Optical Interference: Newtons Rings

A.R Walsh

School of Physics, University of Bristol.

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Utilising the phenomena of Newton's Rings we calculated the refractive index and the radius of convergence for 6 different lenses with a fixed wavelength light and a travelling microscope. We then used Newton's Rings and this information to ascertain a value for the refractive index of water at 1.21542 ± 0.00676 with a 8% accuracy to the accepted value of 1.33[9].

INTRODUCTION

The phenomena caused by light have been widely document in the past, and we continue to be intrigued, finding new applications for a variety of the phenomena that occur during the propagation of light. One such phenomena is referred to as Newtons rings after Sir Issac Newton [1643 - 1727] [1] who originally analysed the effect. Although named after Newton, the first description of this effect was given by Robert Hooke [1635 - 1703] [2] in his book "Micrographia"[3]. We can observe when we place a convex lens, convex side down, on a flat glass surface, a series of concentric light and dark coloured bands.This phenomena is caused by the interference of light rays caused by the refraction of light through the lens[4]. Newton provided a thorough investigation of the cause of this interference pattern and showed that the rings were caused by the constructive and destructive interference of light waves bouncing off the bottom of the lens and the flat glass plate in his 1704 treatise "Opticks"[5]. The constructive interference producing bright rings and in turn the dark rings caused by the destructive interference of the two waves meeting. The rings are concentric and alternate from dark to light producing a striped pattern emanating from a central light point.

Newtons rings have practical applications in a modern world and the effect is often used for testing the uniformity of polished surfaces[4] as well as determining factors such as the wavelength of light, the curvature of a lens and the refractive index of the lens or a medium. In this experiment we attempted to apply some of these applications and measure the radius curvature and refractive index of multiple lenses as well as attempting to determine the refractive index for water. In addition we used the thin lens equation[6](7) first algebraically presented by Edmund Halley [1693 - 1742][7] to determine the focal length of our chosen lenses. This will prove important for our calculations. Also we will make uses of the lens makers equation [6](10) to help us find the refractive index of our medium, in this case a small drop of water. We attempt this experiment to assess the validity of the equations and to find a value for the properties of our lenses and a rough value for the refractive index of water.

THEORY

As light travels through multiple mediums its speed is altered depending on that which it is travelling through. When traveling through air, the speed of light is faster that when traveling through a denser material. We can produce a variable referred to as the ”refractive index” of an object from the speed of light in a vacuum and the speed of light through a medium using the equation:

$$\mu = \frac{c}{v} \tag{1}$$

As light travels from air to a thicker medium, its speed is slowed. If this occurs when the light is travelling at an angle to the new medium, the light beam will be slowed along its width. The part of the beam that meets the medium first will slow before the rest of the beam. This causes a bend in the light known as refraction and is why the value μ is referred to as the refractive index of the medium.

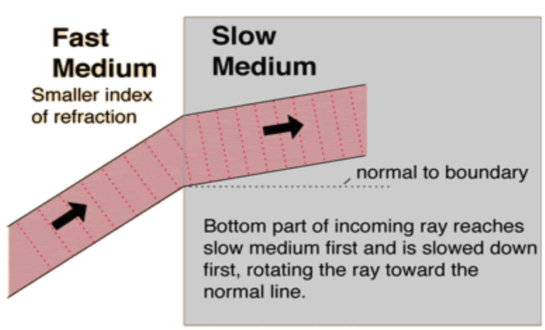

FIG. 1. Refraction occurring at a medium boundary

When we uses a convex lens, as the light approaches the boundary, we can be sure that the rays will not be perpendicular to the lens as a convex lens is curved insuring all light beams are refracted on as they travel from one medium to another. We will uses glass lenses for out experiment. This produces another effect where not all the light travelling through the lens will come out the other side. A portion of this light will be reflected at the boundary and bounced back up towards the source of the light. If we uses a light that is producing approximately parallel light beams, this can be induced by positioning our light source suitably far away from our equipment ensuring that all the light beams that reach us are parallel, we can use a glass panel at 45 degrees to the light rays to reflect them downwards towards a lens. This will now produce refraction and reflection in the lens and we will observe the reflected beams coming back through the lens. The addition of a glass plate to rest the lens on will mean that the refracted beams will be reflected by this plate, bounce back up, be refracted again when re-entering the lens and be produced in addition to the originally reflected beams. These beams will interfere with each other as they combine leaving the lens. Due to the additional distance the refracted waves have travelled, these beams will be of a different phase than the reflected beams. Some of these beams will be of the same phase as those that have been reflected and will combine constructively creating a brighter light. In contrast, some of these beams will be out of phase producing a much darker light. These alternating phases produce a pattern of dark and light concentric rings visible at the top of the lens.

