Charge to Mass

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Objective:

In this experiment, an electron accelerates in a magnetic induction field B, generated by the current flowing through a pair of Helmholtz coils. The electron experience a force given by $F = ev \times B$ when travelling through a potential difference, thus we will be using the formula $\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}}$ to calculate the charge to mass ratio. The magnetic field caused by the Helmholtz coil is $B_c = \left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 nI}{R}$. However, external fields are coming from the earth and all instruments around us. So when calculating the magnetic field B using the second formula, that B would include both the magnetic induction field from the Helmholtz coils and the environment. In this experiment, we will run two trials, one with a constant current and changing voltage and one with a constant voltage and a changing current. Both both trials, we will measure the radius 12 different times with 12 different inputs and then calculate the charge to mass by graphing out the relationship and finding a line of best fit. After the calculation, the calculated charge to mass ratio will be compared to the expected value taken from the internet. Meanwhile, we can also investigate how the current and voltage can affect the charge to mass ratio.

Methods:

Tools:

- Helmholtz coils
- Glass bulb containing an electron gun and hydrogen gas at low pressure
- Electron fun filament
- Multiple VDC Power supplies
- Varying voltage DC supply
- Wires
- Rheostat
- Multi-meters
- Self-illuminated scale and plastic reflector

Logs:

The wiring diagram and photos of the set-up are on the fourth and fifth pages of the lab instruction. First, take a few wires and connect the instrument using Figure 2 from the lab menu. In summary, a 10VDC source, rheostat and an ammeter should be connected in a circle and connected to the left of the Helmholtz coils; a 6.3VDC source should be powering the electron gun filament, and lastly, a varying voltage DC source connected to the anode with a voltmeter in parallel to the varying voltage DC source. Once everything is connected properly, turn all instruments on and the lights off. The filament of the electron gun must be turned on for 30 seconds or more before the anode voltage, and it should never be turned off until the anode voltage is off. Otherwise, the tube may be damaged. Make sure the trajectory of the electron is in a vertical circle and not a helix-like shape for better accuracy, then place the self-illuminates scale in front of the Helmholtz coil and make sure the scale is parallel to the trajectory.

Once everything is ready, it is time for the experiments. It is also worth noting that the voltmeter accumulates changes over time, so it is at best to complete the experiment before the voltmeter is all charged up and resulting in a fluctuating result. For the first trial, we will keep the voltage the same and change the current only. The voltage can not be too low since low voltages can affect the experiment, which we will explore more later in the lab. Adjust the current with the rheostat, and measure the diameter of the circular electron trajectory 12 times with different current readings. After that is done, repeat a similar procedure for trial two, for which we will adjust the voltage instead of current using the varying DC power supply.

Record all useful information on a computer for further analysis. Since the Helmholtz coils are an ellipse, and the formula assumes it is a circle, we will be using the average radius of the Helmholtz coils by taking the average diameter of the longest side and the shortest side of the ellipse.

Equations:

List of variables:
r = radius of the electron bean R = calculated radius of the Helmholtz coils n = number of turns in each coil $\mu_0 = 4\pi * 10^{-7}\ Wb\ A^{-1}\ m^{-1}$ s = slope b = the y-intercept

Equation(1), percentage difference:

$$percentage\ difference = \frac{|experimental\ value - expected\ value|}{expected\ value} * 100\% \qquad (1)$$

Equation(2), to calculate average:

$$\overline{X} = \frac{\sum X}{N} \qquad (2)$$

Equation(3), finding the uncertainty of an average:

$$\sigma_{\overline{X}} = \frac{S}{\sqrt{N}} \tag{3}$$

Equation(4), to find the uncertainty of a reciprocal:

$$\sigma\left(\frac{1}{a}\right) = \frac{\sigma_a}{a^2} \tag{4}$$

Equation(5), error propagation for addition:

$$u(a+b) = \sqrt{u(a)^2 + u(b)^2} \tag{5}$$

Equation(6), error propagation for multiplication:

$$u(a*b) = a*b\sqrt{\left(\frac{u(a)}{a}\right)^2 + \left(\frac{u(b)}{b}\right)^2} \tag{6}$$

