Gyroscope

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Objective:

The objective of this lab is to calculate the gravitational acceleration of the Earth using an air gyroscope. We will be levitating a truncated metal sphere on an air bed such that the metal sphere can be torsion free. As we spin the metal sphere along its axis of rotational symmetry, there will be an angular momentum point in the opposite direction of the truncated part of the metal sphere. The truncated part of the metal sphere has a downward gravitational force that causes a torque in the horizontal plane, and thus making the metal sphere to precess alone the horizontal plane. We know the relationship between the precession angular velocity and the angular momentum to be $\Omega = \frac{L}{I\omega}$. Since we can calculate Ω by measuring the time it takes for the metal sphere to precess one circle, and we know that L is related to g since it is the gravitational force that is causing the metal sphere to precession, we can therefore calculate g. However, since there is also additional torque from the air jets from the bottom, we will need to let the metal sphere to precess in both clockwise and counter-clockwise direction and take its average.

Methods:

Tools:

- Stroboscope
- Laser
- An air gyroscope that includes:
  - A truncated metal sphere that has been magnetized perpendicular to its axis of rotational symmetry.
  - Coil that produces a 60Hz alternating magnetic field with field lines in the vertical direction.
  - A hole that fits the truncated metal sphere and air jets at the bottom to keep the metal sphere levitated.
- Ruler
- Wall
- Pen/pencil
- Scale
- Calibre

Logs:

First, plug in all the equipment in a dark room where you can see the strobe lights and the laser, and make sure there is a wall or something close by that you can use as a marker later. Turn on the support air jet that keeps the metal sphere levitated at a reasonable level, do not turn it up too much because it creates torque that will increase the angular velocity.

Turn on the driving air jet while having a ruler holding against the truncated part of the metal sphere. In the beginning, turn the air valve for the driving air relatively high, so the ball can start spinning fast and then when you remove the ruler, the metal sphere will wobble less. Once you let go of the ruler and see that the metal sphere is not wobbling as much, turn down the driving air. You can use the laser to help check if the metal sphere is wobbling. When the laser is being reflected off the truncated part of the metal sphere, if it is not wobbling, the laser reflected the wall would only be travelling in a straight line. If it is wobbling, then the laser will be making circles on the wall and depending on how much it is wobbling, the size of that circle will increase the worse the wobbling is.

Then adjust the strobe light and have it start at around 25Hz. We want to start from a lower number that is not a factor of 60 instead of starting at 60 Hz because there is the possibility that the ball is running at a multiple or a factor of 60, and if we start directly at 60, we might not be able to notice it. After our trials, we discovered that it was possible to tell when the strobe light was flashing at twice the frequency of the air gyroscope. In this case , it looked like two different sets of markings on the ball overlapped each other. However, we could tell apart the sets of markings because they appeared to wobble slightly differently from each other, letting us know that the ball was rotating at half the required frequency .

Once the metal sphere is spinning around 60Hz, the ball will appear not spinning under a 60Hz strobe light. Once the metal sphere is spinning at a frequency slightly higher than 60Hz, stop the strobe light and turn on the metal coil to the max as it decreases to 60Hz. If the metal sphere locks in with the coils, the metal sphere will start spinning at a constant speed. Otherwise, it will start spinning under the 60Hz light again because it slows down without the driving air jet.

After you lock in the metal sphere, make sure the laser is pointing at the metal sphere, and the reflection of the laser is on a wall or something else you can use as a marker. Then, record the time it takes for each precession. It is vital that no one is touching the air gyroscope or the laser when you are recording the precession time. Otherwise, you will need to repeat this entire experiment.

This experience will take a long time to get it working, so if anything goes wrong, restart the precess.

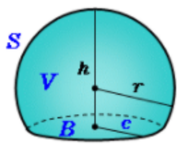
Once you finish the experiment, measure the weight of the metal sphere with a scale, and use the calibre to measure the sphere's diameter with and without the truncated part.

