Measuring h/k and Validating Wein's Law using Black Body Radiation

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Analysing the black body radiation from a tungsten lamp, a measurement of the ratio h/k was taken as $4.43 \pm 0.03 \times 10^{-11}$. Plots of intensity against wavelength were compared across 12 increasing temperatures and were graphically shown to follow Wein's Displacement law.

INTRODUCTION

The mechanisms for electromagnetic wave emission from an object are fundamental to the mathematical and physical interpretation of the world. Understanding and being able to apply methods derived to calculate and measure the effects of theses emissions is essential in a range of analysis from stellar observation[1] to thermodynamics[2].

Early observations of the propagation of electromagnetic waves from objects can be seen in Gustav Kirchhoff's[1824-1887][3] analysis of thermal emissions of heated objects. Following from this Max Planck[1858-1947][4] developed a working mathematical model for the emission of electromagnetic spectra. Understanding ways in which these models can be applied to physical applications is paramount to utilising them to produce accurate observational results of various phenomena.

In this experiment we used Planck's model and work from Wilhelm Wein[1864-1928][5] on the relationship between the peak wavelength of an emission spectra and the temperature of the system. We determine a value for the ratio of h/k where h is planks constant and k is Boltzmann's constant and then produce a plot of black body radiation intensity as a function of wavelength for a black body source at a range of temperatures, using results to validate Wein's law. We show clear and reasoned uses for Planck's laws and equations to demonstrate methods in which they can be used to describe a black body system.

THEORY

The electromagnetic emission of a object in thermal equilibrium is isotropic in nature. This means that the radiation emitted does not vary depending on the direction of emission. The energy density and the frequency distribution depend only on the equilibrium temperature and will be measured the same in any direction. We utilise this property of an object in thermal equilibrium to measure the emissions of a heated object assuming it is acting like a black body. Planck's analysis lead to the equation

$$p(\lambda) = \frac{8\pi hc}{\lambda^5(exp(\frac{hc}{\lambda kT}) - 1)}, \tag{1}$$

where $p(\lambda)$ is the intensity of emission at the wavelength λ, h is Planck's constant, λ is the wavelength of emitted light, c is the speed of light, k is

Boltzmann's constant and T is the temperature of the source of emission. We use this equation to produce a graph of intensity against wavelength for a variety of temperatures of an object.

Wein's displacement law[6] states that:

$$\lambda_{max}T = 2.898 \times 10^{-3}, \tag{2}$$

where λ_{max} is the peak wavelength emitted by the source and T is the temperature of said source. Following Wein's law, the value for the peak wavelength of the curve should decrease as the temperature of the system increases. At a wavelength where $\lambda \leq \lambda_{max}$ it can be seen that

$$exp(\frac{hc}{\lambda kT}) >> 1, \tag{3}$$

thus we can simplify our equation (1) to

$$p(\lambda) \propto \lambda^{-5}(exp(-\frac{hc}{\lambda kT})). \tag{4}$$

In order to determine the temperature of our source we used a lamp with a known current and voltage running through it. By using a tungsten filament lamp we can use the known relationship between temperature and resistance of the lamp[7]

$$R = R_0(1 + 5.2 \times 10^{-3}T + 4.5 \times 10^{-7}T^2), \tag{5}$$

where R is the resistance of the lamp, R_0 is the resistance of the lamp at 0°C and T is the temperature of the lamp in °C. Utilising a prism we can split a outputted beam from a thermal source into its constituent wavelengths and measure the intensity at each wavelength.

Following equation (1) the plot of the intensity against the wavelength produces a Boltzmann distribution curve with peaks following Wein's law.

EXPERIMENTAL METHOD

The intensity with respect to angle can be measured using the set up shown in FIG.1. A light source was channeled into a collimator to produce parallel rays. A chopper is used to produce a intermittent output of these parallel rays with a reference signal sent to the detector, this ensures that only the signal collimated from the lamp is measured by the detector. The parallel rays are separated by the prism into their varying wavelengths, each wavelength then is focused by a telescope onto the signal detector. The angle of the detector and telescope was adjusted to alter the wavelength collected by the telescope and focused onto the detector.