Equation(7), propagation of uncertainty using quadriture, addition (z = x + y):

$$u(z) = \sqrt{u(x)^2 + u(y)^2} \tag{7}$$

Equation(8), propagation of uncertainty using quadriture, multiplication (z = xy):

$$u(z) = z\sqrt{\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(y)}{y}\right)^2} \tag{8}$$

Equation(9), propagation of uncertainty of a power ($z = x^a$ *where a is some constant*):

$$u(z) = ax^{a-1}u(x) \tag{9}$$

Equation(10), force equation of electron through a magnetic field

$$F = ev \times B = m\frac{v^2}{r} \tag{10}$$

Equation(11), the A non-relativistic approximation for the particle acceleration through a potential difference V

$$eV = \frac{1}{2}mv^2 \tag{11}$$

Equation(12), rearranging the force equation

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{V}} \tag{12}$$

Equation(13), given definition of k:

$$k = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 n}{R} \tag{13}$$

Equation(14), given formula for I_0:

$$I_0 = \frac{B_e}{k} \tag{14}$$

Equation(15), components of B:

$$B = B_c + B_e \tag{15}$$

Equation(16), given formula that represents the relationship between V, r, $B_c$and B_e:

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}(E_c - E_e) \tag{16}$$

UNCERTAINTIES:

Equation(17),

$$u(k) = \frac{1}{\sqrt{2}}\left(\frac{4}{5}\right)^{\frac{3}{2}}\mu_0 n\frac{\sigma(R)}{R^2} \tag{17}$$

Equation(18),

$$u\left(\frac{1}{\sqrt{V}}\right) = \frac{1}{2}V^{-\frac{3}{2}}u(V) \tag{18}$$

Equation(19),

$$u(B_e) = B_e\sqrt{\left(\frac{u(b)}{b}\right)^2 + \frac{m}{e}\left(u\left(\sqrt{\frac{e}{m}}\right)\right)^2 + \frac{u(V)^2}{4V}} \tag{19}$$

Equation(20),

$$u(B_c) = B_c\sqrt{\left(\frac{u(R)}{R}\right)^2 + \left(\frac{u(I)}{I}\right)^2} \tag{20}$$

Equation(21),

$$for\ the\ dI\ trial: \quad u\left(\sqrt{\frac{e}{m}}\right) = \sqrt{\frac{e}{m}}\sqrt{\left(\frac{u(s)}{s}\right)^2 + \left(\frac{u(V)}{4V}\right)^2} \tag{21}$$

Equation(22),

$$for\ the\ dV\ trial: \quad u\left(\sqrt{\frac{e}{m}}\right) = \sqrt{\frac{e}{m}}\sqrt{\left(\frac{u(s)}{s}\right)^2 + \frac{u(B_c)^2 + u(B_e)^2}{(B_c + B_e)^2}} \tag{22}$$

Equation(23),

$$u\left(\frac{e}{m}\right) = 2\sqrt{\frac{e}{m}}\,u\left(\sqrt{\frac{e}{m}}\right) \tag{23}$$

Results:

Table 1: Basic Information		
Name:	Value:	Uncertainty: (with both measurement and instrumental errors)
Expected charge to mass ratio of electron	1.758×10^{11}	N.A.
Magnetic Declination in Toronto (www.magnetic-declination.com/)	$5*10^{-5}$ Tesla	N.A.
n	130	N.A.
Measured horizontal diameter:	$0.310m$	0.005m
Measured verticle diameter::	$0.305m$	0.005m
Horizontal Radius:	$0.310/2 = 0.155m$	0.003m
Vertical Radius:	$0.305/2 \approx 0.153m$	0.003m
R	$(0.155 + 0.153)/2 \approx 0.154m$	0.003m

