

Dependence of LBNE long-baseline neutrino oscillation sensitivities on knowledge of neutrino oscillation parameters

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Abstract

The sensitivity of long-baseline neutrino oscillation experiments depends significantly on the true values of neutrino oscillation parameters. The purpose of this paper is to document the dependence of LBNE's sensitivity to determination of the neutrino mass hierarchy, discovery of CP violation, and determination of the θ_{23} octant on the central values of neutrino oscillation parameters and on how well these parameters are determined by external constraints. This information will be useful for adjusting expectations for LBNE sensitivity as new measurements of oscillation parameters become available and for understanding the value of additional pre-LBNE measurements of oscillation parameters. We find that xxx.

1 Introduction

It is well-established experimentally that the flavor composition of neutrinos change as they propagate[1]. In the three-neutrino framework, the three flavor states (ν_e, ν_μ, ν_τ) are superpositions of the three mass states (ν_1, ν_2, ν_3). The PMNS[2, 3, 4] matrix describes this mixing with three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$), which have all been measured experimentally, and a CP-violating phase, δ_{CP} , which is unknown. Measurement of a value of δ_{CP} not equal to zero or pi would be the first observation of CP violation in the neutrino sector. The differences between the mass states, Δm_{21}^2 and $|\Delta m_{31}^2|$, have also been measured. The case in which Δm_{31}^2 is greater than zero is referred to as “normal hierarchy” (NH) and the opposite case is called “inverted hierarchy” (IH); the true neutrino mass hierarchy has not yet been determined. The measured value of θ_{23} is near to 45° ; if it is exactly 45° , which would indicate that ν_μ and ν_τ have equal contributions from ν_3 , that could be evidence for a previously unknown symmetry. The θ_{23} octant, i.e., whether θ_{23} is less than, greater than, or equal to 45° , is unknown.

These three measurements – determination of the neutrino mass hierarchy (MH), discovery of CP violation (CPV), and determination of the θ_{23} octant – are among the primary physics goals of LBNE, the Long Baseline Neutrino Experiment [5]. LBNE will use a combined analysis of ν_e appearance and ν_μ disappearance in a muon-neutrino beam to study neutrino oscillation. The experiment will consist of a muon-neutrino beam from Fermilab to Sanford Underground Research Facility (SURF), a baseline of 1300 km, and a massive underground far detector. The experiment will make use of protons from Fermilab's Main Injector and a new muon-neutrino beam line will be constructed. The far detector will be a liquid argon time projection chamber (LAr TPC) with an integrated photon detection system. For the sensitivities presented in this paper, we study both a 10-kt and a 35-kt fiducial-volume far detector. We assume a 1.2-MW, 80-GeV beam with six years of exposure, divided equally between neutrino and antineutrino running. We also present sensitivities as a function of exposure in kt.MW.years.

2 Neutrino Oscillation in the Three-flavor Paradigm

The oscillation probability of $\nu_{\mu,e} \rightarrow \nu_{e,\mu}$ through matter, in a constant density approximation, keeping terms up to second order in $\alpha \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2|$ and $\sin^2 \theta_{13}$, is [6, 1]:

$$P(\nu_{\mu} \rightarrow \nu_e) \cong P(\nu_e \rightarrow \nu_{\mu}) \cong P_0 + \underbrace{P_{\sin \delta}}_{\text{CP violating}} + P_{\cos \delta} + P_3 \quad (1)$$

where

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta], \quad (2)$$

$$P_{\sin \delta} = -\alpha \frac{8J_{cp}}{A(1-A)} \sin \Delta \sin(A\Delta) \sin[(1-A)\Delta], \quad (3)$$

$$P_{\cos \delta} = \alpha \frac{8J_{cp} \cot \delta_{CP}}{A(1-A)} \cos \Delta \sin(A\Delta) \sin[(1-A)\Delta], \quad (4)$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta), \quad (5)$$

$$(6)$$

and where

$$\Delta = \Delta m_{31}^2 L / 4E, \quad (7)$$

$$A = \sqrt{3} G_F N_e 2E / \Delta m_{31}^2, \text{ and} \quad (8)$$

$$J_{CP}^{\text{PMNS}} \equiv \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos \theta_{13} \sin \delta_{CP}. \quad (9)$$

In the above, the CP phase δ_{CP} appears (via J_{cp}) in the expressions for $P_{\sin \delta}$ (the CP-odd term) which switches sign in going from $\nu_{\mu} \rightarrow \nu_e$ to the $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$ channel, and $P_{\cos \delta}$ (the CP-conserving term) which does not. The matter effect also introduces a neutrino-antineutrino asymmetry, the origin of which is simply the presence of electrons and absence of positrons in the Earth.

A global fit to all available experimental data is described in [7]; the central values, 1σ and 3σ ranges of the fit, and 1σ precision are shown in Table 1. The best-fit values are somewhat different for normal and inverted hierarchy. The fit results for each parameter in the global fit are shown graphically as $N\sigma$ contours in Fig. 1.

3 Calculation of Experimental Sensitivities using GLoBES

The sensitivity of LBNE to the mass hierarchy, CP violation, and the octant of θ_{23} are calculated using the GLoBES [8, 9] package to simulate the detector response using “true” values of the oscillation parameters based on the global fit, smearing based on a simple resolution or on a GENIE[10] simulation, and detector efficiency values based on results from ICARUS[11] and earlier simulation efforts. The oscillation parameters used in GLoBES are shown in Table 2. The detector efficiencies and the energy resolution for ν_e charged current (CC) and ν_{μ} CC interactions used in GLoBES are shown in Table 3. The smearing matrices for ν_{μ} neutral current (NC) and ν_{τ} interactions are shown in Fig. 2.

