Introduction to Binary Field Density

Binary Fields $GF(2)$ of Densities $\delta(f) = \frac{1}{2}$ *and* $\frac{1}{3}$

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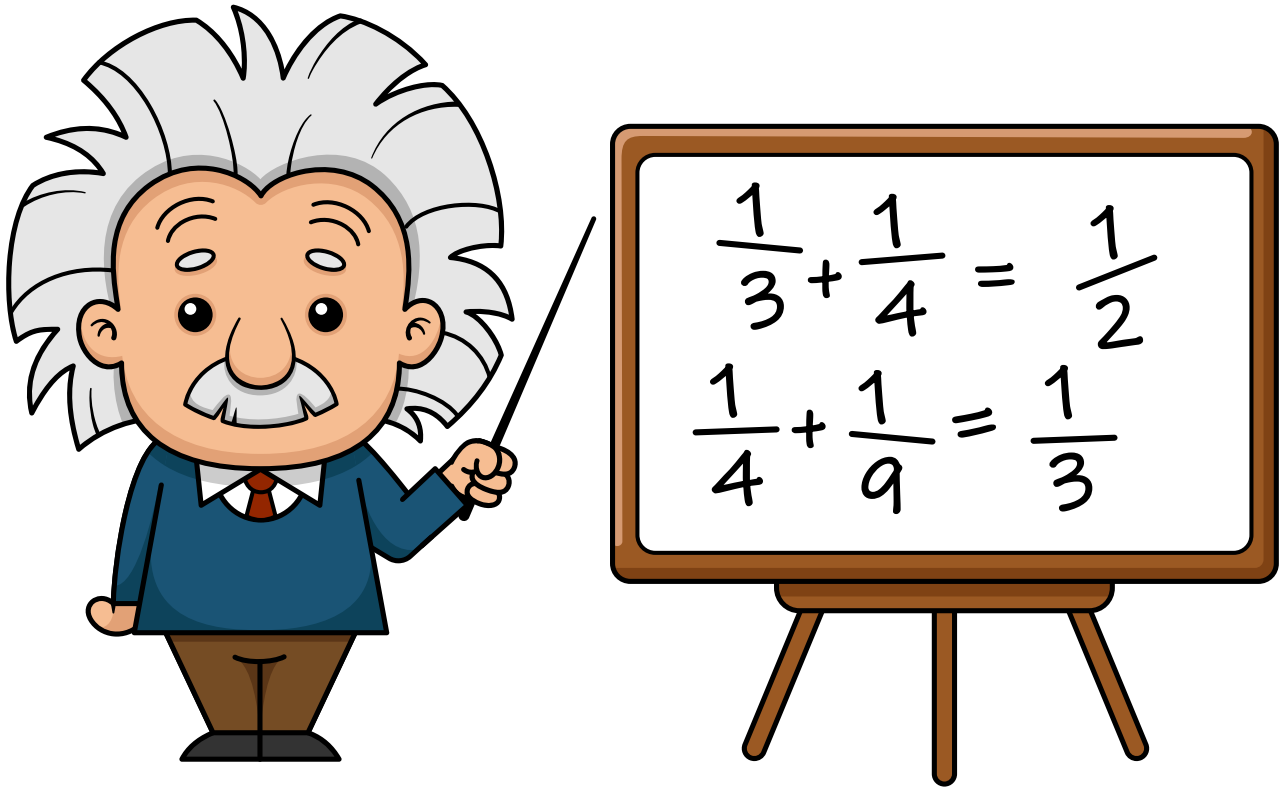
Unit Fractions