Table 2: Change in Current (dI) Trial

Voltage (V)	**Voltage Uncertainty (V)**	**Current (A)**	**Current Uncertainty (A)**	**Diameter (m)**	**Diameter Uncertainty (m)**	**Radius (m)**	**Radius Uncertainty (m)**
170	1	1.006	0.001	0.107	0.002	0.054	0.001
170	1	1.047	0.001	0.104	0.002	0.052	0.001
170	1	1.110	0.001	0.098	0.002	0.049	0.001
170	1	1.161	0.001	0.094	0.002	0.047	0.001
170	1	1.199	0.001	0.092	0.002	0.046	0.001
170	1	1.259	0.001	0.089	0.002	0.045	0.001
170	1	1.323	0.001	0.084	0.002	0.042	0.001
170	1	1.397	0.001	0.079	0.002	0.040	0.001
170	1	1.426	0.001	0.077	0.002	0.039	0.001
170	1	1.540	0.001	0.072	0.002	0.036	0.001
170	1	1.604	0.001	0.069	0.002	0.035	0.001
170	1	1.713	0.001	0.065	0.002	0.033	0.001

First, we need to calculate B_c for each I since we are changing I for each measurement:

$$B_c = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 n I}{R}$$

The to calculate the ratio, the general formula will be used

$$given \quad \frac{1}{r} = \sqrt{\frac{e}{2m}} \frac{1}{\sqrt{V}} (E_c + E_e)$$

$$(rewrite\ into\ y = mx + b)$$

$$\frac{1}{r} = \sqrt{\frac{e}{2m}} \frac{1}{\sqrt{V}} E_c + \sqrt{\frac{e}{2m}} \frac{1}{\sqrt{V}} E_e$$

After graphing the above equation, knowing the ratio is the slope of the line of best fit:

$$slope \ = \ \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}$$

$$\frac{e}{m} = \left(slope*\sqrt{2V}\right)^2$$

And since b is related to B_e from the previous equation:

$$b \ = \ \sqrt{\frac{e}{2m}}\frac{1}{\sqrt{V}}E_e$$

$$B_e = \frac{b\sqrt{2V}}{\sqrt{\frac{e}{m}}}$$

Sample Calculation: to calculate B_c (let $I \ = \ 1.006A$):

$$B_c = \left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 nI}{R}$$

$$B_c = \left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{4\pi*10^{-7}\ Wb\ A^{-1}\ m^{-1}(130)(1.006A)}{0.154m}$$

$$B_c \approx 0.00076\ Tesla$$

Sample Calculation: to to calculate the uncertainty of B_c (let $I \ = \ 1.006A$):

$$u(B_c) = B_c\sqrt{\left(\frac{u(R)}{R}\right)^2 + \left(\frac{u(I)}{I}\right)^2}$$

$$u(B_c) = 0.00076\ Tesla\sqrt{\left(\frac{0.003m}{0.154m}\right)^2 + \left(\frac{0.001A}{1.006A}\right)^2}$$

$$u(B_c) = 1.2419308085929146*10^{-5}$$

$$u(B_c) \approx 0.00001$$

Sample Calculation: to calculate the charge to mass ratio:

$$\frac{e}{m} = \left(slope*\sqrt{2V}\right)^2$$

$$\frac{e}{m} = \left((22525.248539960703)*\sqrt{2(170V)}\right)^2$$

$$\frac{e}{m} = 172511519407.58066$$

$$\frac{e}{m} \approx 1.7*10^{11}$$

Sample Calculation: to calculate B_e

$$B_e = \frac{-b\sqrt{2V}}{\sqrt{\frac{e}{m}}}$$

$$B_e = \frac{-1.3628333834683806\sqrt{2*170}}{415345.0606514789}$$

$$B_e = -6.0502479297871396*10^{-5}$$

$$B_e \approx -6*10^{-5}$$

Sample Calculation: to calculate the uncertainty of sqrt(charge to mass ratio) for dI

$$for\ the\ dI\ trial: \quad u\left(\sqrt{\frac{e}{m}}\right) = \sqrt{\frac{e}{m}}\sqrt{\left(\frac{u(s)}{s}\right)^2 + \left(\frac{u(V)}{4V}\right)^2}$$

$$u\left(\sqrt{\frac{e}{m}}\right) = 415345.0606514789\sqrt{\left(\frac{1068.6795204666075}{22525.248539960703}\right)^2 + \left(\frac{1}{4(170)}\right)^2}$$

