How to *survive* in case of a multi-modal problem? An investigation into a diversity-based multi-objective approach.

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***Abstract*— Highly multi-modal optimization functions pose a unique set of problems for the design and implementation of evolutionary algorithms. The ability to incorporate a diversity preservation methodology within a survivor selection mechanism allows for a more refined exploration and exploitation balance. The resulting diversity-based multi-objective survivor selection mechanism has been proposed and investigated within recent bodies of literature. This study is an answer to initial testing of the diversity-based multi-objective survivor selection scheme done by Segura et. al. The testing of diversity-based multi-objective survivor selection was extended to two other multimodal problems in order to verify the alleged superiority of the novel method and to gain better insight into its appropriate applications. It was observed that the computational cost required for application of such a method can be justified for highly multi-modal problems that lack a strong structure. This study contributes but does not complete the further testing required before diversity-based multi-objective survivor selection can be adopted as a standard diversity preservation tool.**

I. INTRODUCTION

Darwinian inspired evolutionary algorithms (EAs) have the unique ability to provide reasonably good solutions to problem classes by balancing problem specificity and solution generality. Since its inception in the 1940s, many elaborations have been introduced to the EA framework depicted in Figure 1 [1]. Particularly, more recent literature suggests that an intelligent survivor selection mechanism has a more drastic effect on the entire optimization process than previously realized, since the fixing of improper solutions strengthens the population for subsequent generations [2].

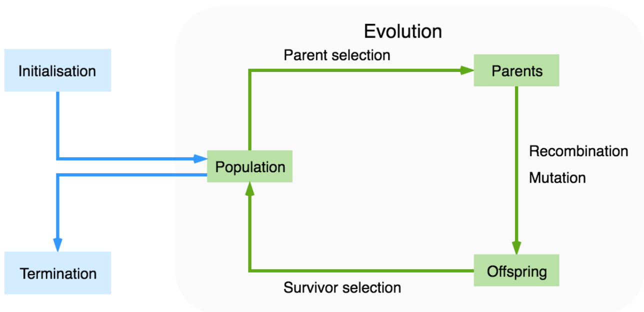


Fig. 1: General flowchart EA schematic.

These findings pose interesting implications for the constant tension between diversity preservation and evolution present in multimodal problems, which has long been established as an exploration vs. exploitation problem [3]. The level at which a diversity preservation mechanism acts distinguishes it as either an implicit or explicit approach. If the mechanism maintains diversity through the genotypic or phenotypic space, it is considered an explicit approach. Meanwhile, the implicit approach maintains diversity within the algorithmic space —the environmental plane where evolution occurs [1]. This paper will focus on explicit diversity preservation as achieved through survivor selection.

Segura et. al have investigated the ability to better balance the needs of exploration and exploitation through the introduction of a dynamic, stopping criterion input within survivor selection. The additional consideration of diversity, in terms of the distance to closest neighbor (DCN), turns the three newly presented survivor selection methodologies into diversity-based multi-objective evolutionary algorithms (MOEAs). The last algorithms elaborate upon the MULTI survivor selection scheme, the first algorithm introduced [4]. In answer to the limited testing of MULTI survivor selection, we will apply it to the Katsuura and Schaffers F7 problems, and compare our results to the similar but more conventional survivor selection method of deterministic crowding in order to verify this methods alleged superiority.

II. METHODS

A. Algorithm description

In order to allow for an equitable comparison of the selected survivor selection methodologies, all other EA components were kept constant among the various EAs tested. Given the problems ten-dimensional search space of real-values ranging from -5 to 5, a real-valued representation was selected because of its identity mapping between the genotypic and phenotypic spaces. The resulting population was initialized randomly to fairly sample the entire search space. Linear rank-based selection was used to assign each population member a probability to enter the mating pool. From this newly created mating pool, stochastic universal sampling (SUS) considered each members relative fitness as well as the inherent randomness of evolution made to identify the future parents. Whole arithmetic recombination selection was selected by literary convention

[1]. The testing of various mutation methodologies revealed uncorrelated self-adaptive mutation with 10 step sizes to be the best, since its self-evolving capabilities translated well among the various environmental landscapes. To make the EA design compatible with deterministic crowding, the number of offspring generated needed to be equal to the population size. To allow for future comparisons, the ratio of offspring to parents was kept consist among all tested EAs. This evolutionary cycle was iterated for the Schaffers F7 function and the Katsuura function $100,000$ and $1,000,000$ times respectively.