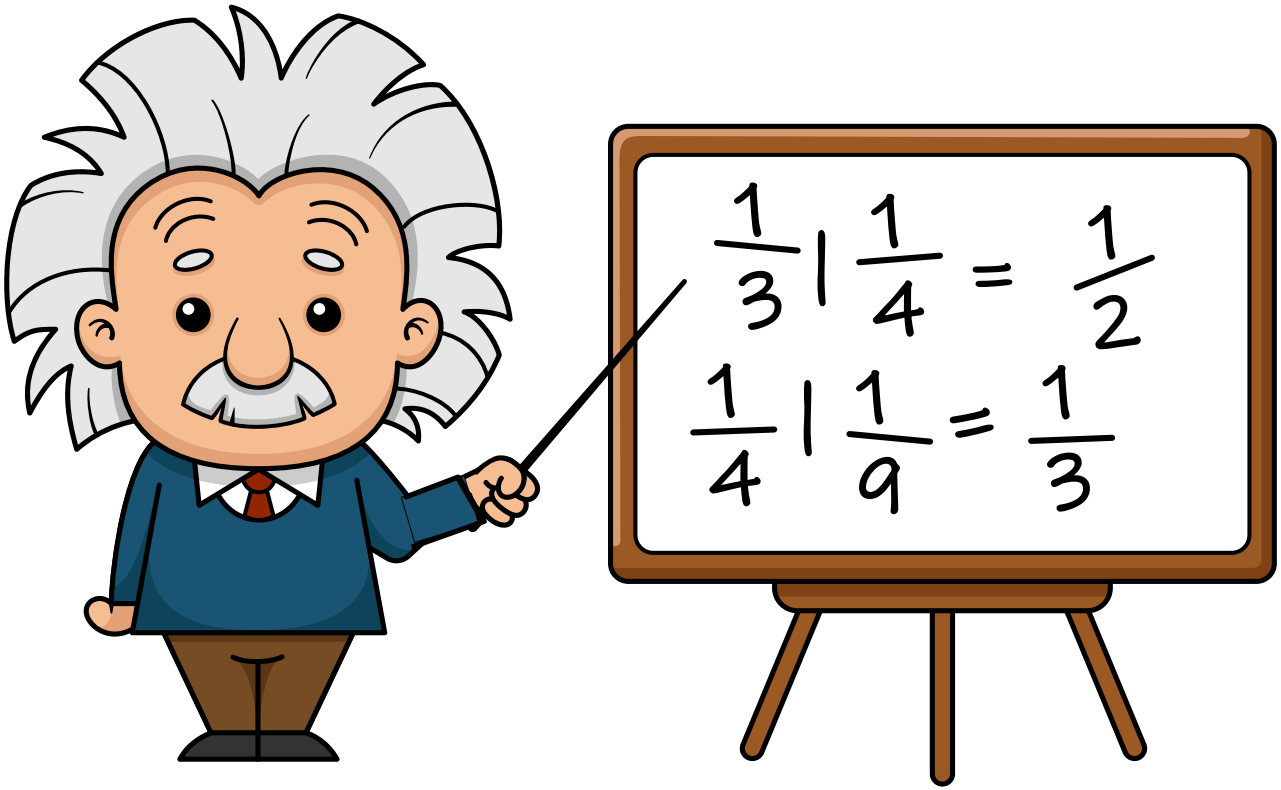
Unit fractions



Vector Graphics by Vecteezy

Unit Fractions

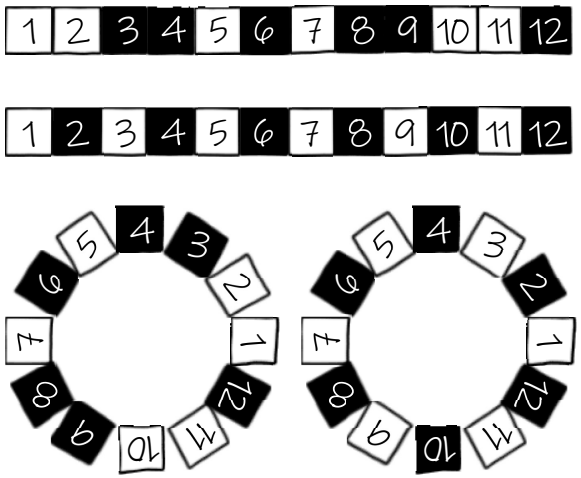
Unit fractions



Vector Graphics by Vecteezy

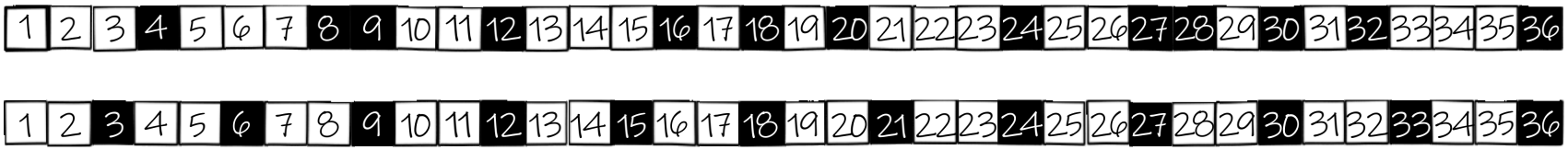
Visual Proof

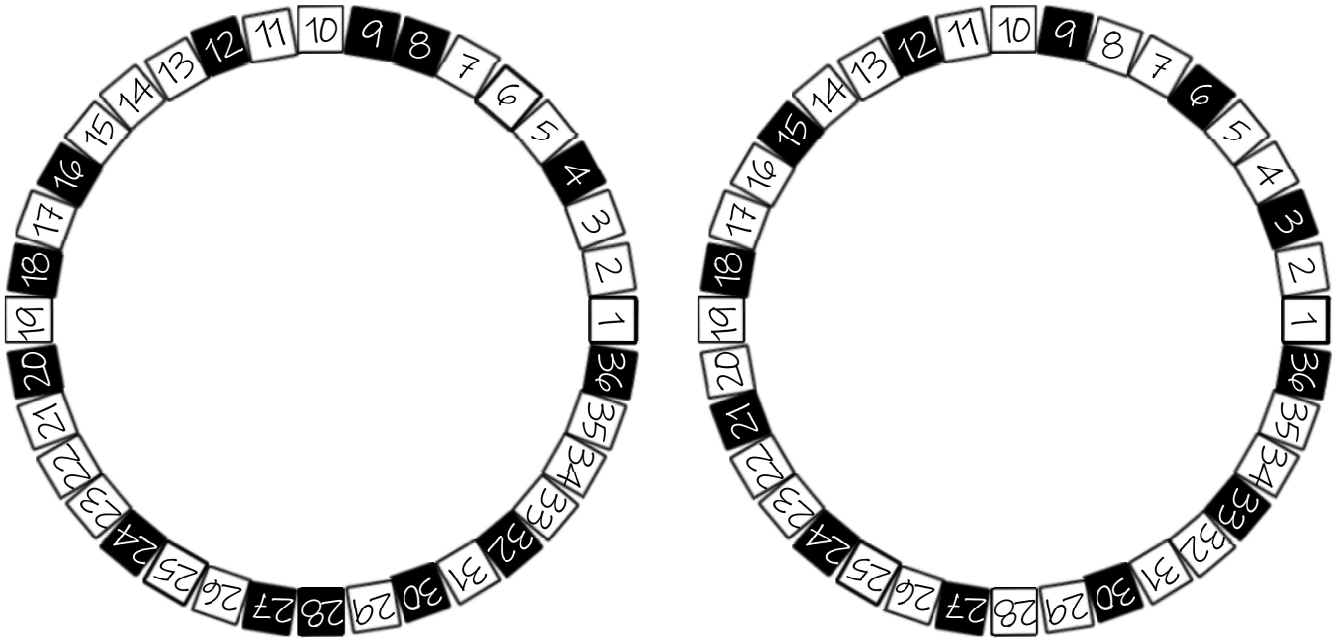
Visual representation of density $\delta = \frac{1}{2}$



Visual Proof

Visual representation of density $\delta = \frac{1}{3}$





Decelerated Alternating Sum

Decelerated alternating series

$$\frac{1}{(1-2a_3-2a_4+2a_{12})}\sum_{n=1}^{\infty}\chi(n;3,4)\ a_n$$

$$\textit{where } \chi(n;3,4)=\begin{cases} 1, \text{if } n \bmod 3 \neq 0 \ \& \ n \bmod 4 \neq 0 \\ -1, \text{if } n \bmod 3 = 0 \mid n \bmod 4 = 0 \end{cases}$$

Decelerated Alternating Sum

Decelerated alternating series

$$\frac{1}{(1-2a_3)}\sum_{n=1}^{\infty}\chi(n;3)\ a_n$$

$$\textit{where } \chi(n;3)=\begin{cases}1, \text{if } n \bmod 3 \neq 0\\ -1, \text{if } n \bmod 3 = 0\end{cases}$$

$$\frac{1}{(1-2a_4-2a_9+2a_{36})}\sum_{n=1}^{\infty}\chi(n;4,9)\ a_n$$

$$\textit{where } \chi(n;4,9)=\begin{cases}1, \text{if } n \bmod 4 \neq 0 \ \&\ n \bmod 9 \neq 0\\ -1, \text{if } n \bmod 4 = 0 \mid n \bmod 9 = 0\end{cases}$$

Appendix: Data

Tabular data for density $\delta = \frac{1}{2}$
https://github.com/AlexWeslowski/Integer-Sequence/

