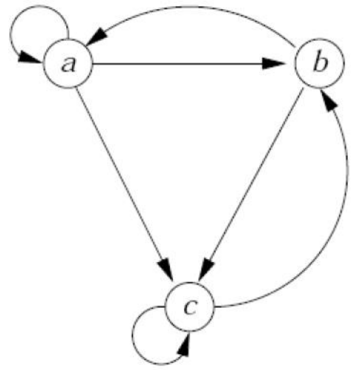
1. Compute the PageRank of each vertex in the following figure:

   (1) assuming no random teleport

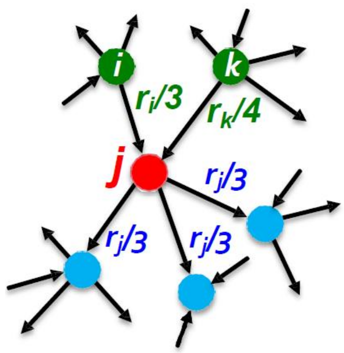
   (2) assuming β= 0.8



Define a "rank" r_j for page j

$$r_j = \sum_{i \to j} \frac{r_i}{\mathrm{d_i}}$$

$$r_j = r_i/3 + r_k/4$$



Answer:

(1) The transition matrix for the graph is:

$$\begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix}$$

By equation method ($\boldsymbol{Mr} = \boldsymbol{r}$), we get

$$\frac{1}{3}r_1 + \frac{1}{2}r_2 + 0 = r_1$$

$$\frac{1}{3}r_1 + 0 + \frac{1}{2}r_3 = r_2$$

$$\frac{1}{3}r_1 + \frac{1}{2}r_2 + \frac{1}{2}r_3 = r_3$$

$$r_1 + r_2 + r_3 = 1$$

and the result $\boldsymbol{r} = \left[\frac{3}{13}, \frac{4}{13}, \frac{6}{13}\right]^T$

(1) (2) assuming β= 0.8

$$r_j = \sum_{i \to j} \beta \frac{r_i}{d_i} + (1-\beta)\frac{1}{N}$$

$$\sum_i r_i = 1$$

$$0.8 \times \left(\frac{1}{3}r_1 + \frac{1}{2}r_2 + 0\right) + 0.2 \times \frac{1}{3} = r_1$$

$$0.8\left(\frac{1}{3}r_1 + 0 + \frac{1}{2}r_3\right) + 0.2 \times \frac{1}{3} = r_2$$

$$0.8\left(\frac{1}{3}r_1 + \frac{1}{2}r_2 + \frac{1}{2}r_3\right) + 0.2 \times \frac{1}{3} = r_3$$

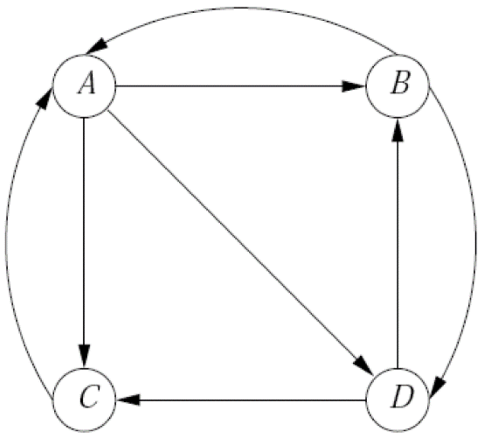
$$r_1 + r_2 + r_3 = 1$$

and the result $\boldsymbol{r} = \left[\frac{7}{27}, \frac{25}{81}, \frac{35}{81}\right]^T$

2. Use three iterations to compute the topic-sensitive PageRank for the graph of the following figure, assuming the teleport set is (suppose β=0.8):

(1) A only.

(2) A and C.



Answer:

(1) The transition matrix **M** of this Figure is:

$$\begin{bmatrix} 0 & 1/2 & 1 & 0 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 0 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A} = \beta \cdot \mathbf{M} + (1-\beta)\,[1/N]_{N\times N}$$

$$0.2 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{r}$$

$$\boldsymbol{r}^{\text{new}} = \boldsymbol{A} \cdot \boldsymbol{r}^{\text{old}}$$

$$\boldsymbol{r}' = \begin{bmatrix} 0 & 2/5 & 4/5 & 0 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 0 & 0 & 2/5 \\ 4/15 & 2/5 & 0 & 0 \end{bmatrix} \boldsymbol{r} + \begin{bmatrix} 1/5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}1.0\\0.0\\0.0\\0.0\end{bmatrix},\begin{bmatrix}0.2\\0.2666\\0.2666\\0.2666\end{bmatrix},\begin{bmatrix}0.52\\0.16\\0.16\\0.16\end{bmatrix},\begin{bmatrix}0.392\\0.2026\\0.2026\\0.2026\end{bmatrix},\begin{bmatrix}0.4432\\0.1856\\0.1856\\0.1856\end{bmatrix},\begin{bmatrix}0.4227\\0.1924\\0.1924\\0.1924\end{bmatrix}\cdots\cdots\begin{bmatrix}0.4285\\0.1904\\0.1904\\0.1904\end{bmatrix}$$

(2)

$$\boldsymbol{r}' = \begin{bmatrix}0 & 2/5 & 4/5 & 0\\4/15 & 0 & 0 & 2/5\\4/15 & 0 & 0 & 2/5\\4/15 & 2/5 & 0 & 0\end{bmatrix}\boldsymbol{r} + \begin{bmatrix}1/10\\0\\1/10\\0\end{bmatrix}$$

$$\begin{bmatrix}1.0\\0.0\\0.0\\0.0\end{bmatrix},\begin{bmatrix}0.1\\0.2666\\0.3666\\0.2666\end{bmatrix},\begin{bmatrix}0.5\\0.1333\\0.2333\\0.1333\end{bmatrix},\begin{bmatrix}0.34\\0.1866\\0.2866\\0.1866\end{bmatrix},\begin{bmatrix}0.404\\0.1653\\0.2653\\0.1653\end{bmatrix},\begin{bmatrix}0.3784\\0.1738\\0.2738\\0.1738\end{bmatrix}\cdots\cdots\begin{bmatrix}0.3857\\0.1714\\0.2714\\0.1714\end{bmatrix}$$