Three different EA designs, only distinguishable by their survivor selection mechanisms, were developed. A baseline, generational model EA served as a control, since generational model EAs preclude the need for survivor selection mechanism. Crowding was deemed as the most comparable standard to evaluate the effectiveness of the multi-dynamic survivor selection scheme, since in both survivor selection mechanisms one individual arises as a survivor after competing with a similarly fit group of individuals. Once again, the focus of this study is to compare a novel survivor selection mechanism with an accepted standard; therefore, there was no need to determine an optimal type of crowding.

	General EA Design
Representation	Real-Valued Representation
Recombination	Whole Arithmetic
Recombination Probability	$p_r = 1$
Mutation	uncorrelated Self-Adaptive Mutation
Mutation probability p_m	$p_m = 1$
Parent Selection	Linear Rank-based Selection, SUS
Number of Parents	2
Number of Offspring	2
Survivor selection	*specified in Table II*
Initialization	Random
Terminal Condition	Restricted number of fitness evaluations

TABLE I: General description of all EAs used. Further specifications in Table II and III.

The three different survivor selection mechanisms that were implemented are displayed in Table II. Note that we always select from Parents ∪ Offspring. We briefly explain the three mechanisms.

	Survival Selection
Baseline EA	Generational Model
Crowding EA	Deterministic Crowding
MOEA	Diversity-based multi-objective selection

TABLE II: The survivor selection mechanisms selected for comparison.

1) Generational model: In this variant we do not perform survivor selection on but just replace all the parents with their offspring.

2) Deterministic crowding: In deterministic crowding, as first proposed by Mahfoud, parents are paired with their children in a way that minimizes the sum of distances between them (pair parents p_1 and p_2 in such a way with their children o_1, o_2 that $dist(p_1, o_1) + dist(p_2, o_2) < dist(p_1, o_2) + dist(p_2, o_1)$) [5]. We use Euclidean distance for this purpose. The parent and child in a pair compete with each other and the fittest individual survives. In case of equal fitness, the child survives.

3) Diversity-based MOEA survivor selection: Diversity-based multi-objective ('MULTI') survivor selection simultaneously optimizes fitness and distance to the closest neighbour. The procedure is shown in the pseudocode in Figure 2. As shown in the pseudocode, we select individuals randomly from the non-dominated front. This non-dominated front is calculated with respect to the the DCN (distance to the closest neighbour) and fitness value. In figure 3 the non-dominated front is illustrated. Again we used Euclidean distance. The best individual in terms of fitness, is always the first member of the new population. The DCN values are always calculated with respect to the individuals already chosen for the new population [4]. For the duration of this paper, the algorithm with diversity-based multi-objective selection will be referred to as MOEA.

Algorithm 1 MULTI Survivor Selection Scheme

CurrentMembers = Population ∪ Offspring
Best = Individual with best $f(x)$ in CurrentMembers
NewPop = { Best }
CurrentMembers = CurrentMembers - { Best }
while (|NewPop| < N) **do**
    Calculate DCN of CurrentMembers, considering as reference NewPop
    ND = Non-dominated individuals of CurrentMembers (without repetitions)
    Selected = Randomly select an individual from ND
    NewPop = NewPop ∪ Selected
    CurrentMembers = CurrentMembers - {Selected}
end while
Population = NewPop

Fig. 2: Pseudocode for the diversity-based MOEA selection procedure as described in [4]

B. *Parameter tuning*

Initial rough parameter tuning tests were conducted to identify which existing parameters significantly influenced the MBF. From this initial testing, population size, the slope of linear rank-based selection ($1 < s \leq 2$), the initialized step size (σ), and the lower threshold for self-adaptive mutation step size (ϵ_0) were identified as critical. Initial rough parameter tuning also provided a range of values for each parameter where the optimum was more likely to be found. A grid search was then conducted to fine tune these parameters in order to account for the effects of parameter interaction. For fine tuning the parameters, the model was simulated by 50 iterations for each combination of parameters to account

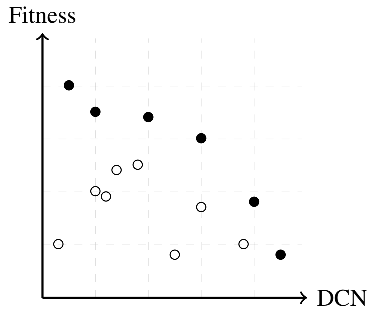


Fig. 3: Illustration of the non-dominated front. The black individuals are the ones that are not dominated in terms of Fitness and DCN.

for the stochasticity involved in heuristic algorithms. The following values were obtained:

	Generational	Crowding	MOEA
Population Size	120	2000	130
Selection Pressure	1.95	1.85	1.80
Lower Threshold for σ	0.0005	0.0	0.0001
Initialized Step Size	0.0007	0.01	0.01

TABLE III: Parameter tuning results on the Katsuura function.

