FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION

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Report

on the practical task No. 4

“Algorithms for unconstrained nonlinear optimization. Stochastic and metaheuristic algorithms”

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Goal

The use of stochastic and metaheuristic algorithms (Simulated Annealing, Differential Evolution, Particle Swarm Optimization) in the tasks of unconstrained nonlinear optimization and the experimental comparison of them with Nelder-Mead and Levenberg-Marquardt algorithms.

Formulation of the problem

I. *Generate the noisy data* (x_k, y_k), *where* $k = 0, \dots, 1000$, *according to the rule:*

$$y_k = \begin{cases} -100 + \delta_k, f(x_k) < -100, \\ f(x_k) + \delta_k, -100 \le f(x_k) \le 100, \\ 100 + \delta_k, f(x_k) > 100, \end{cases} x_k = \frac{3k}{1000},$$

$$f(x) = \frac{1}{x^2 - 3x + 2},$$

where $\delta_k \in N(0,1)$ *are values of a random variable with standard normal distribution. Approximate the data by the rational function*

$$F(x, a, b, c, d) = \frac{ax + b}{x^2 + cx + d}$$

by means of least squares through the numerical minimization of the following function:

$$D(a, b, c, d) = \sum_{k=0}^{100} (F(x_k, a, b, c, d) - y_k)^2.$$

*To solve the minimization problem, use Nelder-Mead algorithm, Levenberg-Marquardt algorithm and **at least two** of the methods among Simulated Annealing, Differential Evolution and Particle Swarm Optimization. If necessary, set the initial approximations and other parameters of the methods. Use* $\varepsilon = 0.001$ *as the precision; at most 1000 iterations are allowed. Visualize the data and the approximants obtained **in a single plot**. Analyze and compare the results obtained (in terms of number of iterations, precision, number of function evaluations, etc.).*

II. *Choose at least 15 cities in the world having land transport connections between them. Calculate the distance matrix for them and then apply the Simulated Annealing method to solve the corresponding Travelling Salesman Problem. Visualize the results at the first and the last iteration. If necessary, use the city dataset from*
https://people.sc.fsu.edu/~jburkardt/datasets/cities/cities.html

Brief theoretical part

There are some stochastic and metaheuristic algorithms inspired by nature. Usually they are good in global optimization but require adjusting hyperparameters. What is more, such algorithms can process complex functions that has no opportunity to calculate first/second-order derivatives. Therefore, on average these methods take more time than others.

Particle Swarm Optimization *(PSO) simulates swarm movement where each individual is affected by all others. It helps to get out of local extremum.*

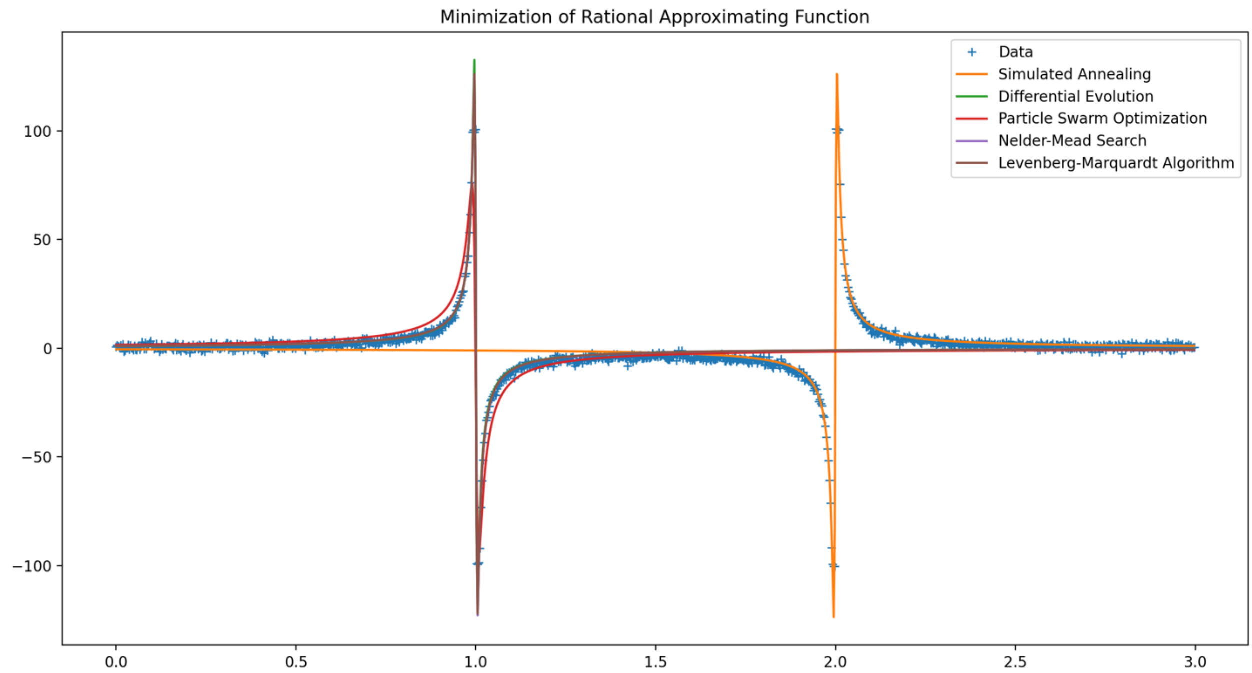
Simulated annealing *is a metaheuristic algorithm that solves the optimization problem similar to the process of annealing in metallurgy. Step by step, it decreases "energy" of system.*