The angle was related to the wavelength detected by use of a calibration curve. A mercury lamp produces a discreet set of spectra with known wavelengths as shown in appendix FIG.A1. Utilising this feature of a mercury lamp the intensity of the signal was measured with respect to angle with a detector sensitivity of 2mV. To better define the infrared window of these measurements the detector sensitivity was set to 5mV for the infrared section and measurements were repeated. The peak intensities represents the spectral emission lines of the mercury lamp and their associated angles.

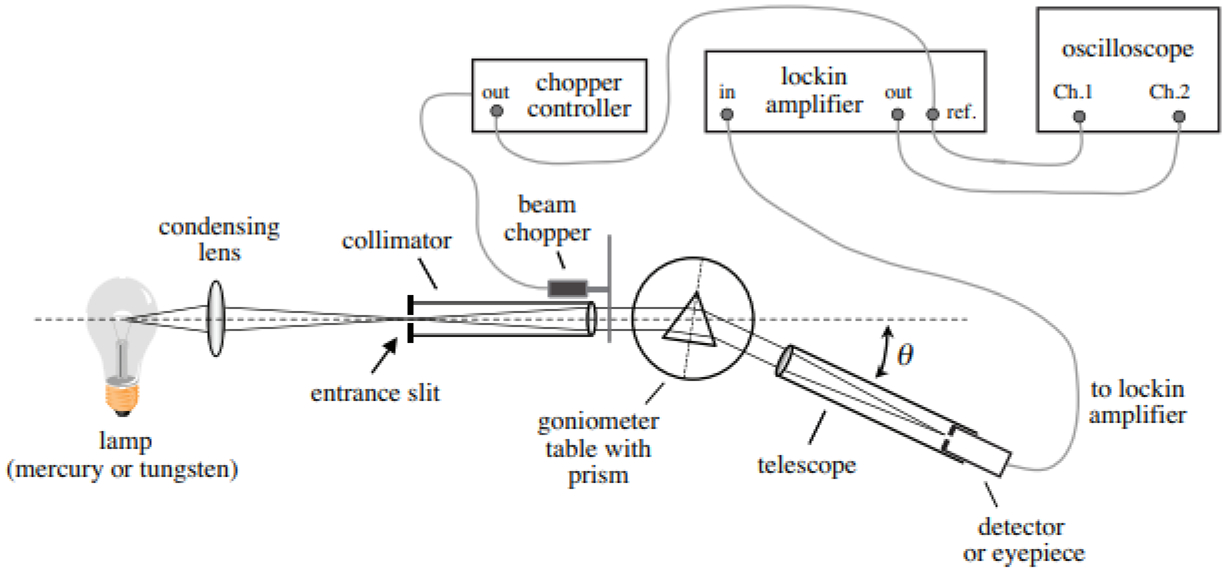

FIG. 1. Setup for measuring intensity with angle.

Construction of a calibration curve using our emission angles and their associated wavelengths can be fit with the equation

$$\theta(\lambda) = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}, \quad (6)$$

a known optical dispersion of glass, the Cauchy expression[8].

Using equation (4) and assuming that the measured intensity is proportional to the output intensity we can see that

$$G(\lambda, T) \propto (exp(-\frac{hc}{\lambda kT})) \quad (7)$$

where $G(\lambda, T)$ is the measured intensity. Using a tungsten filament lamp, the resistance of the lamp was measured when no current was flowing and the lamp was at room temperature 20°. Using the know relationship (5) with the values $R = R_{20}$ and $T = 20$ a value for R_0 was calculated. A known voltage and current was run through the lamp which behaves as a black body. Using relationship (5) and our value for R_0 along with the equation

$$V = IR, \quad (8)$$

we calculated the temperature of the lamp. Using this we then measured the intensity of the lamp at a set angle and altered the voltage and current to change the temperature for a range of temperatures between 1000K and 2500K. Utilising equation (7) we displayed a linear relationship on a graph of $\ln(G(\lambda, T))$ against $1/T$ with a gradient of

$$\frac{-hc}{\lambda k}. \quad (9)$$

Thus λ was found from the calibration curve and the set angle. A calculated value for h/k then was derived.

