trend of this curve with the form of change in the score of the game. When the curve is constant positive in a certain period, the player's score in the corresponding period of the curve is larger than that of the opponent, then it can be shown that the change of the form of winning and losing in the game is not random, and there is the influence of momentum.

Second, the following are recommendations for players based on Model 2. First of all, players need to realize that events occur during the match that affect the momentum, and these events may positively or negatively affect the momentum. According to the conclusions of Model 2, the events that positively influence momentum are whether or not to serve, whether or not to win the previous set, and the number of points in the lead. When these events occur, players are advised to adopt the following strategies: stay focused, prevent errors due to overconfidence and letting their guard down, and make sure to do their best on every shot. Capitalize on momentum by attempting more aggressive serves and forcing your opponent to make errors. Stay patient and don't rush into a situation just because it's advantageous, but solidly expand your advantage.

Similarly, when faced with events that are not conducive to momentum, such as an intolerable error or a break serve point by the opponent, players can adopt the following strategies to cope and regain the rhythm of the match: first, quickly adjust the mindset, don't let the error or loss affect your confidence, and stay calm. Then analyze the cause of the error, whether it is caused by technical errors or mental fluctuations, and make targeted adjustments. After that, adjust the strategy, you can adopt a more solid and reduce the number of mistakes. It is important to strengthen self-encouragement and self-confidence; not to give up lightly in the face of unfavorable momentum, and to strengthen the resilience of the game.

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