Differential Evolution *is simulation of actual Evolution. It bases on mutation, recombination, selection and other natural effects. This algorithm like PSO requires proper hyper-parameters setup to gain its best speed.*

Results

Three meta-heuristic algorithms were chosen to solve the problem – differential evolution (scipy Python library), particle swarm optimization (pyswarms Python library) and simulated annealing. Scipy implementations of Nelder-Mead and Levenberg-Marquardt algorithms were used for comparison.

The figures of functions and solutions by different methods are demonstrated below.



Simulated Annealing arguments: 1.034550, -2.068660, -3.999220, 3.998458

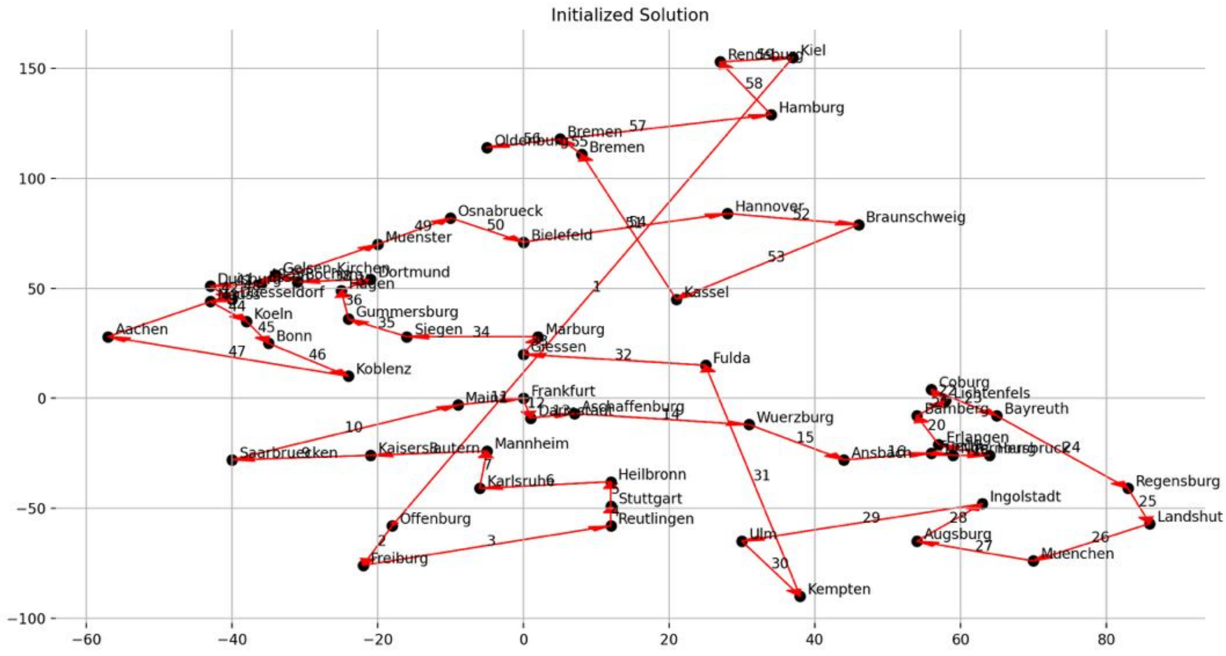
Differential Evolution arguments: -0.973931, 0.974189, -2.000000, 1.000015

