Coordinate Geometry

A Matrix approach

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Problem

A circle passes through the points $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \end{bmatrix}$ and it's Center lies on the line $\begin{bmatrix} -1 & 4 \end{bmatrix} \mathbf{x} + 3 = 0$.

Find the Radius of this Circle.

Introduction

The Given Problem can easily be solved using the following property:
The Perpendicular Bisector of a Chord Made by Joining Two Points on a Circle **Must** *pass through the Center of that Circle*

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- Find the intersection of this perpendicular with the given line.
- Now that we have the center of the circle, calculate distance from either of the given points and thus obtain the radius.

Solution

- We have $A = \begin{bmatrix} 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \end{bmatrix}$ and the line : $\begin{bmatrix} -1 & 4 \end{bmatrix} \mathbf{x} + 3 = 0$.

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- $\mathbf{x} = \begin{bmatrix} -7 \\ -3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 4 \end{bmatrix}^{-1}$

Results

From the Executed Python Program, we come to know that :

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- The Center of the Circle is : $\begin{bmatrix} 6.2 & 0.8 \end{bmatrix}$
- The Radius of the Circle is : 4.741307836451879 units

Plot

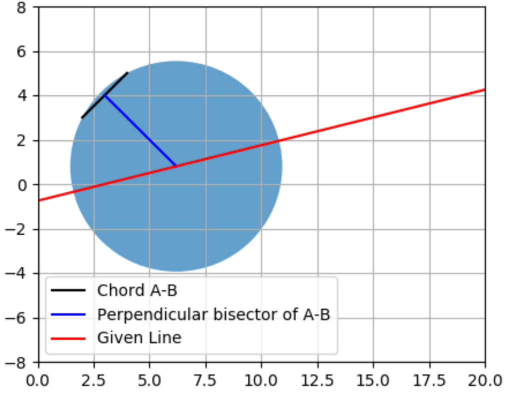

Figure: The Plotted Diagram of the lines meeting at the center of the circle