	Generational	Crowding	MOEA
Population Size	50	150	150
Selection Pressure	1.80	1.10	2.00
Lower Threshold for σ	0.0	0.0	0.0001
Initialized Step Size	0.8	0.3	0.03

TABLE IV: Parameter tuning results on the Schaffers F7 function.

The crowding method of survivor selection focuses at exploring multiple optimum hills simultaneously to find the global one. In case of *Katsuura* function, the number of sub-optimal hills are very high as compared to *Schaffers F7* function. Hence, it is reasonable to expect that crowding will require a larger population size to have signification proportion of population located at multiple hills. The above mentioned argument justifies the optimal population size of 2000 for *Katsuura* and 150 for *Schaffers F7* in case of crowding based selection.

In general, the MOEA algorithm explicitly has an objective to maintain diversity. Hence it can be postulated that the optimal population size for MOEA will not increase dramatically with the complexity of the optimization problem at hand. The optimal data produced from fine tuning is in agreement with our above postulate. The optimal population size for MOEA seems to be more robust as compared to others as it is not much problem specific.

C. Experimental setup

Function optimization by its nature requires scoring EAs by the mean best fitness (MBF). Similarly, the superior survivor selection mechanism can be accurately defined by the average best fitness score it produces. Thus, the MBF over 100 iterations was used to assess all the given EAs. To gain insight in the diversity, we also calculated kept track of the average distance to the closest neighbour (DCN) in the population.

III. RESULTS

The average MBF values for the different EA instances with their standard deviations (SD) are displayed in Figure 4. A one-way ANOVA test was performed to compare the performance of the three different algorithms. On both Katsuura and Schaffers F7, there is a significant difference within the three algorithms ($p < .000001$). Two-tailed t-tests are performed for pairwise comparison. All differences are significant ($p < .00001$), except for the difference between the Generational EA and the Crowding EA on Katsuura ($p = .086$). This is what we could expect given the big standard deviations. For both functions, we have a significant winner: the MOEA for Katsuura, with $MBF = 9.121$ and the Generational model for Schaffers F7, with $MBF = 9.999$. Although not all statistical test conditions may be satisfied, these statistics can be seen as reliable, based on the low p-values in combination with the relatively large data set and the known robustness of the tests.

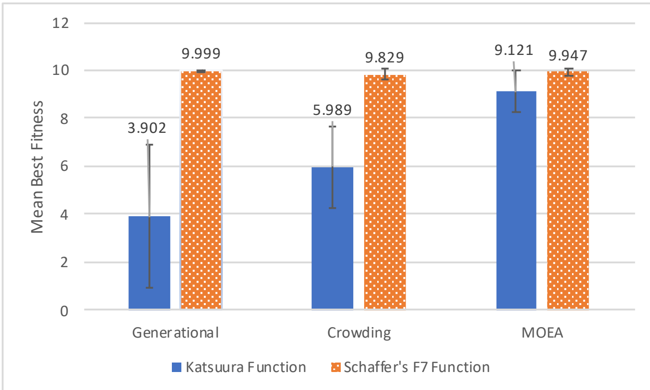


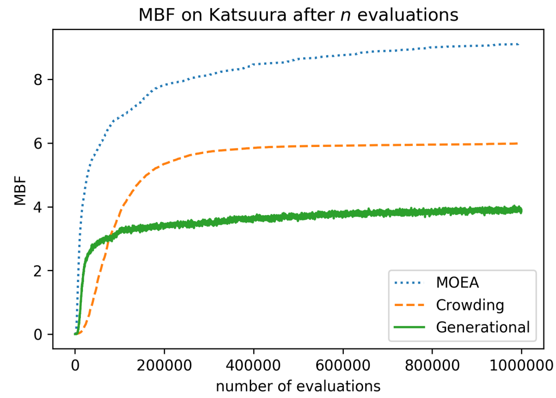
Fig. 4: MBF as a function of survivor selection methodology. The standard deviations for the Generational, Crowing, and MOEA models are 2.998, 1.704, and 0.869 for the Katsuura function respectively. Likewise, the standard deviation values for Schaffer's F7 function are: 0.003, 0.223, and 0.165.