Estimates of experimental sensitivity are obtained in GLoBES by simultaneously fitting the $\nu_{\mu} \rightarrow \nu_{\mu}$, $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}$, $\nu_{\mu} \rightarrow \nu_e$, and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$ oscillated spectra, examples of which are shown in Figure 3. In these calculations, experimental sensitivity is quantified using $\Delta\chi^2$ parameters, which are determined

Table 1: Blah

Parameter	Hierarchy	Best Fit	1σ Range	3σ Range	1σ Precision
$\delta m^2/10^{-5} \text{ eV}^2$	NH/IH	7.54	7.32-7.80	6.99-8.18	2.6%
$\sin^2 \theta_{12}/10^{-1}$	NH/IH	3.08	2.91-3.25	2.59-3.59	5.4%
$\Delta m^2/10^{-3} \text{ eV}^2$	NH	2.44	2.38-2.52	2.22-2.66	3.0%
	IH	2.40	2.33-2.47	2.17-2.61	3.1%
$\sin^2 \theta_{13}/10^{-2}$	NH	2.34	2.16-2.56	1.77-2.97	8.5%
	IH	2.39	2.18-2.60	1.78-3.00	8.5%
$\sin^2 \theta_{23}/10^{-1}$	NH	4.25	3.98-4.54	3.57-6.41	11.1%
	IH	4.37	$4.08-4.96 \oplus 5.31-6.10$	3.63-6.59	12.3%
δ/π	NH	1.39	1.12-1.72	—	—
	IH	1.35	0.96-1.59	—	—

Table 2: Oscillation parameter inputs used in GLoBES sensitivity calculations. Central values are taken from the global fit in [7]. Angles are in radians. Relative uncertainty is defined using $1/6$ of the full 3σ range from [7]. In the case of θ_{13} , the relative uncertainty is based on the current systematic uncertainty in $\sin^2 2\theta_{13}$ from Daya Bay [12] as an approximation of the ultimate precision of the current generation of reactor experiments.

Parameter	Hierarchy	Central Value	Relative Uncertainty
$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	NH/IH	7.54	3%
θ_{12}	NH/IH	0.588	3%
$\Delta m_{31}^2/10^{-3} \text{ eV}^2$	NH	2.48	3%
	IH	-2.36	3%
θ_{13}	NH	0.154	3%
	IH	0.155	3%
θ_{23}	NH	0.710	7%
	IH	0.722	7%
δ_{CP}	NH/IH	scan	no constraint

by comparing the predicted spectra for various scenarios. These quantities are defined, differently for neutrino MH, CP-violation, and octant sensitivity, to be:

$$\Delta\chi_{MH}^2 = |\chi_{MH^{test=IH}}^2 - \chi_{MH^{test=NH}}^2|, \quad (10)$$

$$\Delta\chi_{CPV}^2 = \min(\Delta\chi_{CP}^2(\delta_{CP}^{test} = 0), \Delta\chi_{CP}^2(\delta_{CP}^{test} = \pi)), \text{ where} \quad (11)$$

$$\Delta\chi_{CP}^2 = \chi_{\delta_{CP}^{test}}^2 - \chi_{\delta_{CP}^{true}}^2, \text{ and} \quad (12)$$

$$\Delta\chi_{Octant}^2 = |\chi_{\theta_{23}^{test} > 45^\circ}^2 - \chi_{\theta_{23}^{test} < 45^\circ}^2|. \quad (13)$$

These sensitivities are evaluated separately for true NH and IH. Since the true value of δ_{CP} is unknown, a scan is performed over all possible values of δ_{CP}^{true} . For the case of the octant sensitivity, the value of θ_{23} in the “wrong” octant is constrained only to have a value within the “wrong” octant, i.e., it is not required to have the same value of $\sin^2 2\theta_{23}$ as the true value. The individual χ^2 values are calculated

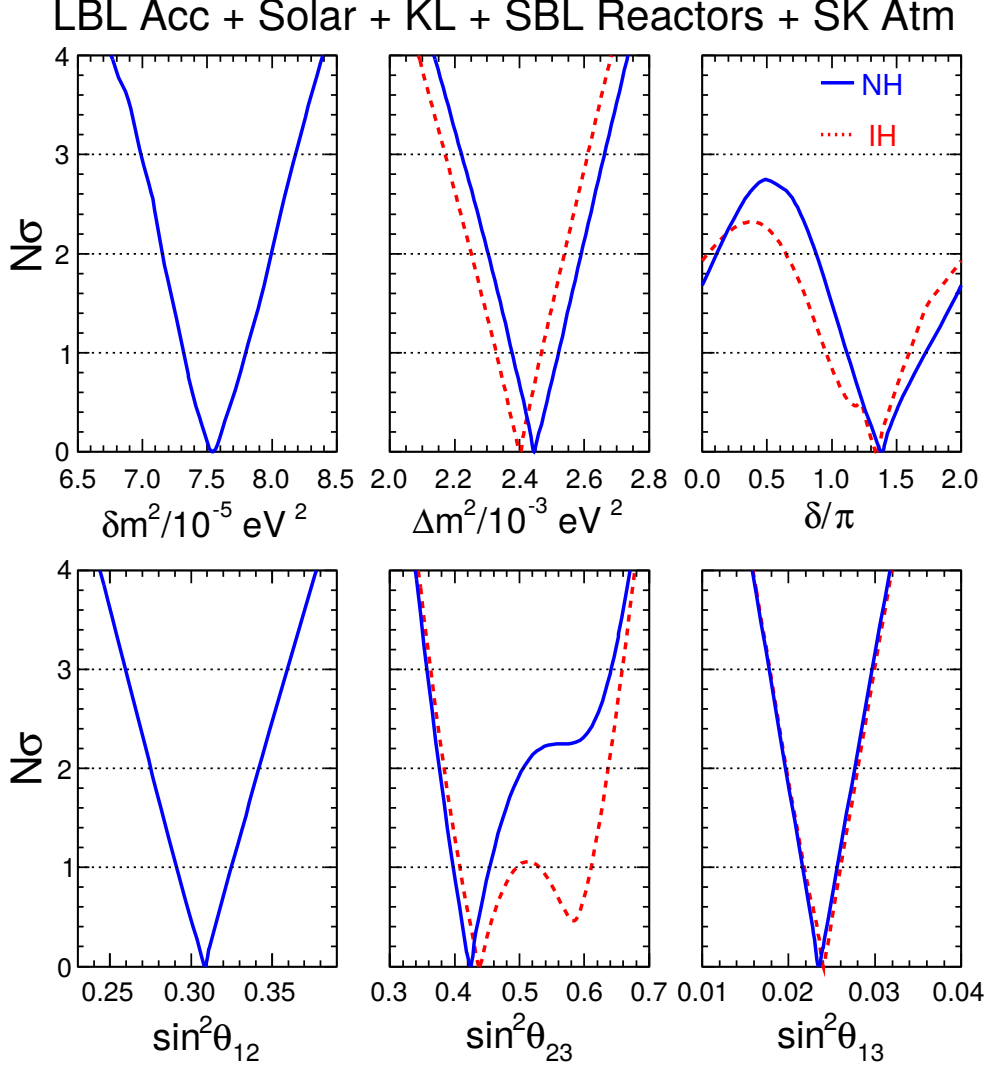


Figure 1: Results of the 2013 global analysis from Capozzi *et al.* shown as $N\sigma$ bounds on the six parameters governing three ν flavor oscillations. Blue (solid) and red (dashed) curves refer to NH and IH, respectively. Figure is from [7].

using

$$\chi^2(\mathbf{n}^{true}, \mathbf{n}^{test}, f) = 2 \sum_i^{N_{reco}} (n_i^{true} \ln \frac{n_i^{true}}{n_i^{test}(f)} + n_i^{test}(f) - n_i^{true}) + f^2, \quad (14)$$

where \mathbf{n} are event rate vectors in N_{reco} bins of reconstructed energy and f represents a nuisance parameter to be profiled. Nuisance parameters include the values of mixing angles, mass splittings, and signal and background normalization. The nuisance parameters are constrained by Gaussian priors; in the case of the oscillation parameters, the Gaussian prior has standard deviation determined by taking 1/6 of the 3σ range allowed by the global fit [13].