Equations:

Definition for each symbol:

R = radius of the metal sphere
ξ = radius without the truncated part
δ = distance of the truncated part
ω = angular velocity
Ω = precession angular velocity
I = moment of inertia

c = radius of the truncated section
h = diameter without the truncated part



Equation(1), percentage difference:

$$percentage\ difference = \frac{|experimental\ value - expected\ value|}{expected\ value} * 100\% \tag{1}$$

Equation(2), to calculate average:

$$\overline{X} = \frac{\sum X}{N} \tag{2}$$

Equation(3), to find the uncertainty of a reciprocal:

$$\sigma\left(\frac{1}{a}\right) = \frac{\sigma_a}{a^2} \tag{3}$$

Equation(4), error propagation for addition:

$$u(a+b) = \sqrt{u(a)^2 + u(b)^2} \tag{4}$$

Equation(5), error propagation for multiplication:

$$u(a*b) = a*b\sqrt{\left(\frac{u(a)}{a}\right)^2 + \left(\frac{u(b)}{b}\right)^2} \tag{5}$$

Equation(6), to calculate the radius of the truncated section (https://keisan.casio.com/exec/system/1223382199)

$$c = \sqrt{h(2R-h)} \tag{6}$$

Equation(7), to calculate the volume of a truncated sphere (https://keisan.casio.com/exec/system/1223382199)

$$V = \frac{\pi}{6}h\left(3c^3 + h^2\right) \tag{7}$$

Equation(8), formula to calculate the density

$$\rho = mass/V \tag{8}$$

Equation(9), to calculate the truncated distance

$$\delta = R - \xi \tag{9}$$

Equation(10), the formula for ϵ

$$\epsilon = \frac{\delta}{R} = 1 - \frac{\xi}{R} \tag{10}$$

Equation(11), formula for precession angular frequency

$$\Omega = \frac{L}{I\omega} = \frac{2\pi}{T} \tag{11}$$

Equation(12), the formula for torque

$$L = g\rho\frac{\pi * R^4}{4}\epsilon^2(2-\epsilon)^2 \tag{12}$$

Equation(13), the formula for moment of inertia

$$I = \rho\frac{\pi * R^5}{10}(2-\epsilon)^2\left(\epsilon^2 + \epsilon + \frac{2}{3}\right) \tag{13}$$

Equation(14), the formula for $\frac{L}{I}$:

$$\frac{L}{I} = g\frac{5\epsilon^2}{2R(2-\epsilon)^2\left(\epsilon^2 + \epsilon + \frac{2}{3}\right)} \tag{14}$$

Equation(15), the formula for angular frequency

$$\omega = 2\pi f \tag{15}$$

Equation(16), the formula to calculate g

$$g = \frac{4\pi\omega R(2-\epsilon)\left(\epsilon^2 + \epsilon + \frac{2}{3}\right)}{5\epsilon^5 T} \tag{16}$$

Equation(17), propagation of uncertainty using quadriture, addition (z = x + y):

$$u(z) = \sqrt{u(x)^2 + u(y)^2}$$

Equation(18), propagation of uncertainty using quadriture, multiplication (z = xy):

$$u(z) = z\sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(y)}{y}\right)^2}$$

Equation(19), propagation of uncertainty of a power ($z = x^a$ *where a is some constant*):

$$u(z) = ax^{a-1}u(x)$$

Equation(20), finding the uncertainty of an average:

$$\sigma_{\overline{X}} = \frac{S}{\sqrt{N}}$$

Torque and Moment of Inertia equation derivation:

