Fintech545-HW3
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Problem 1

Calculate and compare the expected value and standard deviation of price at time t, we apply the three pricing method in class note:

1. Classical Brownian Motion

$$P_t = P_{t-1} + r_t$$

2. Arithmetic Return System

$$P_t = P_{t-1}\left(1 + r_t\right)$$

3. Log Return or Geometric Brownian Motion

$$P_t = P_{t-1}e^{r_t}$$

First, we simulate each return equation, set sigma=0.2, P0=100, t=1. Apply three pricing formula above, we get the output below:

```
Classical price: E(P) = 100.00, SD(P) = 0.20
Arithmetic price: E(P) = 100.00, SD(P) = 20.00
Geometric price: E(P) = 102.02, SD(P) = 20.20
```

From output, classical pricing formula have the same expected value as arithmetic pricing formula, the standard deviation of arithmetic price formula is much the same as geometric pricing formula. The standard deviation of classical pricing formula is quiet lower than the other two methods. Overall,, the generated prices are almost the same.

Problem 2

Set Alpha=0.05. We pick the pd.pct_change() to do the same work as return_calculate() in this week's code. Choose stock META , calculate mean of return and remove it and the return is shown below:

```
1      0.015175
2     -0.020165
3     -0.040761
4     -0.007446
5     -0.019774
         ...
244   -0.011850
245   -0.002503
246    0.029899
247   -0.042725
248   -0.030022
Name: META, Length: 248, dtype: float64
```

2.1 Using Normal Distribution

Assume the returns are normal distribution, we apply the probability function of normal distribution and get the output below:

```
VaR normal distribution: 0.06546917484881118
```

2.2 Using Normal Distribution with expected weight matrix

Apply the df.ewm() method, calculate the variance, we get the output below:

```
VaR normal distribution with EWMA variance: 0.0901898406731005
```

2.3 Using MLE fitted T distribution

Using stats package, fit the return by MLE t distribution, we get output below:

```
VaR MLE fited T distribution: 0.057579649751533055
```

2.4 Using fitted AR(1)

By importing ARIMA model, we fit the data for AR(1) model, simulate the returns and get the output below:

```
VaR AR(1): 0.06569973786653496
```

2.5 Using historical simulation

Using the historical data, simulate returns, set number of simulation as 1000, we get the output below:

```
VaR historical simulation: 0.05590681367337082
```

Form the result above, we could see that calculate var using Normal Distribution with expected weight matrix has the highest output, while using historical data calculating var has the lowest output.

Problem 3

First, import two files and using pct_change() to calculate return for each stock. Next, we split the portfolio in three groups and dealing with data in DailyPrices.csv to fing the stock names, returns, holdings, current value and weights for every portfolio.

To calculate portfolio VaR, I first implement Delta Normal VaR following the algorithm in class note:

$$VaR(\alpha) = -\ PV * F_X^{-1}(\alpha) * \sqrt{\nabla R^T \Sigma \nabla R}$$

We first find portfolio value(PV) by multiple current values and holdings matrix. Next, the quantile part as 1.65. For the third part, we first find the grandient vector of portfolio return(R), Since we assume linear instruments and onely 1 asset is affected my 1 price, thus, the gradient vector element is the price of stock devide by protfolio value times holding. The Sigma is generated by exponentially weighted matrix function.

Above all, we get the 4 VaR value:

```
Portfolio A VaR: $5670.7075

Portfolio B VaR: $4494.9984

Portfolio C VaR: 3786.9260

Total VaR: 13578.2836
```

For second method, I implement Normal Monte Carlo method. We apply the algorithm in class note:

1. Calculate Current Portfolio Value
2. Simulate N draws from the assumed distribution.
3. Calculate new prices from the returns (using chosen methodology)
4. Price each asset for all N draws.
5. Calculate portfolio Value for each N Draws
6. Using the simulated portfolio values, find the α% of the distribution.
7. Calculate VaR.

To simulate returns, we first calculate the covariance matrix for assets return. Then set

simulate 10000 draw from multivariate normal distribution. Then price the asset and calculate the VaR,we get the output below:

```
Portfolio A VaR: 8032.5840

Portfolio B VaR: 6732.9929

Portfolio C VaR: 5638.2563

Total Portfolio VaR: 20004.1749
```

Comparing two method VaR outputs, we can find that all the output values of Monte Carlo method are greater than Delta Normal VaR, which indicates using Monte Carlo method, we would expect more loss. However, using Monte Carlo method can help dealing with non-linear return issue. That makes Monte Carlo method is more accurate than Delta Normal method.