$$u\left(\sqrt{\frac{e}{m}}\right) = 25337.70358243841$$

$$u\left(\sqrt{\frac{e}{m}}\right) \approx 30000.$$

Sample Calculation: to calculate the uncertainty of the ratio

$$u\left(\frac{e}{m}\right) = 2\sqrt{\frac{e}{m}}\,u\left(\sqrt{\frac{e}{m}}\right)$$

$$u\left(\frac{e}{m}\right) = 2(415345.0606514789)(25337.70358243841)$$

$$u\left(\frac{e}{m}\right) = 21047780062.43415$$

$$u\left(\frac{e}{m}\right) \approx 2*10^{10}$$

Sample Calculation: to calculate the uncertainty for B_e

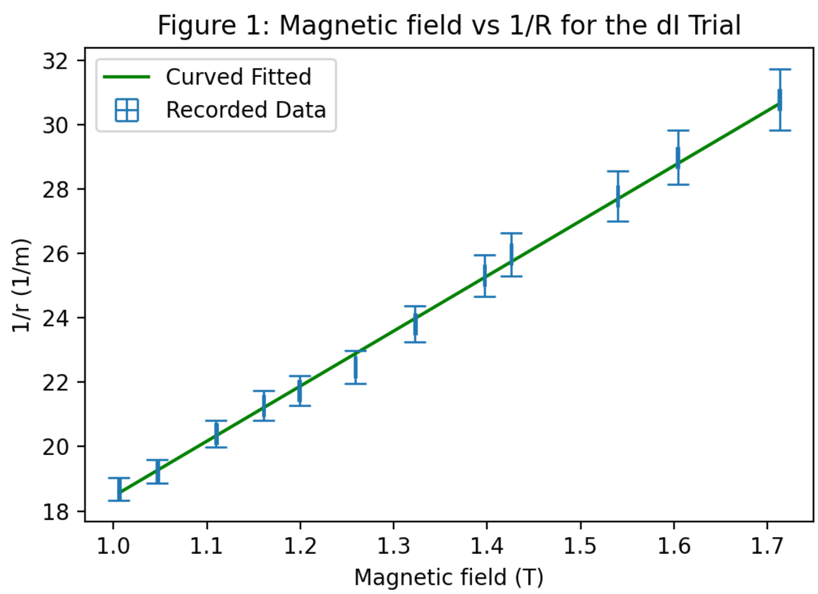
$$u(B_e) = B_e\sqrt{\left(\frac{u(b)}{b}\right)^2 + \frac{m}{e}\left(u\left(\sqrt{\frac{e}{m}}\right)\right)^2 + \frac{u(V)^2}{4V}}$$

$$u(B_e) = -6.0502479297871396*10^{-5} \sqrt{\left(\frac{0.9823996341768743}{1.3628333834683806}\right)^2 + \frac{(25337.70358243841)^2}{172511519407.58066} + \frac{(1)^2}{4(170)}}$$

$$u(B_e) = 4.383061411045349*10^{-5}$$

$$u(B_e) = 4*10^{-5}$$

Here is a plot of the relationship found between B_c and the reciprocal of the radius found for the electrons:



By varying current, the charge to mass ratio of the electron was found to be $172511519408 \pm 21047780062\ C/kg$. Furthermore, the external magnetic field was found to be $-6.0502479*10^{-5} \pm 4.383061\ T$. The actual charge to mass ratio of an electron is $1.758*10^{11} C/kg$, the percentage error of the charge to mass ratio is 1.87%. Thus, the charge to mass ratio found is very close to the expected value. The actual external magnetic field in Toronto is around $5*10^{-5} T$, so the percentage difference is around 21%. It is interesting to note that the y-uncertainty increases as magnetic field increases. This is because r is inversely proportional to the magnetic field, but the uncertainty in r is constant. The ratio between the two is used to calculate the uncertainty of 1/r, leading to this effect.

