Problem Set 5 Ans

Problem 1

a)

命题为假.

例如$A = \{1, 2, 3\}, B = \{2, 3\}, C = \{3, 4\}$时,

有$A \cap C = B \cap C = \{3\}$,但$A \neq C$

b)

命题为假.

例如$A = \{1, 2, 3\}, B = \{3, 4\}, C = \{2, 3, 4\}$时,

有$A \cup B = A \cup C = \{1, 2, 3, 4\}$,但$B \neq C$

c)

命题为假.

例如$A = \{1\}, B = \{2\}$时,

$\therefore 2^{A \cup B} = \{\{1\}, \{2\}, \{1, 2\}\}$

$\therefore 2^A \cup 2^B = \{\{1\}\} \cup \{\{2\}\} = \{\{1\}, \{2\}\}$

d)

命题为真.

$$
\begin{aligned}
2^{A\cap B} &= \{x|x \subseteq A\cap B\} \\
&= \{x|\forall y(y\in x \to y\in A\cap B)\} \\
&= \{x|\forall y(y\in x \to y\in A \wedge y\in B)\} \\
&= \{x|\forall y(y\notin x \vee (y\in A \wedge y\in B))\} \\
&= \{x|\forall y((y\notin x \vee y\in A) \wedge (y\notin x \vee y\in B))\} \\
&= \{x|\forall y((y\in x \to y\in A) \wedge (y\in x \to y\in B))\} \\
&= \{x|\forall y(y\in x \to y\in A) \wedge \forall y(y\in x \to y\in B)\} \\
&= \{x|x\subseteq A \wedge x\subseteq B\} \\
&= \{x|x\in \{x|x\subseteq A\} \wedge x\in \{x|x\subseteq B\}\} \\
&= \{x|x\in 2^A \wedge x\in 2^B\} \\
&= 2^A \cap 2^B
\end{aligned}
$$

$\therefore 2^{A\cap B} = 2^A \cap 2^B$

Problem 2

结论:

$A=\emptyset$或$B=\emptyset$

证明:

$\because A\times B=\emptyset$

$\therefore A\times B = \{(x,y)|x\in A\wedge y\in B\}=\emptyset$

$\therefore \exists x\exists y(x\in A\wedge y\in B)\equiv F$

$\therefore \forall x\forall y(x\notin A\vee y\notin B)\equiv T$

$\therefore$ 不存在$x\in A$或不存在$y\in B$

$\therefore A=\emptyset$或$B=\emptyset$

Problem 3

a) $\{-1,0,1\}$

b) $\emptyset$

Problem 4

a)

$$
\begin{aligned}
A - B &= \{x | x \in A \land x \notin B\} \\
&= \{x | x \in A \land x \in \overline{B}\} \\
&= A \cap \overline{B}
\end{aligned}
$$

b)

$$(A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A$$

Problem 5

a)

不能.

例如$A = \{1, 2, 3\}, B = \{1, 2\}, C = \{3, 4\}$时,

有$A \cup C = B \cup C = \{1, 2, 3, 4\}$,但$A \neq B$

b)

不能.

例如$A = \{1, 2, 3\}, B = \{2, 3\}, C = \{3, 4\}$时,

有$A \cap C = B \cap C = \{3\}$,但$A \neq B$

c)

能.

Problem 6

$$\begin{aligned} A \subseteq B &\equiv \forall x(x \in A \to x \in B) \\ &\equiv \forall x(x \notin B \to x \notin A) \\ &\equiv \overline{B} \subseteq \overline{A} \end{aligned}$$

$$\therefore A \subseteq B \equiv \overline{B} \subseteq \overline{A}$$

Problem 7

a)

$$A \oplus A = (A - A) \cup (A - A) = \emptyset$$

b)

$$A \oplus U = (A \cup U) - (A \cap U) = U - A = \overline{A}$$

Problem 8

a)

$$\because A_i = \{1, 2, 3, \ldots, i\}, i = 1, 2, 3, \ldots$$

$$\therefore 当 i \geq 2 时, A_{i-1} \subseteq A_i, A_{i-1} \cup A_i = A_i$$

$$\therefore \bigcup_{i=1}^{n} A_i = A_n = \{1, 2, 3, \ldots, n\}$$

b)