LBNE's sensitivity to MH and CPV is based on observation of the neutrino-antineutrino asymmetry,

$$A_{CP} = \frac{P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}. \quad (15)$$

Table 3: Signal and background efficiencies, normalization uncertainties, and energy resolutions, which are obtained from ICARUS and earlier simulation efforts, used in the GLoBES sensitivity calculations.

Parameter	Value Used for LBNE Sensitivities
	For ν_e -CC appearance
ν_e -CC efficiency	80%
ν_μ -NC mis-identification rate	1%
ν_μ -CC mis-identification rate	1%
Other background	0%
Signal normalization error	1%
Background normalization error	5%
	For ν_μ -CC disappearance
ν_μ -CC efficiency	85%
ν_μ -NC mis-identification rate	1%
Other background	0%
Signal normalization error	5%
Background normalization error	10%
	Neutrino Energy Resolutions
ν_e CC energy resolution	$15\%/\sqrt{E(\text{GeV})}$
ν_μ CC energy resolution	$15\%/\sqrt{E(\text{GeV})}$

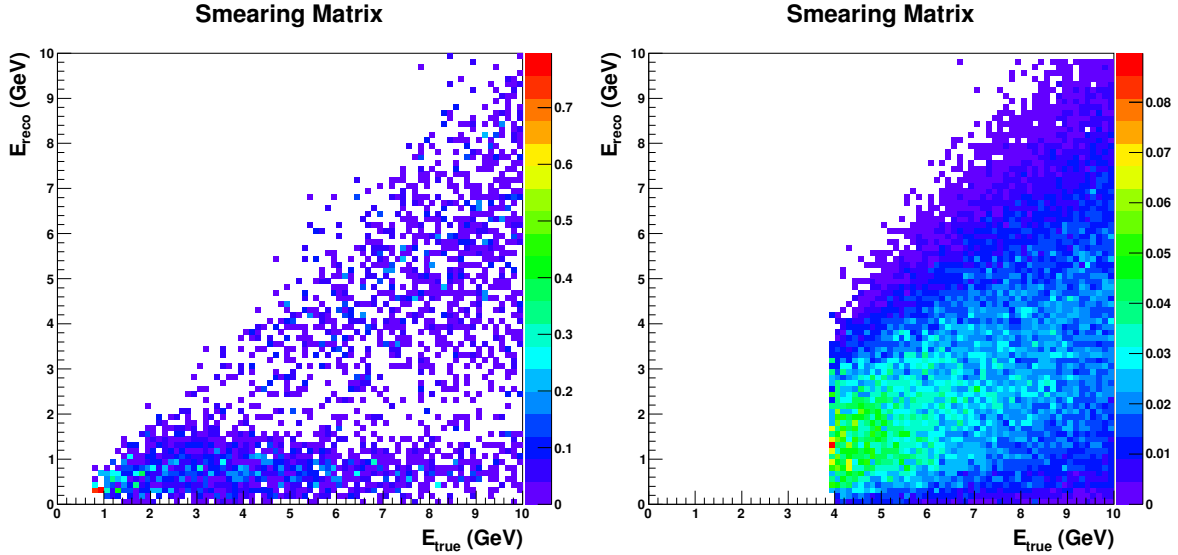


Figure 2: ν_μ NC (left) and ν_τ energy smearing matrices, produced using a GENIE simulation, used in the GLoBES sensitivity calculations.

This asymmetry contains contributions both from the different effective masses of neutrinos and antineutrinos as they propagate in matter and from CP violation related to δ_{CP} . Figure 4 shows the total asymmetry at 1300 km for the first and second oscillation nodes, for both NH and IH. There is a nearly degenerate value for the asymmetries at $\delta_{CP} \sim \pi/2$, true NH and at $\delta_{CP} \sim -\pi/2$, true IH in the first oscillation node. This is the reason for the decrease in MH sensitivity for long-baseline

