Industrial Robotics Assignment - Exercise 5 Report

1. Introduction

This report focuses on the kinematic analysis and motion planning of a 6-DOF anthropomorphic manipulator (ENGINEAI-SE01 style). The robot arm consists of a waist, shoulder, elbow, and a 3-DOF spherical wrist. We perform Forward Kinematics (FK), Workspace analysis, Differential Kinematics (Jacobian), Inverse Kinematics (IK), and Trajectory Planning.

2. Kinematic Modeling (Exercise 5.1)

The robot is modeled using the Product of Exponentials (PoE) formula.

Parameters:

- Link Lengths: $L_1 = 0.15$ m, $L_2 = 0.1$ m, $L_3 = 0.25$ m, $L_4 = 0.2$ m.
- Home Configuration (M): We define the home configuration as the arm fully extended vertically along the z_s-axis.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 + L_3 + L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Screw Axes ($\mathcal{S}_i$):

Defined in the Space Frame $\{s\}$ located at the base.

1. **Joint 1 (Waist)**: Rotation about z_s. $\omega_1 = [0, 0, 1]^T, q_1 = [0, 0, 0]^T$.
2. **Joint 2 (Shoulder)**: Rotation about y_s. $\omega_2 = [0, 1, 0]^T, q_2 = [0, 0, L_1]^T$.
3. **Joint 3 (Elbow)**: Rotation about y_s. $\omega_3 = [0, 1, 0]^T, q_3 = [0, 0, L_1 + L_2]^T$.
4. **Joint 4 (Wrist 1)**: Rotation about z_s. $\omega_4 = [0, 0, 1]^T, q_4 = [0, 0, L_1 + L_2 + L_3]^T$.
5. **Joint 5 (Wrist 2)**: Rotation about y_s. $\omega_5 = [0, 1, 0]^T, q_5 = [0, 0, L_1 + L_2 + L_3]^T$.
6. **Joint 6 (Wrist 3)**: Rotation about x_s. $\omega_6 = [1, 0, 0]^T, q_6 = [0, 0, L_1 + L_2 + L_3]^T$.

The Forward Kinematics is given by:

$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} e^{[\mathcal{S}_2]\theta_2} \dots e^{[\mathcal{S}_6]\theta_6} M$$

Workspace:

Using a Monte Carlo method with joint limits ($\theta_1 \in [-90^\circ, 90^\circ]$, $\theta_2 \in [-70^\circ, 90^\circ]$, $\theta_3 \in [-90^\circ, 90^\circ]$), we visualized the reachable workspace (see Matlab Figure 1).

3. Jacobian Analysis (Exercise 5.2)

The Space Jacobian $J_s(\theta)$ maps joint velocities to the end-effector spatial twist: $\mathcal{V}_s = J_s(\theta)\dot{\theta}$.

The columns of the Jacobian are given by:

$$J_{s1} = \mathcal{S}_1$$

$$J_{si} = \mathrm{Ad}(e^{[\mathcal{S}_1]\theta_1} \dots e^{[\mathcal{S}_{i-1}]\theta_{i-1}})\mathcal{S}_i, \quad i = 2 \dots 6$$

We implemented a custom script to compute this without JacobianSpace. For the configuration $\theta = (50, 10, -15, 30, 30, -15)^\circ$, the resulting end-effector velocity twist was calculated (see code output).

4. Inverse Kinematics (Exercise 5.3)

We implemented a **Newton-Raphson** numerical solver.

- **Update Rule**: $\theta_{k+1} = \theta_k + J^\dagger(\theta_k)\mathcal{V}_{err}$
- **Target 1 ($\mathbf{x}_e$)**: Position $(0.2, 0.1, 0.3)$, Orientation defined by Euler Angles $(10^\circ, 30^\circ, 20^\circ)$.
- Target 2 (T_{se}): Given matrix.
  The solver successfully converged for both targets.

5. Trajectory Planning (Exercise 5.4 & 5.5)

Point-to-Point (P2P):

A cubic time-scaling path was generated between $\mathbf{x}_e$ and T_{se} ($T_f = 0.5$ s). The joint velocity analysis shows that a duration of 0.5 s results in velocities exceeding the $30°/s$ limit. To satisfy the constraint $|\dot{\theta}| \leq 30°/s$, the duration T_f must be significantly increased (approx. factor of 6-10 based on peak velocity).

Multi-Waypoint:

A trajectory passing through Home $\rightarrow \mathbf{x}_e \rightarrow T_{se} \rightarrow$ Home was generated ($T_f = 25$ s). The IK was solved at each step, and the joint profiles were plotted, confirming smooth motion within limits.