Combined Modular Fields

Sets of Numbers Whose Combined Multiples Cover Either $\frac{1}{2}$ or $\frac{1}{3}$ of the Positive Integers

Alex Weslowski
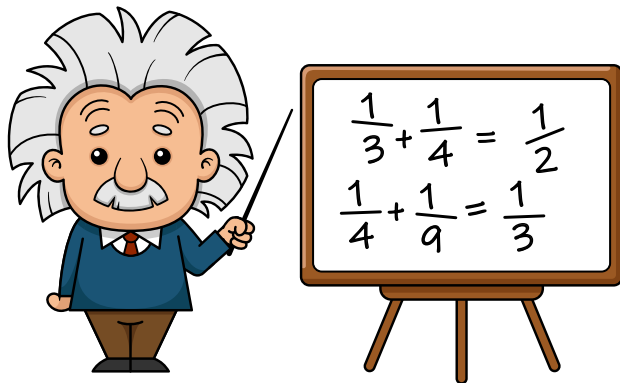
April 13, 2023

```
f"number.theory.{'447867046214735262'[::-1]}@gmail.com"
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Unit Fractions

Unit fractions

Vector Graphics by Vecteezy

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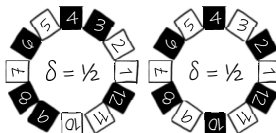## Unit Fractions

Unit fractions
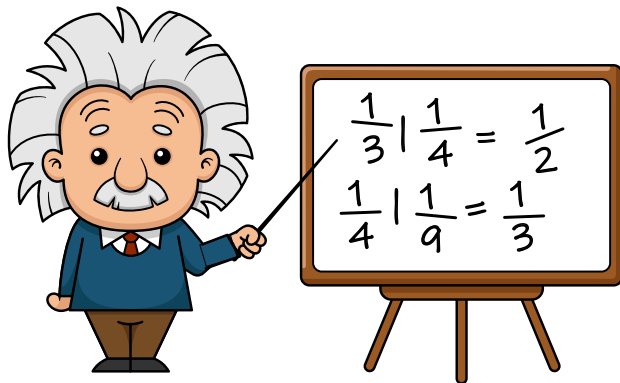
Vector Graphics by Vecteezy

Visual Proof

Visual representation of density $\delta = \frac{1}{2}$

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Visual Proof

Visual representation of density $\delta = \frac{1}{3}$

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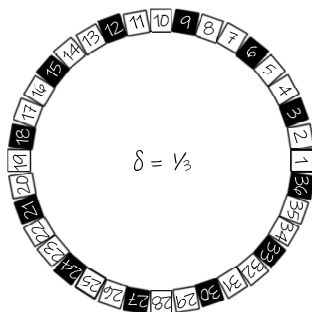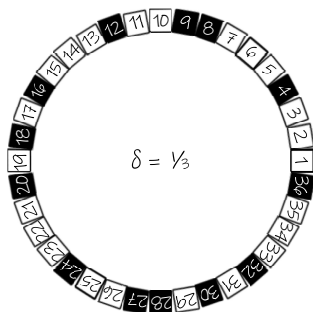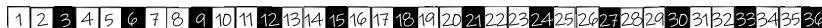
Decelerated Alternating Sum

Decelerated alternating series

$$\frac{1}{(1 - 3a_3 - 3a_4 + 3a_{12})} \sum_{n=1}^{\infty} \chi(n; 3, 4) \, a_n$$

$$\text{where } \chi(n; 3, 4) = \begin{cases} 1, \text{if } n \bmod 3 \neq 0 \ \& \ n \bmod 4 \neq 0 \\ -2, \text{if } n \bmod 3 = 0 \mid n \bmod 4 = 0 \end{cases}$$

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Decelerated Alternating Sum

Decelerated alternating series

$$\frac{1}{(1 - 2a_3)} \sum_{n=1}^{\infty} \chi(n; 3) \, a_n$$

$$\text{where } \chi(n; 3) = \begin{cases} 1, \text{if } n \bmod 3 \neq 0 \\ -1, \text{if } n \bmod 3 = 0 \end{cases}$$

$$\frac{1}{(1 - 2a_4 - 2a_9 + 2a_{36})} \sum_{n=1}^{\infty} \chi(n; 4, 9) \, a_n$$

$$\text{where } \chi(n; 4, 9) = \begin{cases} 1, \text{if } n \bmod 4 \neq 0 \ \& \ n \bmod 9 \neq 0 \\ -1, \text{if } n \bmod 4 = 0 \mid n \bmod 9 = 0 \end{cases}$$

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Combined Modular Fields

Appendix: Data

Tabular data for density $\delta = \frac{1}{2}$
https://github.com/AlexWeslowski/Integer-Sequence/

[2]	2	[3, 5, 23, 64, 176]	3886080
[3, 4]	12	[3, 5, 22, 64, 331]	6990720
[3, 7, 8]	168	[5, 6, 7, 11, 29, 256]	17149440
[3, 5, 16]	240	[3, 7, 22, 23, 29, 128]	39443712
[3, 7, 10, 32]	6720	[3, 7, 20, 22, 26, 256]	61501440
[3, 5, 31, 32]	14880	[3, 5, 17, 511, 512]	66716160
[3, 5, 17, 256]	65280	[3, 7, 20, 22, 29, 256]	68597760
[3, 7, 11, 32, 80]	591360	[3, 5, 17, 341, 1024]	89041920
[3, 7, 11, 40, 64]	591360	[3, 7, 11, 52, 64, 130]	99939840
[3, 7, 10, 62, 64]	833280	[3, 7, 11, 64, 65, 104]	99939840
[3, 7, 10, 64, 71]	954240	[5, 6, 9, 11, 14, 19, 128]	101122560
[3, 7, 11, 32, 155]	1145760	[3, 7, 11, 50, 64, 160]	118272000
[3, 7, 11, 29, 320]	2143680	[3, 7, 10, 55, 64, 176]	130099200
[3, 7, 10, 41, 256]	2204160	[3, 7, 11, 38, 95, 160]	133425600