$$\because A_i = \{1, 2, 3, \ldots, i\}, i = 1, 2, 3, \ldots$$

$$\therefore 当 i \geq 2 时, A_{i-1} \subseteq A_i, A_{i-1} \cap A_i = A_{i-1}$$

$$\therefore \bigcup_{i=1}^{n} A_i = A_1 = \{1\}$$

Problem 9

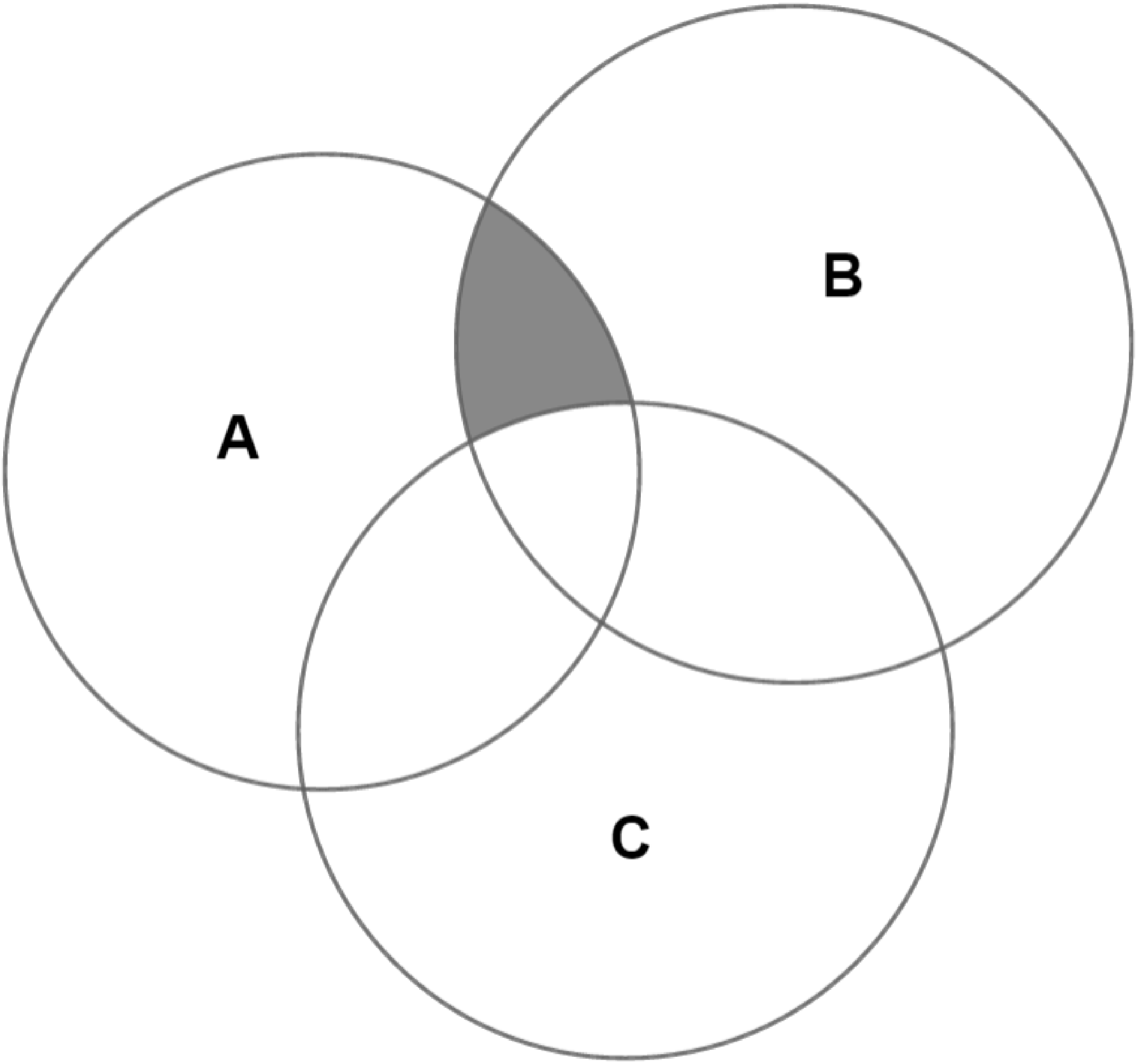
$$\begin{aligned}(A-B)\oplus B &= ((A-B)\cup B)-((A-B)\cap B)\\ &= A\cup B-\emptyset\\ &= A\cup B\end{aligned}$$