Table 3: Change in Voltage (dV) Trial							
Voltage (V)	**Voltage Uncertainty (V)**	**Current (A)**	**Current Uncertainty (A)**	**Diameter (m)**	**Diameter Uncertainty (m)**	**Radius (m)**	**Radius Uncertainty (m)**
106	1	1.045	0.001	0.077	0.002	0.039	0.001
111	1	1.045	0.001	0.081	0.002	0.041	0.001
117	1	1.045	0.001	0.084	0.002	0.042	0.001
121	1	1.045	0.001	0.086	0.002	0.043	0.001
126	1	1.045	0.001	0.088	0.002	0.044	0.001
131	1	1.045	0.001	0.090	0.002	0.045	0.001
136	1	1.045	0.001	0.092	0.002	0.046	0.001
141	1	1.045	0.001	0.094	0.002	0.047	0.001
146	1	1.045	0.001	0.096	0.002	0.048	0.001
151	1	1.045	0.001	0.097	0.002	0.049	0.001
156	1	1.045	0.001	0.100	0.002	0.050	0.001
161	1	1.045	0.001	0.101	0.002	0.051	0.001

First, we need to calculate B_c for using the constant I:

$$|B_c| = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 n I}{R}$$

Then, use the general formula to calculate the ratio.

$$given \qquad \frac{1}{r} = \sqrt{\frac{e}{2m}}(-E_c + E_e)\frac{1}{\sqrt{V}}$$

Knowing the ratio is the slope of the line of best fit, to calculate the charge to mass using B_c and the B_e after graphing the above equation:

$$slope = \sqrt{\frac{e}{2m}}(-E_c + E_e)$$

$$\frac{e}{m} = \left(\frac{\sqrt{2}*slope}{-E_c + E_e}\right)^2$$

Sample Calculation: to calculate the charge to mass ratio for dV:

$$\frac{e}{m} = \left(\frac{\sqrt{2}*slope}{-E_c + E_e}\right)^2$$

$$\frac{e}{m} = \left(\frac{\sqrt{2}*(312.21945773782153)}{-(0.0007932019376306557) + (0.00006050247929787139 6)}\right)^2$$

$$\frac{e}{m} = 267506826401.96432$$

$$\frac{e}{m} \approx 2.7*10^{11}$$

Sample Calculation: to calculate the uncertainty of 1/sqrt(V)

$$u\left(\frac{1}{\sqrt{V}}\right) = \frac{1}{2}V^{-\frac{3}{2}}u(V)$$

$$u\left(\frac{1}{\sqrt{V}}\right) = \frac{1}{2}(106)^{-\frac{3}{2}}(1)$$

$$u\left(\frac{1}{\sqrt{V}}\right) = 0.0004581537$$

$$u\left(\frac{1}{\sqrt{V}}\right) \approx 0.0005$$

Sample Calculation: to calculate the uncertainty of sqrt(charge to mass ratio) for dV

$$for\ the\ dV\ trial: \quad u\left(\sqrt{\frac{e}{m}}\right) = \sqrt{\frac{e}{m}}\sqrt{\left(\frac{u(s)}{s}\right)^2 + \frac{u(B_c)^2 + u(B_e)^2}{(B_c + B_e)^2}}$$