By altering the voltage and current through the tungsten lamp we produced a range of temperatures between 1500K and 3000K. For each temperature the angle of the detector was adjusted in increments of 0.1°. The intensity of the signal detected was recorded for each angle increment. Due to the nature of our setup and the slope of the calibration curve, at angles further from the parallel, the same detector slit width will allow a wider range of wavelengths into the detector producing an inflated signal. We can calibrate out this discrepancy by multiplying the measured intensity by the derivative of the calibration curve

$$G_{calibrated}(\lambda) = G(\lambda).(\delta\theta/\delta\lambda). \tag{10}$$

The angle measurement was converted to wavelength using equation (6) and the values of A, B, C from our calibration data.The results across a range of temperatures were plotted and the relationship between peak frequency and temperature was examined.

RESULTS

Utilising the defined spectra of a mercury lamp, measurements for angle in intervals of 0.1°and detected signal were recorded for a detector sensitivity of 2 mV between 96°and 105.5°. A clear set of peaks is displayed in the visible light spectrum but further data with a detector sensitivity of 5 mV between angles of 96.5°and 99.5°was taken to further distinguish the peaks in the infrared range. These results were plotted as voltage (mV) against angle (°) as shown in FIG.2.

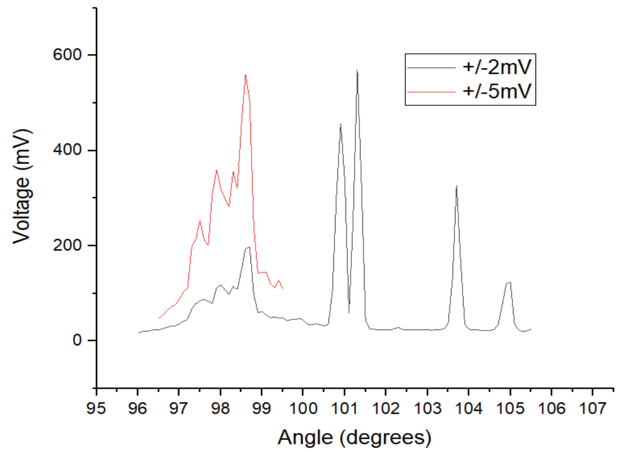

FIG. 2. Measured intensity against angle for mercury spectra.

The distinguished peaks of FIG.2 can be assigned to the spectral wavelengths of the mercury lamp as shown in TABLE.I

Colour	**Angle (°)**	**Wavelength (nm)**
Violet	105	404.7
Blue	103.7	435.8
Blue-green (faint)		491.6
Green	101.3	546.1
Yellow	100.9	578
Red (broad)		690.5-709
infrared 1	98.6	1014
infrared 2	98.3	1129
infrared 3	97.9	1360
infrared 4	97.5	1707

TABLE I. Spectra colour, angle of peak and associated wavelength.

From this table the relationship angle vs wavelength is plotted and equation (6) used to form a fit shown in FIG.3

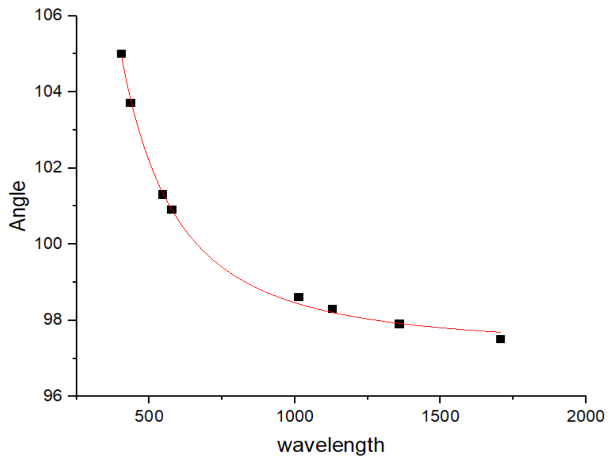

FIG. 3. Angle against wavelength with calibration curve.