$\therefore (A-B)\oplus B = A\cup B$

Problem 10

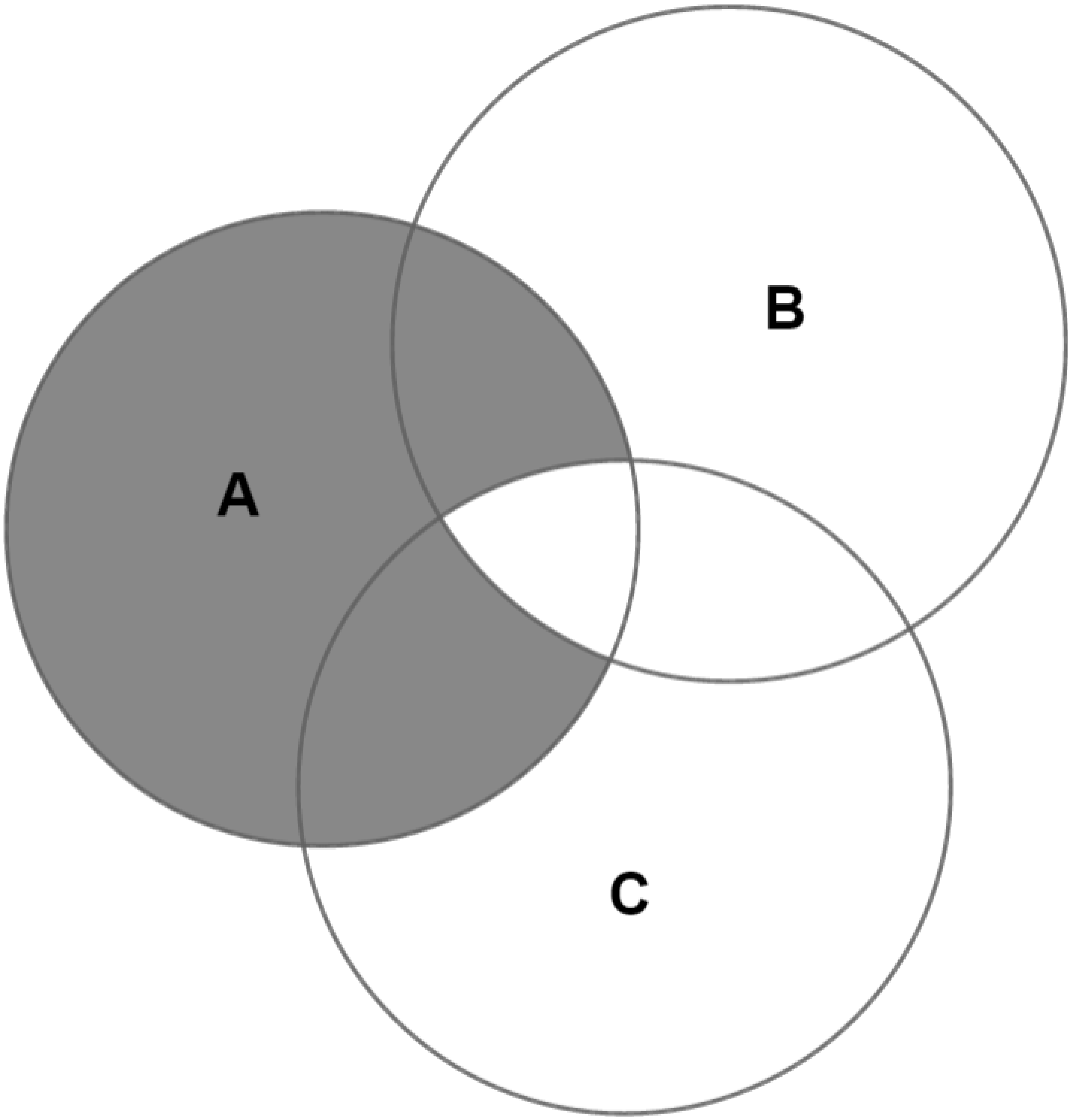
a)

