Problem Set 6

Problem 1

a) 正确

b) 错误

c) 错误

d) 正确

Problem 2

不妨假定 $A - B \neq \emptyset,$ 取 $a \in A - B, b \in B$

$\therefore (a, b) \in A \times B$

$\because a \notin B$

$\therefore (a, b) \notin B \times A$

$\therefore A \times B \neq B \times A$

Problem 3

a) $R^{-1} = \{(b, a) | (a, b) \in R\} = \{(b, a) | a \text{整除} b\}$

b) $R^{-1} = \{(a, b) | (a, b) \notin R\} = \{(a, b) | a \text{不整除} b\}$

Problem 4

$\because R = \{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\},$
$\quad S = \{(2, 1), (3, 1), (3, 2), (4, 2)\}$

$\therefore S \circ R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

Problem 5

$\because R = R^{-1}, S = S^{-1}$

$\therefore (R \circ S)^{-1} = S^{-1} \circ R^{-1} = S \circ R$

$\because R \circ S \subseteq S \circ R$

$\therefore R \circ S \subseteq (R \circ S)^{-1}$

$\because$ 显然 $|R \circ S| = |(R \circ S)^{-1}|$, 即元素个数相等

$\therefore R \circ S = (R \circ S)^{-1}$

$\therefore (R \circ S)^{-1} \subseteq R \circ S$

$\therefore S \circ R \subseteq R \circ S$

$\therefore R \circ S = S \circ R$

Problem 6

(1)

b的论文导师的论文导师是a的时候.

(2)

b的n辈论文导师是a的时候.

Problem 7

a) $R_1 \cup R_2 = \{(a, b) | a \equiv b (mod 3) \vee a \equiv b (mod 4)\}$

b) $R_1 \cap R_2 = \{(a, b) | a \equiv b (mod 3) \wedge a \equiv b (mod 4)\}$

c) $R_1 - R_2 = \{(a, b) | a \equiv b (mod 3) \wedge a \not\equiv b (mod 4)\}$

d) $R_2 - R_1 = \{(a, b) | a \not\equiv b (mod 3) \wedge a \equiv b (mod 4)\}$

e) $R_1 \oplus R_2 = \{(a, b) | (a \equiv b (mod 3) \wedge a \not\equiv b (mod 4)) \vee (a \not\equiv b (mod 3) \wedge a \equiv b (mod 4))\}$

Problem 8

a) 有$2^{4^2} = 2^{16} = 256 \times 256 = 65536$种关系.

b) 有$2^{(4^2-1)} = 2^{15} = 256 \times 128 = 32768$种关系.

Problem 9

a) $R \circ S = \{(x,y)|y = 2x + 5\}$

b) $R \circ S = \{(x,y)|y = x^2 + 3x - 3\}$

c) $R \circ S = \{(x,y)|y = 2^{\sqrt{x}} \vee y = 2^{-\sqrt{x}}\}$

d) $R \circ S = \{(x,y)|y = 2^{x^2}\}$

Problem 10

a)

$$
\begin{aligned}
(R_1 \cup R_2)^{-1} &= \{(y,x)|(x,y) \in R_1 \cup R_2\} \\
&= \{(y,x)|(x,y) \in R_1 \vee (x,y) \in R_2\} \\
&= \{(y,x)|(y,x) \in R_1^{-1} \vee (y,x) \in R_2^{-1}\} \\
&= R_1^{-1} \cup R_2^{-1}
\end{aligned}
$$

b)

$$
\begin{aligned}
(R_1 \cap R_2)^{-1} &= \{(y,x)|(x,y) \in R_1 \cap R_2\} \\
&= \{(y,x)|(x,y) \in R_1 \wedge (x,y) \in R_2\} \\
&= \{(y,x)|(y,x) \in R_1^{-1} \wedge (y,x) \in R_2^{-1}\} \\
&= R_1^{-1} \cap R_2^{-1}
\end{aligned}
$$