$[2]$	2	$[3, 5, 23, 64, 176]$	3886080
$[3, 4]$	12	$[3, 5, 22, 64, 331]$	6990720
$[3, 7, 8]$	168	$[5, 6, 7, 11, 29, 256]$	17149440
$[3, 5, 16]$	240	$[3, 7, 22, 23, 29, 128]$	39443712
$[3, 7, 10, 32]$	6720	$[3, 7, 20, 22, 26, 256]$	61501440
$[3, 5, 31, 32]$	14880	$[3, 5, 17, 511, 512]$	66716160
$[3, 5, 17, 256]$	65280	$[3, 7, 20, 22, 29, 256]$	68597760
$[3, 7, 11, 32, 80]$	591360	$[3, 5, 17, 341, 1024]$	89041920
$[3, 7, 11, 40, 64]$	591360	$[3, 7, 11, 52, 64, 130]$	99939840
$[3, 7, 10, 62, 64]$	833280	$[3, 7, 11, 64, 65, 104]$	99939840
$[3, 7, 10, 64, 71]$	954240	$[5, 6, 9, 11, 14, 19, 128]$	101122560
$[3, 7, 11, 32, 155]$	1145760	$[3, 7, 11, 50, 64, 160]$	118272000
$[3, 7, 11, 29, 320]$	2143680	$[3, 7, 10, 55, 64, 176]$	130099200
$[3, 7, 10, 41, 256]$	2204160	$[3, 7, 11, 38, 95, 160]$	133425600

Appendix: Data

Integer sequence for density $\delta = \frac{1}{2}$
https://github.com/AlexWeslowski/Integer-Sequence/

2, 12, 168, 240, 6720, 14880, 65280, 591360, 591360, 833280, 954240, 1145760, 2143680, 2204160, 3886080, 6990720, 17149440, 39443712, 61501440, 66716160, 68597760, 89041920, 99939840, 99939840, 101122560, 118272000, 130099200, 133425600, 187031040, 209932800, 229152000, 236651520, 262416000, 272912640, 366213120, 367933440, 421048320, 426961920, 435240960, 514631040, 522762240, 546712320, 546712320, 623293440, 870481920, 1045524480, 1080330240, 1454046720, 1565921280, 1733698560, 1788695040, 1923018240, 2067394560, 2067394560, 2195128320, 2196606720, 2393825280, 2606714880, 2650306560, 2658163200, 2685281280, 2709611520, 2722473600, 2915404800, 3429888000, 3644256000, 4294901760, 4638036480, 4665239040, 4918911360, 5311299840, 5379306240, 6259545600, 7608944640, 9835668480, 17012244480, 22160307840, 42365266944

Appendix: Data

Tabular data for density $\delta = \frac{1}{3}$
https://github.com/AlexWeslowski/Integer-Sequence/

$[3]$	3	$[7, 9, 11, 30, 76]$	1580040
$[4, 9]$	36	$[7, 9, 11, 32, 80]$	1774080
$[4, 10, 21]$	840	$[7, 9, 11, 40, 64]$	1774080
$[5, 7, 36]$	1260	$[7, 9, 10, 48, 62]$	1874880
$[4, 11, 45]$	1980	$[7, 9, 10, 48, 71]$	2147040
$[7, 9, 10, 32]$	20160	$[7, 9, 10, 52, 69]$	2260440
$[8, 10, 11, 12, 21]$	221760	$[7, 9, 10, 62, 64]$	2499840
$[8, 10, 11, 12, 23]$	242880	$[7, 9, 10, 46, 87]$	2521260
$[7, 9, 12, 15, 34]$	385560	$[7, 9, 11, 24, 155]$	2577960
$[7, 9, 12, 16, 50]$	604800	$[7, 9, 10, 64, 71]$	2862720
$[7, 9, 11, 30, 48]$	997920	$[7, 9, 11, 32, 155]$	3437280
$[7, 9, 11, 40, 48]$	1330560	$[7, 9, 10, 33, 172]$	3575880
$[7, 9, 10, 48, 51]$	1542240	$[7, 9, 11, 29, 240]$	4823280
$[7, 9, 10, 33, 75]$	1559250	$[7, 9, 10, 41, 192]$	4959360

Appendix: Data

Integer sequence for density $\delta = \frac{1}{3}$
https://github.com/AlexWeslowski/Integer-Sequence/

3, 36, 840, 1260, 1980, 20160, 221760, 242880, 385560, 604800,
997920, 1330560, 1542240, 1559250, 1580040, 1774080, 1774080,
1874880, 2147040, 2260440, 2499840, 2521260, 2577960, 2862720,
3437280, 3575880, 4823280, 4959360, 6431040, 8225280, 9574740,
10834560, 12735360, 13110240, 13464000, 14002560, 17962560,
18532800, 22619520, 23587200, 23746800, 26308800, 27692280,
31600800, 33929280, 36516480, 46494000, 53222400, 57380400,
58618560, 59028480, 59028480, 66528000, 66673152, 71051904,
74366208, 76396320, 85542912, 87816960, 89147520, 95413248,
96163200, 99380736, 101969280, 137192832, 155925000, 158096400,
187488000, 192780000, 194232720, 195022080, 214704000, 237144600,
248648400, 275425920, 285538176, 291349080, 295384320, 303367680,
303367680, 319334400, 328382208, 354816000, 390297600, 395010000