Measuring the Young's Modulus of a Metal Bar

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The young's modulus of an material is a important measure of a materials properties often sought by engineers and scientists. The determination of this value using a simple mass and bar can be conducted to a relatively precise degree. Using this method a value for an unknown material bar was measured to be $1.02 \times 10^{11} N/M^{-2} \pm 0.00046 \times 10^{11}$

INTRODUCTION

When force is applied to a object we can often determine that the object's shape is effected. This is clearly evident in the case of a thin wire where it can easily be deformed and bent in a way that appears irreversible. This is known as inelastic deformation. When a small force is added and the object can return to its original state once the force is removed, this defines elastic deformation.[2] The Measure of the ease of deforming a material is related to a value called the young's modulus. Young's modulus defines a property of a material.

Originally postulated by Leonhard Euler [1] but named for Thomas Young, the Young's modulus measures the ratio of stress to strain for a material. This quantity is useful to look at as it gives a value for the relative stiffness of a material. Some materials have a different young's modulus for their compressive and tensile strengths however in most materials these values are the same[2]. It can be attempted to show a representative value for the Young's modulus of a metal bar using a set of visual measurements.

By supporting a bar across two points and adding mass to the center of this bar, a visual change in the position of the center of the bar can be observed and then thus measured. In this Experiment a value for the young's modulus of a bar will be calculated however there will inevitably be a large amount of uncertainty with this value due to the human inaccuracy and error involved with determining a value using visual methods. The value will be comparable to other materials measured in the same way and thus can act as a good way of comparing this value of Young's modulus between materials.

THEORY

As a force is applied to the bar this induces a stress in the material. Stress is a measure of the force over the cross sectional are and is represented by:

$$stress = \frac{F}{A} \tag{1}$$

F is the force applied to stretch or compress the material, and A is the cross sectional area of the material. In addition, if a bar is being pulled between two forces, a strain is applied to the material which is a measure of the percentage change in length for the project along the axis of the forces:

$$strain = \frac{\Delta L}{L} \tag{2}$$

Here, delta L is the extension of the bar and L is the original length of the bar. The relationship between these values can be graphed as a graph of stress against strain which forms the following relationship with several key points shown in FIG.1. While the

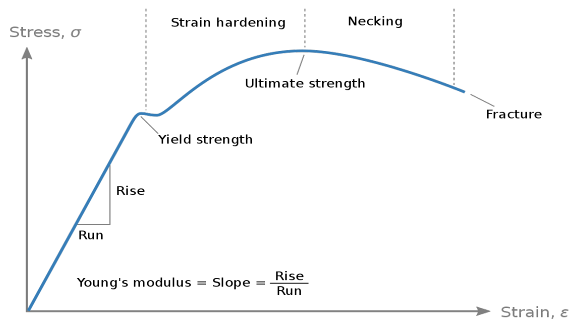

FIG. 1. Graph of stress against strain relationship showing key points of reference

relationship between the stress and the strain is linear, the young's modulus can be defined as the stress of the material over the strain of the material following:

$$Young'sModulus = \frac{stress}{strain} \tag{3}$$

or

$$Young'sModulus = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \tag{4}$$

This equation is used when measuring the extension of material being stretched. For the needs of this experiment however it requires an equation that incorporates the fact the bar is being depressed about the center[4]:

$$x = \frac{mgL^3}{4wd^3E} \tag{5}$$

Using this equation when m is the mass on the bar, g acceleration due to gravity, w is the width of the bar, d is the depth of the bar, E is the young's modulus of the bar and x is the depression. Re-arrange with some basic algebra manipulation and find:

$$mg = x\frac{4wd^3E}{L^3} \tag{6}$$

This gives us an equation that can easily be plotted after measuring values.

