Brown University
Department of Economics

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Problem set 1. The Solow Growth Model

There are four questions in this problem set. Try to answer all questions below. Each question worth is specified at the beginning of the question. You are not allowed to copy or share your answers with other students. You are free to discuss questions with your peers, but please submit your own responses (i.e., not copied from others).

Question 1. Income ratios in the Solow model (30 pts)

Consider a version of the Solow model **with** population growth (but **without** productivity growth) discussed in Lecture 2. Production function is Cobb-Douglas with $\alpha = 1/3$. Consider two economies, Richland (R) and Poorland (P). In Richland, savings rate is 2 times larger than in Poorland: $s_R = 2 \cdot s_P$. Moreover, the sum of population growth rate and depreciation rate in Richland is 2 times smaller than in Poorland: $(n_R + \delta_R) = 0.5 \cdot (n_P + \delta_P)$. Productivity levels, A, are the same in these two countries.

(a) Derive the equation for the steady state level of capital per worker, k^*. Do it 'in general', not just for R or for P (hint: we did something very similar in our math section).

(b) Combine the expression for the steady state capital per worker from (a) with the production function per worker, to derive output per worker in a steady state, y^*.

(c) Define the ratio of steady state output per worker between the two countries as y_R/y_P. Calculate this ratio given the parameters in this problem (hint: you should arrive at a whole number).

(d) Assume that Richland is represented by the US. What was the savings rate in the US in the year 2016 or around? What was the population growth rate in the US around these years? Find a country in the world with savings rate approximately half of that in the US, and population growth rate approx. double that of the US. You can allow for reasonably large deviations from exact "half" and exact "double". What is the ratio of GDP per capita between the US and this country? Do you think the Solow model does a good job predicting income per capita differences in this case? Why or why not? (Hint: you can find data for savings and population growth rates on the websites we referenced in our lectures. Note that such data often comes from either the World Bank, the United Nations, or the IMF, and you can find lots of interesting data on their websites.)

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Question 2. 'Catch-up' growth rates in the Solow model (30 pts)

Consider a version of the Solow model **with** population growth (but **without** productivity growth) discussed in Lecture 2. Production function is Cobb-Douglas with $\alpha = 1/3$. In this problem you will have to prove the result about the rates of 'catch-up' growth in transition to the steady state. Namely, let's prove that the rate of growth of capital per worker g_{k_t} is increasing in A, increasing in s, decreasing in n, and decreasing in k_t. We will go step by step:

(a) Find output per worker in this model as a function of capital per worker (and parameters). Next, derive the function for investment per worker.

(b) Using the dynamics of the aggregate capital K_{t+1}, and population L_{t+1}, express the dynamics of capital per worker $\Delta k_t = k_{t+1} - k_t$ as a function of k_t and parameters (hint: we did exactly that in Lecture 2, but in a more general form).

(c) Next, find the growth rate of capital per worker $g_{k_t} = \frac{\Delta k_t}{k_t} = \frac{k_{t+1} - k_t}{k_t}$ using part (b). Express g_{k_t} as a sum (or a difference) of two components. Depict these two components you got on a graph where you put k_t on the horizontal axis, and values of the two components on the vertical axis (hint: one component should be a horizontal line, as it does not depend on the value of k_t).

(d) Show that a higher A increases g_{k_t}. Show that a higher s increases g_{k_t}. Illustrate this on a graph from part (c): show how curves shift in response to an increase in A; and on a separate graph how curves respond to an increase in s.

(e) Show that a higher n decreases g_{k_t}. Show that a higher k_t decreases g_{k_t}. Illustrate this on a graph from part (c): show how curves shift in response to an increase in n; and on a separate graph how curves respond to an increase in k_t.

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Question 3. Will capital flow from rich countries to poor countries? (20 pts)

Consider a version of the Solow model **without** population growth and **without** productivity growth. Assume that international capital flows (such as foreign direct investments) depend on returns to capital across countries in the world. Namely, investors want to invest in those countries where returns are higher. By returns to capital we mean the additional output (and hence money) that investment produces, i.e., the MPK - marginal product of capital. If we add these simplified international capital flows to the Solow model, what would your opinion be on the following claim: "Capital will flow from richer countries to poorer countries, because returns (in terms of the MPK) to each additional unit of capital invested are higher in poorer countries". (Hint: this problem is related to the 'law of diminishing returns', and to the various reasons why a country can be poor).

Question 4. Epidemics and wars in the Solow Model (20 pts)

Consider a version of the Solow model **without** population growth and **with** technological progress. Assume that the economy is initially in (or very close to) the steady state.

(a) In period t, the economy is struck by an epidemic: L_t decreases by X% in a single period. (hint: what does it imply for the initial change in k_t?) Show graphically, on the main diagram of the Solow model (the one depicting investment per worker, depreciation per worker, and output per worker), how this decrease in population would affect capital per worker and income per worker immediately after the shock (in the same period), and in the long-run? Explain the intuition behind these effects and the dynamic adjustment of this economy.

(a) In period t, the economy is affected by war: both capital and labor decreased, but K_t decreased by X% more than does L_t. (hint: what does it imply for the initial change in k_t?) Show graphically, on the main diagram of the Solow model, how this event would affect capital per worker and income per worker immediately after the shock (in the same period), and in the long-run? Explain the intuition behind these effects and the dynamic adjustment of this economy.