To find the Torque acting on the gyroscope, we must first find its centre of gravity, as this is where the gravitational force will act on the ball. Since the gyroscope is symmetric around the axis of rotation, we center of gravity is somewhere on the axis of rotation. Thus, we only need to find the x coordinate for the centre of gravity, assuming the x-axis is the axis of rotation. Let the positive direction be towards the truncated bit, and negative x be away from the truncated bit. Furthermore, let the origin be the geometric centre of the sphere. Now, say we are at some point x on the x-axis. Then, the cross sectional-area of the sphere at this point can be found using the area of a circle equation:

$$A(x) = \pi\left(R^2 - x^2\right)$$

Now, we find the mass dm between an infinitesimally small interval dx on the axis of symmetry. This can be approximated by assuming the shape between the interval is a cylinder, which will be more accurate the smaller dx is. Thus, dm is approximately related to dx in the following way:

$$dm = \rho\pi\left(R^2 - x^2\right)\, dr$$

The centre of mass at coordinate x is equal to:

$$x_{CM} = \frac{1}{M}\int x\, dm$$

Let $\epsilon = 1 - \frac{\xi}{R}$. *Then,* $\xi = R(1-\epsilon)$. Substituting dm for dr and summing over the entire sphere, we can calculate x_{CM}:

$$x_{CM} = \frac{1}{M}\int_{-R}^{R(1-\epsilon)} \rho\pi x\left(R^2 - x^2\right)\, dr$$

$$= \frac{1}{M}\int_{-R}^{R(1-\epsilon)} \rho\pi x R^2 - \rho\pi x^3\, dr$$

$$= \frac{1}{M}\rho\pi\left[\frac{1}{2}x^2R^2 \quad - \frac{1}{4}x^4\right]_{-R}^{R(1-\epsilon)}$$

$$= \frac{1}{M}\rho\pi\left[\frac{1}{2}R^2(R(1-\epsilon))^2 \quad - \frac{1}{4}(R(1-\epsilon))^4 - \frac{1}{2}R^4 \quad + \frac{1}{4}R^4\right]$$

$$= \frac{1}{M}\rho\pi R^4\left[\frac{1}{2}(1-\epsilon)^2 \quad - \frac{1}{4}(1-\epsilon)^4 - \frac{1}{2} \quad + \frac{1}{4}\right]$$

$$= \frac{1}{M}\rho\pi R^4\left(-\epsilon^2 + \epsilon^3 - \frac{1}{4}\epsilon^4\right)$$

$$= \frac{-1}{4M}\rho\pi R^4\epsilon^2\left(4 - 4\epsilon + \epsilon^2\right)$$

$$= \frac{-1}{4M}\rho\pi R^4\epsilon^2(2-\epsilon)^2$$

Finally, since the axis of precession runs through the centre of the sphere, and the force of gravity is equal to Mg, the magnitude of the torque acting on the sphere is equal to the magnitude of x_{CM} times Mg:

$$L = Mg * |x_{CM}|$$

$$= \frac{1}{4}g\rho\pi R^4\epsilon^2(2-\epsilon)^2, \text{ as desired.}$$

Now, we find the moment of inertia of the truncated sphere. Note that the moment of inertia for a disc is $I = \frac{1}{2}MR^2$, where R is the radius of the disc. Thinking of the sphere as a group of discs with infinitesimally small depth centred around the axis of rotation, we can integrate over all of the discs to find the moment of inertia of the entire sphere. In particular, a disc of infinitesimally small depth dr at a displacement r from the centre of the sphere has moment of inertia dI given by:

$$dI \ = \ \frac{1}{2}M_{disc}R_{disc}^2$$

$$= \frac{1}{2}\rho\pi R_{disc}^4 \, dr$$

$$= \frac{1}{2}\rho\pi\left(R^2 - r^2\right)^2 \, dr$$

Thus, integrating dI over the entire sphere, we get:

$$I = \int dI$$

$$= \int_{-R}^{R(1-\epsilon)} \frac{1}{2}\rho\pi\left(R^2 - r^2\right)^2\, dr$$

$$= \frac{1}{2}\rho\pi\left(\int_{-R}^{R(1-\epsilon)} R^4 - 2R^2r^2 + r^4 dr\right)$$