EXPERIMENTAL METHOD

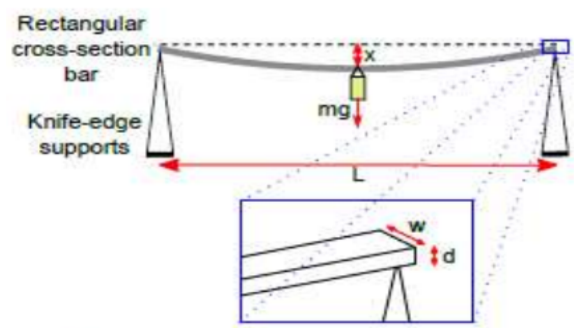

FIG. 2. Diagram of the equipment set up for experiment y [5]

A diagram of the experimental equipment set up is shown in FIG.2. A solid metal bar is used and suspended between two knife point edges allowing the full distance of the bar to be utilised in this experiment. With the addition of a travelling microscope it was set in the position so it can focus on a point in the middle of the bar. As more weight is added to the bar the new height of a central point can be measured. The difference between the original height and the new slightly more depressed high can be found to give us a value for the depression of the bar equal to x. The error on this depression value can be calculated with equation (7)[6]

$$\delta F = \sqrt{(\delta A)^2 + (\delta B)^2} \tag{7}$$

Next the values for the bar being used in order to gain information needed to later to extract the value of E can be measured. Measuring equipment can be used to find the length L of the bar $\pm 0.05cm$, the width w of the bar $\pm 0.005cm$ and the depth d of the bar$\pm 0.005cm$.

With these measurements the the mass can be multiplied by the acceleration due to gravity to give a value of weight for each addition of mass. Next the errors on these measurements can be assessed. The standard deviation on the height measurements can be measured and subsequently the error on each value for h. This may not work for all values of h as some measurements may be consistent despite our knowledge that errors must probably have occurred in our measurements even if these error belong simply to the equipment. Thus we can take an approx error for these values being the error on the equipment for this measurement.

Using the values obtained from above a graph of weight against depression can be produced. Including a fit line, the gradient of this graph is calculated. With the gradient the experimental value for E can be extrapolated using:

$$Gradient = \frac{4wd^3E}{L^3} \tag{8}$$

And thus E can be calculated with:

$$E = \frac{Gradient * L^3}{4wd^3} \tag{9}$$

Finally a value for the error can be produced by propagating our known errors through equation (9) using the standard partial differentiation method.

RESULTS

The value of L was measured with a meter rule. It was found to have a value of $49.95cm \pm 0.05cm$. Values for the depth and width were measured with vernier calipers and were found to have the values$0.520cm \pm 0.005cm$ and $0.345cm \pm 0.005cm$ respectively.

A(X)	B(Y)	C(Y)	D(Y)	E(Y)	F(Y)	G(Y)
Mass	height of mi	height of mi	height of mi	mean heigh	error of height	error in height
g	cm	cm	cm	cm	cm	cm
0	3.51	3.51	3.51	3.51	0.005	0.005
50	3.45	3.45	3.45	3.45	0.005	0.005
100	3.38	3.37	3.38	3.376	0.0058	0.0058
150	3.31	3.305	3.31	3.308	0.0029	0.0029
200	3.24	3.23	3.24	3.236	0.0058	0.0058
250	3.17	3.16	3.17	3.16	0.0058	0.0058
300	3.1	3.1	3.1	3.1	0.005	0.005
350	3.04	3.04	3.04	3.036	0.005	0.005
400	2.96	2.96	2.96	2.93	0.005	0.005
450	2.9	2.9	2.9	2.9	0.005	0.005

FIG. 3. Table of measurements for height of bar and the mean and error associated with each.

Multiple measurements for the depression of each mass were measured and recorded in Figure 3.The error on these calculations was also calculated by the standard deviation and the error of the measuring equipment.The depression was calculated for each value shown in Figure 4 along with the error on depression and weight of the mass hanging on the bar with its error also.

H(Y)	I(Y)	J(xEr±)	K(Y)	L(Y)
error in depression(cm)	depression	error in depression	weight	error in weight
cm	m	m	N	N
sqrt(((0.005)^2)+((col(H))^2))	(3.51-col(G))*10^-2	col(L)*10^-2	(col(A)*10^-3)*9.8	(0.5*10^-3)*9.8
0.00707	0	4.9E-5	0	0.0049
0.00707	6E-4	4.9E-5	0.49	0.0049
0.00766	0.00134	4.9E-5	0.98	0.0049
0.00578	0.00202	4.9E-5	1.47	0.0049
0.00766	0.00274	4.9E-5	1.96	0.0049
0.00766	0.0035	4.9E-5	2.45	0.0049
0.00707	0.0041	4.9E-5	2.94	0.0049
0.00707	0.00474	4.9E-5	3.43	0.0049
0.00707	0.0058	4.9E-5	3.92	0.0049
0.00707	0.0061	4.9E-5	4.41	0.0049

FIG. 4. Table of depression, weight and their errors

The Weight can now be plotted against the depression of the bar with all the errors in place to ascertain a approximate value for the gradient of the graph. Then using this value fore the gradient a approximate value for the Young's Modulus of the bar can be derived.

