Advanced exercise: a fast, precise approximation to the natural exponential

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Advanced exercise: a fast, precise approximation to the natural exponential

- We are going to explore how to combine different methods we have learnt to obtain a fast and precise approximation for the exponential e(x)
- We are going to start from the code present in the working directory Taylor0: exp.c and tester.c
- tester.c: it calculates the exponential by a number of methods for a number (to be provided) of random points x between -10.0 and 10.0
- exp.c contains the subroutines to calculate the exponential

Advanced exercise: a fast, precise approximation to the natural exponential

- tester.c also calculates execution times for the different methods, and provides an average error with respect to the standard exponential function

- The code in Taylor0 performs a simple Taylor expansion: at what degree?

- The code in Pade0 performs a simple Pade expansion: what order of approximant is used?

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- First task: check their performance (execution time and error)

- Second task: reduce the range of testing from [-10,10] to [-0.5,0.5]: what has changed? Has the execution time improved? And the error?

- Third task: exploit the insight from the second task and modify the Pade` code exploiting the other tricks: see following slide

Advanced exercise: a fast, precise approximation to the natural exponential

- Our Pade` approximation works best in a neighbourhood of 0. The strategy is now to exploit this fact to use these approximation only in a range around 0, also when calculating e(x) for x in [-10,10]

- Moreover, we are going to work with integers when possible

- In particular, as an intermediate result we are going to calculate an exponential of 2^y, exploiting the fact that $e^x = 2^{\log(\exp(x))} = 2^{x \log(e)}$, where log is the logarithm in base 2: $\log_2(x)$

Advanced exercise: workflow to calculate e^x

To calculate e^x

- We calculate $y = x \log_2(e)$
- Then we calculate 2^y:
  1. Split **y** in integer part **ipart** and non-integer part **fpart**
  2. We calculate 2^{ipart} as integer and we use a Pade` approximation[a] for 2^{fpart} in [-0.5,0.5]
  3. $2^y = 2^{ipart} * 2^{fpart}$
- $e^x = 2^y$

[a] See next slide for details

Advanced exercise: workflow to calculate e^x

Pade` approximation for 2^{fpart}:

We are going to use the following Pade` approximant:

$$p(x) = a_0 x^5 + a_1 x^3 + a_2 x$$

$$q(x) = x^4 + b_1 x^2 + b_2$$

$$f(x) = 1 + 2\frac{p(x)}{q(x) - p(x)}$$

where

a_0 = *2.30933477057345225087e-2*

a_1 = *2.02020656693165307700e1*

a_2 = *1.51390680115615096133e3*

b_1 = *2.33184211722314911771e2*

b_2 = *4.36821166879210612817e3*

Advanced exercise:

Which is the fastest method to calculate exp(x)?

Which is the most accurate?