School of Mechanical Engineering
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Connected and Autonomous Vehicles Systems

Kalman Filter

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Kalman Filter

Introduction

The standard equations of the Kalman Filter are divided into two main steps:

1. **Prediction** (estimates the current state based on the previous state and the system model)
2. **Update** (corrects the prediction using the new measurement)

Vehicle position model: $\mathrm{x}(t) = v \times \mathrm{t}$

Example of 5s prediction for a vehicle driving at 20±1m/s:

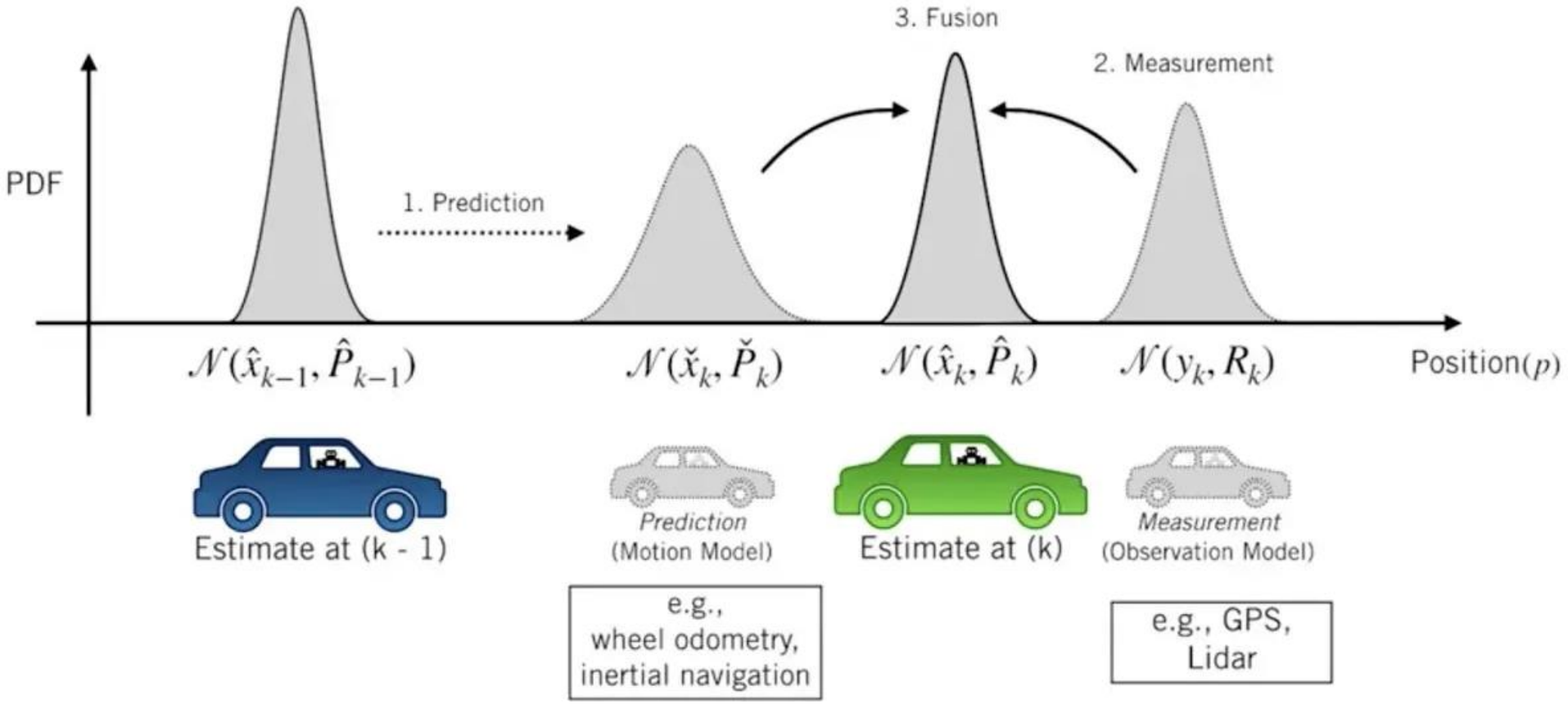
$$\mathrm{x}(5)_{min} = v_{min} \times \mathrm{t} = 19m/s \times 5s = 95\mathrm{m}$$

$$\mathrm{x}(5)_{max} = v_{max} \times \mathrm{t} = 21m/s \times 5s = 105\mathrm{m}$$

Predicted position depends on speed measurement accuracy.

But speed is estimated from **position** (GPS) or **distance** (encoder, IMU) and **time**.

The Kalman Filter | Prediction and Correction



Kalman Filter

Estimation of position and velocity of an autonomous vehicle, by combining sensor data and a vehicle position model.

1. Prediction Step

This step estimates the current state based on the previous state and the system model.

State prediction: $\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + Bu_k$

Where: $\hat{x}_{k|k-1}$: predicted state at time k
$\hat{x}_{k-1|k-1}$: previous state estimate
A: state transition matrix
B: control input matrix
u_k: control input

Covariance prediction: $P_{k|k-1} = AP_{k-1|k-1}A^T + Q$

Where: $P_{k|k-1}$: predicted estimate covariance
Q: process noise covariance

2. Update Step

This step corrects the prediction using the new measurement.

State update: $\hat{x}_{k|k} = \hat{x}_{k|k-1} + \boldsymbol{K_k}(z_k - H\hat{x}_{k|k-1})$

Where: z_k: measurement at time k

Kalman Gain: $\boldsymbol{K_k} = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1}$

Where: K_k: Kalman gain H: observation matrix R: measurement noise covariance

Covariance update: $P_{k|k} = (I - K_kH)P_{k|k-1}$

These equations form the basis of the linear Kalman Filter. For nonlinear systems, in autonomous vehicles, the Extended Kalman Filter is often used.

ANY QUESTIONS ???