$$= \frac{1}{2}\rho\pi\left[R^4 r - \frac{2}{3}R^2r^3 + \frac{1}{5}r^5\right]_{-R}^{R(1-\epsilon)}$$

$$= \frac{1}{2}\rho\pi\left[R^4(R(1-\epsilon)) - \frac{2}{3}R^2(R(1-\epsilon))^3 + \frac{1}{5}(R(1-\epsilon))^5 + R^5 - \frac{2}{3}R^5 + \frac{1}{5}R^5\right]$$

$$= \frac{1}{2}\rho\pi R^5\left[1 - \epsilon - \frac{2}{3}(1-\epsilon)^3 + \frac{1}{5}(1-\epsilon)^5 + 1 - \frac{2}{3} + \frac{1}{5}\right]$$

$$= \frac{1}{2}\rho\pi R^5\left[\frac{16}{15} - \frac{4}{3}\epsilon^3 + \epsilon^4 - \frac{1}{5}\epsilon^5\right]$$

$$= \frac{1}{10}\rho\pi R^5(2-\epsilon)\left(\epsilon^4 - 3\epsilon^3 + \frac{2}{3}\epsilon^2 + \frac{4}{3}\epsilon + \frac{8}{3}\right)$$

$$= \frac{1}{10}\rho\pi R^5(2-\epsilon)^3\left(\epsilon^2 + \epsilon + \frac{2}{3}\right),\ \textit{as desired.}$$

Results:

Table 1: Measurements		
	Diameter of the metal sphere	**Diameter without the truncated part (h)**
	0.05055m	0.04862m
	0.05056m	0.04864m
	0.05055m	0.04864m
Average:	0.05055m	0.04863m

Table 2: Basic information		
Name:	**Values:**	**Uncertainty:**

Expected value for g (g value of the Burton Tower)	9.804253 m/s^2	N.A.
Mass of the metal sphere	0.5325kg	0.0001kg
Diameter of the metal sphere	0.05055m	0.00002m
Diameter without the truncated part (h)	0.04863m	0.00002m
Full radius of the metal sphere (R)	0.05055/2 =0.02528m	0.00001m
Radius without the contracted part (ξ)	0.04863 - R = 0.02336m	0.00003m
Distance of the truncated part (δ)	R - ξ = 0.00192m	0.00004m
ϵ	$\delta / r\ = 0.07596$	N.A.
Volume of the metal sphere	6.7*10^-5 m^3	1*10^-6m^3
Density of the metal sphere ρ	7906.7 kg/m^3	0.2 kg/m^3

Sample Calculation for Equation(6), Equation(7) and Equation(8):

$$c = \sqrt{h(2R-h)} \tag{17}$$
$$c = \sqrt{0.04863m(2*0.02528m - 0.04863m)}$$
$$c \approx 0.00966m$$

$$V = \frac{\pi}{6}h\left(3c^3 + h^2\right) \tag{18}$$
$$V = \frac{\pi}{6}h\left(3(0.00966m)^3 + 0.04863m\right)^2)$$
$$V = 6.7348*10^{-5}\ m^3$$
$$V \approx 6.7*10^{-5}\ m^3$$

$$\rho = mass / V \tag{19}$$
$$\rho = 0.5325kg / 6.7348*10^{-5}\ m^3$$
$$\rho = 7906.7\ kg / m\text{^}3$$
$$\rho \approx 7906.7\ kg / m\text{^}3$$

