Why is Shrinkage Appealing?

- Why should we shrink the LS estimate?
- Isn't unbiasedness a nice property?
- Consider a simple estimation problem: $Z_1, \ldots, Z_n$ iid $\sim \mathsf{N}(\theta, \sigma^2)$. What's the MSE of $\bar{Z}$ and what's the MSE of $\frac{1}{2}\bar{Z}$?

$$\begin{aligned} \mathsf{MSE}(\bar{Z}) &= \mathbb{E}(\bar{Z} - \theta)^2 = \frac{\sigma^2}{n} \\ \mathsf{MSE}\left(\frac{1}{2}\bar{Z}\right) &= \mathbb{E}(\bar{Z} - \theta)^2 = \frac{\theta^2}{4} + \frac{1}{4}\frac{\sigma^2}{n} \end{aligned}$$

- Shrinkage may introduce bias but can also reduce variance, which could lead to an overall smaller MSE.