Lasso with $p > n$

- When $\mathbf{X}$ is of full rank, the Lasso solution, the minimizer of a convex function over a convex set, is unique since the 1st term is a strictly convex function.
- When $p > n$ or when $\mathbf{X}$ is not of full rank, the 1st term is no longer strictly convex. Then $\hat{\boldsymbol{\beta}}^{\text{lasso}}$ may be
  - unique if $\mathbf{X}_S$ is of full rank where S is selected variable set, or
  - not unique, however $\mathbf{X}\hat{\boldsymbol{\beta}}^{\text{lasso}}$ and $|\hat{\boldsymbol{\beta}}^{\text{lasso}}|$ are still unique.
- For more discussion on the uniqueness of Lasso, check this paper.