## T1 - Steering by wire

Prototype of a racing car will have "by-wire" steering system presented in Figures T1a and T1b installed. Calculate main parameters of this steering system, as described below, based on a hydraulic actuator for a car with weight on a steering axle of 1200 kg and tyres 205/55R16. Assume ratio of E/B=0.4, circular shape of tyre patch and wheel steering angle of +/-35º. Use arm radius equal to radius of the wheel.

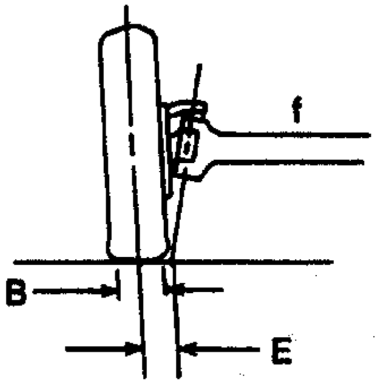



Figure T1a: Kingpin offset and tyre patch width.

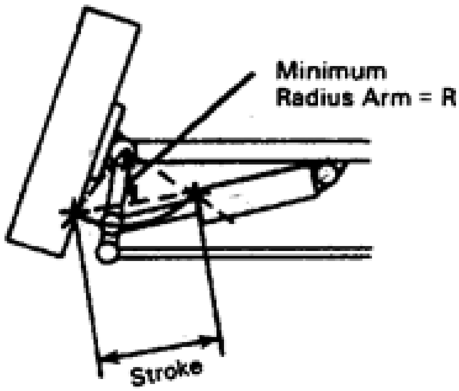



Figure T1b: Configuration of hydraulic actuator.

Calculate:

a. Calculate kingpin torque ($T_k$) assuming coefficient of friction μ=0.44 [566.9 Nm]

b. Determine minimum cylinder force ($F_c$) necessary to turn the wheel when vehicle is stationary [3405.6 N]

c. Calculate minimum cylinder area ($A_c$) if the hydraulic pump can deliver pressure of 5 MPa [0.00068111 $m^2$]

d. Determine cylinder stroke (S) required to achieve steering angle of +/-35º [0.2331 m]

e. Calculate swept volume of hydraulic oil required to turn wheels from -35º to +35º [0.000 158 769 $m^3$]

## T1 - Steering by wire - Solution:

Known values:

$m_v = 1200\text{kg}$ $\quad\mu = 0.44$ $\quad g = 9.81\frac{m}{s^2}$

$\alpha = 35\text{deg}$ $\quad\frac{E}{B} = 0.4$ $\quad P = 5\text{MPa}$

### a) Calculate the kingpin torque

Assuming circular tire patch and tire size 205/55 R16

Tyre width:

$B = 0.205m$ $\quad B = 205\text{mm}$

Kingpin offset:

$E = 0.4 \cdot B = 0.4 \cdot 0.205m$ $\quad E = 0.4 \cdot B = 0.082m$

$W = m_v \cdot g = 1200\text{kg} \cdot 9.81\frac{m}{s^2}$ $\quad W = m_v \cdot g = 11772N$

Kingpin Torque:

$$T_K = W \cdot \mu \cdot \sqrt{\frac{B^2}{8} + E^2} = 11772N \cdot 0.44 \cdot \sqrt{\frac{(0.205m)^2}{8} + (0.082m)^2} = 566.8644Nm$$

### b) Determine the Cylinder Force

Fc = Cylinder Force (N)

R = Minimum Radius Arm (NOTE: different from arm length)

(For 16" wheel arm should be no more than 8", therefore if arm is 8"

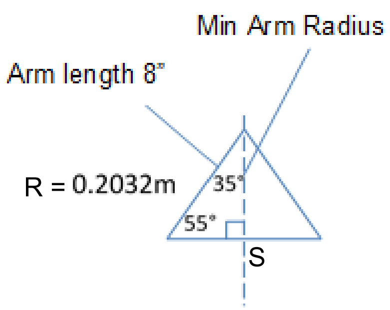
Minimum Radius Arm will be 8" x cos(35º) therefore

R=8"* x cos(35º)=6.55"=0.1665m)