Table 3: Result Table				
Clock-wise trials				
Trial:	**Time(s):** $\pm 0.01s$	**Converted Time(s):** $\pm 0.01s$	**Calculated g (m/s^2):**	**Calculated g's Uncertainty(m/s^2):**
1*	09:32.33	572.33	10.44	0.05
2*	09:36.59	576.59	10.36	0.05
3	10:23.92	623.92	9.58	0.05
4	10:27.01	627.01	9.53	0.05
5	10:27.38	627.38	9.52	0.05
6	10:26.25	626.25	9.54	0.05
7	10:26.48	626.48	9.54	0.05
8	10:27.76	626.48	9.52	0.05
9	10:25.60	625.6	9.55	0.05
10	10:25.58	625.58	9.55	0.05
Counter-clock-wise trials				
Trial:	**Time(s):** $\pm 0.01s$	**Converted Time(s):** $\pm 0.01s$	**Calculated g (m/s^2):**	**Calculated g's Uncertainty(m/s^2):**
1	10:36.99	636.99	9.38	0.05
2	10:29.00	636.99	9.50	0.05
3	10:36.22	636.99	9.39	0.05

* Outliers (We will not be using the data from the first two trials from the clock-wise trials, since the pressure of the support air jet, the ones that keeps the metal sphere levitated, was too high)

Table 3 shows the period we measured for each trial of the experiment, for both the clockwise and counter-clockwise directions. Furthermore, it shows the gravitational acceleration calculated using each period. The gravitational acceleration can be found using the precessional period in the following way:

By equation (11), $\Omega = \frac{L}{I\omega}$ where Ω is the angular velocity of the precession, L is torque, I is moment of inertia and ω is the angular velocity of the rotating sphere. As well, note that $\Omega = \frac{2\pi}{T}$ where T is the period of the precession, since the angular velocity is constant. However, we also have equations for L and I, which were derived earlier in this report (equations 12 and 13). Substituting these equations together, we get:

$$\Omega = \frac{L}{I\omega}$$

$$\Omega = \frac{L}{I}*\frac{1}{\omega}$$

$$\frac{2\pi}{T} = \frac{1}{\omega}*g*\frac{5\epsilon^2}{2R(2-\epsilon)^2\left(\epsilon^2+\epsilon+\frac{2}{3}\right)}$$

$$g = \frac{4\pi\omega R(2-\epsilon)\left(\epsilon^2+\epsilon+\frac{2}{3}\right)}{5\epsilon^5 T}$$

ω is equal to $2*\pi*60 \approx 377\ rad/s$, since the frequency of the rotation of the sphere was held at 60Hz. Furthermore, R was measured to be 0.02528m, and ϵ is equal to 0.07596. Thus, using the above equation we can calculate the theoretical gravitational acceleration from each measurement of the period T.

Sample Calculation for Equation (16):

$$g = \frac{4\pi\omega R(2-\epsilon)\left(\epsilon^2+\epsilon+\frac{2}{3}\right)}{5\epsilon^5 T} \tag{20}$$

$$g = \frac{4\pi(120\pi)(0.02528m)(2-0.07596)\left(0.07596^2+0.07596+\frac{2}{3}\right)}{5(0.07596)^5(572.33s)}$$

$$g \approx 10.44\ m/s^2$$

The uncertainty for each gravitational acceleration was propagated using equations (17), (18), and (19).

Sample Calculation for the uncertainty of ϵ:

$$u(\epsilon) = \epsilon\sqrt{\left(\frac{u(R)}{R}\right)^2 + \left(\frac{u(\xi)}{\xi}\right)^2}$$

$$u(\epsilon) = 0.07596\sqrt{\left(\frac{0.00001}{0.02528}\right)^2 + \left(\frac{0.00003}{0.02336}\right)^2}$$

$$u(\epsilon) \approx 0.00010$$

Sample Calculation for the uncertainty of g:

$$u(g) \;=\; g\sqrt{\left(\frac{u(R)}{R}\right)^2 + \left(\frac{u(\epsilon)}{\epsilon}\right)^2 + \frac{(2\epsilon * u(\epsilon))^2 + u(\epsilon)^2}{\left(\epsilon^2 + \epsilon + \frac{2}{3}\right)^2} + \left(2\frac{u(\epsilon)}{\epsilon}\right)^2 + \frac{u(T)}{T}}$$

