How to solve a one-dim Lasso?

Define $f(x) = (x-a)^2 + \lambda|x|$ where $a \in \mathbb{R}^1$ is a number and $\lambda > 0$. How to find x^* that minimizes $f(x)$?

The solution x^* should satisfy the following equation

$$0 = \frac{\partial}{\partial x}(x^* - a)^2 + \lambda \frac{\partial}{\partial x}|x^*| = 2(x^* - a) + \lambda z^*$$

where z^* is the sub-gradient of the absolute value function evaluated at x^*, which equals to sign(x^*) if $x^* \neq 0$, and any number in $[-1, 1]$ if $x^* = 0$.

So the minimizer of $f(x) = (x-a)^2 + \lambda|x|$ is given by

$$x^* = S_{\lambda/2}(a) = \mathsf{sign}(a)(|a| - \lambda/2)_+ = \begin{cases} a - \lambda/2, & \text{if } a > \lambda/2; \\ 0, & \text{if } |a| \leq \lambda/2; \\ a + \lambda/2, & \text{if } a < -\lambda/2; \end{cases}$$

$S_{\lambda/2}(\cdot)$ is often referred to as the soft-thresholding operator.

When the design matrix $\mathbf{X}$ is orthogonal, the lasso solution is given by

$$\hat{\beta}_j^{\textsf{lasso}} = \begin{cases} \textsf{sign}(\hat{\beta}_j^{\textsf{LS}})(|\hat{\beta}_j^{\textsf{LS}}| - \lambda/2) & \textsf{if } |\hat{\beta}_j^{\textsf{LS}}| > \lambda/2 \\ 0 & \textsf{if } |\hat{\beta}_j^{\textsf{LS}}| \leq \lambda/2. \end{cases}$$

A large λ will cause some of the coefficients to be exactly zero. So lasso does both "variable (subset) selection" and (soft) "shrinkage."