$R = 8\text{in}$ $\quad R = 0.2032m$

$$R_{\min} = R \cdot \cos(\alpha) = 0.2032m \cdot \cos\left(35 \cdot \frac{\pi}{180}\right) = 0.1665m$$

$$F_c = \frac{T_K}{R_{\min}} = \frac{566.8644N}{0.16645m} = 3405.5793N$$




### c) Calculate the Cylinder Area

P = Pressure of hydraulic pump

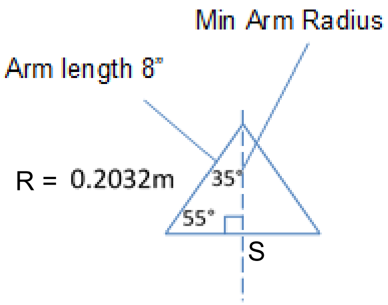
$P = 5 \cdot 10^6\text{Pa}$

$$A_C = \frac{F_c}{P} = \frac{3405.5793N}{5 \cdot 10^6 Pa} = 0.00068111m^2 = 681.1159\text{mm}^2$$

and cylinder diameter is: $A_C = \pi \cdot r^2$

Therefore: $r = \sqrt{\frac{A_C}{\pi}} = \sqrt{\frac{0.00068111m^2}{\pi}} = 0.0147m$

### d) Determine the Cylinder Stroke




Therefore:

$S = 2 \cdot R \cdot \cos(55\text{deg}) = 2 \cdot 0.2032m \cdot \cos(55\text{deg}) = 0.2331m$

### e) Calculate the Swept Volume

Diameter of the cylinder

$D_B = 2 \cdot r = 2 \cdot 0.0147m = 0.02944m$

$$V_S = \frac{\pi \cdot {D_B}^2}{4} \cdot S = \frac{\pi \cdot (0.02944m)^2}{4} \cdot 0.2331m = 0.0001587691m^3 = 158.7691\text{cm}^3$$

Swept Volume in litres:

$V_S = 0.1588\text{dm}^3$

Alternatively Volume can be calculated from Area x Stroke (without radius calculations)

$V_s = A_C \cdot S = 0.0001587691m^3$

**T2 - LIDAR Processing Speed**

A LIDAR sensor has a field of view of 180 degrees horizontally and 30 degrees vertically, and its horizontal angular resolution is 0.2 degrees and vertical angular resolution of 1.0 degree. The maximum frame rate is 20 Hz. Calculate how many points can be updated per second.
[540000 points/second]

**T2 - LIDAR Processing Speed - Solution:**

Number of 'point cloud' will be:
$N_C = (180 / 0.2) \times (30 / 1.0) = 900 \times 30 = 27000$ scanning points
With 20 updates per second a total number of points updates per second
$T_C = N_C \times 20 = 27000 \times 20 = 540000$ points/second

**T3 - Camera Processing Speed**

Image data obtained from the camera with resolution of 1920x1080 pixels and frame rate of 30 fps will be processed by the on-board computer with a single processor with 4 cores; each data point will take 100 processor cycles (ticks) to process, and other functions will take another 600,000 cycles, what should be the minimum processor speed in MHz. [1555.35 MHz]

**T3 - Camera Processing Speed - Solution:**

Camera resolution: 1920x1080 = 2 073 600 pixels (2MP)
Number of pixels to process $N_p = 1920 \times 1080 \times 30 = 2073600 \times 30 = 62208000$ pixels
$CPU_{cycl} = (N_p \times 100) + 600\,000 = 6{,}221{,}400{,}000$ cycles/second
Speed per core = 6,221,400,000 / 4 = 1,555,350,000 = 1.55GHz
This is theoretical speed of camera data processing, more realistic value will be 2GHz as the CPU will be doing other thing as well.

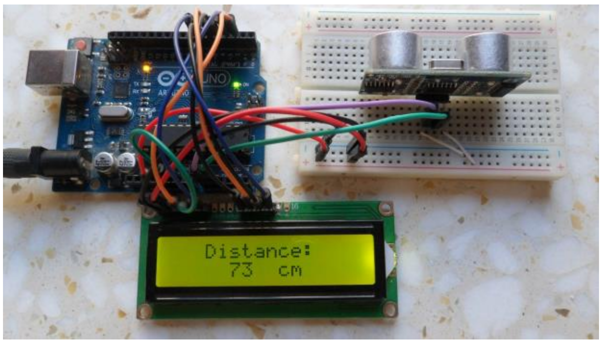
**T4 - Ultrasound sensor**

Calculate the minimum and maximum frequency of signal updates that can be obtained from the ultrasound sensors working with the following parameters:

Range of the detection: $d_{US}$ from 0.3m to 8m

Speed of sound: $c_s$=343 m/s

Experimental tests with an ultrasound sensor connected to the Arduino Uno suggest that the maximum frequency for 0.3 m is 10% lower than the theoretical value. Please explain why the theoretical speed is not possible to achieve in real-life applications.
[min 21Hz, max 571Hz]




**T4 - Ultrasound sensor - Solution:**
The sound wave emitted by the sensor will have to travel 8m, bounce back from the obstacle, and then come back to the sensor. Therefore, the distance travelled will be 2 x range.