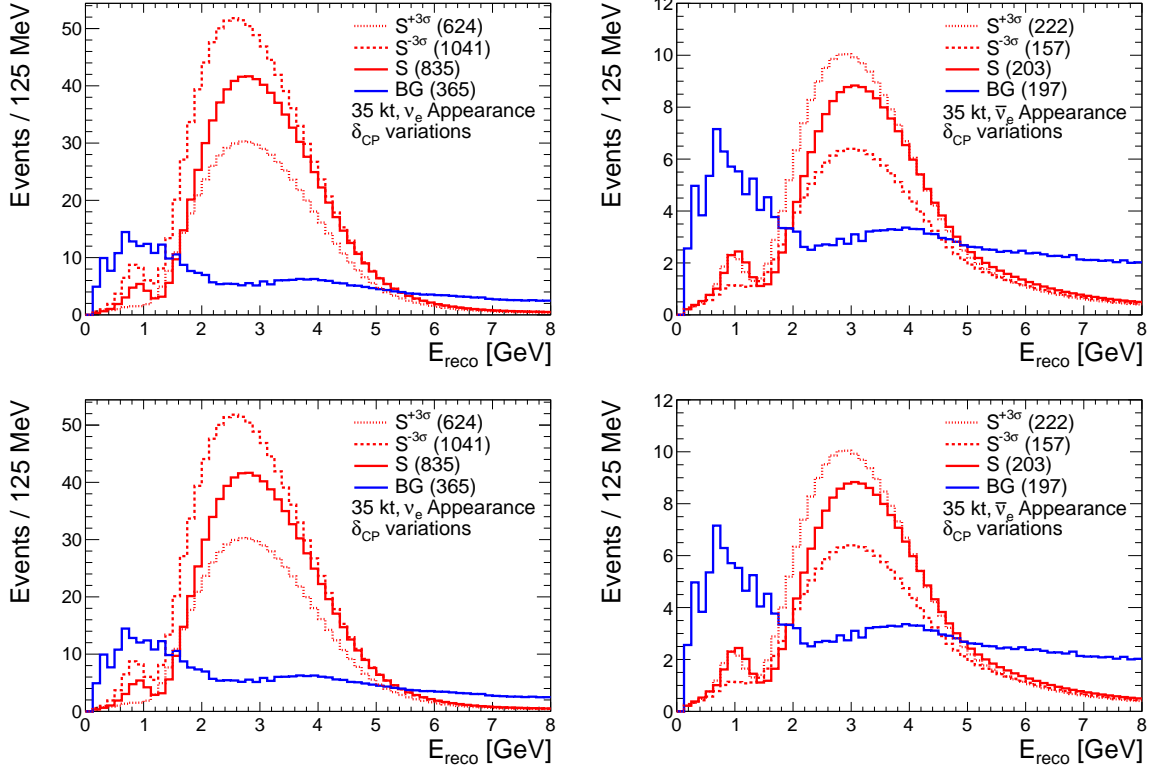


Figure 3: FIXME: Get real disappearance plots. Examples of spectra produced by GLoBES for a 35-kt LArTPC with 3 years of exposure in an 80-GeV, 1.2-MW beam, for the case of true NH. ν_μ disappearance (top left), $\bar{\nu}_\mu$ disappearance (top right), ν_e appearance (bottom left), and $\bar{\nu}_e$ appearance (bottom right). The effect of varying the true value of δ_{CP} from $-\pi/2$ to zero to $\pi/2$ is shown. The total background, which does not depend on δ_{CP} , is overlaid on the signal. The total number of signal and background events are indicated in parenthesis.

experiments in the region near $\delta_{CP} \sim \pi/2$. Shorter baseline experiments also see degradation of CPV sensitivity in this region [?], but in LBNE, the MH will be resolved with sufficient precision that the CPV sensitivity is not strongly affected by degeneracies. Fig. 4 also highlights the critical importance of experimental access to the second oscillation node, where the CP asymmetry is larger, so no such degeneracy with the matter effect is present.

4 Sensitivity Dependence on θ_{13}

Figure 5 shows the variation in ν_e and $\bar{\nu}_e$ appearance signal when the true value of θ_{13} is varied within its 3σ allowed range. The effect of increasing θ_{13} is an increase in event rate with no change to the spectral shape. This can be understood by inspection of Equation 1, in which P_0 includes a factor of $\sin^2 2\theta_{13}$. Figure 6 shows the variation in MH and CPV sensitivity when θ_{13} is varied within its 1σ and 3σ allowed range. In each case, the nominal relative constraints on all oscillation parameters, including θ_{13} , are applied and the normalization constraints are the nominal 1% for signal and 5% for background. The increased statistics from increasing θ_{13} provides a modest increase in MH sensitivity for both true NH and IH. The CPV sensitivity does not improve significantly. The slightly larger increase in sensitivity in IH compared to NH can be understood because the statistics in true IH are significantly lower than in true NH, so increasing the sample size has a larger effect. FIXME: Seems

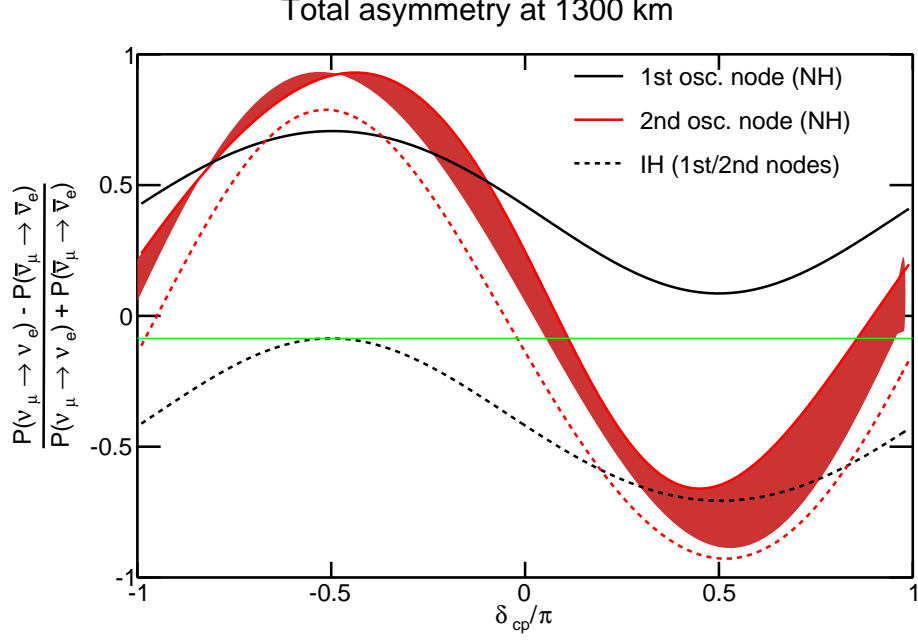


Figure 4: Neutrino-antineutrino asymmetry as a function of δ_{CP} for the first (black) and second (red) oscillation nodes, for NH (solid) and IH (dashed), at the LBNE baseline of 1300 km.

believable but is this true?

5 Sensitivity Dependence on θ_{23}

Figure 7 shows the variation in ν_e and $\bar{\nu}_e$ appearance signal when the true value of θ_{23} is varied within its 3σ allowed range. Increasing the true value of θ_{23} results in an increase in ν_e appearance signal; this can be understood by inspection of Equation 1, where P_0 includes a factor of $\sin^2 \theta_{23}$. Figure 8 shows the variation in MH and CPV sensitivity when θ_{23} is varied within its 1σ and 3σ allowed range. In each case, the nominal relative constraints on all oscillation parameters, including θ_{23} , are applied and the normalization constraints are the nominal 1% for signal and 5% for background. It is clear from these figures that LBNE sensitivity to MH and CPV depends rather strongly on the true value of θ_{23} . As θ_{23} increases, sensitivity to CPV decreases while sensitivity to MH increases. Starting from Equation 1, one can show that the CP asymmetry measured by LBNE depends directly on θ_{23} :

$$\mathcal{A}_{CP} \sim \frac{\cos \theta_{23} \sin 2\theta_{12} \sin \delta_{CP}}{\sin \theta_{23} \sin \theta_{13}} \left(\frac{\Delta m_{21}^2 L}{4E_\nu} \right) + \text{matter effects.} \quad (16)$$

This asymmetry decreases with increasing θ_{23} ; sensitivity decreases because the quantity being measured is smaller. The decrease in CP asymmetry also explains the increase in MH sensitivity. The limiting factor to MH sensitivity in the region where $\delta_{CP} > 0$ is the degeneracy between CP and matter asymmetries, described in Section 2. The decrease in CP asymmetry with increasing θ_{23} partially breaks this degeneracy, increasing MH sensitivity. This effect is illustrated by the spectra in Fig. 9. The explicit dependence of the MH sensitivity on θ_{23} is shown in Fig. ??.