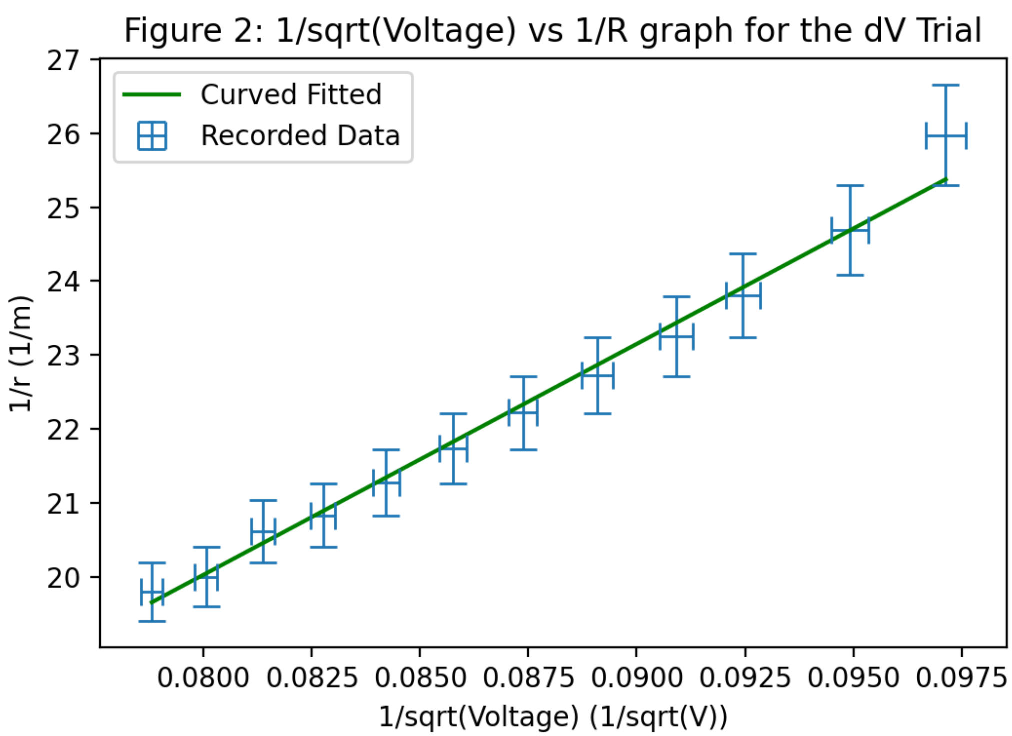
$$u\left(\sqrt{\frac{e}{m}}\right) = -517210.62092919584*$$

$$\sqrt{\left(\frac{26.12505205214014}{312.21945773782153}\right)^2 + \frac{1.2419308085929146*10^{-5^2} + 4.383061411045349*10^{-5^2}}{\left(0.00076 - 6.0502479297871396*10^{-5}\right)^2}}$$

$$u\left(\sqrt{\frac{e}{m}}\right) = -53973.60017042035$$

$$u\left(\sqrt{\frac{e}{m}}\right) = -50000.$$

Here is a plot of the relationship found between $1/\sqrt{V}$ and the reciprocal of the radius found for the electrons:



By varying voltage, the charge to mass ratio of the electron was found to be $267506826402 \pm 55831438516\ C/kg$. The actual charge to mass ratio of an electron is

$1.758*10^{11} C/kg$, so the percentage error of the charge to mass ratio is 52.16%. Thus, the charge to mass ratio found is very far from the expected value. It is interesting to note that the linear relationship in the plot seems to disintegrate for the smaller voltages. This is especially true for the very last point in the plot (which corresponds to the smallest voltage, since we are plotting $1/\sqrt{V}$), which is far above the line of best fit compared to other values. The most likely explanation for this deviation from linearity is explained in the analysis section below.

Table 4: Final result

Trial	**Name**	**Value (Tesla)**	**Uncertainty (Tesla)**	**Percentage Difference:**
dI	Charge to Mass ratio	$1.7*10^{11}$	$2*10^{10}$	1.87%
	B_e	$-6*10^{-5}$	$4*10^{-5}$	21.00%
dV	Charge to Mass ratio	$2.7*10^{11}$	$6*10^{10}$	52.16%
dV - without lowest one voltage	Charge to Mass ratio	$2.4*10^{11}$	$6*10^{10}$	38.72%
dV - without lowest two voltage	Charge to Mass ratio	$2.3*10^{11}$	$6*10^{10}$	33.31%
dV - without lowest three voltage	Charge to Mass ratio	$2.3*10^{11}$	$7*10^{10}$	29.92%
dV - without lowest four voltage	Charge to Mass ratio	$2.3*10^{11}$	$7*10^{10}$	29.01%

Sample Calculation: percentage difference for dI charge to mass ratio

$$percentage\ difference = \frac{|experimental\ value - expected\ value|}{expected\ value} * 100\%$$

$$percentage\ difference = \frac{|172511519407.58066 - 1.758 \times 10^{11}|}{1.758 \times 10^{11}} * 100\%$$

$$percentage\ difference = 1.87\%$$

Analysis:

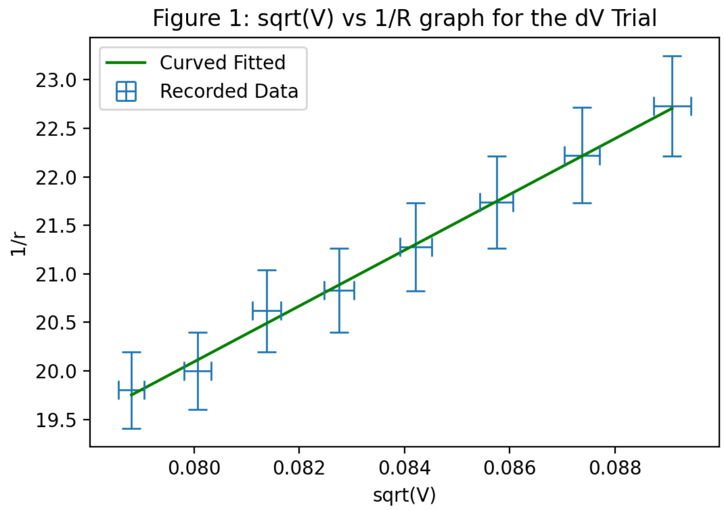
1. Explain how you used the self-illuminated scale and a plastic reflector to eliminate problems of parallax

Since the path of the electrons are within a spherical chamber, we are unable to directly measure the radius of the circular orbit directly with a ruler without there being parallax, since we cannot place the ruler directly in the middle of the sphere. However, the self-illuminated scale projects an image of a ruler onto the plastic reflector, which can then be adjusted until the projected image appears to be on the same plane as the circular motion. Thus, the projection of the ruler can be used measure the path of the electrons without there being any parallax errors.

2. Investigate the anomalous behaviour of the electron trajectory in the case of low accelerating voltage V and high current in the coils I (resulting in a strong magnetic field B).

The reason that the electron trajectory becomes anomalous when the accelerating voltage V is small compared to the current I of the coils is due to the fact that this leads to the radius of the circular motion of the electrons being small as well. This, in turn results in the electron trajectory not being centred around the centre of the sphere. However, the magnetic field becomes varies as one gets further away from the centre. This doesn't matter when the electron trajectory is centred, because the electrons are always a constant away from the centre so they still experience constant magnetic field (this is just the definition of circular motion). However, when the trajectory is not centred around the sphere the magnetic field changes as the electrons move around their trajectory, leading to error in the measured radius. In particular, the ratio between the magnetic field at the centre and the field a distance ρ from the centre (for $0.2R < \rho < 0.5R$) is given by: $\frac{B(\rho)}{B(o)} = 1 - \frac{\rho^4}{R^4\left(0.6583 + 0.29\frac{\rho^2}{R^2}\right)^2}$. Note that the the ratio decreases as ρ increases, so the magnetic field decreases the further from the centre one is. Thus, the electron trajectory experiences a lower magnetic field as it gets further away from the centre, and a higher field closer to the centre, leading to erroneous radii measurements. This error can be resolved simply by removing the small voltage measurements from the calculations to find the charge to mass ratio, since they only cause error.

We repeat the calculations from the voltage subsection of the results section, but with the 4 smallest voltages removed. Here is a plot of the data:



By varying voltage and discarding the smallest 4 voltages, the charge to mass ratio of the electron was found to be $226811408964 \pm 80296554709\ C/kg$. The actual charge to mass ratio of an electron is $1.758 * 10^{11} C/kg$, so the percentage error of the charge to mass ratio is 29.01%. Thus, the charge to mass ratio found is much closer to the expected value (more than 20%!) than without removing the smallest voltages. However, it is still very far off from the expected charge to mass ratio of an electron.

> ***3. Evaluate the influence of nearby ferromagnetic materials and other sources of magnetic fields on the electron trajectory (for example, bring a cellphone near the glass bulb). Is it significant enough to affect the measurements?***

The nearby sources of magnetic fields do not emit large enough fields to affect our measurements. For example, cell phones emit around $2 * 10^{-6}$T at most, which gets exponentially smaller as one gets further away from the phone. This is much too small to have any effect on the measurements. For comparison, the external field strength we found using the experiment was 30 times larger than this. The only thing that might emit non-trivial magnetic fields in the laboratory we conducted the experiment in would be the circuitry and electronics of the building itself. However, this should stay relatively constant throughout the experiment, and thus not affect the measurements. In fact, this may have been the cause of the higher than expected external magnetic field strength.

