Radius of the Earth

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Objective:

This experiment aims to measure the radius of the Earth by using a Sodin gravimeter to measure the differences in the gravitational acceleration as one goes up the Burton Tower. The gravimeter was originally used to detect the oil shale and domes underneath the surface. Thus it is engineered to have a high sensitivity to any changes in gravitational acceleration. Because of that, we will be measuring the Δg between each floor of the Burton tower, since they are mostly identical, with a height difference of 395 cm and floor mass of $1*10^6$kg. The gravitational force is inversely proportional to the distance, with the formula: $g = G * \frac{m_{earth}}{R^2}$, and the Δg is related to g by this formula: $\frac{\Delta g}{g} = -2*\frac{\Delta R}{R}$. Using the difference of Δg between floor, we can then calculate the radius of the Earth using the slope of the graph that represents the relationship between the difference in height (ΔR) and difference in gravitational acceleration (Δg).

Methods:

Tools:

- Sodin Gravimeter
- Burton Tower

Logs:

This experiment requires measurements from the gravimeter starting from the basement to the 15th floor of the Burton Tower. To use the gravimeter, you first need to pull up the light switch and level the meter using the adjustable screws at the meter's base. Then, look through the eyepiece and try to alight the bean to the center verticle line by rotating the knob on the top of the device. The value shown on the top of the meter is in the unit of milligals per division. Because of that, we will need to multiply that value by the meter constant to get the reading in milligals. Once you get the reading from all 16 floors, repeat the same process two more times at a similar time on a different day. Remember, since the gravimeter is highly sensitive to masses nearby, try to station the gravimeter at the same position relative to each successive floor to increase the accuracy.

One of the uncertainties we need to worry about in this experiment is the measurement uncertainty. To calculate that, we recorded the reading of the gravimeter at the same location 4 different times by un-levelling the meter after every measurement. The measurement uncertainty would be the average of the four different measurements. There are also other uncertainties/ sources of errors in this experiment that should be consider later in the report.

Equations:

Equation(1), convert the reading from the gravimeter to Δg with the unit m/s^2,, where k is the gravimeter constant.

$$\frac{(reading\ from\ gravimeter) * k}{100,000} = \Delta g\ in\ \frac{m}{s^2} \tag{1}$$

Equation(2), the relationship between the gravitational acceleration and distance:

$$g = G * \frac{m}{R^2} \tag{2}$$

Equation(3), the relationship between Δg and g:

$$\frac{\Delta g}{g} = -2 * \frac{\Delta R}{R} \tag{3}$$

Equation(4), percentage difference:

$$percentage\ difference = \frac{|experimental\ value - expected\ value|}{expected\ value} * 100\% \tag{4}$$

Equation(5), to calculate the influence from the floors above and beneath in the Burton Tower:

$$g_{correction} = \sum_{i=4}^{n} G * M_{floor} * \left(\frac{1}{((i-2)r_{floor})^2} + \frac{1}{((16-i)r_{floor})^2} \right) \tag{5}$$

Equation(6), to calculate average:

$$\overline{X} = \frac{\sum X}{N} \tag{6}$$

Equation(7), centrifugal force:

$$f_{centrifugal} = m\omega^2 r \tag{7}$$

Equation(8), gravitational force by the Earth:

$$F_g = m*g \tag{8}$$

Equation(9), to find the uncertainty of a reciprocal:

$$\sigma\left(\frac{1}{a}\right) = \frac{\sigma_a}{a^2} \tag{9}$$

Equation(10), error propagation for addition:

$$u(a+b) = \sqrt{u(a)^2 + u(b)^2} \tag{10}$$

Equation(11), error propagation for multiplication:

$$u(a*b) = a*b\sqrt{\left(\frac{u(a)}{a}\right)^2 + \left(\frac{u(b)}{b}\right)^2} \tag{11}$$

Results:

Table 1: Basic Information	
Expected value for Toronto (https://rechneronline.de/earth-radius/)	6368000 m
Instrumental Uncertainty	0.2 mgal
Measurement Uncertainty	0.2 mgal
Gravitation acceleration at the bottom of the Burton Tower	9.804253
Gravimeter Constant	0.10055
Gravitational force exerted on Earth by the Sun	5928.89 m/s^2
Gravitational force exerted on Earth by the Moon	33.1777m/s^2
Weight of the Sodin gravimeter (according to the manual)	4 kg