In the graphs in Figure 5, we show the MBF values and the average DCN against the number of evaluations.

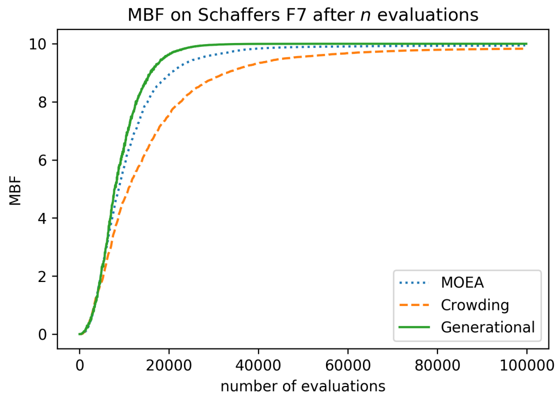
IV. DISCUSSION

A. General analysis of performance

A clear difference can be seen between the results obtained in the Schaffers F7 and Katsuura optimization problems. Although the performance of different algorithms cannot be generalized to all (multi-modal) optimization problems, we can conclude that: the MOEA only yielded significantly better results in the environment landscape imposed by the Katsuura function, which is known to be

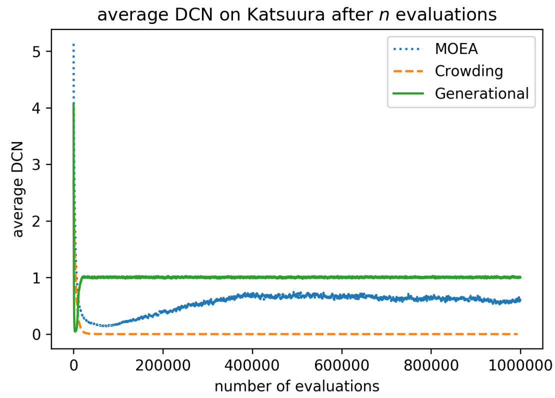


(a)

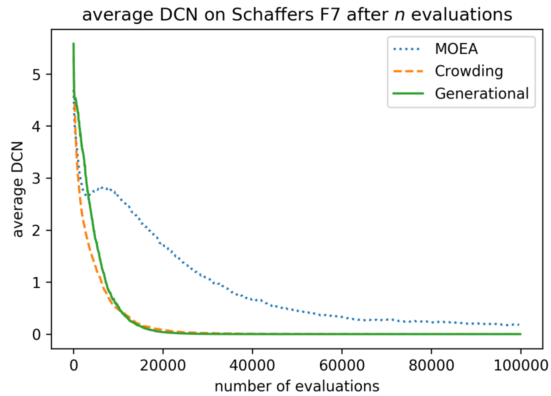


(b)

Fig. 5: Mean best fitness values over 100 iterations on Katsuura (a) and Schaffers F7 (b).



(a)



(b)

Fig. 6: Average DCN values over 100 iterations on Katsuura (a) and Schaffers F7 (b).

highly multi-modal and non-differentiable. Furthermore, Figure $5a$ shows that the MOEA's average MBF for Katsuura continues to increase after $1,000,000$ evaluations. In tandem, a comparision of Figures $5a$ and $6a$ reveals that a high level of diversity preservation, shown by the consistently high DCN values, is an unnecessary delay in optimization for the Katsuura function.

In contrast, Schaffer's F7 function provides a more structured environmental landscape that contains less optima. As a result, all the tested survivor methodologies were able to deliver good results. The generational model's near perfect results can be attributed to its more 'forgetful' nature. The consistent removal of previous generations reduces the overall selection pressure and allows for constant diversity generation in a way that is uniquely different from standard and novel diversity preservation techniques. As a result, this may be one of the instances in which more 'forgetful' algorithms yield the best results [1]. The MOEA's low MBF, seen in Figure $6b$ is most likely a consequence of the algorithm's need to balance multiple objectives, which prevents it from solely optimizing the population's overall fitness.

