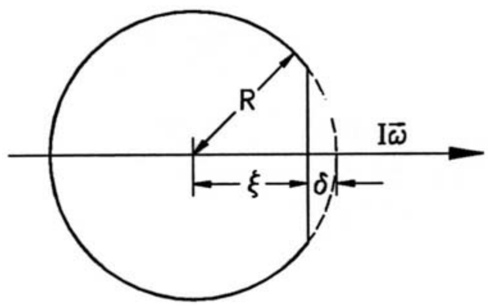
‘g’ USING PRECESSION OF AN AIR GYROSCOPE

THE EXPERIMENT

In this experiment, the gyroscopic rotator is a very accurately machined sphere which has had a segment sliced off to leave a flat region. In use, the truncated sphere rotates with its axis of rotational symmetry horizontal. It is driven at (60.0 ± 0.1) Hz (synchronized to Hydro’s 60 Hz frequency standard) by a surrounding coil connected to the A.C. mains. (The coil produces a 60 Hz alternating magnetic field with field lines in the vertical direction. The truncated sphere has been magnetized perpendicular to its axis of rotational symmetry, i.e. along an equatorial diameter.) Thus the rotator and coil is essentially a synchronous motor with the rotor free which permits the angular momentum vector $I\vec{\omega}$ to change its direction freely. This is because of the torsion-free support of the air suspension.



$\vec{L}$ Acts out of the page.

The truncation, or missing slice of matter at the “polar cap” results in an imbalance of the sphere, i.e., a torque $\vec{L}$, which acts in the horizontal plane, and causes a change in $I\vec{\omega}$. As this torque is perpendicular to $I\vec{\omega}$ at all times there is a pure precession of the sphere with precession angular velocity Ω given by $\Omega = \frac{L}{I\omega}$. Note that this precession is about a vertical axis, so that the angular momentum $I\vec{\omega}$ always stays in the horizontal plane.

Assuming a uniform density of the sphere, L and I may be calculated in terms of R the radius, ρ the density of the truncated sphere and ξ the distance from the centre of the flat to the centre of the sphere (or equivalently δ, the amount the sphere is truncated). (As part of your experimental writeup, you must do these calculations.) Both L and I are double integrals. Note that the torque $\vec{L}$ results from the force of gravity acting downward through the centre of mass and the supporting force from the air jet which acts upwards through the centre of the sphere. Your calculations should produce the following result: Calling: $\delta = R - \xi$,

We can calculate: $\varepsilon = \frac{\delta}{R} = 1 - \frac{\xi}{R}$, with g = acceleration due to gravity

Then
$$L = g \cdot \rho \cdot \frac{\pi R^4}{4} \cdot \varepsilon^2 \cdot (2-\varepsilon)^2 \tag{1}$$

$$I = \rho \cdot \frac{\pi R^5}{10} \cdot (2-\varepsilon)^3 \cdot (\varepsilon^2 + \varepsilon + \frac{2}{3}) \tag{2}$$

So that:
$$\frac{L}{I} = g \cdot \frac{5}{2R} \cdot \frac{\varepsilon^2}{(2-\varepsilon)(\varepsilon^2 + \varepsilon + \frac{2}{3})} \tag{3}$$

The measurement of Ω is made by driving the sphere up to speed with the side air jet. Initial wobble may be damped out by gently touching the centre of the flat of the rotor with a pencil or pen. Use a strobe to estimate the sphere's rotation. Be sure you calibrate the strobe against the built-in 120 Hz vibrating reed. Accelerate the rotor to an angular speed slightly higher than 3600 rpm (60 Hz), cut the air to the driving jet and as the rotor slows down through the 3600 rpm point, switch the coil to max, position. The sphere will lock to the field of the coil, unless the phase is badly off or the frequency is even slightly off. Make sure that the rotor is not rotating at some multiple or sub-multiple of 60 Hz. If the rotor fails to lock-in, try again. For easy locking, it is important to have the air bed quite level and ω quite constant and near 3600 rpm. Once locked, it is very stable.

Once the driving jet is cut off, the sphere precesses freely with a period T ~ 10 minutes It is best to reduce the support air jet somewhat to reduce any torque from it i.e., turn down the air until the noise changes pitch. The period is easily measured by reflecting a light beam from a laser off of the flat onto the wall or a screen and timing passages of the spot. The first few precessions will not be correct as transients in the motion die out the period will tend to a constant value.

Knowing ω and Ω and the geometry of the rotor, obtain as precise a value for g as you can. This will necessitate measuring the precession in both directions (i.e. for both senses of ω) in order to eliminate any torque from the air suspension due to departure from level.

A caution: Do not turn the air valve for the air suspension too high. The air pressure gauge should not go off scale; otherwise you will pop the hoses.

In performing your experiment, it is important to identify the principal source of error in your determination of g and thus do everything you can to minimize the error from this source.

➔ Python programming (PHY224/324 only); Use `Python` to get the volume of the truncated sphere. In calculating g, write a Python program, including functions to calculate the error.

(gmg/gdl – 1971, jbv 1990)