Estimated weight of the gravimeter's core	0.5 kg

Sample Calculation: Equation(2), the relationship between the gravitational force and the distance:

$$g_{Sun-Earth} = G * \frac{m_{sun}}{R_{sun}^2}$$

$$g_{Sun-Earth} = 6.67*10^{-11} Nm^2 kg^{-2} * \frac{2.0*10^{30} kg}{(1.50*10^8 km)^2}$$

$$g_{Sun-Earth} = 5928.89\ m/s^2$$

Table 2: Experimental Result				
Floor Number	Height (m) ±0.01m:	Measurement of Trial 1 ±0.02:	Measurement of Trial 2 ±0.02:	Measurement of Trial 3 ±0.02:
B	N/A	663.5	667.1	665.9
1	0	647.9	652.6	650.1
2	3.95	633.6	635.3	633.7
3	7.90	621.4	623.0	621.8
4	11.85	609.4	611.8	612.0
5	15.80	597.6	601.4	599.9
6	19.75	585.5	587.7	587.8
7	23.70	574.9	577.7	575.5
8	27.65	564.7	563.5	564.6
9	31.60	549.2	554.5	552.6
10	35.55	539.7	540.5	539.2
11	39.50	528.7	529.4	N/A
12	43.45	516.2	517.6	N/A

13	47.40	503.8	503.3	N/A
14	51.35	493.2	494.3	N/A

To get the heights of each measurement relative to the ground floor, we multiplied the difference of the floors numbers in question with the height difference of consecutive floors in Burton Tower (3.95m). To get the relative gravitational field strengths from our raw data (Table 2), we multiplied each measurement with the k-constant of the Gravimeter we used (0.10055 mgal) and divided by 100000 to convert from mgal to m/s^2. The resulting data was plotted to find the relationship between height and relative gravitational field strength for all three trials , shown in figures 1, 2, and 3.

Sample Calculation: Equation(1), convert the reading from the gravimeter to Δg with the unit m/s^2, where k is the gravimeter constant.

Let reading from gravimeter = 609.4 and k = 0.10055

$$\frac{(reading\ from\ gravimeter) * k}{100,000} = \Delta g\ in\ \frac{m}{s^2}$$

$$\frac{609.4 * 0.10055}{100,000} \approx 0.00061275\, m/s^2$$

Using linear regression, the line of best fit was found for all three trials. As explained in the introduction, the ratio between change gravitational acceleration and change in height can be used to calculate the radius R of the earth, using Equation (3). This ratio is by definition the slope of our plot, so by plugging it into the equation we were able to calculate the experimental radius of the earth.

Sample calculation of equation(3): using the relationship between Δg and ΔR to calculate R:

Assuming after plotting all the data and finding the line of best fit, the slope of the line of best fit is -2.959e-06

$$\frac{\Delta g}{g} = -2 * \frac{\Delta R}{R}$$

$$\frac{\Delta g}{\Delta R} = -2g * \frac{1}{R}$$

$$-2.959 * 10^{-6} = -2g * \frac{1}{R}$$

$$\frac{-2.959*10^{-6}}{-2g} = \frac{1}{R}$$
$$R \approx 6626360.497$$

The radius of the earth was found to be $6.430*10^6 \pm 7*10^3$ for the Trial one, $6.300*10^6 \pm 7*10^3$m for Trial two, and $6.288*10^6 \pm 1*10^4$m for Trial Three. Averaging these results, **the radius of the earth was found to be $6.342*10^6 \pm 5*10^3$m,** which is remarkably close to the actual radius of the earth at Toronto of 6368000 m (the percent difference between these values is only 0.41%). The radius of the earth at Toronto was gotten using this website (https://rechneronline.de/earth-radius/). The uncertainty of the slopes was found using curve fit as well as the measurement uncertainties in table 2. This error was then propagated using equation (9) to find the actual uncertainty of the radius.

