Brown University
Department of Economics

Alexander Yarkin
Summer 2021

Problem set 2. Endogenous Growth, Malthusian, and the UGT models

There are four questions in this problem set. Try to answer all questions below. Each question worth is specified at the beginning of the question. You are not allowed to copy or share your answers with other students. You are free to discuss questions with your peers, but please submit your own responses (i.e., not copied from others).

Question 1. Population, technological progress, and the effects of R&D investment (25 pts)

Consider the version of the model we studied in Lecture 3 (part II) with idea production parameters $\nu = 1, \phi = 0.5, \lambda = 0.5$, and output production parameter $\sigma = 2/3$. Moreover, assume that initially, the economy has R&D employment share $\gamma_A = 0.05$. Population growth rate is $n = 0.03$.

(a) Derive the equation for g_{A_t}: the growth rate of A (ideas, TFP). Describe the meaning of this equation: what does the growth rate of ideas depend on and why?

(b) Find the growth rate of output per capita on the Balanced Growth Path, g_y^{BGP} (hint: it should be a number less than 1). What can we say about the BGP in the case when $\phi = 1$?

(c) In period t_0, the government decides to stimulate innovations by providing more funds for labor to work in the R&D sector, so γ_A increases from 0.05 to 0.1. How will this change affect (i) the rate of TFP growth g_{A_t} in period t_0; and (ii) the growth rate of TFP on the BGP, g_A^{BGP}? Draw a graph sketching the dynamics of g_{A_t} (hint: on the horizontal axis you would have time t, and on the vertical axis you would have g_{A_t}; note that initially, before the policy change, g_{A_t}=g_A^{BGP})

(d) How will this policy change affect (i) the level of log income per capita $ln(y_t)$ in period t_0; and (ii) the growth rate of income per capita on the BGP? Draw a graph sketching the dynamics of ln (y_t) (hint: on the horizontal axis you would have time t, and on the vertical axis you would have ln (y_t); note that initially, before the policy change, $g_{y_t} = g_y^{BGP}$, so the growth rate of y is constant).

Brown University
Department of Economics

Alexander Yarkin
Summer 2021

Question 2. Malthusian model: the effects of costs of raising children, and the effects of epidemics (30 pts)

Consider a version of the Malthusian model we studied in Lecture 4. Namely, assume that $A = 10, X = 40, \alpha = 1/2$ in the production function, and that parents spend one half of their incomes on raising children, and that the cost of raising one child is 1 unit.

(a) Derive the function for output per capita. Is output per capita decreasing or increasing in the size of population?

(b) Express population in period $t + 1$, L_{t+1}, as a function of population in period t, L_t. Depict this function on a graph with L_t on the horizontal axis, and L_{t+1} on the vertical axis.

(c) Derive the steady-state level of population, L^* (hint: your answer should be a whole number).

(d) Now suppose that because of a more sedentary lifestyle (people start to move around less), there is a decrease in the cost of raising children ($\rho\downarrow$), so now one child 'costs' 0.5 units. How does this change affect the steady state level of population, L^*? (hint: again, your answer should be a whole number). Illustrate on the graph from part (b).

(e) Derive the initial steady-state level of incomes per capita, y^*. How does it react to a decrease in the cost of raising children? Show on a separate graph.

(f) Now suppose that in period t_0 there is an epidemic that kills 40% of the population (of the initial steady-state value). Describe what happens to population and output per worker over time following this event (both at the time of the epidemic, and in the long-run). Use graphs, equations, and intuitive explanations where needed. It's not necessary to calculate precise values in this part of the problem.

Alexander Yarkin

Question 3. Short 'essays' about technological progress (25 pts)

(a) Do you expect the pace of technological progress to increase, decrease, or remain roughly the same over your lifetimes? Explain your answers using the concepts, theories, and evidence we have learned so far. (10 pts)

(b) We discussed that market competition affects R&D and innovation incentives in several distinct ways. Through the lens of theories and evidence we studied in Lecture 3, what do you think the government competition policy in the pharmaceuticals industry should be? (hint: think of international pharmaceuticals industry as an example of industry with several quite large companies, quite high market entry barriers, and large profits) (15 pts)

(hint: in both of these questions, there is no '100% correct' answer, but please explain your line of reasoning, and try to rely on concepts and ideas we discussed in the course)

NOTE: Please limit the length of your replies to the max of 1 page (14pt., single spaced, standard margins) for each question. But please answer as precisely and as detailed as needed within these limits.

Question 4. The Unified Growth Theory and the role of population growth (20 pts)

As we have seen in Module 4, the role of population in economic growth historically is quite different from its role nowadays. In this question, please use Module 4 materials to describe how the role of population in economic growth changes over time as economies go through various stages of development.

(a) Start with the Malthusian period. How would you describe the relationship between population levels and incomes per capita? How would you describe the relationship between population growth rates and incomes per capita? (hint: recall that the relationship is bi-directional: population affects incomes, and incomes affect fertility)

(b) In the Post-Malthusian regime, what is the crucial novelty in how population matters for development? Why do population and incomes per capita grow together? (hint: think about the acceleration of technological progress and why it happens)

(c) Why does population growth begin to decline in the second phase of the demographic transition? Namely, why do we see incomes per capita continue growing even faster, while population growth rates fall?

(d) Contrast the effects of population growth on GDP per capita in the Solow model with what you have just described. What is the main difference between the Solow model and the UGT regarding population growth and its effects?

For this question, you are encouraged to use graphs, words, pictures from lectures or from elsewhere to substantiate your answer.