What's with the negative parallaxes?

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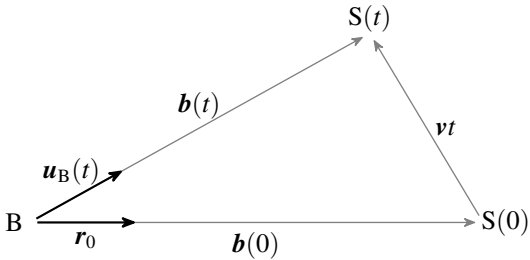
Our view of sources in the sky



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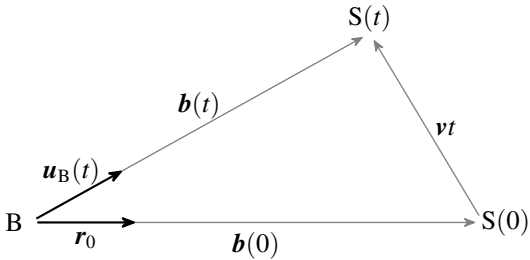

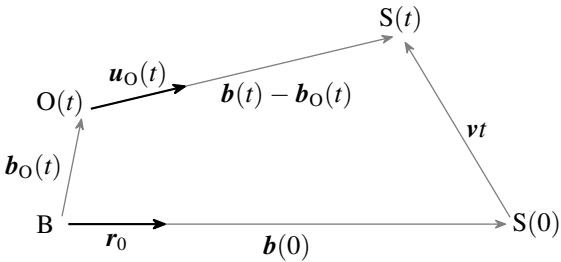

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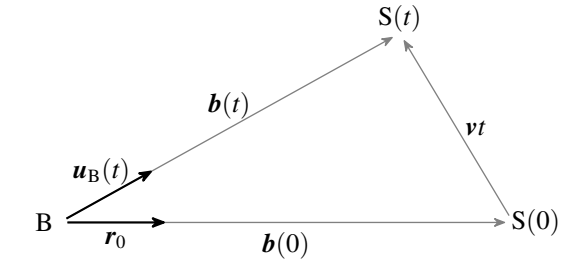
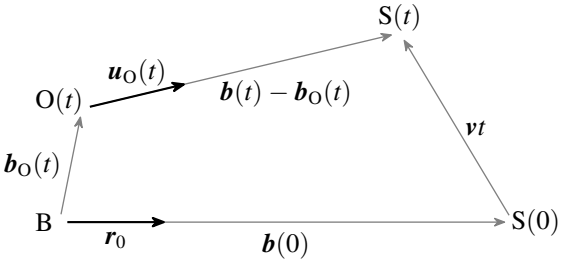
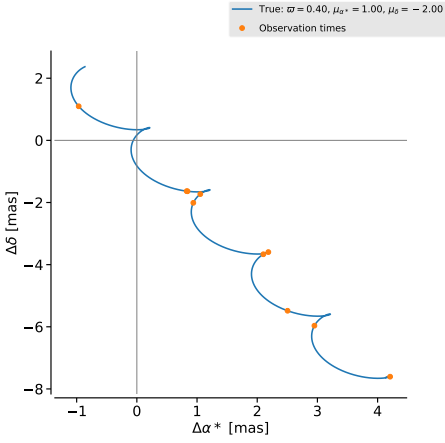


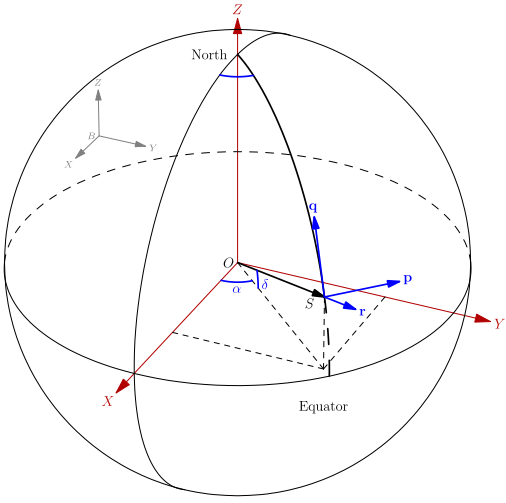


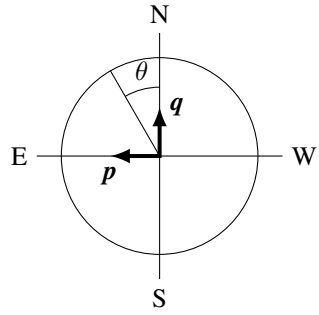
$$\boldsymbol{u}_{\mathrm{B}}(t) = \langle \boldsymbol{b}(t_0) + \boldsymbol{v}(t - t_0) \rangle$$

$$\boldsymbol{u}_{\mathrm{O}}(t) = \langle \boldsymbol{b}(t_0) + \boldsymbol{v}(t - t_0) - \boldsymbol{b}_{\mathrm{O}}(t) \rangle$$