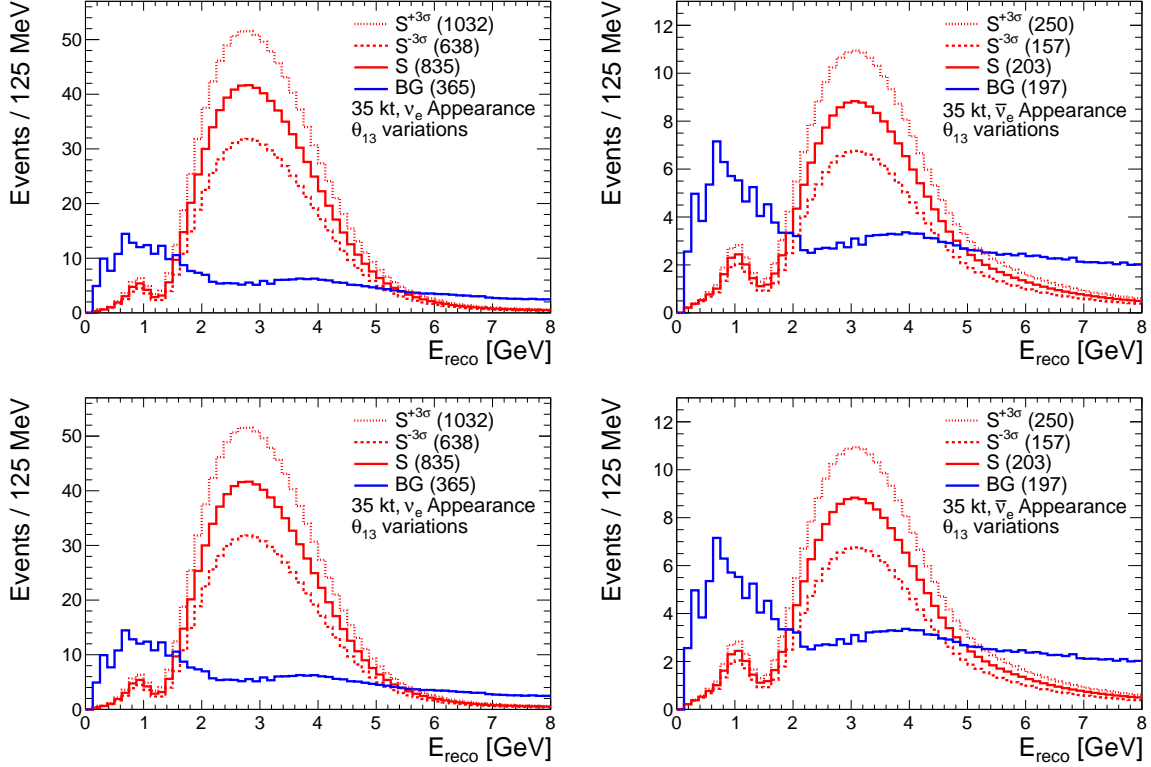


Figure 5: FIXME Real IH plots. Variations in θ_{13} : ν_e appearance (left) and $\bar{\nu}_e$ appearance (right) spectra produced by GLoBES for a 35-kt LArTPC with 3 years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effect of varying the true value of θ_{13} within its 3σ allowed range is shown. The total background, which does not depend on value of θ_{13} (CHECKME), is overlaid on the signal. The total number of signal and background events are indicated in parenthesis.

6 Sensitivity Dependence on Δm_{31}^2

Figure 10 shows the variation in ν_e and $\bar{\nu}_e$ appearance signal when the true value of Δm_{31}^2 is varied within its 3σ allowed range. Figure 11 shows the variation in MH and CPV sensitivity when Δm_{31}^2 is varied within its 1σ and 3σ allowed range. In each case, the nominal relative constraints on all oscillation parameters, including Δm_{31}^2 , are applied and the normalization constraints are the nominal 1% for signal and 5% for background.

7 Sensitivity Dependence on δ_{CP}

The variation in ν_μ disappearance, $\bar{\nu}_\mu$ disappearance, ν_e appearance, and $\bar{\nu}_e$ appearance with variations in the true value of δ_{CP} is shown in Fig. 3.

8 Conclusions

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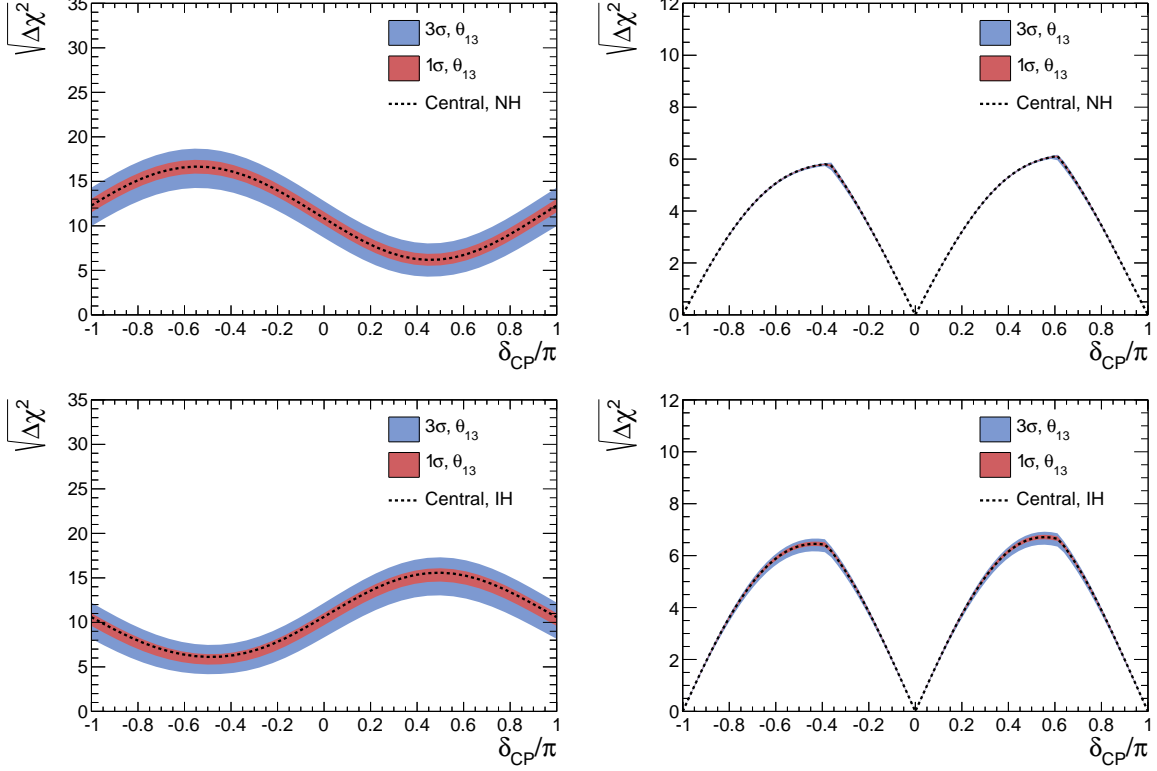


Figure 6: Variations in θ_{13} : MH (left) and CPV (right) sensitivities produced by GLoBES for a 35-kt LArTPC with 3+3 ($\nu + \bar{\nu}$) years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effects of varying the true value of θ_{13} within its 1σ (red band) and 3σ (blue band) allowed ranges are shown.