Conclusion:

The purpose of this investigation was to determine the charge to mass ratio of an electron by analysing their trajectory through a magnetic field. Our results are that by varying the magnitude of the magnetic field, the charge to mass ratio was found to be $1.7*10^{11} \pm 2*10^{10}$C/kg. Furthermore, the magnitude of the external magnetic field along the z-axis was found to be $6*10^{-5} \pm 4*10^{-5}$ T. The expected value for the charge to mass ratio of electrons is $1.758*10^{11} C/kg$, so the percent error of the gravitational acceleration was 1.87%. This percent errors is very low, suggesting that the result of this investigation was quite accurate to its theoretical value. Furthermore, the expected value falls into the uncertainties of the charge to mass ratio found, suggesting that this investigation reliably found the correct ratio. As well, the expected value for the external magnetic field was around 5*10^-5 T, which also fits in the uncertainties of the found external magnetic field. However, the error on the calculated external magnetic field was very high, being almost as large as the actual value. Thus, the calculated magnetic field was not precise.

By varying the accelerating voltage of the electrons, the charge to mass ratio was found to be $2.3*10^{11} \pm 7*10^{10}$ C/kg, which is much higher than the expected value (29% error). However, the error on this calculation was high enough that the expected value is still within uncertainties of the calculated value, so even though the ratio is inaccurate it doesn't necessarily mean the investigation itself was unreliable. The reason for such a high error comes from the fact that we were forced to use our calculated external magnetic field to find the second charge to mass ratio. Since the calculated magnetic field was itself imprecise, it is not surprising that the calculated charge to mass ratio using it ended up being quite inaccurate. In conclusion, by varying the magnetic field strength affecting the trajectory of electrons we were able to find the charge to mass ratio of an electron to a high level of accuracy and reasonable level of precision, suggesting this investigation succeeded well in its goal.

Most of the errors in this lab are an result of measurement errors and calculation errors. The instrumental errors are not significant in comparison. For the measurement of the trajectory radius is a good example of the measurement error, ideally, the electron's trajectory has to be verticle and in the shape of a circle. If it is slanted or in the shape of a helix, then the reading will be off. On top of that, the measurement was taken with a self-illuminated scale and plastic reflector because the electron trajectory is in a glass bulb, so the position and angle of the self-illuminated scale or the glass bulb itself can affect the reading of the measurement if they weren't 't placed properly.

The other source of error came from the calculation error, for example, the radius of the Helmholtz coils. The uncertainties came from its radius. As mentioned before, the Helmholtz coil is in the shape of an ellipse instead of a circle, and in all the formulas used, the magnetic field was calculated with the assumption that the shape of the Helmholtz coils be a circle. Because of that, we used the average radius of the Helmholtz coils. To calculate the radius of the Helmholtz coils, we had to measure at least what we thought was the longest diameter and the shortest diameter. Then we need to take the average and more calculations to get the final result. Because of the complicated process involves many calculation, the uncertainty will increase. Not only that, but the high uncertainty from the radius was also used to calculate the magnetic field B_e in the dI trial, and the same one was used later in the dV trial too. That is why the uncertainty for the magnetic field is so high.

Some methods of improvement include getting a better voltmeter that does not charge up over time, so the team can obtain a better measurement of the voltage instead of having it fluctuate. Another way to improve the lab is to limit the calculation uncertainties. For example, when calculating the radius of the Helmholtz coil, if we used a string and wrapped it around the circumference and used that to estimate a radius, that value would have a much lower uncertainty, and later result in a lower uncertainty for both the magnetic field B_e and B_c. Lastly, in the dV calculation, if we were to calculate B_e using the same method we used in dI, we would need to use the charge to mass ratio, what we were trying to find in the end, to find the B_e, which is impossible to do without introducing a very high uncertainty. So if we were able to use a device to measure the magnetic field coming from the environment, it could increase the accuracy of the result, and to better understand how the environment can affect the reading.