Sample calculation: Finding the uncertainty of the radius of the earth:

The slope of one of the plots was found to be -2.9607e-06 with an error of 5.8701e-09. Then the uncertainty of the radius is:

$$u(r) = -2g * u\left(\frac{1}{\frac{\Delta g}{\Delta R}}\right)$$
$$= -2g\frac{u\left(\frac{\Delta g}{\Delta R}\right)}{\left(\frac{\Delta g}{\Delta R}\right)^2}$$
$$= -2 * 9.804253*\frac{5.8701*10^{-9}}{-2.9607*10^{-6}}$$
$$= 13131$$

There are a few interesting things to note about the plots above. Mainly, it appears as if the first few points on each plot are actually non-linear, and instead curve slightly upwards. This is especially noticeable in the first data point, which is noticeably higher than expected for all three plots. This is actually to be expected, and is caused by the basement floor. The basement is essentially a large cavity of empty space in the earth, leading to lower than expected gravitational acceleration readings on every floor other than the basement. This in part is what leads to the first reading being larger than expected compared to every other reading. Furthermore, the basement floor is much larger than every other floor, and is actually larger than 3.95m in height, leading to the floors directly above it having non-linear gravitational acceleration readings as well. The effect of the basement on the floors decreases according to the inverse-square law, since the gravitational field of an object decreases proportionally to the inverse-square of distance from the object. Thus, the upper floors are negligibly affected by the basement, and should still follow the expected linear relationship we desire. In conclusion, to get more accurate results we will discard the first 3 floors and basement, as they are all non-negligibly effected by the lack of mass in the basement.

Another correction that needs to be made comes from the fact that the floors of the Burton Tower also emit gravitational fields. The mass of a floor in the tower is approximately 10^6kg, which can actually affect the measurements of the Gravimeter by a non-trivial amount. So, the added gravitational acceleration caused by the mass of the floors as one goes up the tower for needs to be taken into account and corrected for in the data.

$$g_{correction} = \sum_{i=4}^{n} G*M_{floor}*\left(\frac{1}{((i-2)r_{floor})^2} + \frac{1}{((16-i)r_{floor})^2}\right) \tag{5}$$

One can think of the total effect of the floors on g as a summation of the effect of each individual floor on the Gravimeter. With this model, each floor affects g according to equation (2), where m is the mass of a floor (10^6kg) and R is the height difference of the two floors. It is also important to note that we do not include the ground floor in the summation, since it can be considered part of the earth's mass.

By calculating the change in the summation when the fifth floor is the focal point instead of the fourth floor, we can find how much the gravitational acceleration changes our measurement of g between these floors and account for it. As it turns out, the summation for the two floors is exactly the same, except for one term which differs. This is caused by the fact that one can think of moving up one floor as removing the top floor of the building, and pasting it beneath the bottom floor of the building (the resulting building is the same in both cases as long as each floor has the same mass, and the person ends up on the same floor of the building either way). Thus, the change in the gravitational acceleration due to the floors is caused purely by the term that gets replaced. Now, suppose that we are measuring the change in g from floor n-1 to floor n. The only floor in the summation for floor n-1 that doesn't have a corresponding floor in the summation for floor n is the rooftop. This is because for there to be a corresponding floor, the floor in question would have to be one floor higher than the roof. Top floor would have decreased the reading on floor n-1 by (assuming the roof is floor 15):

$$g_{floor\ above} = G*M_{floor}*\left(\frac{1}{((16-n)r_{floor})^2}\right), \tag{12}$$

Thus, in effect the lack of the roof floor increases the relative g reading on floor n by the value above, so we must compensate by subtracting this amount from our g reading on floor n.

The term in the summation for floor n that replaces the term above is caused by the gravitational field of the second floor. For an identical term to exist in the summation of floor n-1, term in question would have to be caused by a floor directly below floor 2. However, we have already explained that floor 1 isn't included in the summations because it can be considered part of the earth, so no such term can exist in the summation. The gravitational field caused by the second floor at floor n is:

$$g_{floor\ below} = G*M_{floor}*(\frac{1}{((n-2)r_{floor})^2} \quad (13)$$

This term effectively increases the gravitational acceleration measured on floor n compared to floor n-1, since the 2nd floor is below floor n (we are assuming n>2). Thus, to correct for this new term
we must subtract it from our reading of floor n. Thus, the total correction that we subtract from our reading of g:

$$g_{correction} = G*M_{floor}*\left(\frac{1}{((i-2)r_{floor})^2} + \frac{1}{((16-i)r_{floor})^2}\right)$$

However, this equation only holds if we are correcting the measurement at floor n relative to the g-value of floor n-1. Say for example we must correct for floor n relative to floor n-2. Then two floors are swapped in the term summation, and we must account for both. We can make this correction by subtracting the correction required to when moving to floor n-1 from n-2, and then subtracting the correction required when moving to floor n from floor n-1. Putting all of this together, we get the following equation for the Burton tower with floor 4 as our reference floor (where n is the floor number):

$$g_{correction} = \sum_{i=4}^{n} G*M_{floor}*\left(\frac{1}{((i-2)r_{floor})^2} + \frac{1}{((16-i)r_{floor})^2}\right)$$