In Figure $6a$, the average DCN for the Katsuura function decreases rapidly towards zero in crowding. The sharp, sudden drop in diversity can be attributed to the high selection pressures created by both SUS and crowding. SUS can select multiple clones [of the fittest individuals] as parents. When two clones, a parent and a child, are inputted into the crowding mechanism, both are included in the new population. Once more clones exist within the population, this process is more likely to be repeated until the clones have overtaken the entire population. Within this framework, the effects of mutation are deemed negligible. Since the MOEA has an additional explicit objective, to optimize the DCN value, its limited population remains relatively unaffected by the high selection pressures. Meanwhile, the generational model only experiences selection pressure from SUS; thus, it does not face the same problem in maintaining diversity as crowding or MOEA do. In general, the high degree of multimodality exhibited in the Katsuura environmental landscape causes premature convergence to occur when compared to Figures $6a$ and $6b$. Thus, the population often becomes trapped within some local optima, as observed in the the crowding and generational models'

MBF values.

Surprisingly, the plot in Figure $6a$ shows a higher average DCN in the generational model than the MOEA model for the Katsuura landscape. This could be an effect of self-adaptive mutation: higher mutation step-sizes are rewarded more for their exploratory nature. As a consequence, the resulting children are farther from their parents within the genotypic space, but may not necessarily be anywhere closer to the global optimum. Whereas with a more recombination-driven approach, less diversity would be observed; however, the children would be more likely to steadily approach an optima. The effects of this can be seen in the generational model's results: the high standard deviation observed illustrates that the search is not always focused on the right peaks.

Ultimately, diversity does not play an influential role as originally thought in the both optimization problems. In spite of the clear difference between in average DCN between the crowding and generational models, the MOEA model still out-performs both in Katsuura. In Schaffer's F7, the large gap in average DCN between the generational and MOEA models does not translate into a large gap in MBF. Rather, the two models yield similar results.

B. Limitations of our specific EA design

Although all algorithms were tuned to be most suitable for the methods analyzed, no claims of the superiority of survivor selection mechanisms —outside of the specific EA designs outlined —can be supported. For example, crowding could be more efficient with another (or, possibly, no) parent selection mechanism.

In short, each survivor selection mechanism works best with a certain set of other mechanisms for certain problems. The specificity and intricacy of these interactions makes it impossible to draw any broader comparisons than those previously discussed. Finally, only two multi-modal problems were analyzed. In order to more a certain statement about the efficacy of the MOEA model, more highly multi-modal and non-differentiable optimization problems would need to be tested. [In order to see whether the observations of MOEA's success for the Katsuura function does hold.]

C. Note about efficiency

Even though the analysis of computational effort is outside the scope of this paper, it is worth noting that even though MOEA seems promising, it is perhaps not most efficient approach for all problems due to the computational cost required to obtain the DCN for all population members.

Thus, a more computationally efficient implementation of DCN calculation would make the testing of the MOEA model more appealing. Consider a problem similar to Schaffer's F7 function, even if the MOEA performs just as good or slightly better than the generational model, it may still be better to use the generational model because it yields solutions more quickly. Another implication of such a version of MOEA is the ability to assess more evaluations within a given time frame. Another option could be a compromise: to only partially apply the diversity-based multi-objective survivor selection scheme within an EA, and to utilize an easier survivor selection mechanism the rest of the time.

These musings do not hold for the Katsuura optimization problem as the other algorithms failed to locate a reasonably high optimum. The early plateauing of the generational and crowding models shows that an infinite evaluations would likely not significantly improve their results. Therefore, until a more computational attractive alternative is investigated, MOEA's computational power cost is clearly worth the results obtained.

D. Broader Implications

Although similar to crowding conceptually, MOEA can yield significantly different results within certain problems. As a result, there is a potential for this novel mechanism to become more broadly accepted within the Evolutionary Computing community as it provides the ability to address certain problems that other survivor selection mechanisms may not be able to.

V. CONCLUSION

Diversity-based MOEAs present promise for addressing multi-modal problems. However, before this new tool can be fully adopted, additional testing of its capabilities must take place. First of all, its ability to address highly multi-modal problems needs to be verified in plethora. As main purpose of the multi survivor selection scheme is to ensure diversity preservation throughout the progression of evolution, it needs to be more thoroughly compared to other explicit and implicit diversity preservation mechanisms before its appropriate usage can be fully understood. Due to the intricacies of EA design, an investigation of this method's potential side effects is needed to better understand what other mechanisms are most compatible with it and if any mechanism incompatibilities exist.

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