$A\cap(B-C)$



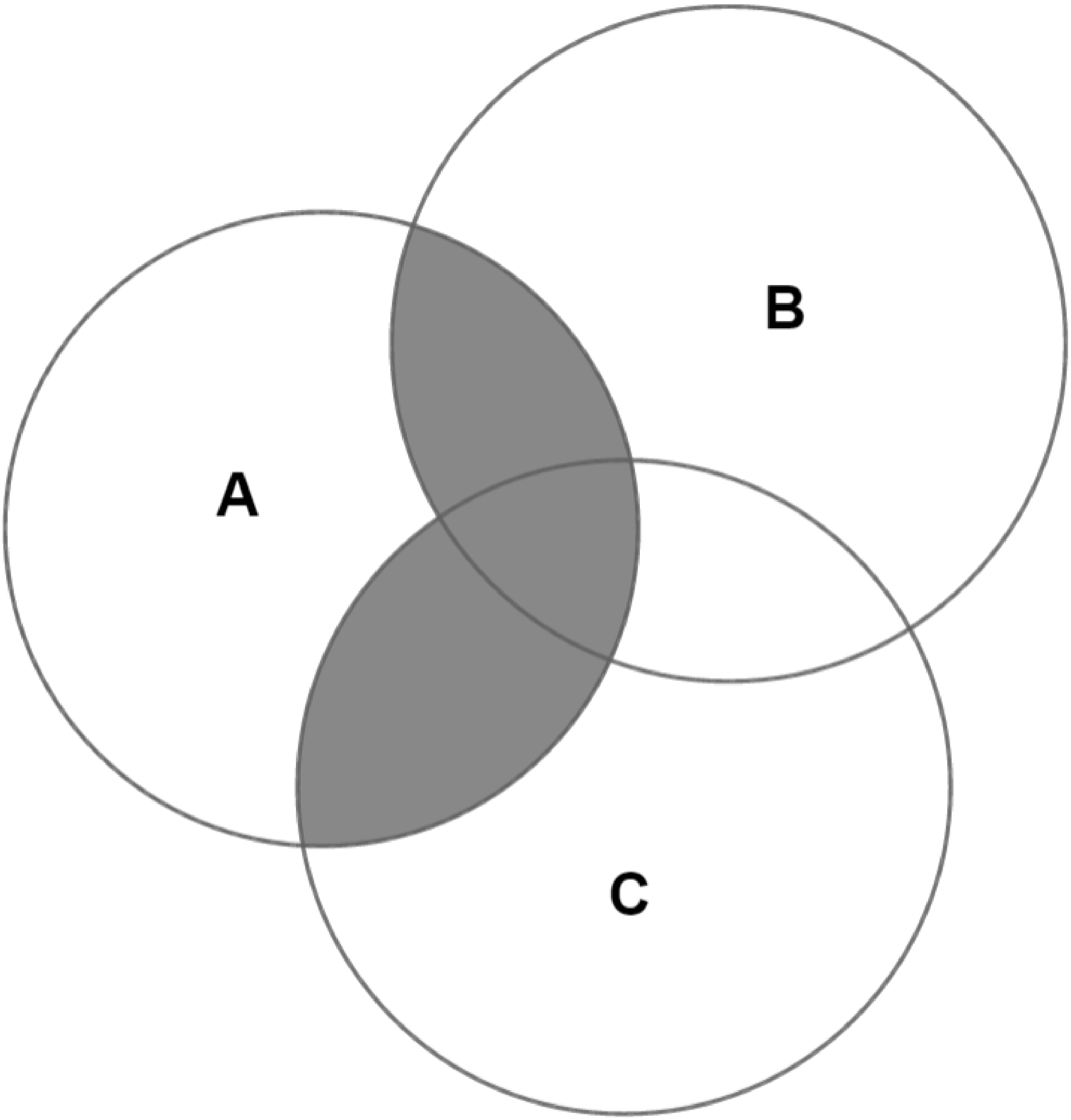
b)

$(A \cap \overline{B}) \cup (A \cap \overline{C})$



c)

$(A \cap B) \cup (A \cap C)$


A
B
C