The values for A, B and C from equation (6) were found from FIG.3 and are shown in TABLE.II.

Letter	**Value**
A	97.28571 ± 0.10654
B	1162774.10497 ± 99646.20166
C	1.42478E10 ± 1.57464E10

TABLE II. Equation (6) constants and their value from FIG.3.

Equation (7) suggests a relationship between $G(\lambda, T)$ and T. Selecting a stable angle of 98.6°the voltage across a tungsten lamp was incremented from 1.65V to 12.52V with the current changing between 1.35A to 3.8A. Equation (5) gives values of T between 1288°and 2885°. A plot of $\ln(G(\lambda, T))$ against $1/T$ shows a negative linear fit with gradient (9) in FIG.4. Using the gradient of FIG.4 and our calibration curve with angle 98.6 to find the associated wavelength 952nm, h/k was found to be $4.43 \pm 0.03 \times 10^{-11}$.

Working with fixed temperatures between 1500K and 3000K the intensity of light was measured at an-

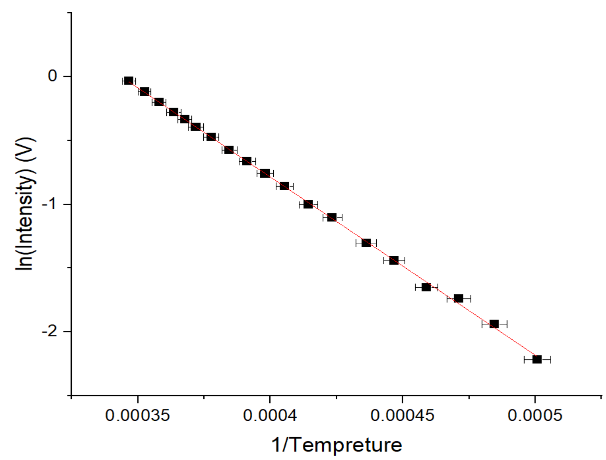

FIG. 4. $\ln(intensity)$ against $1/T$ with linear relationship.

gles between 101°and 96.1°for 12 increasing temperatures. Wavelength was found from angle and the intensity vs wavelength was plotted. Equation (6) was used to calibrate this data to produce a more accurate plot producing FIG.5.

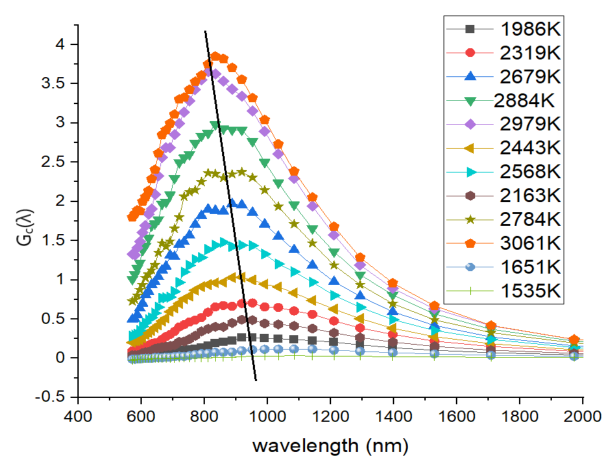

FIG. 5. Boltzmann distributions of calibrated intensity against wavelength with fit through peaks.

DISCUSSION

Produced from FIG.4, a result of $4.43 \pm 0.03 \times 10^{-11}$ was obtained. Quoting known values of k[9] and h[10] we can produce a value of h/k of approximately 4.8×10^{-11}. The result found is 92% of the known result. It is clear that the errors in this experiment have been underestimated. Possible additional source of error include the use of equation (5) and it's reliability in the chosen temperature range along with the the accuracy of the wavelength calculated from the calibration curve at the angle 98.6°.

