PHY324 Pendulum Project - 2023

1 Project Description

This document gives you some theory, and then describes what you are required to do, including what your report should include. The brief version is that you are to build a simple pendulum at home (or use the in-lab equipment in MP235) and determine to what extent the model of damped harmonic motion predicts the behaviour of your pendulum. **You should not expect that the model is perfect.** In fact, you are specifically tasked with finding what are the limitations of the model with respect to your apparatus.

While everything in this document is important, please pay extra attention to the last section on Collaboration as it contains **advice on how to avoid getting into serious academic trouble for plagiarism.**

2 Learning Goals

The primary learning goals for this project include:

- Designing your own procedure for achieving a goal.
- Refining your procedure in order to reduce your final uncertainty.
- Clearly and concisely communicating the essential elements of your procedure.
- Deciding upon and communicating a *quantitative* metric by which to judge whether an experimental result agrees with a theory.
- Incorporating feedback to improve the experiment.

This is not meant to imply that you can get away with sloppy graphs, poor uncertainty analysis, poor writing, etc. Those are still important things to do well. However, we assume you have practised those skills before, whereas the above list of skills are likely new (or newish) for you, at least in the context of experimental physics.

3 Theoretical Model

Say you have a mass m attached to a string of negligible mass. The other end of the string is attached to something immovable, making a fixed pivot point. The string has length L, and the distance from the centre of mass to the point where the string is attached is D. If we further assume that you release the mass so that the pendulum swings in planar (2-D) motion somewhere on Earth, and the initial angle of release was θ_0, then I predict that the motion of your pendulum will obey the following equation:

$$\theta(t) = \theta_0 e^{-t/\tau} \cos\left(2\pi \frac{t}{T} + \phi_0\right)$$

where $\phi_0 = 0$ (the phase constant) if you start your video at the precise moment you release the mass. τ is the time constant of decay (similar to a half-life), and T is the period of oscillations of the pendulum. Note that all angles are measured in radians, and the zero point is when the pendulum hangs straight down while motionless. Implicit in this prediction is that your pendulum is symmetric, i.e. it behave the same no matter which side you release the mass from. **This may not be true depending on how you attach your string to your pivot point!** Make sure you explicitly check whether your pendulum is symmetric.

Furthermore, I predict that neither τ nor T depend on t. Finally, I predict that

$$T = 2(L + D)^{1/2}$$

and thus T does not depend on m, θ_0 or ϕ_0. **Note that changing m can change D (heavier masses are typically larger in size) and L (the string is elastic so can stretch).** This means that m may have an indirect impact on T. You will need to take care in how you investigate this. My prediction assumes $D^2 \ll L^2$, so if your string length gets small enough then my prediction should start to fail.

These predictions come from the mathematical model of damped harmonic motion. Note also that the pendulum is assumed to be symmetric in the sense that the behaviour should be the same whether you release the mass from the 'left' side or the 'right' side. You should explicitly verify this. There are ways to make an asymmetric pendulum; please try to avoid doing this.

Note that no predictions are made about how τ depends on $(L + D)$ or m. You are expected to find an empircal function which works. That means you need not explain why your best fit function is theoretically sound. You only need to show that the goodness of fit for your chosen function is reasonable.

4 What do you do?

- Build a pendulum at home (or use the equipment in MP235). L, m and θ_0 must all be changeable. Also, θ_0 should be able to go up to $\pm\pi/2$.
- Verify or refute the claim that the period T is independent of amplitude θ_0. You can do this for a single length and mass combination. Make sure you get to large angles (close to $\pi/2$).
- Provide a quantitative estimate of the asymmetry of your pendulum. You can do this as part of the work in the previous bullet point.
- Verify or refute the claim that the decay is exponential, and determine the time constant τ. You can do this for a single length and mass combination.
- Verify or refute the claim that the period depends on L as stated: $T = 2(L + D)^{1/2}$ (provided that $D^2 \ll L^2$).
- Verify or refute the claim that the period is independent of the mass (except insofar as the mass may inadvertently affect L and/or D).

- Investigate the effect of $L + D$, m, and θ_0 on τ. If you find a trend, attempt to find a function which fits your data. You do not need to justify your function beyond the fact that it seems to fit your data. As no prediction was given for this, there is nothing to verify or refute.

You will submit a complete draft of your paper for peer review. Your paper will be reviewed by three other students, and you will review three papers. You need to use the marking rubric on each paper you review, and provide at least three constructive points of feedback. Constructive feedback tells your peer what changes you think will significantly improve their grade.

At the end of the semester you will submit your final report for marking by the TAs. You are expected to use any feedback you have received to improve your experiment, including taking new data. Focus on making your uncertainties small, and documenting how you achieved this.