$$u(g) \;=\; g\sqrt{\left(\frac{0.00001}{0.02528}\right)^2 + \left(\frac{0.00010}{0.07596}\right)^2 + \frac{(2*0.07596*0.00010)^2 + 0.00010^2}{\left(0.07596^2 + 0.07596 + \frac{2}{3}\right)^2} + \left(2*\frac{0.00010}{0.07596}\right)^2 + \left(\frac{0.01}{572.33}\right)^2}$$

$$u(g) \;=\; 0.05\ m/s^2$$

Table 4: Calculated Averages			
	Value (kg/m^2)	**Uncertainty (kg/m^2)**	**Percentage difference**
Average g clockwise with outlier	9.71	0.03	0.91%
Average g clockwise without outlier	9.54	0.02	2.67%
Average g counter clock wise	9.42	0.03	3.87%
Average g with outliers	9.569	0.003	2.40%
Average g without outliers	9.483	0.002	3.27%

Sample Calculation for Equation():

$$percentage\ difference = \frac{|experimental\ value - expected\ value|}{expected\ value} * 100\% \tag{21}$$

$$percentage\ difference = \frac{|9.71 m/s^2 - 9.80\ m/s^2|}{9.80\ m/s^2} * 100\%$$

$$percentage\ difference = 0.91\%$$

Analysis:

The gravitational acceleration g was found to be **9.54 ± 0.02 m/s^2** for the clockwise trials, and **9.42 ± 0.03 m/s^2** for the counterclockwise rotational trials. Averaging these two results, the true gravitational acceleration was found to be **9.48 ± 0.02m/s^2**, which is very close to the actual gravitational acceleration in the Burton Tower of 9.804m/s^2 (the percent difference between these values is only 3.27%).

There are a few interesting things to note about the results of our investigation. First of all, in table 3 the first few periods were much higher than the other results. This is due to the fact that we initially put the pressure of the air jet multiple times higher than necessary, resulting in a large uncontrolled torque on the sphere. After the second trial, we reduced the air pressure to reduce this error, resulting in a drop in measured periods. As well, we have excluded the first two trials from most of our calculations. There is also another potential reason why the first trials have a much larger period, which is the fact that the sphere was wobbling much more for the first few counterclockwise and clockwise trials. Over time, the magnetic coils stabilized the sphere, decreasing the amount it wobbled. However, at first, the wobbling was still very prevalent, and could have led to some increased error in the first couple of trials. Unfortunately, we were only able to get three measurements for counterclockwise rotations due to time constraints (and some issues with the instruments we used). Thus, the effect of wobbling could still have been fairly prevalent for these trials since not much time had passed since the ball was initially stabilized. In conclusion, there is likely some error in the counterclockwise rotation trials, since the the wobbling of the sphere had not yet fully smoothed during the trials we took.

It is interesting to note that the periods found for the clockwise trial were consistently smaller than the periods found for the counterclockwise trial. This is due to the fact that the air-jet used to levitate the ball exerts a torque, which affects the net-torque and thus period measured. When the sphere was rotating clockwise, the torque was in the same direction as the torque of gravity, leading to a larger angular velocity and thus lower periods measured. However, when the sphere was rotating counterclockwise, the torque opposed the torque of gravity, leading to a lower angular velocity and thus higher periods measured. We accounted for this systematic error by taking periods for both clockwise and counterclockwise rotations, since theoretically they should both both be offset from the actual period by equal but opposite amounts. Thus, by averaging our findings from each, the offset should be cancelled out, leading to a gravitational acceleration calculated with little to no error due to the torque of the air-jet.