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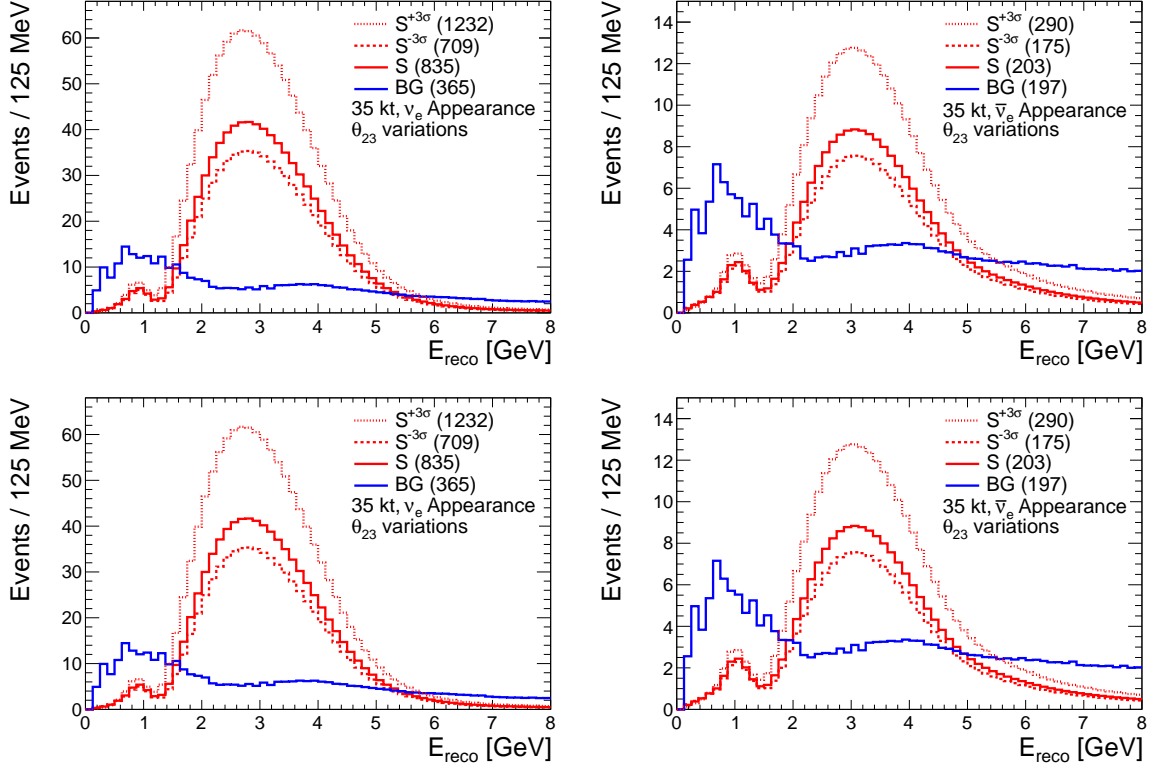


Figure 7: FIXME Real IH plots. Variations in θ_{23} : ν_e appearance (left) and $\bar{\nu}_e$ appearance (right) spectra produced by GLoBES for a 35-kt LArTPC with 3 years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effect of varying the true value of θ_{23} within its 3σ allowed range is shown. The total background, which does not depend on value of θ_{23} (CHECKME), is overlaid on the signal. The total number of signal and background events are indicated in parenthesis.

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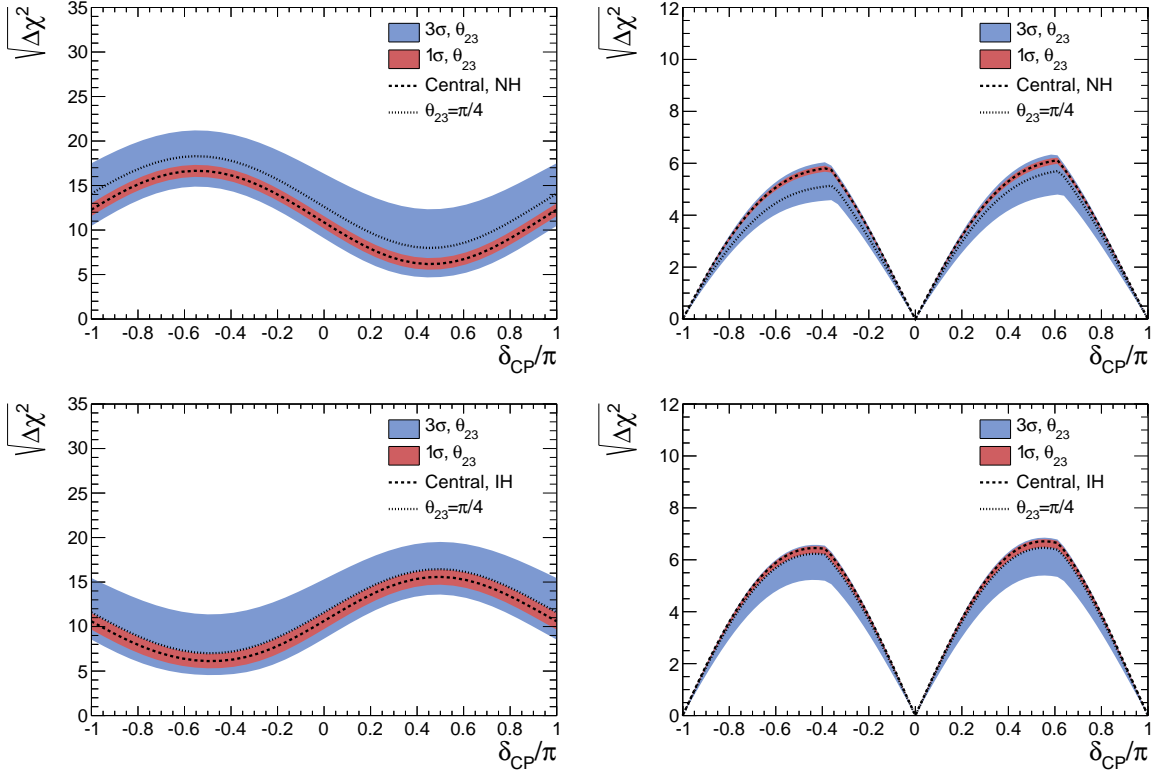