When the gap between the lens and the glass plate is d_n interference at the dark rings occurs when:

$$2d_n = n\lambda_0 \tag{2}$$

Where n is the ring number from the center and λ_0 is the wavelength of the light in free space. If we assume a different medium then air, the wavelength of the light becomes $\lambda_0 = \mu\lambda$ so the wavelength in the medium λ can be substituted for in equation (2). This thus become:

$$2\mu d_n = n\lambda_0 \tag{3}$$

From elementary geometry we can see that if the lens has a long focal length the radius of curvature will thus be large and then the radius of the rings is given as:

$$t_n^2 = d_n(2r_0 - d_n) \tag{4}$$

However we can assume that $d_n << r_0$ as the radius of curvature is bound to be much larger than the radius of the small rings formed by refraction. Thus:

$$2d_n = \frac{t_n^2}{r_0} \tag{5}$$

Then:

$$\mu t_n^2 = n\lambda_0 r_0 \tag{6}$$

This equation thus defines Newton's Rings, and can be used to find the radius of curvature for the lenses.

EXPERIMENTAL METHOD

The first step in determining the radius of curvature for each of our lenses was producing a value for the focal length. This was easily accomplished by placing a light source, our lens and a screen all in line with each other and adjusting the distance between them until the image on the screen comes into focus. This is a frequently used technique for determining the focal point of a lens using the thin lens equation[6]:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{7}$$

To ensure accuracy of this technique we repositioned the apparatus multiple times to produce a range of result. We then calculated the average focal length for each lens.

Next we proceeded to find the radius of curvature of each lens.

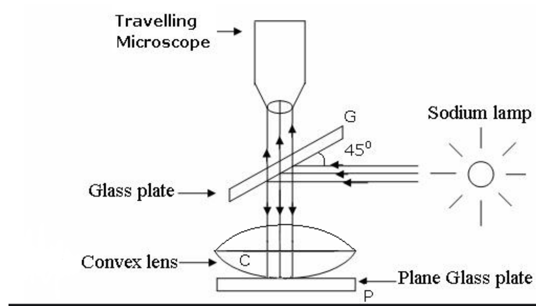

FIG. 2. Set up and equipment for experiment

A diagram of the set up is shown in FIG.2. 6 different convex lenses were used in this experiment to produce values for their refractive index's and radius of curvature's. The lenses were placed on a glass plate. A sodium light was set up on the other side of the room to ensure that the beams of light that reach the equipment were parallel. The experiment was conducted in a dark room to ensure no interference from other light sources in the area. An additional glass plate was suspended, using a clamp, at 45 degrees from the light source to reflect the light down towards the lens sitting below it as seen in FIG.2. A travelling microscope was used to look through the 45 degree plate and observe the newtons rings formed on the surface of the lenses.

Now we had the apparatus set up we placed the first lens labelled lens 1 onto the surface of the glass plane and positioned the travelling microscope at the center of a clear set of newtons rings. We noted down the position of the center of these rings as a reference point to refer to later if needed. We then moved out microscope to the center of the thickness of the first dark ring on the left and recorded the position of the microscope before moving it to the center of the thickness for the same ring on the right. We then repeated this process for each ring going out from the center until ring 10. This same method was repeated for the other 6 lenses. After Compiling this data we then calculated the radius of each ring from the equation:

$$R = \frac{(Ringcenter + x) - (ringcenter - x)}{2} \quad (8)$$

We can now use equation (6) in the form:

$$y = mx + c \quad (9)$$

Where $y = \mu t_n^2, c = 0$ and $x = n\lambda_0$ to find $m = r_0$. We did this by plotting the graphs and finding the values of m as the gradient of the line of best fit. To determine the refractive index of each lens the lens makers equation was used:

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2}) \quad (10)$$

As we used bi-convex lenses, $R_1 = R_2$ thus:

$$\frac{1}{f} = (n-1)(2\frac{1}{r_0}) \quad (11)$$

Then re-arranging for the refractive index n:

$$n = \frac{r_0}{2f} + 1 \quad (12)$$

Next we used our values to determine the refractive index of a new medium placed between the lens and the glass surface. For the purpose of this experiment we utilised a drop of water to give us a new medium to investigate. We set up the equipment in much the same way as shown in FIG.2 however we placed a drop of water on the glass plate before placing the lens on top of this.

We then repeated the process for finding the radius of the Newtons Ring's formed on the surface of the lens. In order to determine the refractive index of the water that was placed between the lens and the plate we reused equation (6) but substituted different values into (9). We made $y = n\lambda_0 r_0, c = 0$ and $x = t_n^2$, and plotted our data. Then using a line of best fit we calculated the gradient of the graph and thus the refractive index for water.