Conclusion:

The purpose of this investigation was to determine the gravitational acceleration of the earth by finding the precessional period of an air gyroscope. Our results are that the radius of the earth was found to be gravitational acceleration was found to be **9.48 ± 0.02m/s^2**. The expected value for the gravitational acceleration of the earth where we conducted the experiment was 9.804m/s^2, so the percent error of the gravitational acceleration was 3.27%. This percent errors is fairly low, suggesting that the result of this investigation was quite accurate to its theoretical value. However, the expected value of 9.804 doesn't fall into the uncertainties of the gravitational acceleration found, so there is a discrepancy between the actual and theoretical radius. In particular, this suggests there was some systematic error in our investigation that we didn't account for. It is interesting to note that the gravitational acceleration that we found including the outlier data points from the first trials (0.569m/s^2) was closer to the actual gravitational acceleration than without the outliers. However, this is likely just caused by the fact that the increased air pressure led to higher torque and thus a higher calculated gravitation acceleration, ironically somewhat countering whatever systematic error led to the discrepancy. In the end, the gravitational acceleration was calculated both fairly accurately to its actual value and with fairly high precision, so in conclusion this experiment mostly succeeded in it's goal.

There are a lot of sources of error in this lab. The main one that we encountered was the supporting air jet that helped keep the metal sphere levitated. In the beginning, our supporting air jet was almost twice as large as the recommended level. As a result, the first two recorded times were for the clockwise trial were almost 1 minute faster than the average. The support air-jet also applies an external force to the metal sphere from the bottom, creating more torque to the metal sphere. The stronger the air pressure from the supporting air jets, the higher the torque. Even though this can be resolved by averaging the time from the clockwise trial and the counter-clock-wise trial, this will also increase the uncertainty significantly, making the result unreliable. On the same note, since we performed the clockwise trial and the counter-clockwise trial on two different days, the magnitude of the supporting air-jet might be different. Because of that, our final value would not be accurate since we can not completely cancel the torque coming from the bottom by averaging out the effects from the two trials.

The other major source of error is the wobbling of the metal sphere as it spins along its axis of rotational symmetry. During our experiment, we held a ruler against the truncated part of the metal sphere with the air jet on high at the beginning, so the metal sphere would start spinning against the ruler without wobbling as much. But as I let go of the ruler, any small movements can make it wobble again. Even though the metal sphere will stabilize after a while, it is almost impossible to calibrate it such that it does not wobble at all.

There are also some measurement errors from measuring the dimension of the truncated sphere and the precession time. We used the calibre to measure the dimension of the truncated sphere, but it was hard to eyeball if the calibre was going through the center and were measuring the full diameter/truncated distance. We used the lapping function from our phone to measure the precession time and recorded each lap's time whenever we saw the laser hits the marker we left on the wall. That is not accurate since the accuracy of that measurement depends on our reaction time.

There are quite a few things that can be done to increase the accuracy of this experiment. First of all, perform both the clockwise and counter-clockwise trial on the same day and ensure the magnitude of the supporting air jet is at the recommended level. By doing this, we can precisely cancel out the effects coming from the air jet by averaging out the readings from both days if the magnitude of the air jet is never changed. Not only that, record around ten laps from each trial. In our experiment , we were only able to record three laps for the counter-clockwise trial. Because of that, there is not enough data to prove if our values are accurate or not, nor if the sphere was wobbling.

To eliminate the measurement errors from the measurement of the truncated sphere's dimension, we can use a piece of paper and wrap it around the sphere to get a better result of the circumference, since we can tell if the tape is in the center by checking if the paper is slanted or not as it is wrapping around the metal sphere. Using that circumference, we can accurately calculate the radius knowing the relationship $C = 2\pi r$. And for the truncated part, we can use a compass and a rule to measure the diameter of the truncated part, using that, we can calculate c and use Equation 6 to calculate h and thus ξ. To eliminate the error from the time lap, we can use a photoelectric sensor, Arduino and some code to help us record the time whenever the laser shines on the photoelectric sensor. That way, the reading for each precession time will be much more accurate.