Sample Calculation: *Equation(2), to calculate the influence from the floors above and beneath the 4th floor in the Burton Tower:*

$$g_{5th\ floor\ correction} = \sum_{i=4}^{4} G*M_{floor}*\left(\frac{1}{((4-2)r_{floor})^2} + \frac{1}{((16-n)r_{floor})^2}\right)$$

$$= \left(6.67*10^{-11}N*kg^{-2}*m^2\right)\left(10^6 kg\right)\left(\frac{1}{((4-2)3.95m)^2} + \frac{1}{((16-4)3.95m)^2}\right)$$

$$= 5.1032*10^{-7}\ N$$

Figure 4: the height (dr) vs. gravitational acceleration (dg) of Trial One

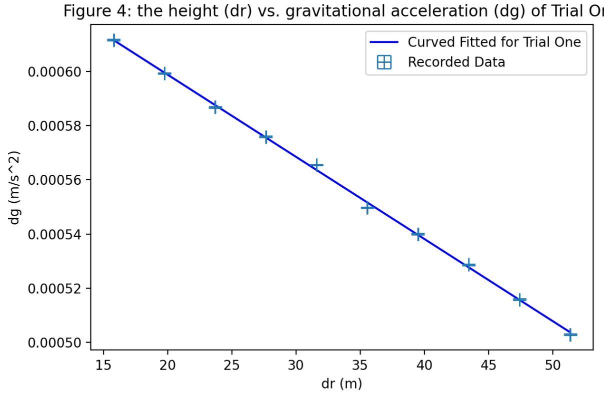


Figure 5: the height (dr) vs. gravitational acceleration (dg) of Trial Two

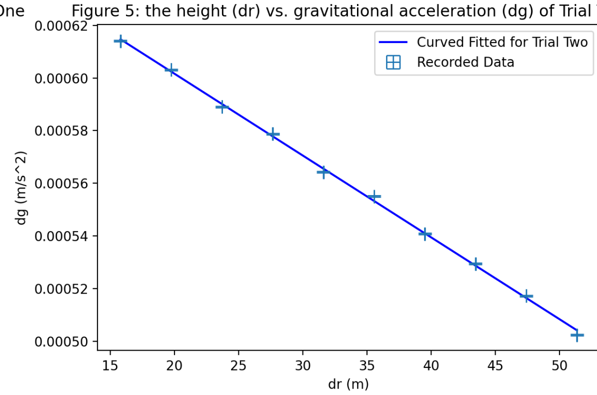
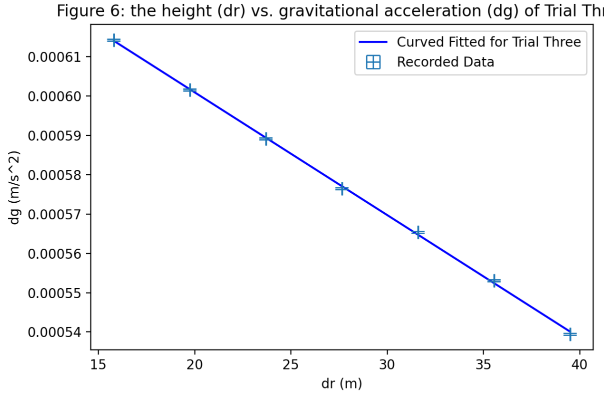


Figure 6: the height (dr) vs. gravitational acceleration (dg) of Trial Three



Figures 4, 5, and 6 are a plot of the three trials with the floor corrections added. The radius of the earth using corrections was found to be $6.46*10^6 \pm 1*10^4$ for the first trial, $6.31*10^6 \pm 1*10^4$m for trial two, and $6.29*10^6 \pm 2*10^4$m for trial 3. Averaging these results, **the radius of the earth using corrections was found to be $6.35*10^6 \pm 1*10^4$m,** which is even closer to the actual radius of the earth at Toronto of 6368000 m (the percent difference between these values is only 0.25%). The uncertainty of the slopes was found using curve fit as well as the measurement uncertainties in table 2. The error of the corrected measurements was then propagated using equations 9, 10 and 11.