SYSTEMATIC AND RANDOM ERRORS

The measured values all have errors associated with them. To begin, the error on the measurements for the focal length come mainly from human errors. The position in which the image is in focus ranges across a wide envelope so the best way of determining the error on our values was to find the error on the mean across the three different sets of lengths we calculated. The error on the mean is calculated from the equation:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2} \tag{13}$$

The next error was that associated with the radius of the dark fringes. We managed to eliminate the error associated by the measurement of the middle point of the central bright fringe by determine the radius simply from the two values for the position of the dark fringes. The next step was to determine the error on the travelling microscope that can be attributed to 0.01mm. Two values for the radius were then added and divided by two. We can propagate this error through these equations by using the rules of error propagation:

$$F = A + B \mid (\Delta F)^2 = (\Delta A)^2 + (\Delta B)^2 \tag{14}$$

$$\frac{F}{Constant} \mid \frac{\Delta F}{Constant} \tag{15}$$

Propagating the error through these equation gave us an error of 0.007. Next we had to square the value of the radius to produce a suitable value for plotting. When we squared the error we used:

$$F = \frac{AB}{C} \mid (\frac{\Delta F}{F})^2 = (\frac{\Delta A}{A})^2 + (\frac{\Delta B}{B})^2 + (\frac{\Delta C}{C})^2 \tag{16}$$

Thus the error becomes dependent on each of the values for r_0 and was calculated in our tables. After being graphed, the error on the gradient was calculated through the graphing software and produced on the graph.

We then calculated the refractive index of each of the lenses using equation (12). There were two errors present going into the equation in the form of the error on the radius of curvature and the error of the focal length. In order to propagate these errors through equation (12), we used equation (16) and:

$$F = A * c_1 \mid \Delta F = \Delta A * c_1 \tag{17}$$

The value 1 is a constant outside of the equation with no error and thus can be disregard. We can create a temporary value of X and make $X = \frac{r_0}{f}$. Then by finding the error on X we can find the error on n by using equation (17) and multiplying by $\frac{1}{2}$.

Finally the error on the refractive index of water is a combination of the error on the refractive index and the error on the radius of curvature on the lens. The error on the refractive index is once more calculated by the graphing software.

RESULTS

To measure our values of focal length for each lens we measured the distances 3 separate times readjusting the positions of the apparatus each time to ensure that the results were different. We took a mean of our three values to gain a foal length we can work with and took the error on the mean with equation (13). These values are shown in FIG.3.

Lens number	Mean m	SE of mean m
1	0.17301	1.4063E-4
2	0.10697	6.52276E-4
3	0.24908	7.37609E-4
4	0.34566	0.05479
5	0.22624	0.02296
6	0.26189	0.00137

FIG. 3. Focal length of each lens

We took our measurements for the radius of the lenses and produced graphs following the format of equation (9). Here the graph of lens 1 is used as an example in FIG.4

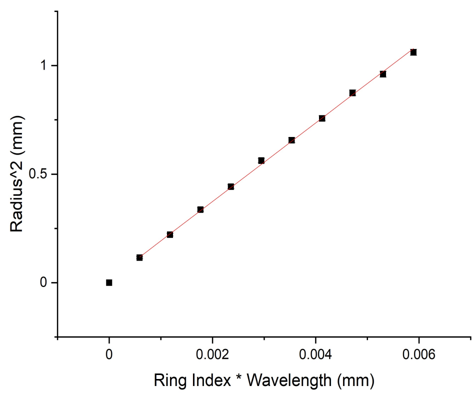

FIG. 4. Graph for lens 1

The value for the radius of curvature of the lens is the same value of the gradient of the graph and is calculated by the graphing solution. After producing these graphs for all 6 of our lenses we produced a table of the radius of curvature of each of the lenses with their associated errors calculated through by the software from the error on the radius of the concentric dark rings.

Lens Number	Raduis of curvature mm	Error on R mm
1	181.2	1.72
2	112.7	1.33
3	261.8	2.04
4	412.1	2.62
5	258.7	2.02
6	422.2	4.28

FIG. 5. Radius of curvature and focal length for each lens

We then calculate our refractive index from the values obtained in FIG.5. Using equation (12) we produced values for the refractive index of all 6 lenses including their errors as shown is FIG.6

Lens Number	refractive index (col(E)/col(B))^2	Error on refractive index
1	1.52366	0.00499
2	1.5268	0.007
3	1.52554	0.00438
4	1.5961	0.09456
5	1.57174	0.0582
6	1.80607	0.00919