Figure 8: Variations in θ_{23} : MH (left) and CPV (right) sensitivities produced by GLoBES for a 35-kt LArTPC with 3+3 ($\nu + \bar{\nu}$) years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effects of varying the true value of θ_{23} within its 1σ (red band) and 3σ (blue band) allowed ranges are shown. The sensitivities when $\theta_{23} = \pi/4$ (maximal mixing between ν_μ and ν_τ) are also shown.

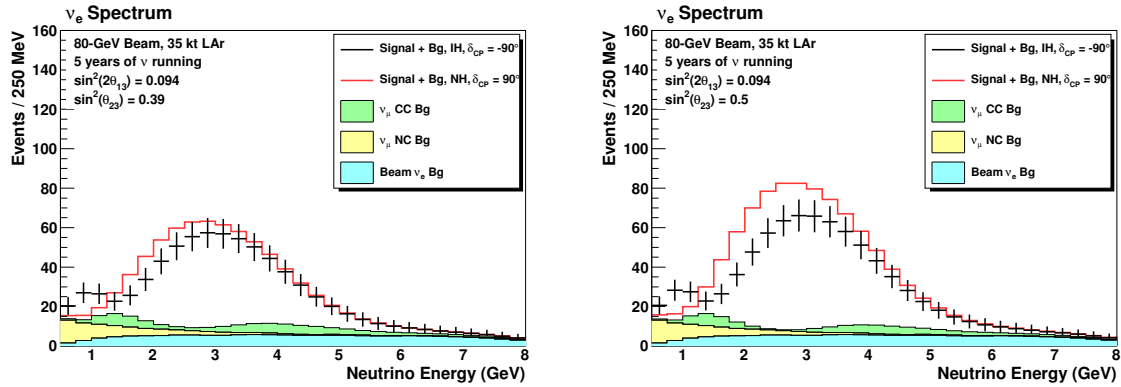


Figure 9: Spectrum for true normal hierarchy and $\delta_{CP} = \pi/2$ (red) compared to the spectrum for true inverted hierarchy and $\delta_{CP} = -\pi/2$ (black), for $\sin^2(\theta_{23}) = 0.39$ (left) and $\sin^2(\theta_{23}) = 0.5$ (right). For the smaller value of θ_{23} shown on the left, the spectra are quite degenerate. For the larger value of θ_{23} shown on the right, the degeneracy is broken. Neutrino event spectra are shown for illustration; the degeneracy is also broken in antineutrino spectra.

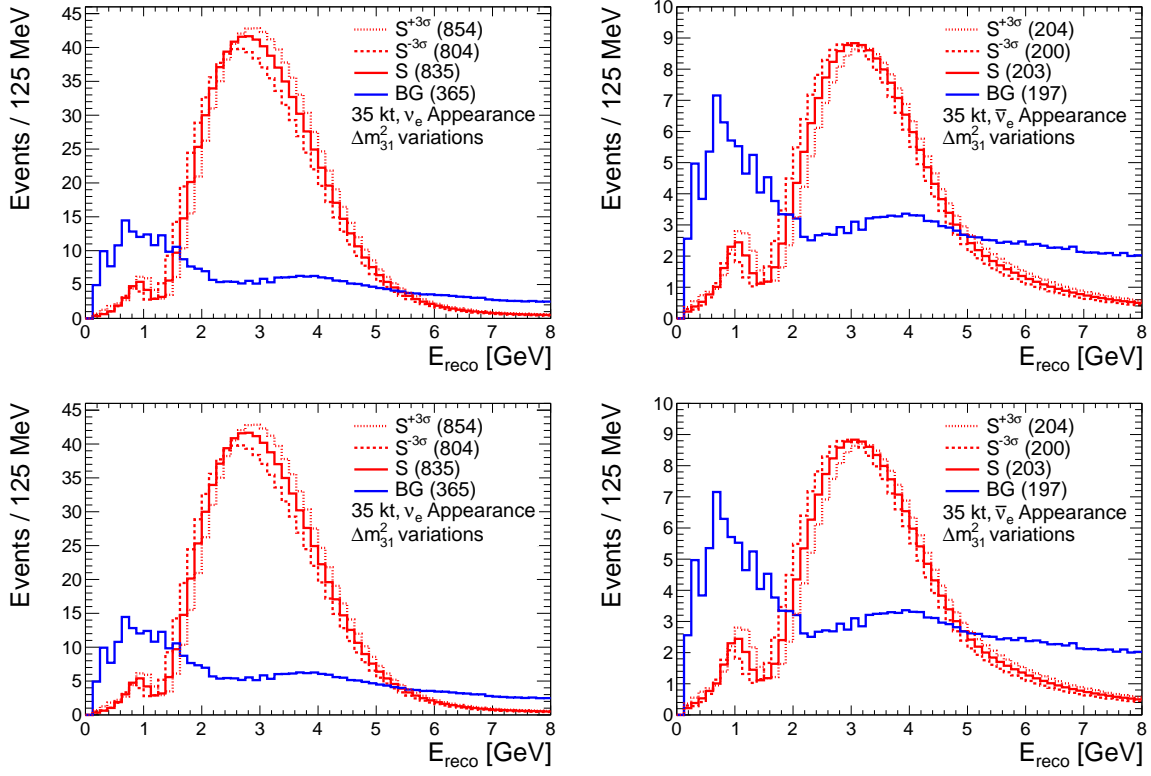


Figure 10: FIXME Real IH plots Variations in δm_{31}^2 : ν_e appearance (left) and $\bar{\nu}_e$ appearance (right) spectra produced by GLoBES for a 35-kt LArTPC with 3 years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effect of varying the true value of Δm_{31}^2 within its 3σ allowed range is shown. The total background, which does not depend on value of Δm_{31}^2 (CHECKME), is overlaid on the signal. The total number of signal and background events are indicated in parenthesis.