**Time of travel for 8m range**

$$d_{US} = \frac{t \cdot c_s}{2} \qquad t = \frac{2 \cdot d_{US}}{c_s} \qquad f = \frac{1}{t}$$

t = (2 x $d_{US}$) / $c_s$ = (2 x 8m) / 343 m/s = 0.046 647 230 s

*f* = 1/t = 1/ 0.046 647 230s = 21.4375Hz

$f_{US}$ = $c_s$ / (2 x range) = 343m/s / 16m = 21.4375Hz

**Time of travel for 0.3m range**

t = (2 x $d_{US}$) / $c_s$ = (2 x 0.3m) / 343 m/s = 0.001 749 271s

*f* = 1/t = 1/ 0.001 749 271s = 571.6666Hz

$f_{US}$ = $c_s$ / (2 x range) = 343m/s / 0.6m = 571.6666Hz

Theoretical speed is not possible to achieve due to the processing time and slow communication protocols (serial COM port, LCD display $I^2C$) used in this application.

## T5 - Encoders

Formula Student autonomous vehicle has encoders installed on the wheels and is driving along left-hand turn of the track.

| | |
|---|---|
| Number of teeth on the encoders: | 200 teeth per revolution |
| Wheel rolling radius: | 0.4 m |
| Vehicle track: | 1.5 m |
| Left wheel encoder signal pulses | 20 pulses per 80ms |
| Right wheel encoder signal pulses | 24 pulses per 80ms |

Calculate the radius of the turn [8.25 m]

## T5 - Encoders Solution

Known values:

$n_i = 20$  $n_o = 24$  $N = 200$  $t = 80\text{ms}$

$r = 0.4m$  $W_t = 1.5m$

ω = 2πn/Nt

Where: ω = angular speed (rad/s)  n = number of pulses

t = sampling period (s)  N = pulses per rotation

**Inner wheel:**

$t = 0.08s$

$$\omega_i = \frac{2\cdot\pi}{t}\cdot\frac{n_i}{N} = \frac{2\cdot\pi}{0.08}\cdot\frac{20}{200}$$

$$\omega_i = \frac{2\cdot\pi}{t}\cdot\frac{n_i}{N} = 7.854(\text{rad/s})$$

$$N_w = \omega_i\cdot\frac{60}{2\cdot\pi}$$

$$N_w = \omega_i\cdot\frac{60}{2\cdot\pi} = 75(\text{rpm})$$

$$V = 2\cdot\pi\cdot r\cdot\frac{N_w}{60}$$

$$V_i = r\cdot N_w = 30\frac{m}{s}$$

$$V_i = 67.1081\text{mph}$$

**Outer wheel:**

$$\omega_o = \frac{2\cdot\pi}{t}\cdot\frac{n_o}{N} = \frac{2\cdot\pi}{0.08}\cdot\frac{24}{200}$$

$$\omega_o = \frac{2\cdot\pi}{t}\cdot\frac{n_o}{N} = 9.4248(\text{rad/s})$$

$$N_w = \omega_o\cdot\frac{60}{2\cdot\pi}$$

$$N_w = \omega_o\cdot\frac{60}{2\cdot\pi} = 90(\text{rpm})$$

$$V = 2\cdot\pi\cdot r\cdot\frac{N_w}{60}$$

$$V_o = r\cdot N_w = 36\frac{m}{s}$$

$$V_o = 80.5297\text{mph}$$

Radius

$$R = \frac{W_t}{2}\cdot\frac{(\omega_o+\omega_i)}{(\omega_o-\omega_i)} = \frac{1.5m}{2}\cdot\frac{(9.4248\ +7.854)}{(9.4248\ -7.854)} = 8.25m$$

$$R = \frac{W_t}{2}\cdot\frac{(V_o+V_i)}{(V_o-V_i)} = \frac{1.5m}{2}\cdot\frac{(80.5297\ +67.1081)}{(80.5297\ -67.1081)} = 8.25m$$