Additionally the accuracy of the intensity measurements could have been systematically recorded incorrectly or the tungsten lamp may not have had time to reach thermal equilibrium before the results were taken. In order to improve on this measurement additional results could be taken for alternate temperature ranges and angles and compared to attempt to identify a range of results to find a more accurate value. More time could be given to allow the lamp to reach a thermal equilibrium.

The calibration curve FIG.3, relies on Cauchy's equation. For values over approximately 2000nm wavelengths the reliability of this equation begins to breakdown. The values measured exceeded the 2000nm wavelength range and thus in the final FIG.5 it can be seen that measurements of intensity past 2000nm wavelength were excluded. This has little bearing on the results however as all peaks lie before the cut off point.

Shown in FIG.5, the distribution of intensity with wavelength appears to obey a Boltzmann distribution curve at a variety of temperatures. With an increase of temperature it can be seen that the peak of the graph increases in intensity and decreases in wavelength as shown by the fit line on FIG.5. This then validates graphically Wein's displacement law (2).

CONCLUSION

To conclude, the value obtained for the ratio h/k of $4.43 \pm 0.03 \times 10^{-11}$ does not include within its error the actual value 4.8×10^{-11}, but is a good approximate. To better calculate the true value additional measurements could be taken at different temperatures and a different angle. In addition the accuracy of equation (5) could be assessed and the system should have ample time to reach thermal equilibrium.

FIG.5 displays a graphical representation of the plot of intensity vs wavelength for varying temperatures and validates Wein's displacement law through the drift in peak wavelength to the left as temperature increase.

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[1] *Use of EM radiation in stellar observation* `https://hubblesite.org/contents/articles/the-electromagnetic-spectrum`

[2] *en.wikipedia Use of EM radiation in thermodynamics* `https://en.wikipedia.org/wiki/Thermal_radiation`

[3] *en.wikipedia Gustav Kirchhoff 1824-1887* `https://en.wikipedia.org/wiki/Gustav_Kirchhoff`

[4] *en.wikipedia Max Planck 1858-1947* https://en.wikipedia.org/wiki/Max_Planck

[5] *en.wikipedia Wilhelm Wein 1864-1928* https://en.wikipedia.org/wiki/Wilhelm_Wien

[6] *en.wikipedia Wein's Law* https://en.wikipedia.org/wiki/Wien%27s_displacement_law

[7] *G.Winters Practical Physics I Laboratory Handbook p.82*

[8] *en.wikipedia Cauchy's Equation* https://en.wikipedia.org/wiki/Cauchy%27s_equation

[9] *en.wikipedia Boltzmann's Constant* https://en.wikipedia.org/wiki/Boltzmann_constant

[10] *en.wikipedia Planck's Constant* https://en.wikipedia.org/wiki/Planck_constant

APPENDIX A

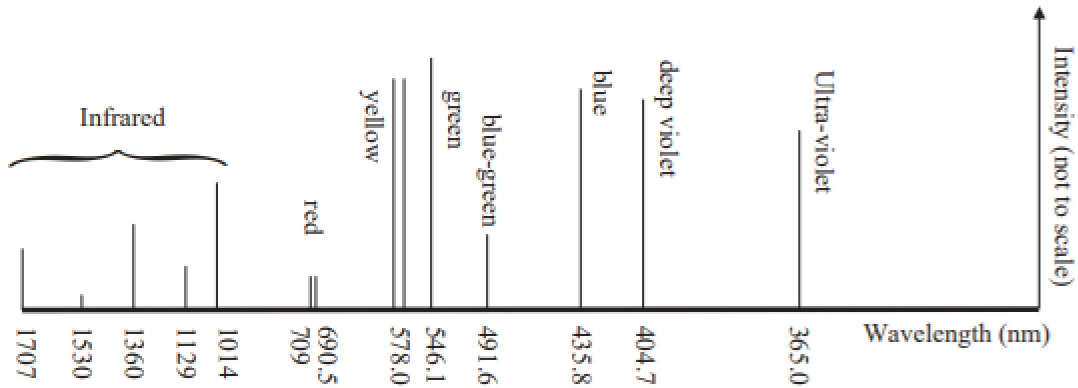

FIG. 1. Spectra of mercury lamp