Directions on the celestial sphere and local plane coordinates



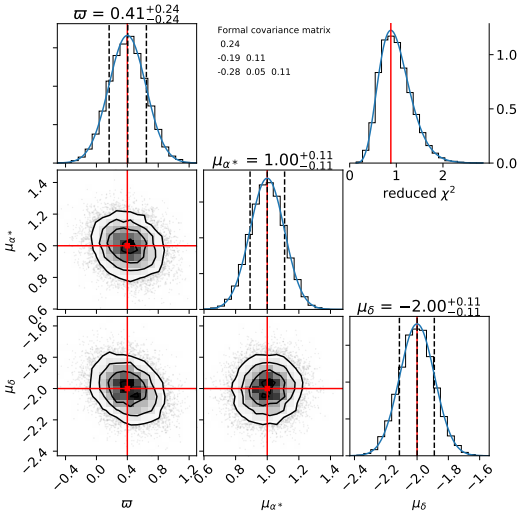


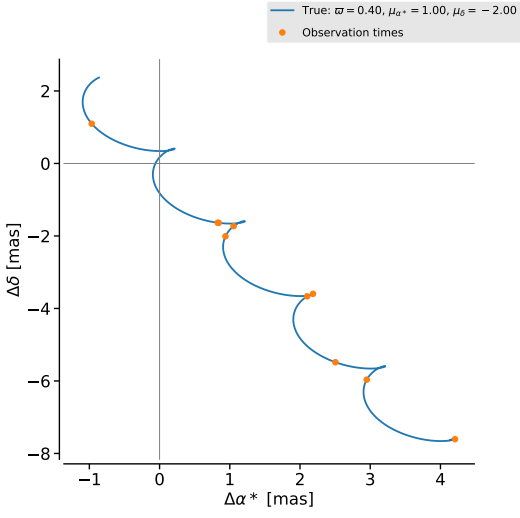
$$\boldsymbol{u}_{\mathrm{O}}(t) = \langle \boldsymbol{r} + (t_{\mathrm{B}} - t_0)(\boldsymbol{p}\mu_{\alpha*} + \boldsymbol{q}\mu_\delta + \boldsymbol{r}\mu_r) - \boldsymbol{b}_{\mathrm{O}}(t)\varpi/A_{\mathrm{u}} \rangle$$

Simplified astrometric model:

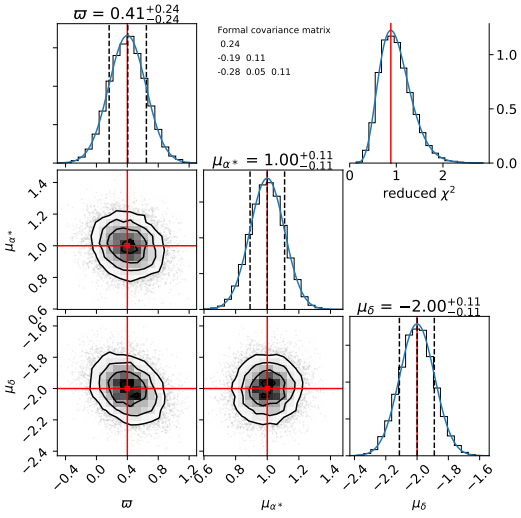
$$\Delta\alpha* \approx (t_{\mathrm{B}} - t_0)\mu_{\alpha*} - \varpi \boldsymbol{p}' \boldsymbol{b}_{\mathrm{O}}(t)/A_{\mathrm{u}}$$
$$\Delta\delta \approx (t_{\mathrm{B}} - t_0)\mu_\delta - \varpi \boldsymbol{q}' \boldsymbol{b}_{\mathrm{O}}(t)/A_{\mathrm{u}}$$

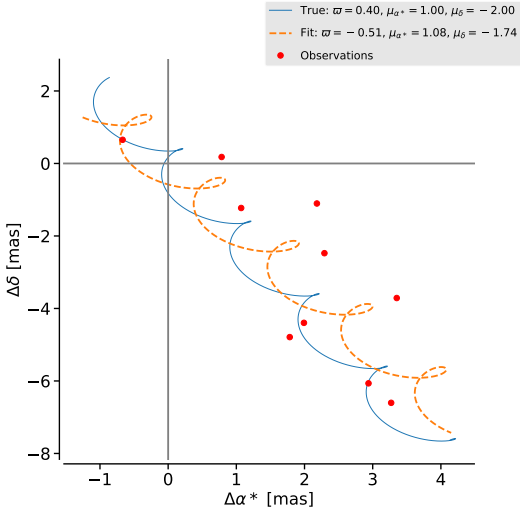
Simulated solutions for parallax and proper motion





Simulated solutions for parallax and proper motion





Conclusion

- Negative parallaxes are an expected outcome in the presence of observational uncertainties comparable to the parallax
- A negative parallax is a perfectly legitimate *measured* value of some true (positive) parallax
  - loosely speaking the epoch astrometry is modelled with the observer going the 'wrong way around the sun'
- Given a correct model for the astrometric observations and normally distributed measurement uncertainties (with zero mean), a measured parallax is an unbiased estimate of the true parallax according to

$$p(\varpi \mid \varpi_{\text{true}}) = \frac{1}{\sigma_\varpi \sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\varpi - \varpi_{\text{true}}}{\sigma_\varpi}\right)^2\right)$$