Your report should have:

- An introduction which summarizes what you did, how you did it, and very briefly mentions your results (specifically any part of the model your data refutes really needs to be mentioned here as that is a key result). An abstract is optional but is recommended.
- A theory or background section which includes every equation which will be referenced later. Specifically, any equation you use to fit your data should probably first appear in the theory section.
- A description of your setup (with a photo) and your methods. This should be written so that someone else can reproduce your experiment. Please do not include bullet-point lists of equipment or step-by-step 'recipes' of how you measured things, as while that makes reproducing your result simple, it misses the valuable thoughts you have put into this experiment. You can assume that the reader knows how to use a ruler, a stopwatch, a protractor, and so on. Instead, **focus on explaining why you made your decisions to use your methods** over other possible choices.
- Your results, with uncertainty analysis. Please put all your results as graphs with lines of best fit whenever possible. Any data tables belong in an appendix unless you cannot figure out a more visual way to display the data. If you forget to include uncertainties in your data you will not get a good mark.
- Your conclusions, which are both numerical (with uncertainties), such as the value of τ, and explicitly linked to confirming/rejecting various facets of the model, such as whether the amplitude decays exponentially. You need to provide quantitative arguments supporting each claim of confirming/rejecting a facet of the model.

The idea is that your report should be something which can be given to a first year student and they should be able to understand what you've done and recreate your experiment in order to verify your findings.

In terms of 'voice', I recommend using the first-person past tense. Example: "I chose to use fishing line instead of regular string because when I switched from 100 g mass to 1 kg mass the fishing line stretched by less than 1 mm compared with a 3 mm stretching observed in the string. I could thus ignore any changes in L as m was changed." If you wish to use a different voice, like third-person present tense, feel free to do so but make sure to keep it consistent.

Expect to have to refine your experiment. Specifically, in the quest to reduce uncertainties, you are likely to find that a result in one part might change how you view your uncertainties elsewhere. If so, you might need to retake data in order to reduce your uncertainties. You might face mundane challenges too, such as getting the pendulum to oscillate in 2-D planar motion instead of 3-D elliptical motion. Recall that you can change your setup however you wish as long as it remains a recognizable pendulum.

Please proof-read your report before submission, and make sure your writing is consistent. A poorly written report will impair its readability, thus will receive a low score despite the quality of the experiment and data analysis.

5 Advice

- Google "physics tracker app" (without the quotation marks).
- Think hard about what you can do to make your uncertainties smaller.
- The fixed-end (pivot point) of the string needs to not move much, even for your heaviest mass.
- One of the first things to check is whether your pendulum is symmetric. If it's not, you might want to redesign it.
- It is important that your pendulum moves in a 2-D plane, not a 3-D ellipse. A simple mass-on-string design is unlikely to work well, as I'm sure you will discover.
- Roughly measure the period (T) and the decay constant (τ) early. If τ is not at least 20 times larger than T, your pendulum has a lot of friction and you might want to redesign it. A ratio of 100 is fine, don't put much effort to improve it beyond this point or your exponential decay data will be tedious.
- To really show the decay is exponential, you're going to need to have your pendulum swing until it loses most of its energy. If your period is long, this could require 10-20 minutes of taking data. If you find that tedious, shorten the period by shortening the string.
- I'm sure you know what the small angle approximation is. You're going to want to find an experimental (not theoretical) version of it for your pendulum. You're also going to want to violate the small angle approximation for at least some of the experiment (specifically the period-amplitude measurements).

- A power series can be a good way to measure small deviations from a theoretical result, especially when the theory claims that something should be constant (independent of some parameter).
- The effect of mass on the pendulum is likely the most difficult part to do well. You really need small uncertainties. It would be wise to have the mass change by one to two orders of magnitude in your quest to find some signal in your data. I recommend exponential increases of mass (like doubling it).
- Do your best to change only 1 parameter at a time. Document how you did this.
- Think hard about what you can do to make your uncertainties smaller. This is so important that I repeated it.
- A main learning goal is to have you make your own decisions. Make sure you document your decisions and the logic behind them!
- All uncertainties must be analyzed with the usual rigour. These are critical. You need to determine how well the model predicts the behaviour of your pendulum, and any observed discrepancies must be compared with the uncertainties.
- Any time you refute a given prediction, you should try to find a better function which fits your data. You don't need to justify your function beyond the fact that it works better (though you do have to quantify what better means). Your better function can be a simple power series if that works for your data.

6 Collaboration

You are encouraged to collaborate with others in terms of brain-storming ideas, sharing insights into design or procedural elements, and so forth. **You must take your own data and write your own report.** Copying data or writing from another student is a violation of academic integrity and can get **BOTH** parties into serious trouble. Similarly, copying text from a website (like Wikipedia) **without citations and references** is plagiarism, which is also a violation of academic integrity.

Remember that a reference is in the bibliography at the end of your paper, while a citation is in the paper itself and indicates where that sentence (or paragraph) comes from. Everything in your bibliography should have a citation in your paper, and every citation needs a clarifying entry in the bibliography. In addition, if you are copying text you **must** use quotation marks. **Note that using quotation marks properly gets you out of trouble (it's no longer plagiarism) but does not help you get a good mark.** Instead, paraphrasing the text you reference is a much better practice and will be rewarded with better marks.

You may use any style for references and citations.