Stopping search: Swarm best position change less than 0.001

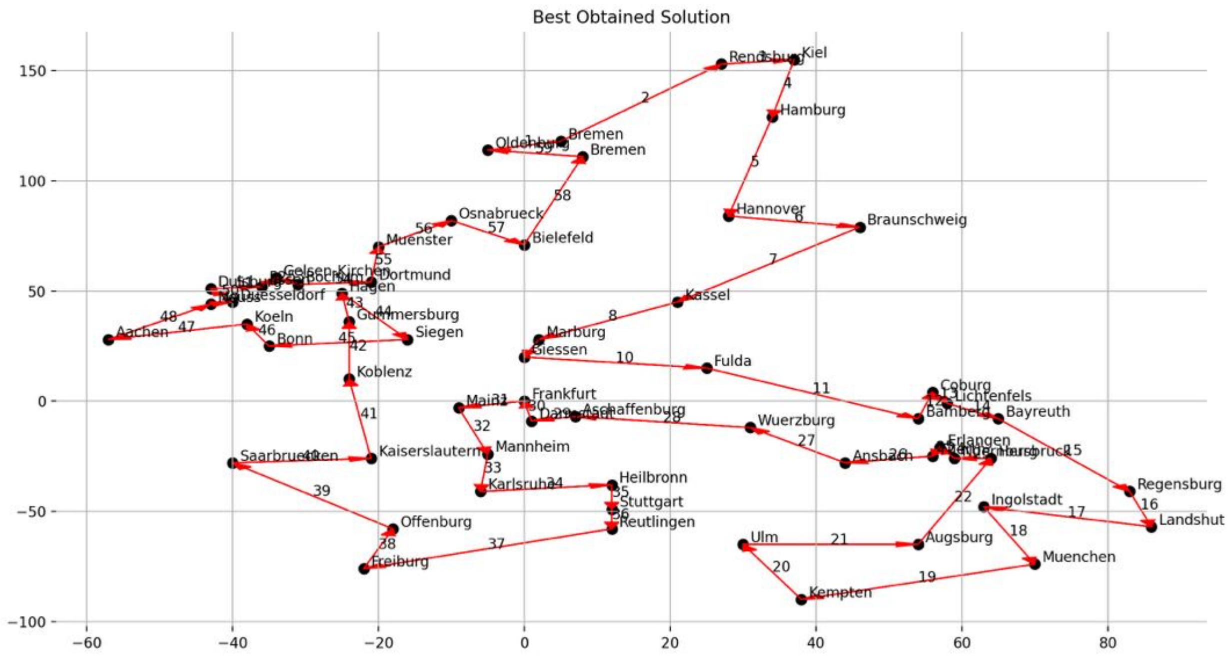
Particle Swarm Optimization arguments: -1.546277, 1.544632, -2.000000, 1.000082

Nelder-Mead Search arguments: -1.001520, 1.002023, -2.000922, 1.000938

Levenberg-Marquardt Algorithm arguments: -1.001461, 1.001941, -2.000834, 1.000850



Initialized solution: 1368.1084230727195



Best obtained solution: 1142.5431198303413

Conclusions

As can be seen from the graph, only the Annealing Simulation method coped with finding the second gap, the worst case in terms of accuracy is the particle swarm Method, possibly due to an error. It was found that particle swarm optimization cannot find the best solution if area near the optimal values is too narrow.The rational approximation of the solution of differential evolution was solved quite accurately by the Levenberg-Marquardt method, the Nelder-Meade method was less accurate. The longest time was the calculation by the method of Simulated Annealing.

Stochastic and metaheuristic algorithms uses much more function calls but they are able to find global minimum. And Nelder-Mead and Levenberg-Marquardt methods, vice versa, takes less function calls but don't guarantee to find global optimum.

The annealing method was also applied to solve the TSP problem for German cities. The expected results obtained.

Appendix

Source code link:
https://github.com/sheograph/Analysis-and-development-of-algorithms