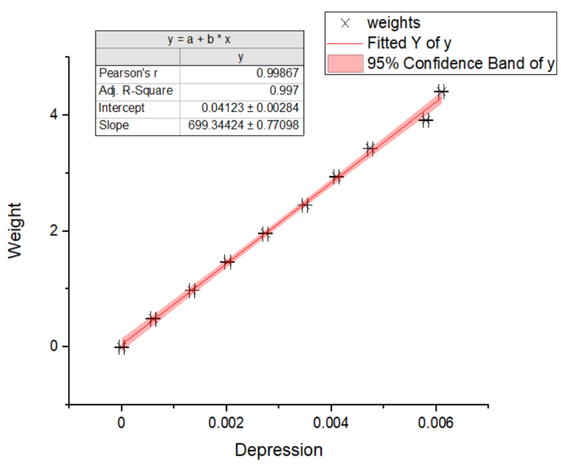

FIG. 5. The graph of weight against depression with the error band included and summary of the slope and intercept values

DISCUSSION

After calculating and propagating through out errors using the standard partial differentiation method, we abstract a value of E as $1.02 \times 10^{11} N/M^{-2} \pm 0.00046 \times 10^{11}$. This value includes all the errors produced by the experiment and is a representation of the precision of the experiment and it's accuracy. There is evidence of a random error with point with weight 3.92N. This value lies short of the expected value that would sit within the fitted line and confidence band. this suggests that there was a mistake in the measurement of one or more of the values of this point and further analysis shows a large abnormality in the depression for this value. Finding the average difference between the depressions of consecutive values to be around 8cm the difference between the depression of mass 350g and mass 400g being around 13 and thus the difference between 400g and 450g being approximately 3 suggests that the measurement of the height at 400g was incorrect.

Further analysis shows that the measurements of the height for weight 400 was systematically wrong suggesting some kind of interference during these measurements. This may have occurred due to a shift in positioning of some equipment or the incorrect reading of the measuring equipment multiple times in a row however unlikely this is to be the case considering measurements were taken independently by two separate individuals. This error however is unlikely to shift our calculated value of the young's modulus too drastically as the fit on our diagram largely disregards this data point as anomalous.

CONCLUSION

In conclusion our error on the young's modulus appears within a reasonable amount of our value giving us an error boundary of 0.01%. Our value for E is within an order of magnitude of what you would expect for a rigid metal such as our bar[7]. This suggests there is no major systematic error in the results and that the value is accurate. The data suggests the proportionality limit was not exceeded and thus it can be assumed that the value of E is consistent.

For a further analysis, lesser increments of mass could be taken and the number of times each measurement is made could be increased for further accuracy. In addition the use of more accurate digital equipment would grant a large advantage in the accuracy of the results as visual measurements are not always particular accurate and in turn are more prone to systematic error due to a persons misunderstanding of equipment. The use of a rod in place of a bar could further reduce inaccuracy with measuring the dimensions of the bar and could thus help to increase the accuracy and precision of this experiment. To conclude a value measured of $1.02 \times 10^{11} N/M^{-2} \pm 0.00046 \times 10^{11}$ was found and this is within a precision of 0.01%.

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[1] Orell Fussli. *The Rational mechanics of Flexible or Elastic Bodies, 1638–1788: Introduction to Leonhardi Euleri Opera Omnia, vol. X and XI, Seriei Secundae.*

[2] Paul. A. Tipler, Gene Mosca. *Physics for Scientists and Engineers* page 209

[3] Wikipedia *https://en.wikipedia.org/wiki/Stress*

[4] University of Bristol *Practical Physics 1, Laboratory Handbook* Page 60

[5] University of Bristol *Practical Physics 1, Laboratory Handbook* Page 59

[6] University of Bristol *Practical Physics 1, Laboratory Handbook* Page 28

[7] University of Cambridge *http://www-materials.eng.cam.ac.uk/mpsite/properties/non-IE/stiffness.html*