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Integer sequence for density $\delta = \frac{1}{2}$
https://github.com/AlexWeslowski/Integer-Sequence/

2, 12, 168, 240, 6720, 14880, 65280, 591360, 591360, 833280, 954240,
1145760, 2143680, 2204160, 3886080, 6990720, 17149440, 39443712,
61501440, 66716160, 68597760, 89041920, 99939840, 99939840,
101122560, 118272000, 130099200, 133425600, 187031040, 209932800,
229152000, 236651520, 262416000, 272912640, 366213120, 367933440,
421048320, 426961920, 435240960, 514631040, 522762240, 546712320,
546712320, 623293440, 870481920, 1045524480, 1080330240, 1454046720,
1565921280, 1733698560, 1788695040, 1923018240, 2067394560,
2067394560, 2195128320, 2196606720, 2393825280, 2606714880,
2650306560, 2658163200, 2685281280, 2709611520, 2722473600,
2915404800, 3429888000, 3644256000, 4294901760, 4638036480,
4665239040, 4918911360, 5311299840, 5379306240, 6259545600,
7608944640, 9835668480, 17012244480, 22160307840, 42365266944

Appendix: Data

Tabular data for density $\delta = \frac{1}{3}$
https://github.com/AlexWeslowski/Integer-Sequence/

[3]	3	[7, 9, 11, 30, 76]	1580040
[4, 9]	36	[7, 9, 11, 32, 80]	1774080
[4, 10, 21]	840	[7, 9, 11, 40, 64]	1774080
[5, 7, 36]	1260	[7, 9, 10, 48, 62]	1874880
[4, 11, 45]	1980	[7, 9, 10, 48, 71]	2147040
[7, 9, 10, 32]	20160	[7, 9, 10, 52, 69]	2260440
[8, 10, 11, 12, 21]	221760	[7, 9, 10, 62, 64]	2499840
[8, 10, 11, 12, 23]	242880	[7, 9, 10, 46, 87]	2521260
[7, 9, 12, 15, 34]	385560	[7, 9, 11, 24, 155]	2577960
[7, 9, 12, 16, 50]	604800	[7, 9, 10, 64, 71]	2862720
[7, 9, 11, 30, 48]	997920	[7, 9, 11, 32, 155]	3437280
[7, 9, 11, 40, 48]	1330560	[7, 9, 10, 33, 172]	3575880
[7, 9, 10, 48, 51]	1542240	[7, 9, 11, 29, 240]	4823280
[7, 9, 10, 33, 75]	1559250	[7, 9, 10, 41, 192]	4959360

Combined Modular Fields

Appendix: Data

Integer sequence for density $\delta = \frac{1}{3}$
https://github.com/AlexWeslowski/Integer-Sequence/

3, 36, 840, 1260, 1980, 20160, 221760, 242880, 385560, 604800,
997920, 1330560, 1542240, 1559250, 1580040, 1774080, 1774080,
1874880, 2147040, 2260440, 2499840, 2521260, 2577960, 2862720,
3437280, 3575880, 4823280, 4959360, 6431040, 8225280, 9574740,
10834560, 12735360, 13110240, 13464000, 14002560, 17962560,
18532800, 22619520, 23587200, 23746800, 26308800, 27692280,
31600800, 33929280, 36516480, 46494000, 53222400, 57380400,
58618560, 59028480, 59028480, 66528000, 66673152, 71051904,
74366208, 76396320, 85542912, 87816960, 89147520, 95413248,
96163200, 99380736, 101969280, 137192832, 155925000, 158096400,
187488000, 192780000, 194232720, 195022080, 214704000, 237144600,
248648400, 275425920, 285538176, 291349080, 295384320, 303367680,
303367680, 319334400, 328382208, 354816000, 390297600, 395010000

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