Table 3: Calculated Radius of the Earth				
		Calculated Radius (m):	Uncertainties (m):	Percentage Difference
With correction	Trial 1	6460000	10000	1.50%
	Trial 2	6310000	10000	0.97%

	Trial 3	6290000	20000	1.10%
Average of the three trials		6350000	10000	0.25%
Without correction	Trial 1	6430000	7000	0.97%
	Trial 2	6300000	7000	0.92%
	Trial 3	6288000	10000	1.26%
Average of the three trials		6342000	5000	0.41%

Sample Calculation: Equation(4), percentage difference of Trial 1 with correction:

Let the experimental value to be 6626360.49749133m, expected value to be 6368000m.

$$percentage\ difference\ =\ \frac{|experimental\ value\ -\ expected\ value|}{expected\ value} * 100\%$$

$$=\ \frac{|6626360.49749133m\ -\ 6368000m|}{6368000m} * 100\%\ \ = 4.0571\%$$

Sample Calculation: Equation(6), to calculate average:

Let X = {6626360.49749133m, 6458343.480692914m, 6483131.418026538m}, for which N = 3

$$\overline{X} = \frac{\sum X}{N}$$

$$\overline{X} = \frac{6626360.49749133m\ +\ 6458343.480692914m\ +\ 6483131.418026538m}{3}$$

$$\overline{X} =\ 6522611.798736927m$$

Questions & Analysis:

1. *The linear portion of your graph should be able to provide data that will give R to about 1% or 2%. However, also of interest is the deviation from linearity near the basement or the fifteenth floor. What does this deviation tell you? At what level would the centrifugal force due to the earth's rotation affect the result of this experiment?*

The Burton Tower is directly connected to the Earth so it will not be affected by Earth's centrifugal force and thus our reading. However, the gravimeter and the core inside is not, thus it has the slight chance of being affected by the centrifugal force. Here we will calculate the centrifugal force of the core of the gravimeter, since this is the part that determines the measurements. Since there are no information on its weight, we will be taking 0.5kg as the weight of the gravimeter's core.

$$F_{centrifugal} = m_{gravimeter's\ core}\omega^2 R_{earth} \tag{7}$$
$$F_{centrifugal} = m_{gravimeter's\ core}\left(\frac{2\pi}{T_{earth}}\right)^2 R_{earth}$$
$$F_{centrifugal} = 0.5kg\left(\frac{2\pi}{86400s}\right)^2 (6483131.4180m)$$
$$F_{centrifugal} \approx\ 0.01714\ N$$

$$F_{Earth-gravimeter}\ =\ m_{gravimeter's\ core}*g \tag{8}$$
$$F_{Earth-gravimeter}\ =\ 0.5kg * \left(-9.81m/s^2\right)$$
$$F_{Earth-gravimeter}\ =\ -4.905\ N$$

$$\frac{|F_{Earth-gravimeter}|}{|F_{centrifugal}|} = \frac{|-4.905\ N|}{|0.01714\ N|} \approx 286$$

From the math above, we can see that centrifugal force will have no affect on the reading since the gravitational force, the force that is pulling the gravimeter and the core of the gravimeter toward the Earth, is approximately 286 times greater than the centrifugal force. Thus neither the altitude nor the centrifugal force will not have a substantial affect on the reading of the gravimeter as long as it is stationed on Earth.

2. How important is the earth's shape?

Since the Earth is not a perfect sphere with its equator being the Radius of the Earth will change depending on the location. We were able to find the Radius of the Earth at Toronto is approximately 6368000m, using this website (https://rechneronline.de/earth-radius/). This radius value is thus the expected/theoretical value of this experiment.

The eccentricity of Earth matters, since the Earth's orbit is not a perfect circle, the distance between the Earth and the Sun will change. But since we performed our measurements within three days, therefore it does not affect the result of this experiment.