FIG. 6. Refractive index and error for each lens

Finally we measured the positions of the edges of the dark rings of interference with water as a medium using lens 5. We then used equation (8) to find the radius of each of these rings and squared this to produce a value we can plot according to equation (9) with the new values for x,y and c. We propagated the error for the radius squared in the same way as before to give us FIG.7

ring order(N) lens 6	Radius ^2 mm	Error on radius ^2 mm
0	0	0
1	0.1089	0.00462
2	0.2209	0.00658
3	0.36	0.0084
4	0.49	0.0098
5	0.6084	0.01092
6	0.7396	0.01204

FIG. 7. Radius of fringes on water with lens 5

Next we found each axis and the errors associated with these using equation (17) and plotted a graph with the gradient being the value for the refractive index of water FIG8. Our final value for the refrac-

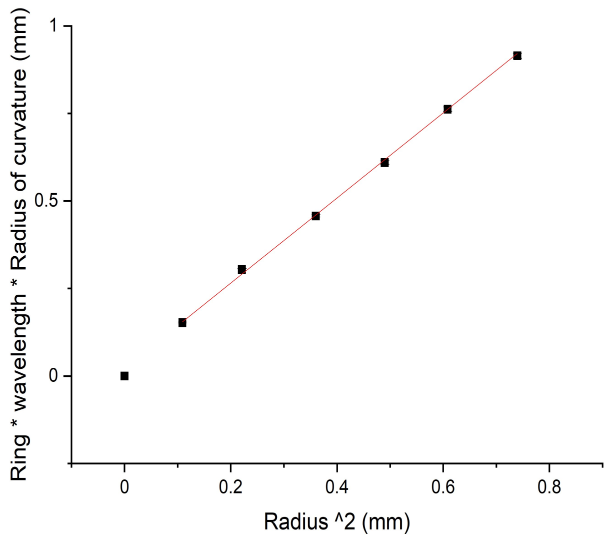

FIG. 8. Graph with slope of refractive index of water

tive index of water was 1.21542 ± 0.00676

DISCUSSION

After calculating and propagating through our errors using a variety of techniques we find a value for the refractive index of water to be 1.21542 ± 0.00676. This value suggests we have conducted an experiment with a high precision but a lower accuracy. The expected value for the refractive index of water is 1.33 [9] thus our experimental value is off by approx 0.11 = 8% of the true value. There are a few reasons our results may be off by this much.

When measuring the displacement of each dark concentric ring this requires the rough measurement of the center of the thickness of each of these rings. The error in this experiment was taken as the error associated to the vernier scale on the travelling microscope but it is very likely that the larger error is a result of the imprecision of measuring the center of the thickness of the rings. The error can be re-evaluated using an alternate method to produce a better error. The measurements could each be taken three times to produce data for a error on the mean however the amount of measurements that would then be necessary would be considerably higher. Another method could have been to measure the edges of each of the rings and then calculate the middle point between the edges, this would be effective as the inaccuracy of measuring the center by hand is eliminated as it is much easier to position the microscope on the edge of each ring. When measuring the radius of the concentric dark rings, when water was used as a medium, the clarity of Newton's Rings was considerably less then our previous observations. The use of water as the medium meant only 6 data points could be taken rather than the 10 we took for our previous observations on our lenses. This further decreases the accuracy of our final result as we have less data to consider.

In addition to the inaccuracy of our final result for the refractive index of water, the validity of our measurements for the calculation of the radius of curvature and refractive index of lens 6 can be called into question. In our production of the graph for lens 6 it was evident that the first two values where massive outliers. Due to the fact we have only taken 10 data points for our graph, having the first two as visible outliers calls into question the validity of the results for lens 6. In addition the calculated error on the radius of convergence for lens 6 is double that of the other values, this once more throws lens 6 into question and for this reason theses results for lens 6 should be reassessed.

CONCLUSION

In conclusion our value for the refractive index of water is slightly lower then the expected value of 1.33 [9] and our error is small enough that the true value does not sit comfortably within our error range. This being said we have identified a few corrections we could undertake to increase the accuracy of our results and on a second attempt at defining the refractive index of water it may be possible to get a better answer then that which we found.

The values obtained however for the radius of curvature of the lenses and the refractive index of our lenses can all be considered as accurate, apart from the values obtained from our measurements of lens 6 that seem inaccurate due to the outliers in the data. Had we the time available the values for lens 6 should be recalculated to gain a more accurate picture of this lens. The measurements of the radius of the dark rings could be reassessed on a later experiment to produce a more accurate result for all our lenses.

The values obtained for the properties of our lenses are with the bounds we would expect for similar type lenses and the refractive index of water measured represents a good approximation of the true refractive index. We have assessed a few improvements we could implement to further our experiment and we have assessed the validity of our results producing a final value of 1.21542 $\pm$ 0.00676 for the refractive index of water an 8% difference from the accepted value of the refractive index of water.

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