## T6 – Vehicle position

The autonomous vehicle is using wheel **Encoders** and **GPS** signal to estimate the position. Encoders provides position measurement at 20Hz with standard deviation of 0.2m.
GPS data is received at a frequency of 1Hz with a standard deviation of 3m.
Vehicle drives at the velocity of 20m/s.

a. Calculate the predicted position of the vehicle after 5s using **Encoders** data and estimate the error [100m +/-2m]
b. Calculate the predicted position of the vehicle after 5s using **GPS** data and estimate the error [100m +/-3m]
c. After 5s GPS reported new position at 104m from initial position. Calculate the predicted position of the vehicle after another 5s (at t=10s) using above signals fussed by Kalman filter [201.23m +/-2.6m]

## T6 – Vehicle position Solution

a) Calculate the predicted position of the vehicle after 5s using **Encoders** data and estimate the error.
The vehicle moves at a constant velocity of 20 m/s for 5 seconds:

$$x_{enc}(t = 5s) = 20\,m/s \times\ 5\,s\ =\ 100m$$

Error estimation:
Each position measurement has standard deviation σ=0.2 m.
Considering that error can accumulate, cumulative standard deviation over number of observations N=20Hz*5s=100:

$$Position\ error\ variance = \sigma_{enc}^2 = \sum_{i=1}^{N}(\sigma^2)$$

$$Position\ error\ variance = \sigma_{enc}^2 = N \cdot \sigma^2 = 100 \cdot (0.2)^2 = 4\ \mathrm{m}^2$$

Standard deviation of position error:

$$\sigma_{enc} = \sqrt{4} = 2\ \mathrm{m}.$$

Predicted Position = $100\ \mathrm{m} \pm 2\ \mathrm{m}$

b) Calculate the predicted position of the vehicle after 5s using **GPS** data and estimate the error.

Position prediction (using the single GPS measurement at $t = 5$s):

$$x_{gps}(t = 5s) = 20\frac{m}{s} \times\ 5\,s = 100\ \mathrm{m}$$

Error (per measurement):

$$\sigma_{\mathrm{gps}} = 3.0\ \mathrm{m}$$

Predicted Position = $100\ \mathrm{m} \pm 3.0\ \mathrm{m}$

c) After 5s GPS reported new position at 104m from initial position. Calculate the predicted position of the vehicle after another 5s (at t=10s) using above signals fussed by Kalman filter.

**<u>Sensors fusion at 5s:</u>**

Encoder at 5s: $x_{enc}^{-}(t = 5s) = 100$ m $\quad P_5^{-} = \sigma_{\mathrm{enc}}^2 = (2.0)^2 = 4.0$

GPS at 5s: $x_{gps}(t = 5s) = 104$ m $\quad R = \sigma_{\mathrm{gps}}^2 = (3.0)^2 = 9.0$

Kalman gain:

$$K = \frac{P_5^{-}}{P_5^{-} + R} = \frac{4}{4+9} = \frac{4}{13} \approx 0.3077$$

Updated (fused) mean and variance:

$$x_5 = x_{enc}^{-} + K\,(x_{gps} - x_{enc}^{-}) = 100 + 0.3077 \cdot (104 - 100) \approx 101.2308\ \mathrm{m}$$

$$P_5 = (1 - K)\,P_5^{-} = (1 - 0.3077) \cdot 4 \approx 2.7692$$

Error: $\sigma_5 \approx \sqrt{2.7692} \approx 1.664\ \mathrm{m}$

**<u>Predict to t=10s:</u>**

Deterministic motion over 5s

$$\Delta x = v \cdot \Delta t = 20 \cdot 5 = 100 \text{ m}$$

$$x_{10}^{-} = x_5 + \Delta x \approx 101.2308 + 100 = 201.2308\ \mathrm{m}$$

Accumulated encoder process noise over the next 5 s:

$$N = 20\ \mathrm{Hz} \times 5\ \mathrm{s} = 100$$

$$Q = N\,\sigma_{\mathrm{enc}}^2 = 100 \cdot (0.2)^2 = 4 \text{ m}^2$$

Predicted variance at 10 s:

$$P_{10}^{-} = P_5 + Q \approx 2.7692 + 4.0 = 6.7692$$

Error: $\sigma_{10} \approx \sqrt{6.7692} \approx 2.60\ \mathrm{m}$

Predicted Position after fusion:

$$x(t = 10s) \approx 201.23 \text{ m} \pm 2.6 \text{ m}$$