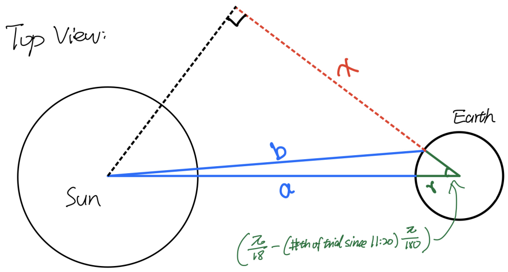


Figure 7: Top view of the Sun and Earth

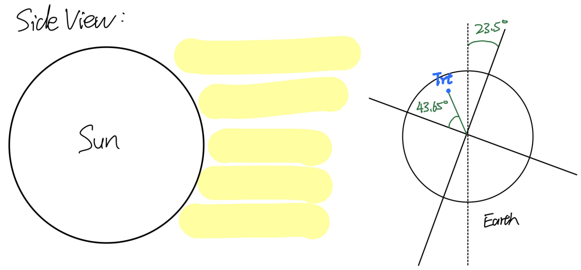


Figure 8: Side view of the Sun and Earth

The gravitational force exerted by the Sun and the moon also has a substantial impact on the reading. We used the website (https://www.theplanetstoday.com/) to estimate the location of the Sun, Moon and the Earth during the day that the measurements were done. We performed Trial One on Oct 26th and both Trial Two and Trial Three on Oct 28th.

Assuming the minimal distance between the Sun and the Earth occurs at 12:00pm. Assuming we performed all of the measurements for floor 4 to floor 14 from 11:20am to 12:00pm with a 4 minutes interval for each floor. Assume the center mass of the Sun, Earth and the Moon are on the same line looking from the side view. Assuming the Earth is rotating alone its verticle axis, not the rotating axis. Assuming looking from the side, the Earth's rotating axis is tilted exactly 23.5 degrees away from the verticle axis, shown in Figure (8).

Since the Sun is always above us around noon, and its gravitational force to the Earth is big, it will pull the core of the gravimeter away from the Earth, thus making our measurements smaller. The minimal distance between the Sun and the Earth happens around noon, which is also the time when the Sun will exert the maximum amount of upward force to the core of the gravimeter, and this force is inversely proportional to the square of the distance, according to Equation(2) .

$$g_{Sun} = +\ G * \frac{m_{Sun}}{R^2_{Sun-Earth}} * cos\left(\frac{\pi}{18} - (the\ \#th\ of\ trial\ since\ 11:20am)\frac{\pi}{180}\right) * cos(23.5° + 43.65°) \tag{14}$$

To estimate the effects from the Sun, we are using Formula (15) to represent the effects base on time, where $\frac{\pi}{18}$ represents the total 40 minutes it took to measure the values from floor 4 to floor 14.

So by 12:00 noon, the effects by the sun will be at max since cos(0) = 1, shown in Figure (7). Since we are assuming the Earth is rotating alone its verticle axis then, and Toronto is not located on the Earth's horizontal axis, then we will need to estimate the effects from the curvature of the Earth. Knowing the Earth's axis is $23.5°$, and the Toronto's latitude is approximately $43.65°$, we know that Toronto is $(23.5° + 43.65°)$ away from Earth's horizontal axis, and thus the effects caused by the location of Toronto can be represented as $cos(23.5° + 43.65°)$.