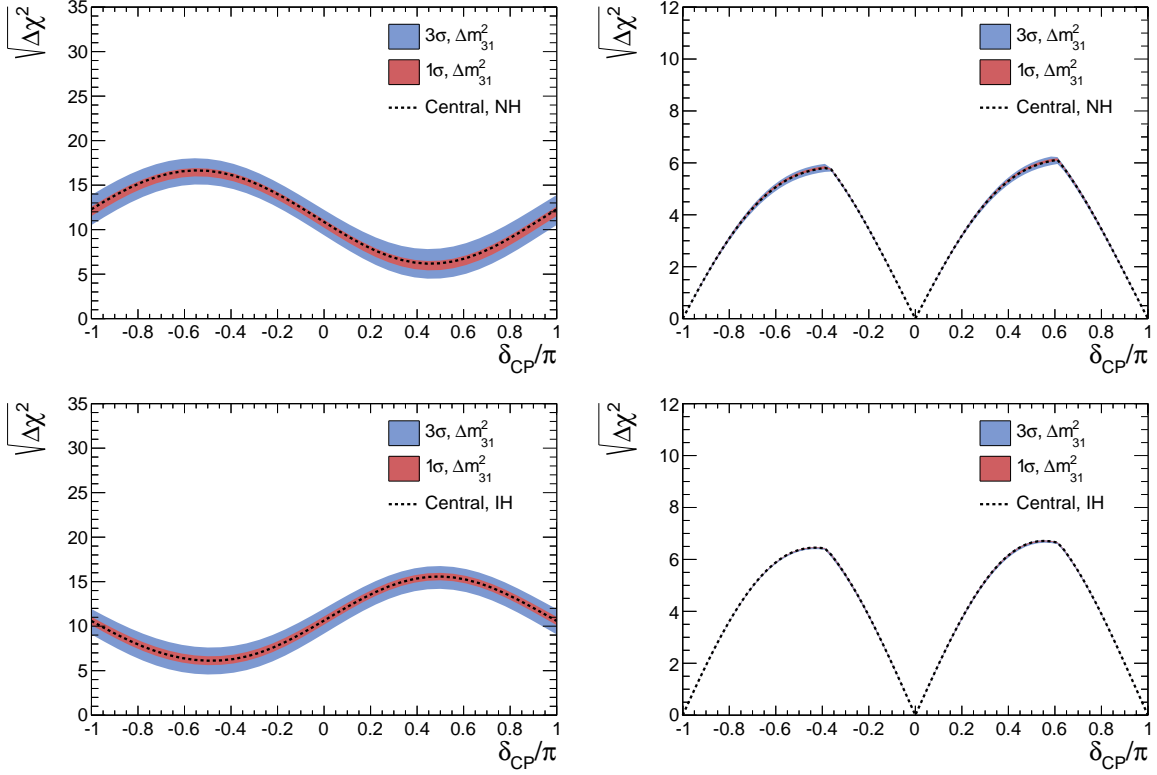


Figure 11: Variations in Δm_{31}^2 : MH (left) and CPV (right) sensitivities produced by GLoBES for a 35-kt LArTPC with 3+3 ($\nu + \bar{\nu}$) years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effects of varying the true value of Δm_{31}^2 within its 1σ (red band) and 3σ (blue band) allowed ranges are shown.

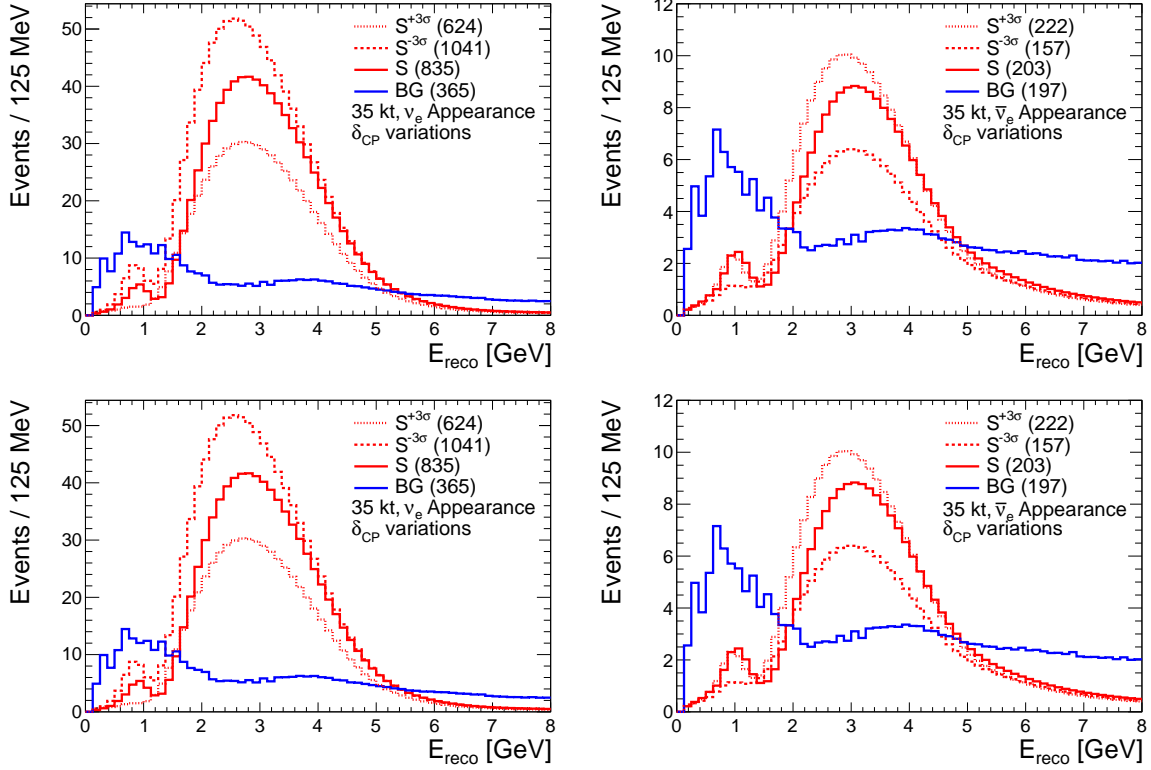


Figure 12: FIXME: Get real IH plots. Variations in δ_{CP} : ν_e appearance (left) and $\bar{\nu}_e$ appearance (right) spectra produced by GLoBES for a 35-kt LArTPC with 3 years of exposure in an 80-GeV, 1.2-MW beam, for true NH (top) and true IH (bottom). The effect of varying the true value of δ_{CP} from $-\pi/2$ to zero to $\pi/2$ is shown. The total background, which does not depend on value of δ_{CP} (CHECKME), is overlaid on the signal. The total number of signal and background events are indicated in parenthesis.