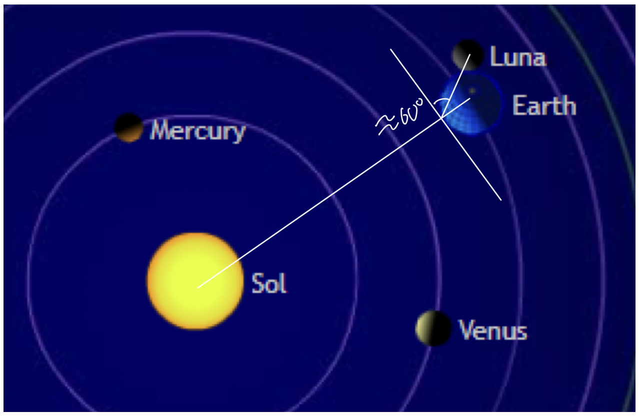


Figure 9: Nov 26th

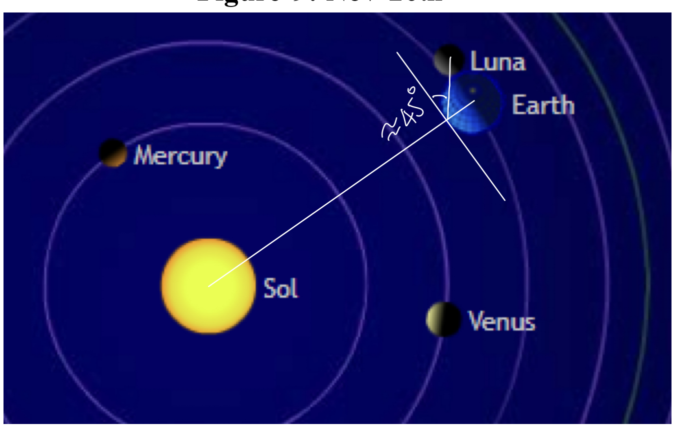


Figure 10: Nov 28th

Then, according to Figure (9) and Figure (10), around noon time, the moon is approximately $60°$ under the horizon on Oct 26th and $45°$ under the horizon on Oct 28th. Since the moon is under our horizon on both days, that means it will exert a downward force to the core of the gravimeter, making the reading slightly bigger. And the effects on Oct 26th will be greater than the effects from the other day. This is accurate from the result of our experiment. From Table (3), you can see our calculated radius from Trial One is larger than the calculated Radius for Trial Two and Trial Three. We are using Formula (16) to model the effects.

$$g_{moon} = -G * \frac{m_{moon}}{R^2_{moon-Earth}} * cos\left(\frac{\pi}{18} - (the\ \#th\ of\ trial\ since\ 11:20am)\frac{\pi}{180}\right) \qquad (15)$$

$$*cos(23.5° + 43.65°)$$

The average gravitation force exerted by the Sun is approximately 5928.89 m/s^2 and the gravitational force exerted by the Moon is approximately 33.1777m/s^2, according to Table (1). Thus, no matter how the Sun, Earth and the Moon are arranged, the Sun will have a much bigger impact on the reading than the Moon.

Figure 11: dr vs dg of Trial One with correction for the sun

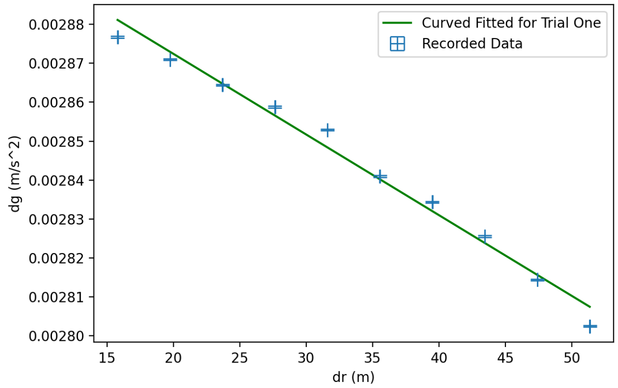


Figure 12: dr vs dg of Trial Two with correction for the sun

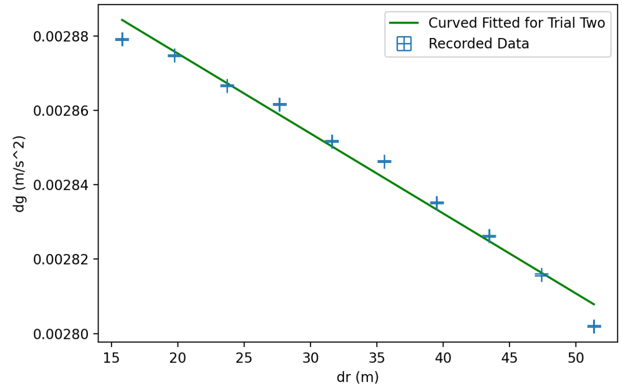
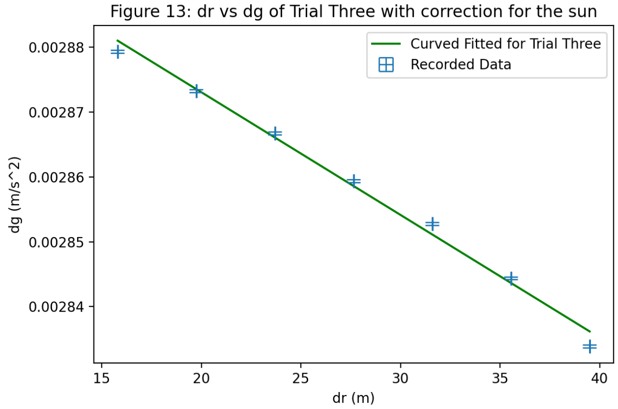


Figure 13: dr vs dg of Trial Three with correction for the sun



However, the effect from the Sun and Moon are not added to the final result tables and graphs because they can be so influential to the result that the new radius of the Earth is meaningless. For example, Figure 11 to Figure 13 shows the new radius if the effects from the Sun are included. According to Table (4), the new average radius $9.6*10^6 \pm 2*10^4 m$ is so big that not only the expected radius, 6368000m, is not within its uncertainty, it is also 1.5 times of the expected value. Because of that, we are not including the influences from the Sun and Moon to the final result. Formula (15) and Formula (16) are just very rough models of how the Sun and Moon can influence the reading, since it is derailing the result significantly, it is therefore not useful to have it added to our final calculation.

Table 4: Calculated Radius of the Earth with correction for the Floors and the Sun			
	Calculated Radius (m):	Uncertainties (m):	Percentage Difference
Trial One	9500000	30000	48.61%
Trial Two	9100000	20000	43.20%
Trial Three	10400000	60000	62.77%
Average	9600000	20000	51.53%

Conclusion:

The purpose of this investigation was to determine the radius of the earth by measuring the relationship between height and gravitational acceleration within the Burton tower. Our results are that the radius of the earth was found to be $6.342*10^6 \pm 5*10^3$m without any corrections, and $6.35*10^6 \pm 1*10^4$m correcting for the gravitational field emitted by the floors of the tower. Furthermore, accounting for the affects of the sun and moon on our measurements as well, the radius was found to be 9600000 ± 20000m. The expected value for the radius of the earth at Toronto is 6368000m. The percent error of the radius without corrections was 0.41%, and the percentage error with corrections was 0.25%. Both of these percent errors are extremely low, suggesting that the results of this investigation were very close to their theoretical values. However, the expected value of 6368000m doesn't fall into the uncertainties of either of the radius calculated (although it comes very close to falling within the range of the corrected value, with a difference of 8km), so there is a discrepancy between the actual and theoretical radius. However, judging by how accurate both calculated radius are to the real value, the most likely explanation is that we simply slightly underestimated the random error of our measurements. In the end, the corrected radius of the earth was slightly closer than the uncorrected radius, so it seems we were justified in making the floor corrections.

The main issue with this investigation is the ridiculously off radius found taking into account the effect of the sun and moon on our measurements. With a percent error 51.5%, there is a very large discrepancy between this radius and the both the other radius found and the expected radius. Judging by the fact that the corrected radius without taking into account the sun and moon was so much closer than the correction including them, it seems that there was some problem with the way we estimated their effects on the data. In conclusion, the extremely high accuracy of the radius calculated with floor corrections suggest that this lab was mostly successful in finding the radius of the earth, even if we were likely unable to correctly account for the effect the sun and moon have on our data.

There are a few big errors with this investigation. The first one has already been discussed, which is the error caused by the fact that we didn't account for the sun and moon in our final radius calculated. However, there is also the error due to the fact that we ignored the basement of the Burton Tower when correcting for the gravitational field of the floors. However, this error has also been discussed earlier in the lab, and was minimized by starting our measurements on the fourth floor, sufficiently far away from the basement. Another small error comes from the fact that the roof of the building is less massive than the other floors, but to account for this we didn't include the measurements on the fourteenth floor in our corrected data. Finally, there is one more error worth mentioning. When conducting the experiment, we measured g using the Gravimeter right next to the elevator shafts of the tower. The tower has four elevator shafts, each of which contain an elevator that weighs multiple tons. Just like with the floors of the building, these elevators emit gravitational fields, and thus can have an affect on the readings from the Gravimeter. However, the elevators are constantly moving up and down the building as well, making it virtually impossible to account for how they'd affect the readings. Thus, they introduced some random error into the experiment.

This experiment could have been improved by eliminating more uncertainties. A more accurate function could have deduced the uncertainty caused by the rotation of the Earth by measuring the reading from the same location for two hours with a 10 minutes interval. Not only that, all three trials should be performed at the same time on three different days. To further investigate the influences from the Sun and the Moon, one could measure the same location for 24 hours once every week for a day. By doing so, the only major changes would be the position of the Sun and Moon. Something else that could have been done was to set a reference spot in the building. Whenever one person measures on a different floor, another person can measure from the reference point. This way, it will be easier to see how the rotation of the Earth affects the reading. Finally, one other improvement that could be made to the experiment would be to conduct the Gravimeter readings further away from the elevator shaft, so that the elevators introduce less of a random error into the measurements. Gravitational acceleration follows an inverse square law away from its source of mass, so we wouldn't have to move the Gravimeter that far before the effect of the elevators became negligible.